

$$h[x_] := \begin{cases} 1 + c_{0,1} x + c_{0,2} x^2 + c_{0,3} x^3 & 0 \leq x \leq 1 \\ c_{1,1} (x-1) + c_{1,2} (x-1)^2 + c_{1,3} (x-1)^3 & 1 < x \leq 2; \\ 0 & \text{True} \end{cases}$$

f[x_] := h[Abs[x]];

AllVars = {c_{0,1}, c_{0,2}, c_{0,3}, c_{1,1}, c_{1,2}, c_{1,3}};

(*Interpolant constraints*)

I1 = f[1]

I2 = f[2]

1 + c_{0,1} + c_{0,2} + c_{0,3}

c_{1,1} + c_{1,2} + c_{1,3}

(*Partition of unity and linear term*)

T0 = CoefficientList[FullSimplify[$\sum_{k=-1}^2 f[x-k]$, x > 0 && x < 1], x]

T1 = CoefficientList[FullSimplify[$\sum_{k=-1}^2 k f[x-k]$, x > 0 && x < 1], x]

{2 + c_{0,1} + c_{0,2} + c_{0,3} + c_{1,1} + c_{1,2} + c_{1,3}, -2 c_{0,2} - 3 c_{0,3} - 2 c_{1,2} - 3 c_{1,3}, 2 c_{0,2} + 3 c_{0,3} + 2 c_{1,2} + 3 c_{1,3}}

{1 + c_{0,1} + c_{0,2} + c_{0,3} + 2 c_{1,1} + 2 c_{1,2} + 2 c_{1,3},
-c_{0,1} - 2 c_{0,2} - 3 c_{0,3} - 3 c_{1,1} - 4 c_{1,2} - 6 c_{1,3}, c_{0,2} + 3 c_{0,3} + c_{1,2} + 6 c_{1,3}, -c_{0,3} - 3 c_{1,3}}

(*Smoothness*)

Dh = Simplify[D[h[x], x], x > 0];

S0 = (Dh /. x → 0) == 0

S1 = Limit[Dh, x → 1, Direction → 1] == Limit[Dh, x → 1, Direction → -1]

S2 = Limit[Dh, x → 2, Direction → 1] == Limit[Dh, x → 2, Direction → -1]

c_{0,1} == 0

c_{0,1} + 2 c_{0,2} + 3 c_{0,3} == c_{1,1}

c_{1,1} + 2 c_{1,2} + 3 c_{1,3} == 0

GenSols = Solve[{

I1 == 0,

I2 == 0,

T0[[1]] == 1,

T0[[2]] == 0,

T0[[3]] == 0,

T1[[1]] == 0,

T1[[2]] == 1,

T1[[3]] == 0,

T1[[4]] == 0,

S0, S1, S2

},

AllVars

]

{ {c_{0,1} → 0, c_{0,2} → - $\frac{5}{2}$, c_{0,3} → $\frac{3}{2}$, c_{1,1} → - $\frac{1}{2}$, c_{1,2} → 1, c_{1,3} → - $\frac{1}{2}$ } }