$$h[x\_] := \begin{cases} 1 + c_{0,1}\,x + c_{0,2}\,x^2 + c_{0,3}\,x^3 + c_{0,4}\,x^4 & 0 \le x \le 1/2 \\ c_{1,1}\,(x-1) + c_{1,2}\,(x-1)^2 + c_{1,3}\,(x-1)^3 + c_{1,4}\,(x-1)^4 & 1/2 < x \le 3/2; \\ 0 & \text{True} \end{cases}$$

```
f[x_] := h[Abs[x]];
AllVars = {c0,1, c0,2, c0,3, c0,4, c1,1, c1,2, c1,3, c1,4};

(*Continuity*)
C1 = Limit[h[x], x → 1/2, Direction → 1] == Limit[h[x], x → 1/2, Direction → -1]
C2 = Limit[h[x], x → 3/2, Direction → 1] == Limit[h[x], x → 3/2, Direction → -1]
```

$$\frac{1}{16}\left(16 + 8\,c_{0,1} + 4\,c_{0,2} + 2\,c_{0,3} + c_{0,4}\right) == \frac{1}{16}\left(-8\,c_{1,1} + 4\,c_{1,2} - 2\,c_{1,3} + c_{1,4}\right)$$

$$\frac{1}{16}\left(8\,c_{1,1} + 4\,c_{1,2} + 2\,c_{1,3} + c_{1,4}\right) == 0$$

```
(*Partition of unity and linear term*)
```

$$T0 = \text{CoefficientList}\left[\text{FullSimplify}\left[\sum_{i=-3}^{3} f[x-i],\ x > 0 \,\&\&\, x < 1/2\right], x\right]$$

$$T1 = \text{CoefficientList}\left[\text{FullSimplify}\left[\sum_{i=-3}^{3} i\, f[x-i],\ x > 0 \,\&\&\, x < 1/2\right], x\right]$$

$$\{1,\ c_{0,1},\ c_{0,2} + 2\,c_{1,2},\ c_{0,3},\ c_{0,4} + 2\,c_{1,4}\}$$

$$\{0,\ -2\,c_{1,1},\ 0,\ -2\,c_{1,3}\}$$

```
GenSols = Solve[{
    C1, C2,
    T0[[2]] == 0,
    T0[[3]] == 0,
    T0[[4]] == 0,
    T0[[5]] == 0,
    T1[[2]] == 1,
    T1[[3]] == 0,
    T1[[4]] == 0
    },
    AllVars
]
```

Solve: Equations may not give solutions for all "solve" variables.

$$\left\{\left\{c_{0,1} \to 0,\ c_{0,3} \to 0,\ c_{0,4} \to -8 - 4\,c_{0,2},\ c_{1,1} \to -\frac{1}{2},\ c_{1,2} \to -\frac{c_{0,2}}{2},\ c_{1,3} \to 0,\ c_{1,4} \to 4 + 2\,c_{0,2}\right\}\right\}$$

```
RegionXY[k_] := {Quotient[k, 2], 1 + Quotient[-k, 2]};
Regions = Table[RegionXY[k], {k, -4, 7}] - 1/2
```

$$\left\{\left\{-\frac{5}{2}, \frac{5}{2}\right\}, \left\{-\frac{5}{2}, \frac{3}{2}\right\}, \left\{-\frac{3}{2}, \frac{3}{2}\right\}, \left\{-\frac{3}{2}, \frac{1}{2}\right\}, \left\{-\frac{1}{2}, \frac{1}{2}\right\}, \left\{-\frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{2}, -\frac{1}{2}\right\}, \left\{\frac{1}{2}, -\frac{3}{2}\right\}, \left\{\frac{3}{2}, -\frac{3}{2}\right\}, \left\{\frac{3}{2}, -\frac{5}{2}\right\}, \left\{\frac{5}{2}, -\frac{5}{2}\right\}, \left\{\frac{5}{2}, -\frac{7}{2}\right\}\right\}$$

```
GenSol = GenSols[[1]];
f[x_, y_] := f[x] f[y];
φ = 1/2;
W[k_] := Piecewise[{{0, k < 0}, {φ^2/2, k == 0}, {1 - (1 - φ)^2/2, k == 1}, {1, True}}];
SumF = Sum[W[i - j] f[x - i, y - j], {i, -5, 6}, {j, -5, 6}] /. GenSol;
SimplifySquare[f_, x0_, y0_] := Simplify[f, x > x0 && x < x0 + 1 && y > y0 && y < y0 + 1];
DSimplifySquare[f_, {x0_, y0_}] := Simplify[D[SimplifySquare[f, x0, y0], {{x, y}}]];
DSumF = ParallelMap[DSimplifySquare[SumF, #] &, Regions];

AnisoInt[df_, {x0_, y0_}] :=
  Simplify[Integrate[Expand[(df.{1, 1})^2], {x, x0, x0 + 1}, {y, y0, y0 + 1}]];
AnisoInts = Parallelize[MapThread[AnisoInt, {DSumF, Regions}]];
Err = Simplify[Total[AnisoInts]]
```

$$\left(9\,621\,843 + 4\,576\,290\, c_{0,2} + 979\,234\, c_{0,2}^2 + 65\,410\, c_{0,2}^3 + 2266\, c_{0,2}^4\right) \Big/ 16\,934\,400$$

```
FreeVars = Variables[Err];
DErr = Simplify[D[Err, {FreeVars}]];
H = D[DErr, {FreeVars}];
Sols = Solve[DErr == 0, FreeVars, Reals];
TableForm[
 {Range[Length[Sols]], Err /. N[Sols], PositiveDefiniteMatrixQ[H /. N[#]] & /@ Sols}ᵀ]
```

1	0.183005	True

```
RootReduce[Sols[[1]]]
```

$$\left\{c_{0,2} \to \mathrm{Root}\left[2\,288\,145 + 979\,234\, \#1 + 98\,115\, \#1^2 + 4532\, \#1^3\, \&,\ 1\right]\right\}$$

```
NSol = N[Sols[[1]]];
FullSol = Join[GenSol /. NSol, NSol]
fo[x_] := f[x] /. FullSol;
Plot[fo[x], {x, -3, 3}, PlotStyle → Black, Background → White]
```

$\left\{c_{0,1} \to 0,\ c_{0,3} \to 0,\ c_{0,4} \to 4.8882,\ c_{1,1} \to -\frac{1}{2},\right.$
$\left. c_{1,2} \to 1.61102,\ c_{1,3} \to 0,\ c_{1,4} \to -2.4441,\ c_{0,2} \to -3.22205\right\}$