```
h[x_] := Piecewise[{
   {1 + c[0,1] x + c[0,2] x^2 + c[0,3] x^3 + c[0,4] x^4, 0 ≤ x ≤ 1},
   {c[1,1] (x - 1) + c[1,2] (x - 1)^2 + c[1,3] (x - 1)^3 + c[1,4] (x - 1)^4, 1 < x ≤ 2},
   {c[2,1] (x - 2) + c[2,2] (x - 2)^2 + c[2,3] (x - 2)^3 + c[2,4] (x - 2)^4, 2 < x ≤ 3},
   {0, True}}];
```

$$h[x\_] := \begin{cases} 1 + c_{0,1}\,x + c_{0,2}\,x^2 + c_{0,3}\,x^3 + c_{0,4}\,x^4 & 0 \le x \le 1 \\ c_{1,1}\,(x-1) + c_{1,2}\,(x-1)^2 + c_{1,3}\,(x-1)^3 + c_{1,4}\,(x-1)^4 & 1 < x \le 2 \\ c_{2,1}\,(x-2) + c_{2,2}\,(x-2)^2 + c_{2,3}\,(x-2)^3 + c_{2,4}\,(x-2)^4 & 2 < x \le 3 \\ 0 & \text{True} \end{cases};$$

f[x_] := h[Abs[x]];

AllVars $= \{c_{0,1}, c_{0,2}, c_{0,3}, c_{0,4}, c_{1,1}, c_{1,2}, c_{1,3}, c_{1,4}, c_{2,1}, c_{2,2}, c_{2,3}, c_{2,4}\};$

(*Interpolant constraints*)

I1 = f[1]

I2 = f[2]

I3 = f[3]

$1 + c_{0,1} + c_{0,2} + c_{0,3} + c_{0,4}$

$c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4}$

$c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4}$

(*Partition of unity and linear term*)

$$\textbf{T0} = \text{CoefficientList}\Big[\text{FullSimplify}\Big[\sum_{k=-2}^{3} f[x-k],\ x > 0\ \&\&\ x < 1\Big],\ x\Big]$$

$$\textbf{T1} = \text{CoefficientList}\Big[\text{FullSimplify}\Big[\sum_{k=-2}^{3} k\, f[x-k],\ x > 0\ \&\&\ x < 1\Big],\ x\Big]$$

$\{2 + c_{0,1} + c_{0,2} + c_{0,3} + c_{0,4} + c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4},$
$-2\,c_{0,2} - 3\,c_{0,3} - 4\,c_{0,4} - 2\,c_{1,2} - 3\,c_{1,3} - 4\,c_{1,4} - 2\,c_{2,2} - 3\,c_{2,3} - 4\,c_{2,4},$
$2\,c_{0,2} + 3\,c_{0,3} + 6\,c_{0,4} + 2\,c_{1,2} + 3\,c_{1,3} + 4\,c_{1,4} + 2\,c_{2,2} + 3\,c_{2,3} + 4\,c_{2,4} + 2\,(c_{1,4} + c_{2,4}),$
$-4\,c_{0,4} - 4\,(c_{1,4} + c_{2,4}),\ 2\,c_{0,4} + 2\,(c_{1,4} + c_{2,4})\}$

$\{1 + c_{0,1} + c_{0,2} + c_{0,3} + c_{0,4} + 2\,c_{1,1} + 2\,c_{1,2} + 2\,c_{1,3} + 2\,c_{1,4} + 3\,c_{2,1} + 3\,c_{2,2} + 3\,c_{2,3} + 3\,c_{2,4},$
$-c_{0,1} - 2\,c_{0,2} - 3\,c_{0,3} - 4\,c_{0,4} - 3\,c_{1,1} - 4\,c_{1,2} - 6\,c_{1,3} - 8\,c_{1,4} - 5\,c_{2,1} - 6\,c_{2,2} - 9\,c_{2,3} - 12\,c_{2,4},$
$c_{0,2} + 3\,c_{0,3} + 6\,c_{0,4} + c_{1,2} + 6\,c_{1,3} + 12\,c_{1,4} + c_{2,2} + 9\,c_{2,3} + 18\,c_{2,4},$
$-c_{0,3} - 4\,c_{0,4} - 3\,c_{1,3} - 8\,c_{1,4} - 5\,c_{2,3} - 12\,c_{2,4},\ c_{0,4} + c_{1,4} + c_{2,4}\}$

```
GenSols = Solve[{
    I1 == 0,
    I2 == 0,
    I3 == 0,
    T0[[1]] == 1,
    T0[[2]] == 0,
    T0[[3]] == 0,
    T1[[1]] == 0,
    T1[[2]] == 1,
    T1[[3]] == 0,
    T1[[4]] == 0
    },
    AllVars
]
```

Solve: Equations may not give solutions for all "solve" variables.

