$$h[x\_] := \begin{cases} 1 + c_{0,1}\, x + c_{0,2}\, x^2 + c_{0,3}\, x^3 + c_{0,4}\, x^4 & 0 \le x \le 1 \\ c_{1,1}\,(x-1) + c_{1,2}\,(x-1)^2 + c_{1,3}\,(x-1)^3 + c_{1,4}\,(x-1)^4 & 1 < x \le 2 \\ 0 & \text{True} \end{cases};$$

```
f[x_] := h[Abs[x]];
AllVars = {c0,1, c0,2, c0,3, c0,4, c1,1, c1,2, c1,3, c1,4};

(*Interpolant constraints*)
I1 = f[1]
I2 = f[2]
```

$1 + c_{0,1} + c_{0,2} + c_{0,3} + c_{0,4}$

$c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4}$

```
(*Partition of unity and linear term*)
```

$$\text{T0} = \text{CoefficientList}\left[\text{FullSimplify}\left[\sum_{k=-1}^{2} f[x-k],\ x > 0 \,\&\&\, x < 1\right], x\right]$$

$$\text{T1} = \text{CoefficientList}\left[\text{FullSimplify}\left[\sum_{k=-1}^{2} k\, f[x-k],\ x > 0 \,\&\&\, x < 1\right], x\right]$$

$\{2 + c_{0,1} + c_{0,2} + c_{0,3} + c_{0,4} + c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4},\ -2\,c_{0,2} - 3\,c_{0,3} - 4\,c_{0,4} - 2\,c_{1,2} - 3\,c_{1,3} - 4\,c_{1,4},$
$2\,c_{0,2} + 3\,c_{0,3} + 6\,c_{0,4} + 2\,c_{1,2} + 3\,c_{1,3} + 6\,c_{1,4},\ -4\,c_{0,4} - 4\,c_{1,4},\ 2\,c_{0,4} + 2\,c_{1,4}\}$

$\{1 + c_{0,1} + c_{0,2} + c_{0,3} + c_{0,4} + 2\,c_{1,1} + 2\,c_{1,2} + 2\,c_{1,3} + 2\,c_{1,4},$
$-c_{0,1} - 2\,c_{0,2} - 3\,c_{0,3} - 4\,c_{0,4} - 3\,c_{1,1} - 4\,c_{1,2} - 6\,c_{1,3} - 8\,c_{1,4},$
$c_{0,2} + 3\,c_{0,3} + 6\,c_{0,4} + c_{1,2} + 6\,c_{1,3} + 12\,c_{1,4},\ -c_{0,3} - 4\,c_{0,4} - 3\,c_{1,3} - 8\,c_{1,4},\ c_{0,4} + c_{1,4}\}$

```
GenSols = Solve[{
    I1 == 0,
    I2 == 0,
    T0[[1]] == 1,
    T0[[2]] == 0,
    T0[[3]] == 0,
    T1[[1]] == 0,
    T1[[2]] == 1,
    T1[[3]] == 0,
    T1[[4]] == 0
    },
    AllVars
]
```

Solve: Equations may not give solutions for all "solve" variables.

$$\left\{\left\{c_{0,4} \to -1 - c_{0,1} - c_{0,2} - c_{0,3},\ c_{1,1} \to -\frac{5}{3} - \frac{5\,c_{0,1}}{3} - \frac{2\,c_{0,2}}{3} - \frac{c_{0,3}}{3},\right.\right.$$
$$\left.\left. c_{1,2} \to 2 + 2\,c_{0,1} + c_{0,2} + c_{0,3},\ c_{1,3} \to -\frac{4}{3} - \frac{4\,c_{0,1}}{3} - \frac{4\,c_{0,2}}{3} - \frac{5\,c_{0,3}}{3},\ c_{1,4} \to 1 + c_{0,1} + c_{0,2} + c_{0,3}\right\}\right\}$$

```
RegionXY[k_] := {Quotient[k, 2], 1 + Quotient[-k, 2]};
Regions = Table[RegionXY[k], {k, -2, 5}]
```

