Prove that there is no polynomial $f(x)$ with integer coefficients, so that $f(7) = 11$ and $f(11) = 13$.

Proof: Consider a polynomial with integer coefficients,

$$f(x) = c_0 + \sum_{i=1}^{k} c_i\, x^i$$

where c_i's are all integers. Then for any a and b,

$$f(a) - f(b) \equiv 0 \bmod (a - b)$$

since,

$$a^k - b^k \equiv 0 \bmod (a - b) \forall k \in \mathbb{Z}_+$$

And hence, there exists no function $f(x)$ such that $f(11) - f(7) \equiv 2 \bmod (4)$.