$$\Big\{\Big\{c_{0,4}\to -1-c_{0,1}-c_{0,2}-c_{0,3},\ c_{1,4}\to -c_{1,1}-c_{1,2}-c_{1,3},$$
$$c_{2,1}\to -\frac{9}{5}-\frac{9\,c_{0,1}}{5}-\frac{4\,c_{0,2}}{5}-\frac{2\,c_{0,3}}{5}-\frac{7\,c_{1,1}}{5}-\frac{2\,c_{1,2}}{5}-\frac{c_{1,3}}{5},$$
$$c_{2,2}\to \frac{12}{5}+\frac{12\,c_{0,1}}{5}+\frac{7\,c_{0,2}}{5}+\frac{6\,c_{0,3}}{5}+\frac{6\,c_{1,1}}{5}+\frac{c_{1,2}}{5}+\frac{3\,c_{1,3}}{5},$$
$$c_{2,3}\to -\frac{8}{5}-\frac{8\,c_{0,1}}{5}-\frac{8\,c_{0,2}}{5}-\frac{9\,c_{0,3}}{5}-\frac{4\,c_{1,1}}{5}-\frac{4\,c_{1,2}}{5}-\frac{7\,c_{1,3}}{5},$$
$$c_{2,4}\to 1+c_{0,1}+c_{0,2}+c_{0,3}+c_{1,1}+c_{1,2}+c_{1,3}\Big\}\Big\}$$

```
RegionXY[k_] := {Quotient[k, 2], 1 + Quotient[-k, 2]};
Regions = Table[RegionXY[k], {k, -4, 7}]
```

{{-2, 3}, {-2, 2}, {-1, 2}, {-1, 1}, {0, 1},
{0, 0}, {1, 0}, {1, -1}, {2, -1}, {2, -2}, {3, -2}, {3, -3}}

```
GenSol = GenSols[[1]];
f[x_, y_] := f[x] f[y];
φ = 1/2;
```

$$W[k\_] := \begin{cases} 0 & k<0 \\ \varphi^2/2 & k==0 \\ 1-(1-\varphi)^2/2 & k==1 \\ 1 & \text{True} \end{cases};$$

$$\text{SumF} = \sum_{i=-5}^{6}\sum_{j=-5}^{6} W[i-j]\, f[x-i,\, y-j]\ /.\ \text{GenSol};$$

```
SimplifySquare[f_, x0_, y0_] := Simplify[f, x > x0 && x < x0 + 1 && y > y0 && y < y0 + 1];
DSimplifySquare[f_, {x0_, y0_}] := Simplify[D[SimplifySquare[f, x0, y0], {{x, y}}]];
DSumF = ParallelMap[DSimplifySquare[SumF, #] &, Regions];
```

```
AnisoInt[df_, {x0_, y0_}] :=
  Simplify[Integrate[Expand[(df.{1, 1})^2], {x, x0, x0 + 1}, {y, y0, y0 + 1}]];
AnisoInts = Parallelize[MapThread[AnisoInt, {DSumF, Regions}]];
Err = Simplify[Total[AnisoInts]]
```

$$\frac{1}{2\,646\,000\,000}\left(14\,670\,634\,896\, c_{0,1}^4 + 606\,507\,296\, c_{0,2}^4 + \right.$$

