$$h[x\_] := \begin{cases} 1 + c_{0,1}\,x + c_{0,2}\,x^2 + c_{0,3}\,x^3 + c_{0,4}\,x^4 & 0 \le x \le 1/2 \\ c_{1,1}\,(x-1) + c_{1,2}\,(x-1)^2 + c_{1,3}\,(x-1)^3 + c_{1,4}\,(x-1)^4 & 1/2 < x \le 3/2 \\ 0 & \text{True} \end{cases};$$

```
f[x_] := h[Abs[x]];
AllVars = {c0,1, c0,2, c0,3, c0,4, c1,1, c1,2, c1,3, c1,4};

(*Continuity*)
C1 = Limit[h[x], x → 1/2, Direction → 1] == Limit[h[x], x → 1/2, Direction → -1]
C2 = Limit[h[x], x → 3/2, Direction → 1] == Limit[h[x], x → 3/2, Direction → -1]
```

$$\frac{1}{16}\left(16 + 8\,c_{0,1} + 4\,c_{0,2} + 2\,c_{0,3} + c_{0,4}\right) == \frac{1}{16}\left(-8\,c_{1,1} + 4\,c_{1,2} - 2\,c_{1,3} + c_{1,4}\right)$$

$$\frac{1}{16}\left(8\,c_{1,1} + 4\,c_{1,2} + 2\,c_{1,3} + c_{1,4}\right) == 0$$

```
(*Partition of unity and linear term*)
```

$$T0 = \text{CoefficientList}\left[\text{FullSimplify}\left[\sum_{i=-3}^{3} f[x-i],\ x > 0\ \&\&\ x < 1/2\right],\ x\right]$$

$$T1 = \text{CoefficientList}\left[\text{FullSimplify}\left[\sum_{i=-3}^{3} i\, f[x-i],\ x > 0\ \&\&\ x < 1/2\right],\ x\right]$$

$$\{1,\ c_{0,1},\ c_{0,2} + 2\,c_{1,2},\ c_{0,3},\ c_{0,4} + 2\,c_{1,4}\}$$

$$\{0,\ -2\,c_{1,1},\ 0,\ -2\,c_{1,3}\}$$

```
(*Smoothness*)
Dh = Simplify[D[h[x], x], x > 0];
S0 = (Dh /. x → 0) == 0
S1 = Limit[Dh, x → 1/2, Direction → 1] == Limit[Dh, x → 1/2, Direction → -1]
S2 = Limit[Dh, x → 3/2, Direction → 1] == Limit[Dh, x → 3/2, Direction → -1]
```

$$c_{0,1} == 0$$

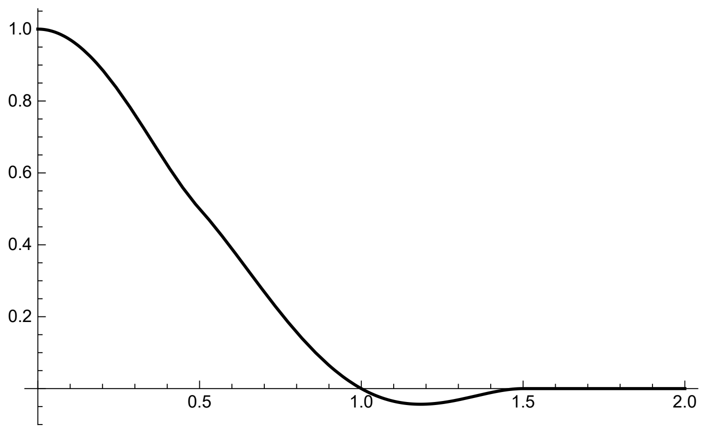
$$c_{0,1} + c_{0,2} + \frac{3\,c_{0,3}}{4} + \frac{c_{0,4}}{2} == c_{1,1} - c_{1,2} + \frac{3\,c_{1,3}}{4} - \frac{c_{1,4}}{2}$$

$$c_{1,1} + c_{1,2} + \frac{3\,c_{1,3}}{4} + \frac{c_{1,4}}{2} == 0$$

```
GenSols = Solve[{
    C1, C2,
    T0[[2]] == 0,
    T0[[3]] == 0,
    T0[[4]] == 0,
    T0[[5]] == 0,
    T1[[2]] == 1,
    T1[[3]] == 0,
    T1[[4]] == 0,
    S0, S1, S2
    },
    AllVars
]
```

$\left\{\left\{c_{0,1} \to 0,\ c_{0,2} \to -3,\ c_{0,3} \to 0,\ c_{0,4} \to 4,\ c_{1,1} \to -\frac{1}{2},\ c_{1,2} \to \frac{3}{2},\ c_{1,3} \to 0,\ c_{1,4} \to -2\right\}\right\}$

```
Plot[f[x] /. GenSols[[1]], {x, 0, 2}, PlotStyle → Black, Background → White]
```




```
RegionXY[k_] := {Quotient[k, 2], 1 + Quotient[-k, 2]};
Regions = Table[RegionXY[k], {k, -4, 7}] - 1/2
```

$\left\{\left\{-\frac{5}{2}, \frac{5}{2}\right\}, \left\{-\frac{5}{2}, \frac{3}{2}\right\}, \left\{-\frac{3}{2}, \frac{3}{2}\right\}, \left\{-\frac{3}{2}, \frac{1}{2}\right\}, \left\{-\frac{1}{2}, \frac{1}{2}\right\}, \left\{-\frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{2}, -\frac{3}{2}\right\}, \left\{\frac{3}{2}, -\frac{3}{2}\right\}, \left\{\frac{3}{2}, -\frac{5}{2}\right\}, \left\{\frac{5}{2}, -\frac{5}{2}\right\}, \left\{\frac{5}{2}, -\frac{7}{2}\right\}\right\}$

```
GenSol = GenSols[[1]];
f[x_, y_] := f[x] f[y];
φ = 1/2;
W[k_] := ⎡ 0                  k < 0
         ⎢ φ^2/2              k == 0
         ⎢ 1 - (1 - φ)^2/2    k == 1 ;
         ⎣ 1                  True
SumF = Sum[W[i - j] f[x - i, y - j], {i, -5, 6}, {j, -5, 6}] /. GenSol;
SimplifySquare[f_, x0_, y0_] := Simplify[f, x > x0 && x < x0 + 1 && y > y0 && y < y0 + 1];
DSimplifySquare[f_, {x0_, y0_}] := Simplify[D[SimplifySquare[f, x0, y0], {{x, y}}]];
DSumF = ParallelMap[DSimplifySquare[SumF, #] &, Regions];

AnisoInt[df_, {x0_, y0_}] :=
  Simplify[Integrate[Expand[(df.{1, 1})^2], {x, x0, x0 + 1}, {y, y0, y0 + 1}]];
AnisoInts = Parallelize[MapThread[AnisoInt, {DSumF, Regions}]];
Err = Simplify[Total[AnisoInts]]
```

$$\frac{208237}{1128960}$$