```
h[x_] := Piecewise[{
  {1 + c0,1 x + c0,2 x^2,          0 ≤ x ≤ 1/2},
  {c1,1 (x - 1) + c1,2 (x - 1)^2,  1/2 < x ≤ 3/2},
  {0,                              True}
}];
f[x_] := h[Abs[x]];
AllVars = {c0,1, c0,2, c1,1, c1,2};

(*Continuity*)
C1 = Limit[h[x], x → 1/2, Direction → 1] == Limit[h[x], x → 1/2, Direction → -1]
C2 = Limit[h[x], x → 3/2, Direction → 1] == Limit[h[x], x → 3/2, Direction → -1]
```

$$\frac{1}{4}\left(4+2\,c_{0,1}+c_{0,2}\right) == \frac{1}{4}\left(-2\,c_{1,1}+c_{1,2}\right)$$

$$\frac{1}{4}\left(2\,c_{1,1}+c_{1,2}\right) == 0$$

```
(*Partition of unity and linear term*)
T0 = CoefficientList[FullSimplify[Sum[f[x - i], {i, -3, 3}], x > 0 && x < 1/2], x]
T1 = CoefficientList[FullSimplify[Sum[i f[x - i], {i, -3, 3}], x > 0 && x < 1/2], x]
```

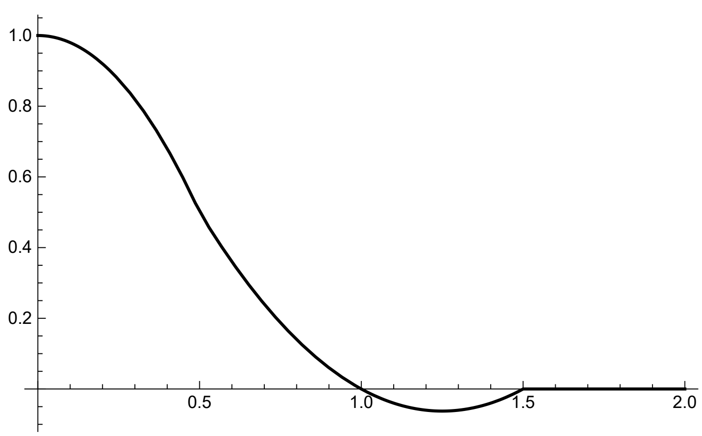
$$\{1,\ c_{0,1},\ c_{0,2}+2\,c_{1,2}\}$$

$$\{0,\ -2\,c_{1,1}\}$$

```
GenSols = Solve[{
    C1, C2,
    T0[[2]] == 0,
    T0[[3]] == 0,
    T1[[2]] == 1
    },
    AllVars
]
```

$$\left\{\left\{c_{0,1}\to 0,\ c_{0,2}\to -2,\ c_{1,1}\to -\frac{1}{2},\ c_{1,2}\to 1\right\}\right\}$$

```
Plot[h[x] /. GenSols[[1]], {x, 0, 2}, PlotStyle → Black, Background → White]
```



```
RegionXY[k_] := {Quotient[k, 2], 1 + Quotient[-k, 2]};
Regions = Table[RegionXY[k], {k, -4, 7}] - 1/2
```

$$\left\{\left\{-\frac{5}{2}, \frac{5}{2}\right\}, \left\{-\frac{5}{2}, \frac{3}{2}\right\}, \left\{-\frac{3}{2}, \frac{3}{2}\right\}, \left\{-\frac{3}{2}, \frac{1}{2}\right\}, \left\{-\frac{1}{2}, \frac{1}{2}\right\}, \left\{-\frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{2}, -\frac{3}{2}\right\}, \left\{\frac{3}{2}, -\frac{3}{2}\right\}, \left\{\frac{3}{2}, -\frac{5}{2}\right\}, \left\{\frac{5}{2}, -\frac{5}{2}\right\}, \left\{\frac{5}{2}, -\frac{7}{2}\right\}\right\}$$

```
GenSol = GenSols[[1]];
f[x_, y_] := f[x] f[y];
φ = 1/2;
```

$$W[k\_] := \begin{cases} 0 & k < 0 \\ \varphi^2/2 & k == 0 \\ 1 - (1-\varphi)^2/2 & k == 1 \\ 1 & \text{True} \end{cases};$$

$$\text{SumF} = \sum_{i=-5}^{6} \sum_{j=-5}^{6} W[i-j]\, f[x-i, y-j] \text{ /. GenSol;}$$

```
SimplifySquare[f_, x0_, y0_] := Simplify[f, x > x0 && x < x0 + 1 && y > y0 && y < y0 + 1];
DSimplifySquare[f_, {x0_, y0_}] := Simplify[D[SimplifySquare[f, x0, y0], {{x, y}}]];
DSumF = ParallelMap[DSimplifySquare[SumF, #] &, Regions];

AnisoInt[df_, {x0_, y0_}] :=
  Simplify[Integrate[Expand[(df.{1, 1})^2], {x, x0, x0 + 1}, {y, y0, y0 + 1}]];
AnisoInts = Parallelize[MapThread[AnisoInt, {DSumF, Regions}]];
Err = Simplify[Total[AnisoInts]]
```

$$\frac{1061}{4608}$$