```mathematica
h[x_] := {
    1 + c01 x + c02 x^2              0 ≤ x ≤ 1
    c11 (x - 1) + c12 (x - 1)^2      1 < x ≤ 2
    c21 (x - 2) + c22 (x - 2)^2      2 < x ≤ 3  ;
                0                       True

f[x_] := h[Abs[x]];

(*Interpolant constraints*)
I1 = f[1]
I2 = f[2]
I3 = f[3]
```

1 + c01 + c02

c11 + c12

c21 + c22

```mathematica
(*Partition of unity and gradient representation*)
T0 = CoefficientList[
   FullSimplify[f[x + 2] + f[x + 1] + f[x] + f[x - 1] + f[x - 2] + f[x - 3], x > 0 && x < 1], x]
T1 = CoefficientList[FullSimplify[
    -2 f[x + 2] - f[x + 1] + f[x - 1] + 2 f[x - 2] + 3 f[x - 3], x > 0 && x < 1], x]
```

$\{2 + c01 + c02 + c11 + c12 + c21 + c22, -2 (c02 + c12 + c22), 2 (c02 + c12 + c22)\}$

$\{1 + c01 + c02 + 2 c11 + 2 c12 + 3 c21 + 3 c22, -c01 - 2 c02 - 3 c11 - 4 c12 - 5 c21 - 6 c22, c02 + c12 + c22\}$

```mathematica
GenSols = Solve[{
    I1 == 0,
    I2 == 0,
    I3 == 0,
    T0[[1]] == 1,
    T0[[2]] == 0,
    T0[[3]] == 0,
    T1[[1]] == 0,
    T1[[2]] == 1,
    T1[[3]] == 0
    },
    {c01, c02, c11, c12, c21, c22}
]
```

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

$\{\{c02 \rightarrow -1 - c01, c12 \rightarrow -c11, c21 \rightarrow -1 - c01 - c11, c22 \rightarrow 1 + c01 + c11\}\}$

```
GenSol = GenSols[[1]];
f[x_, y_] := f[x] f[y];
```

$$
W1[k\_] := \begin{cases} 0 & k < 0 \\ \varphi^2/2 & k == 0 \\ 1 - (1-\varphi)^2/2 & k == 1 \\ 1 & \text{True} \end{cases};
$$

$$
\text{SumF1} = \sum_{i=-5}^{6} \sum_{j=-5}^{6} W1[i-j] \, f[x-i, y-j] \, /. \text{ GenSol};
$$

```
{SumF1a1, SumF1a2, SumF1a3, SumF1a4, SumF1a5, SumF1a6} = Parallelize[{
    Simplify[SumF1, x > 0 && x < 1 && y > 0 && y < 1],
    Simplify[SumF1, x > 0 && x < 1 && y > 1 && y < 2],
    Simplify[SumF1, x > -1 && x < 0 && y > 1 && y < 2],
    Simplify[SumF1, x > -1 && x < 0 && y > 2 && y < 3],
    Simplify[SumF1, x > -2 && x < -1 && y > 2 && y < 3],
    Simplify[SumF1, x > -2 && x < -1 && y > 3 && y < 4]
}];
{DSumF1a1, DSumF1a2, DSumF1a3, DSumF1a4, DSumF1a5, DSumF1a6} = Parallelize[{
    FullSimplify[D[SumF1a1, {{x, y}}]],
    FullSimplify[D[SumF1a2, {{x, y}}]],
    FullSimplify[D[SumF1a3, {{x, y}}]],
    FullSimplify[D[SumF1a4, {{x, y}}]],
    FullSimplify[D[SumF1a5, {{x, y}}]],
    FullSimplify[D[SumF1a6, {{x, y}}]]
}];
{SumF1b1, SumF1b2, SumF1b3, SumF1b4, SumF1b5, SumF1b6} = Parallelize[{
    Simplify[SumF1, x > 1 && x < 2 && y > 0 && y < 1],
    Simplify[SumF1, x > 1 && x < 2 && y > -1 && y < 0],
    Simplify[SumF1, x > 2 && x < 3 && y > -1 && y < 0],
    Simplify[SumF1, x > 2 && x < 3 && y > -2 && y < -1],
    Simplify[SumF1, x > 3 && x < 4 && y > -2 && y < -1],
    Simplify[SumF1, x > 3 && x < 4 && y > -3 && y < -2]
}];
{DSumF1b1, DSumF1b2, DSumF1b3, DSumF1b4, DSumF1b5, DSumF1b6} = Parallelize[{
    FullSimplify[D[SumF1b1, {{x, y}}]],
    FullSimplify[D[SumF1b2, {{x, y}}]],
    FullSimplify[D[SumF1b3, {{x, y}}]],
    FullSimplify[D[SumF1b4, {{x, y}}]],
    FullSimplify[D[SumF1b5, {{x, y}}]],
    FullSimplify[D[SumF1b6, {{x, y}}]]
}];
```

```
DSumF1a1 = Simplify[DSumF1a1 /. φ → 1/2];
DSumF1a2 = Simplify[DSumF1a2 /. φ → 1/2];
DSumF1a3 = Simplify[DSumF1a3 /. φ → 1/2];
DSumF1a4 = Simplify[DSumF1a4 /. φ → 1/2];
DSumF1a5 = Simplify[DSumF1a5 /. φ → 1/2];
DSumF1a6 = Simplify[DSumF1a6 /. φ → 1/2];
DSumF1b1 = Simplify[DSumF1b1 /. φ → 1/2];
DSumF1b2 = Simplify[DSumF1b2 /. φ → 1/2];
DSumF1b3 = Simplify[DSumF1b3 /. φ → 1/2];
DSumF1b4 = Simplify[DSumF1b4 /. φ → 1/2];
DSumF1b5 = Simplify[DSumF1b5 /. φ → 1/2];
DSumF1b6 = Simplify[DSumF1b6 /. φ → 1/2];
```
```
{Err1a1, Err1a2, Err1a3, Err1a4, Err1a5, Err1a6} = Parallelize[{
    Simplify[∫₀¹ ∫₀¹ (DSumF1a1.{1, 1})² ⅆx ⅆy],
    Simplify[∫₁² ∫₀¹ (DSumF1a2.{1, 1})² ⅆx ⅆy],
    Simplify[∫₁² ∫₋₁⁰ (DSumF1a3.{1, 1})² ⅆx ⅆy],
    Simplify[∫₂³ ∫₋₁⁰ (DSumF1a4.{1, 1})² ⅆx ⅆy],
    Simplify[∫₂³ ∫₋₂⁻¹ (DSumF1a5.{1, 1})² ⅆx ⅆy],
    Simplify[∫₃⁴ ∫₋₂⁻¹ (DSumF1a6.{1, 1})² ⅆx ⅆy]
}];
```
```
{Err1b1, Err1b2, Err1b3, Err1b4, Err1b5, Err1b6} = Parallelize[{
    Simplify[∫₀¹ ∫₁² (DSumF1b1.{1, 1})² ⅆx ⅆy],
    Simplify[∫₋₁⁰ ∫₁² (DSumF1b2.{1, 1})² ⅆx ⅆy],
    Simplify[∫₋₁⁰ ∫₂³ (DSumF1b3.{1, 1})² ⅆx ⅆy],
    Simplify[∫₋₂⁻¹ ∫₂³ (DSumF1b4.{1, 1})² ⅆx ⅆy],
    Simplify[∫₋₂⁻¹ ∫₃⁴ (DSumF1b5.{1, 1})² ⅆx ⅆy],
    Simplify[∫₋₃⁻² ∫₃⁴ (DSumF1b6.{1, 1})² ⅆx ⅆy]
}];
```
```
Err1 = FullSimplify[Err1a1 + Err1a2 + Err1a3 + Err1a4 +
    Err1a5 + Err1a6 + Err1b1 + Err1b2 + Err1b3 + Err1b4 + Err1b5 + Err1b6];
```

```
Err = Err1
DErr = FullSimplify[D[Err, {{c01, c11}}]];
H = FullSimplify[D[Err, {{c01, c11}, 2}]];
Sols = Solve[DErr == 0, {c01, c11}];
TableForm[
 {Range[Length[Sols]], Err /. N[Sols], PositiveDefiniteMatrixQ[H /. N[#]] & /@ Sols}ᵀ]
```