$$4\, c_{0,2}^{3}\, (1949851966 + 232907098\, c_{0,3} + 617124018\, c_{1,1} + 290282568\, c_{1,2} + 113366709\, c_{1,3}) +$$
$$6\, c_{0,1}^{3}\, (14010589344 + 4372344264\, c_{0,2} + 1641040132\, c_{0,3} +$$
$$4483994112\, c_{1,1} + 2032526862\, c_{1,2} + 769025431\, c_{1,3}) +$$
$$4\, c_{0,2}^{2}\, (9167147804 + 136084136\, c_{0,3}^{2} + 1130374116\, c_{1,1}^{2} + 2879113734\, c_{1,2} + 253899816\, c_{1,2}^{2} +$$
$$1129878642\, c_{1,3} + 201470916\, c_{1,2}\, c_{1,3} + 40742529\, c_{1,3}^{2} +$$
$$6\, c_{1,1}\, (1009302314 + 175412672\, c_{1,2} + 68040261\, c_{1,3}) +$$
$$c_{0,3}\, (2240602399 + 707391852\, c_{1,1} + 337352727\, c_{1,2} + 133865301\, c_{1,3})) +$$
$$2\, c_{0,1}^{2}\, (8840566888\, c_{0,2}^{2} + 3\, c_{0,2}\, (18937165782 + 2227535596\, c_{0,3} +$$
$$6036848586\, c_{1,1} + 2766513536\, c_{1,2} + 1057275843\, c_{1,3}) +$$
$$3\, (29453209136 + 427108874\, c_{0,3}^{2} + 3668896144\, c_{1,1}^{2} + 9157188606\, c_{1,2} +$$
$$785922744\, c_{1,2}^{2} + 3522204753\, c_{1,3} + 609222194\, c_{1,2}\, c_{1,3} + 120391136\, c_{1,3}^{2} +$$
$$c_{0,3}\, (7159760466 + 2272854293\, c_{1,1} + 1053419843\, c_{1,2} + 409158534\, c_{1,3}) +$$
$$2\, c_{1,1}\, (9864820428 + 1674943894\, c_{1,2} + 636803947\, c_{1,3}))) +$$
$$3\, c_{0,2}\, (47722966\, c_{0,3}^{3} + c_{0,3}^{2}\, (1161284166 + 365577168\, c_{1,1} + 176751868\, c_{1,2} + 71148259\, c_{1,3}) +$$
$$4\, c_{0,3}\, (2345936743 + 287776097\, c_{1,1}^{2} + 66491972\, c_{1,2}^{2} + 294016014\, c_{1,3} + 11009318\, c_{1,3}^{2} + c_{1,2}$$
$$(737796028 + 53640772\, c_{1,3}) + c_{1,1}\, (1536686753 + 271200644\, c_{1,2} + 106899522\, c_{1,3})) +$$
$$2\, (11971578004 + 675849468\, c_{1,1}^{3} + 70686368\, c_{1,2}^{3} + 2452696513\, c_{1,3} +$$
$$174072637\, c_{1,3}^{2} + 4683721\, c_{1,3}^{3} + 4\, c_{1,2}^{2}\, (275830787 + 21080513\, c_{1,3}) +$$
$$c_{1,1}^{2}\, (4931450498 + 933464704\, c_{1,2} + 358152902\, c_{1,3}) + 4\, c_{1,2}$$
$$(1552337394 + 217117737\, c_{1,3} + 8535469\, c_{1,3}^{2}) + 2\, c_{1,1}\, (6452849063 + 219512052\, c_{1,2}^{2} +$$
$$887904374\, c_{1,3} + 34001388\, c_{1,3}^{2} + 24\, c_{1,2}\, (95773077 + 7135123\, c_{1,3})))) +$$
$$3\, (4761552\, c_{0,3}^{4} + c_{0,3}^{3}\, (152378486 + 47832803\, c_{1,1} + 23442953\, c_{1,2} + 9559314\, c_{1,3}) +$$
$$c_{0,3}^{2}\, (1838434268 + 223352272\, c_{1,1}^{2} + 53063372\, c_{1,2}^{2} + 232070089\, c_{1,3} + 9034718\, c_{1,3}^{2} +$$
$$8\, c_{1,2}\, (71850441 + 5430284\, c_{1,3}) + 4\, c_{1,1}\, (296799357 + 53253461\, c_{1,2} + 21295618\, c_{1,3})) +$$
$$2\, c_{0,3}\, (4578199952 + 255032734\, c_{1,1}^{3} + 27775484\, c_{1,2}^{3} + 963184594\, c_{1,3} +$$
$$68786356\, c_{1,3}^{2} + 1921098\, c_{1,3}^{3} + c_{1,2}^{2}\, (426450074 + 33670276\, c_{1,3}) +$$
$$2\, c_{1,1}^{2}\, (940428212 + 178028451\, c_{1,2} + 69391638\, c_{1,3}) +$$
$$c_{1,2}\, (2388891813 + 339841674\, c_{1,3} + 13828463\, c_{1,3}^{2}) + c_{1,1}\, (4908499263 + 169697152$$
$$c_{1,2}^{2} + 691507224\, c_{1,3} + 27092213\, c_{1,3}^{2} + 4\, c_{1,2}\, (441331037 + 33628038\, c_{1,3}))) +$$
$$2\, (8398194146 + 515187536\, c_{1,1}^{4} + 25077936\, c_{1,2}^{4} + 2464110641\, c_{1,3} + 299371476\, c_{1,3}^{2} +$$
$$12762541\, c_{1,3}^{3} + 715571\, c_{1,3}^{4} + 16\, c_{1,2}^{3}\, (12495433 + 2510042\, c_{1,3}) +$$
$$4\, c_{1,1}^{3}\, (475525332 + 234610286\, c_{1,2} + 89137143\, c_{1,3}) +$$
$$4\, c_{1,2}^{2}\, (457033426 + 59089098\, c_{1,3} + 6143001\, c_{1,3}^{2}) +$$
$$2\, c_{1,2}\, (3129991691 + 730471427\, c_{1,3} + 47272023\, c_{1,3}^{2} + 3397734\, c_{1,3}^{3}) +$$
$$2\, c_{1,1}^{2}\, (3892323052 + 325618808\, c_{1,2}^{2} + 508337521\, c_{1,3} + 49153852\, c_{1,3}^{2} +$$
$$6\, c_{1,2}\, (220602207 + 41805493\, c_{1,3})) + 2\, c_{1,1}\, (6514031816 + 102643472\, c_{1,2}^{3} +$$
$$1438433027\, c_{1,3} + 96382698\, c_{1,3}^{2} + 6500334\, c_{1,3}^{3} + 4\, c_{1,2}^{2}\, (156318698 + 30155377\, c_{1,3}) +$$
$$c_{1,2}\, (3693303104 + 486877442\, c_{1,3} + 48105554\, c_{1,3}^{2})))) +$$
$$c_{0,1}\, (5327710384\, c_{0,2}^{3} + 8\, c_{0,2}^{2}\, (6430478624 + 760812647\, c_{0,3} + 2041482552\, c_{1,1} +$$
$$946942227\, c_{1,2} + 365697726\, c_{1,3}) +$$
$$12\, c_{0,2}\, (13370089136 + 195939849\, c_{0,3}^{2} + 1652893894\, c_{1,1}^{2} + 4188688556\, c_{1,2} +$$
$$361115344\, c_{1,2}^{2} + 1627882803\, c_{1,3} + 282820644\, c_{1,2}\, c_{1,3} + 56455961\, c_{1,3}^{2} +$$
$$c_{0,3}\, (3263400916 + 1032163643\, c_{1,1} + 484753168\, c_{1,2} + 190235484\, c_{1,3}) +$$
$$2\, c_{1,1}\, (4457455803 + 760915944\, c_{1,2} + 291922897\, c_{1,3})) +$$
$$3\, (102225866\, c_{0,3}^{3} + c_{0,3}^{2}\, (2523364016 + 794313468\, c_{1,1} + 377723568\, c_{1,2} + 150391009\, c_{1,3}) +$$
$$4\, c_{0,3}\, (5088150143 + 624378947\, c_{1,1}^{2} + 139825522\, c_{1,2}^{2} + 632780814\, c_{1,3} +$$

