

$$h[x_] := \begin{cases} 1 + c_{0,1} x + c_{0,2} x^2 & 0 \leq x \leq 1 \\ c_{1,1} (x-1) + c_{1,2} (x-1)^2 & 1 < x \leq 2; \\ 0 & \text{True} \end{cases}$$

$f[x_] := h[Abs[x]];$

$AllVars = \{c_{0,1}, c_{0,2}, c_{1,1}, c_{1,2}\};$

(*Interpolant constraints*)

$I1 = f[1]$

$I2 = f[2]$

$1 + c_{0,1} + c_{0,2}$

$c_{1,1} + c_{1,2}$

(*Partition of unity and linear term*)

$T0 = \text{CoefficientList}\left[\text{FullSimplify}\left[\sum_{k=-1}^2 f[x-k], x > 0 \&\& x < 1\right], x\right]$

$T1 = \text{CoefficientList}\left[\text{FullSimplify}\left[\sum_{k=-1}^2 k f[x-k], x > 0 \&\& x < 1\right], x\right]$

$\{2 + c_{0,1} + c_{0,2} + c_{1,1} + c_{1,2}, -2 c_{0,2} - 2 c_{1,2}, 2 c_{0,2} + 2 c_{1,2}\}$

$\{1 + c_{0,1} + c_{0,2} + 2 c_{1,1} + 2 c_{1,2}, -c_{0,1} - 2 c_{0,2} - 3 c_{1,1} - 4 c_{1,2}, c_{0,2} + c_{1,2}\}$

$GenSols = \text{Solve}[\{$

$I1 == 0,$

$I2 == 0,$

$T0[[1]] == 1,$

$T0[[2]] == 0,$

$T0[[3]] == 0,$

$T1[[1]] == 0,$

$T1[[2]] == 1,$

$T1[[3]] == 0$

$\},$

$AllVars$

$]$

... **Solve:** Equations may not give solutions for all "solve" variables.

$\{\{c_{0,2} \rightarrow -1 - c_{0,1}, c_{1,1} \rightarrow -1 - c_{0,1}, c_{1,2} \rightarrow 1 + c_{0,1}\}\}$

$RegionXY[k_] := \{\text{Quotient}[k, 2], 1 + \text{Quotient}[-k, 2]\};$

$Regions = \text{Table}[RegionXY[k], \{k, -2, 5\}]$

$\{\{-1, 2\}, \{-1, 1\}, \{0, 1\}, \{0, 0\}, \{1, 0\}, \{1, -1\}, \{2, -1\}, \{2, -2\}\}$

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GenSol = GenSols[[1]];
f[x_, y_] := f[x] f[y];

W[k_] := 
$$\begin{cases} 0 & k < 0 \\ \varphi^2/2 & k == 0 \\ 1 - (1 - \varphi)^2/2 & k == 1 \\ 1 & \text{True} \end{cases}$$


SumF =  $\sum_{i=-3}^5 \sum_{j=-3}^5 W[i-j] f[x-i, y-j] /. \text{GenSol};$ 

SimplifySquare[f_, x0_, y0_] := Simplify[f, x > x0 && x < x0 + 1 && y > y0 && y < y0 + 1];
DSimplifySquare[f_, {x0_, y0_}] := Simplify[D[SimplifySquare[f, x0, y0], {{x, y}}]];
DSumF = ParallelMap[DSimplifySquare[SumF, #] &, Regions];

AnisoInt[df_, {x0_, y0_}] :=
  Simplify[Integrate[Expand[(df.{1, 1})^2], {x, x0, x0 + 1}, {y, y0, y0 + 1}]];
AnisoInts = Parallelize[MapThread[AnisoInt, {DSumF, Regions}]];
Err = Simplify[Total[AnisoInts] /.  $\varphi \rightarrow 1/2$ ]


$$\frac{1}{1440} (752 + 2611 c_{0,1} + 3192 c_{0,1}^2 + 1334 c_{0,1}^3 + 196 c_{0,1}^4)$$


FreeVars = Variables[Err];
DErr = Simplify[D[Err, {FreeVars}]];
H = D[DErr, {FreeVars}];
Sols = RootReduce[Solve[DErr == 0, FreeVars, Reals]];
RootReduce[Sols[[1]]]
TableForm[
  {Range[Length[Sols]], Err /. N[Sols], PositiveDefiniteMatrixQ[H /. N[#] & /@ Sols]^T}
  {c0,1 → Root[2611 + 6384 #1 + 4002 #1^2 + 784 #1^3 &, 1]}
1 0.0494532 True

NSol = N[Sols[[1]]];
FullSol = Join[GenSol /. NSol, NSol]
fo[x_] := f[x] /. FullSol;
Plot[fo[x], {x, -3, 3}, PlotStyle → Black, Background → White]
{c0,2 → -0.378087, c1,1 → -0.378087, c1,2 → 0.378087, c0,1 → -0.621913}

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