```
h[x_] := Piecewise[{
    {1 + c01 x + c02 x^2 + c03 x^3,                       0 ≤ x ≤ 1},
    {c11 (x - 1) + c12 (x - 1)^2 + c13 (x - 1)^3,       1 < x ≤ 2},
    {0,                                                  True}}];

f[x_] := h[Abs[x]];

(*Interpolant constraints*)
I1 = f[1]
I2 = f[2]
```

1 + c01 + c02 + c03

c11 + c12 + c13

```
(*Partition of unity and gradient representation*)
T0 = CoefficientList[FullSimplify[f[x + 1] + f[x] + f[x - 1] + f[x - 2], x > 0 && x < 1], x]
T1 = CoefficientList[FullSimplify[-f[x + 1] + f[x - 1] + 2 f[x - 2], x > 0 && x < 1], x]
```

{2 + c01 + c02 + c03 + c11 + c12 + c13, - 2 c02 - 3 c03 - 2 c12 - 3 c13, 2 c02 + 3 c03 + 2 c12 + 3 c13}

{1 + c01 + c02 + c03 + 2 c11 + 2 c12 + 2 c13,
 - c01 - 2 c02 - 3 c03 - 3 c11 - 4 c12 - 6 c13, c02 + 3 c03 + c12 + 6 c13, - c03 - 3 c13}

```
(*Smoothness*)
Df = Simplify[D[f[x], x], x > 0] /. Abs'[x] → 1
c01 + x (2 c02 + 3 c03 x) /. x → 1
c11 + (2 c12 + 3 c13 (-1 + x)) (-1 + x) /. x → 1
c11 + (2 c12 + 3 c13 (-1 + x)) (-1 + x) /. x → 2
```

$$\begin{cases} c01 + x\,(2\,c02 + 3\,c03\,x) & x \le 1 \\ c11 + (2\,c12 + 3\,c13\,(-1 + x))\,(-1 + x) & 1 < x \le 2 \\ 0 & \text{True} \end{cases}$$

c01 + 2 c02 + 3 c03

c11

c11 + 2 c12 + 3 c13

```
GenSols = Solve[{
    I1 == 0,
    I2 == 0,
    T0[[1]] == 1,
    T0[[2]] == 0,
    T0[[3]] == 0,
    T1[[1]] == 0,
    T1[[2]] == 1,
    T1[[3]] == 0,
    T1[[4]] == 0,
    c01 == 0,
    c01 + 2 c02 + 3 c03 == c11,
    c11 + 2 c12 + 3 c13 == 0
    },
    {c01, c02, c03, c11, c12, c13}
]
```

$$\left\{\left\{c01 \to 0,\ c02 \to -\frac{5}{2},\ c03 \to \frac{3}{2},\ c11 \to -\frac{1}{2},\ c12 \to 1,\ c13 \to -\frac{1}{2}\right\}\right\}$$