{{-1, 2}, {-1, 1}, {0, 1}, {0, 0}, {1, 0}, {1, -1}, {2, -1}, {2, -2}}

```
GenSol = GenSols[[1]];
f[x_, y_] := f[x] f[y];
φ = 1/2;
```

$$W[k\_] := \begin{cases} 0 & k < 0 \\ \varphi^2/2 & k == 0 \\ 1 - (1-\varphi)^2/2 & k == 1 \\ 1 & \text{True} \end{cases};$$

$$\text{SumF} = \sum_{i=-3}^{5} \sum_{j=-3}^{5} W[i-j]\, f[x-i, y-j] \text{ /. GenSol};$$

```
SimplifySquare[f_, x0_, y0_] := Simplify[f, x > x0 && x < x0 + 1 && y > y0 && y < y0 + 1];
DSimplifySquare[f_, {x0_, y0_}] := Simplify[D[SimplifySquare[f, x0, y0], {{x, y}}]];
DSumF = ParallelMap[DSimplifySquare[SumF, #] &, Regions];
```

```
AnisoInt[df_, {x0_, y0_}] :=
  Simplify[Integrate[Expand[(df.{1, 1})^2], {x, x0, x0 + 1}, {y, y0, y0 + 1}]];
AnisoInts = Parallelize[MapThread[AnisoInt, {DSumF, Regions}]];
Err = Simplify[Total[AnisoInts] /. φ → 1/2]
```

$$\frac{1}{6\,350\,400}\Big(28\,242\,002 + 9\,203\,472\, c_{0,1}^4 + 408\,336\, c_{0,2}^4 + 17\,110\,486\, c_{0,3} +$$
$$3\,681\,808\, c_{0,3}^2 + 309\,740\, c_{0,3}^3 + 10\,461\, c_{0,3}^4 + 8\, c_{0,2}^3\,(648\,409 + 79\,850\, c_{0,3}) +$$
$$4\, c_{0,1}^3\,(13\,293\,824 + 4\,151\,754\, c_{0,2} + 1\,567\,813\, c_{0,3}) + 4\, c_{0,2}^2\,(6\,012\,948 + 1\,501\,989\, c_{0,3} + 95\,215\, c_{0,3}^2) +$$
$$2\, c_{0,2}\,(22\,099\,269 + 9\,321\,723\, c_{0,3} + 1\,175\,017\, c_{0,3}^2 + 51\,200\, c_{0,3}^3) +$$
$$4\, c_{0,1}^2\,(27\,705\,822 + 2\,836\,572\, c_{0,2}^2 + 6\,949\,914\, c_{0,3} + 419\,308\, c_{0,3}^2 + 69\, c_{0,2}\,(264\,361 + 31\,361\, c_{0,3})) +$$
$$2\, c_{0,1}\,(47\,116\,758 + 1\,743\,360\, c_{0,2}^3 + 19\,760\,445\, c_{0,3} + 2\,508\,664\, c_{0,3}^2 + 104\,876\, c_{0,3}^3 +$$
$$12\, c_{0,2}^2\,(1\,399\,992 + 168\,155\, c_{0,3}) + 2\, c_{0,2}\,(25\,702\,167 + 6\,445\,542\, c_{0,3} + 395\,537\, c_{0,3}^2))\Big)$$

```
FreeVars = Variables[Err];
DErr = Simplify[D[Err, {FreeVars}]];
H = D[DErr, {FreeVars}];
Sols = Solve[DErr == 0, FreeVars, Reals];
TableForm[
 {Range[Length[Sols]], Err /. N[Sols], PositiveDefiniteMatrixQ[H /. N[#]] & /@ Sols}ᵀ]
```

```
1    0.0493978    True
```

```
RootReduce[Sols[[1]]]
```

```
$Aborted
```

```
NSol = N[Sols[[1]]];
FullSol = Join[GenSol /. NSol, NSol]
fo[x_] := f[x] /. FullSol;
Plot[fo[x], {x, -3, 3}, PlotStyle → Black, Background → White]
```

$\{c_{0,4} \to -0.0675902,\ c_{1,1} \to -0.38799,\ c_{1,2} \to 0.449686,\ c_{1,3} \to -0.129287,$
$c_{1,4} \to 0.0675902,\ c_{0,1} \to -0.617904,\ c_{0,2} \to -0.432006,\ c_{0,3} \to 0.1175\}$