$$
\begin{aligned}
&22\,546\,868\, c_{1,3}^{2} + c_{1,2}\left(1\,601\,822\,453 + 111\,309\,522\, c_{1,3}\right) + \\
&c_{1,1}\left(3\,375\,005\,903 + 581\,045\,794\, c_{1,2} + 226\,491\,672\, c_{1,3}\right)\big) + \\
&2\big(26\,226\,201\,004 + 1\,504\,735\,968\, c_{1,1}^{3} + 151\,476\,568\, c_{1,2}^{3} + 5\,238\,604\,963\, c_{1,3} + \\
&376\,306\,612\, c_{1,3}^{2} + 9\,655\,171\, c_{1,3}^{3} + 96\, c_{1,2}^{2}\left(24\,990\,338 + 1\,858\,287\, c_{1,3}\right) + \\
&2\, c_{1,1}^{2}\left(5\,396\,154\,124 + 1\,030\,280\,202\, c_{1,2} + 391\,850\,101\, c_{1,3}\right) + 2\, c_{1,2} \\
&\left(6\,719\,150\,063 + 940\,706\,774\, c_{1,3} + 35\,651\,988\, c_{1,3}^{2}\right) + 4\, c_{1,1}\big(7\,095\,622\,594 + 239\,418\,376 \\
&c_{1,2}^{2} + 962\,474\,362\, c_{1,3} + 36\,326\,769\, c_{1,3}^{2} + c_{1,2}\left(2\,504\,575\,249 + 184\,853\,551\, c_{1,3}\right)\big)\big)\big)\big)\big)
\end{aligned}
$$

```
FreeVars = Variables[Err];
DErr = Simplify[D[Err, {FreeVars}]];
H = D[DErr, {FreeVars}];
NSols = NSolve[DErr == 0, FreeVars, Reals];
TableForm[
 {Range[Length[NSols]], Err /. NSols, PositiveDefiniteMatrixQ[H /. N[#]] & /@ NSols}ᵀ]
```

```
1    0.029495    True
```

```
NSol = NSols[[1]];
FullSol = Join[GenSol /. NSol, NSol]
fo[x_] := f[x] /. FullSol;
Plot[fo[x], {x, -3, 3}, PlotStyle → Black, Background → White]
```

$\{c_{0,4} \to 0.0338547,\ c_{1,4} \to 0.0165229,\ c_{2,1} \to 0.10093,\ c_{2,2} \to -0.00915814,$
$c_{2,3} \to -0.0413939,\ c_{2,4} \to -0.0503776,\ c_{0,1} \to -0.443271,\ c_{0,2} \to -0.70886,$
$c_{0,3} \to 0.118277,\ c_{1,1} \to -0.54828,\ c_{1,2} \to 0.389883,\ c_{1,3} \to 0.141874\}$