Prove that there is no polynomial $f(x)$ with integer coefficients, so that $f(7)=11$ and $f(11)=13$.

Proof: Consider a polynomial with integer coefficients,

$$f(x)=c_0+\sum_{i=1}^{k} c_i\, x^i$$

where c_i's are all integers. Then for any a and b,

$$f(a)-f(b)\equiv 0 \bmod (a-b)$$

since,

$$a^k-b^k\equiv 0 \bmod (a-b) \forall k\in\mathbb{Z}_+$$

And hence, there exists no $f(x)$ such that $f(11)-f(7)\equiv 2 \bmod (4)$.