$$h[x\_] := \begin{cases} 1 + c_{0,1}\,x + c_{0,2}\,x^2 & 0 \le x \le 1 \\ c_{1,1}\,(x-1) + c_{1,2}\,(x-1)^2 & 1 < x \le 2 \\ c_{2,1}\,(x-2) + c_{2,2}\,(x-2)^2 & 2 < x \le 3 \\ 0 & \text{True} \end{cases};$$

```
f[x_] := h[Abs[x]];
AllVars = {c0,1, c0,2, c1,1, c1,2, c2,1, c2,2};
```

```
(*Interpolant constraints*)
I1 = f[1]
I2 = f[2]
I3 = f[3]
```

$1 + c_{0,1} + c_{0,2}$

$c_{1,1} + c_{1,2}$

$c_{2,1} + c_{2,2}$

```
(*Partition of unity and linear term*)
```

$$\text{T0} = \text{CoefficientList}\left[\text{FullSimplify}\left[\sum_{k=-2}^{3} f[x-k],\ x > 0\ \&\&\ x < 1\right],\ x\right]$$

$$\text{T1} = \text{CoefficientList}\left[\text{FullSimplify}\left[\sum_{k=-2}^{3} k\, f[x-k],\ x > 0\ \&\&\ x < 1\right],\ x\right]$$

$\{2 + c_{0,1} + c_{0,2} + c_{1,1} + c_{1,2} + c_{2,1} + c_{2,2},\ -2\,c_{0,2} - 2\,(c_{1,2} + c_{2,2}),\ 2\,c_{0,2} + 2\,(c_{1,2} + c_{2,2})\}$

$\{1 + c_{0,1} + c_{0,2} + 2\,c_{1,1} + 2\,c_{1,2} + 3\,c_{2,1} + 3\,c_{2,2},$
$\ -c_{0,1} - 2\,c_{0,2} - 3\,c_{1,1} - 4\,c_{1,2} - 5\,c_{2,1} - 6\,c_{2,2},\ c_{0,2} + c_{1,2} + c_{2,2}\}$

```
GenSols = Solve[{
    I1 == 0,
    I2 == 0,
    I3 == 0,
    T0[[1]] == 1,
    T0[[2]] == 0,
    T0[[3]] == 0,
    T1[[1]] == 0,
    T1[[2]] == 1,
    T1[[3]] == 0
    },
    AllVars
]
```

Solve: Equations may not give solutions for all "solve" variables.

$\{\{c_{0,2} \to -1 - c_{0,1},\ c_{1,2} \to -c_{1,1},\ c_{2,1} \to -1 - c_{0,1} - c_{1,1},\ c_{2,2} \to 1 + c_{0,1} + c_{1,1}\}\}$

```
RegionXY[k_] := {Quotient[k, 2], 1 + Quotient[-k, 2]};
Regions = Table[RegionXY[k], {k, -4, 7}]
```

```
{{-2, 3}, {-2, 2}, {-1, 2}, {-1, 1}, {0, 1},
 {0, 0}, {1, 0}, {1, -1}, {2, -1}, {2, -2}, {3, -2}, {3, -3}}
```

```
GenSol = GenSols[[1]];
f[x_, y_] := f[x] f[y];
φ = 1/2;
W[k_] := Piecewise[{{0, k < 0}, {φ^2/2, k == 0}, {1 - (1 - φ)^2/2, k == 1}, {1, True}}];
```

$$W[k\_] := \begin{cases} 0 & k < 0 \\ \varphi^2/2 & k == 0 \\ 1 - (1-\varphi)^2/2 & k == 1 \\ 1 & \text{True} \end{cases};$$

$$\text{SumF} = \sum_{i=-5}^{6} \sum_{j=-5}^{6} W[i-j]\, f[x-i, y-j] \text{ /. GenSol};$$

```
SimplifySquare[f_, x0_, y0_] := Simplify[f, x > x0 && x < x0 + 1 && y > y0 && y < y0 + 1];
DSimplifySquare[f_, {x0_, y0_}] := Simplify[D[SimplifySquare[f, x0, y0], {{x, y}}]];
DSumF = ParallelMap[DSimplifySquare[SumF, #] &, Regions];
```

```
AnisoInt[df_, {x0_, y0_}] :=
  Simplify[Integrate[Expand[(df.{1, 1})^2], {x, x0, x0 + 1}, {y, y0, y0 + 1}]];
AnisoInts = Parallelize[MapThread[AnisoInt, {DSumF, Regions}]];
Err = Simplify[Total[AnisoInts]]
```

$$\frac{1}{1440}\left(4373 + 760\, c_{0,1}^4 + 6438\, c_{1,1} + 3297\, c_{1,1}^2 + 636\, c_{1,1}^3 + 156\, c_{1,1}^4 + 5\, c_{0,1}^3 \left(1051 + 276\, c_{1,1}\right) + 3\, c_{0,1}^2 \left(4381 + 2430\, c_{1,1} + 372\, c_{1,1}^2\right) + c_{0,1} \left(12\,826 + 12\,528\, c_{1,1} + 3897\, c_{1,1}^2 + 456\, c_{1,1}^3\right)\right)$$

```
FreeVars = Variables[Err];
DErr = Simplify[D[Err, {FreeVars}]];
H = D[DErr, {FreeVars}];
Sols = Solve[DErr == 0, FreeVars, Reals];
TableForm[
 {Range[Length[Sols]], Err /. N[Sols], PositiveDefiniteMatrixQ[H /. N[#]] & /@ Sols}ᵀ]
```

1	0.0343838	True

```
RootReduce[Sols[[1]]]
```

$$\{c_{0,1} \to \text{Root}[107\,304\,086\,230\,531\,885 + 599\,287\,446\,548\,831\,718\, \#1 + 1\,400\,958\,868\,626\,252\,747\, \#1^2 + 1\,834\,238\,645\,594\,718\,312\, \#1^3 + 1\,498\,904\,274\,881\,813\,490\, \#1^4 + 798\,582\,300\,168\,995\,568\, \#1^5 + 278\,783\,851\,306\,490\,292\, \#1^6 + 61\,715\,710\,273\,939\,056\, \#1^7 + 7\,883\,190\,806\,676\,480\, \#1^8 + 443\,705\,711\,242\,240\, \#1^9 \,\&,\, 1],$$
$$c_{1,1} \to \text{Root}[18\,431\,582\,450\,625 + 23\,718\,361\,748\,580\, \#1 + 334\,904\,447\,070\,675\, \#1^2 - 37\,839\,981\,285\,332\, \#1^3 - 288\,829\,159\,903\,530\, \#1^4 - 336\,831\,488\,617\,800\, \#1^5 - 632\,563\,845\,450\,300\, \#1^6 + 2\,304\,156\,770\,184\,864\, \#1^7 - 1\,637\,392\,551\,713\,280\, \#1^8 + 354\,964\,568\,993\,792\, \#1^9 \,\&,\, 1]\}$$

```
NSol = N[Sols[[1]]];
FullSol = Join[GenSol /. NSol, NSol]
fo[x_] := f[x] /. FullSol;
Plot[fo[x], {x, -3, 3}, PlotStyle → Black, Background → White]
```

$\{c_{0,2} \to -0.442669,\ c_{1,2} \to 0.596792,\ c_{2,1} \to 0.154123,$
$c_{2,2} \to -0.154123,\ c_{0,1} \to -0.557331,\ c_{1,1} \to -0.596792\}$