$$\frac{1}{1440} \left(4373 + 760\, c01^4 + 5\, c01^3 \left(1051 + 276\, c11\right) + \right.$$
$$3\, c01^2 \left(4381 + 6\, c11 \left(405 + 62\, c11\right)\right) + 3\, c11 \left(2146 + c11 \left(1099 + 4\, c11 \left(53 + 13\, c11\right)\right)\right) +$$
$$\left. c01 \left(12\,826 + 3\, c11 \left(4176 + c11 \left(1299 + 152\, c11\right)\right)\right)\right)$$

1	0.0343838	True
2	0.819269 + 0.493763 i	False
3	0.819269 − 0.493763 i	False
4	1.06321 + 0.252926 i	False
5	1.06321 − 0.252926 i	False
6	1.08507 + 0.215646 i	False
7	1.08507 − 0.215646 i	False
8	0.179691 + 0.00719426 i	False
9	0.179691 − 0.00719426 i	False

```
RootReduce[Sols[[1]]]
```

$\{c01 \to \text{Root}\left[107\,304\,086\,230\,531\,885 + 599\,287\,446\,548\,831\,718\, \#1 + \right.$
$1\,400\,958\,868\,626\,252\,747\, \#1^2 + 1\,834\,238\,645\,594\,718\,312\, \#1^3 +$
$1\,498\,904\,274\,881\,813\,490\, \#1^4 + 798\,582\,300\,168\,995\,568\, \#1^5 + 278\,783\,851\,306\,490\,292\, \#1^6 +$
$61\,715\,710\,273\,939\,056\, \#1^7 + 7\,883\,190\,806\,676\,480\, \#1^8 + 443\,705\,711\,242\,240\, \#1^9 \&,\ 1\right],$
$c11 \to \text{Root}\left[18\,431\,582\,450\,625 + 23\,718\,361\,748\,580\, \#1 + 334\,904\,447\,070\,675\, \#1^2 - \right.$
$37\,839\,981\,285\,332\, \#1^3 - 288\,829\,159\,903\,530\, \#1^4 - 336\,831\,488\,617\,800\, \#1^5 - 632\,563\,845\,450\,300\, \#1^6 +$
$2\,304\,156\,770\,184\,864\, \#1^7 - 1\,637\,392\,551\,713\,280\, \#1^8 + 354\,964\,568\,993\,792\, \#1^9 \&,\ 1\right]\}$

```
Sol = Sols[[1]];
FullSol = N[Join[GenSol /. Sol, Sol]]
fo[x_] := f[x] /. FullSol;
Plot[fo[x], {x, -3, 3}, PlotStyle → Black, Background → White]
```

$\{c02 \to -0.442669,\ c12 \to 0.596792,\ c21 \to 0.154123,$
$c22 \to -0.154123,\ c01 \to -0.557331,\ c11 \to -0.596792\}$