Ejercicio de optimización de supply chain

Rodrigo Maranzana

Enunciado

En una cadena logística existen 3 tipos de nodos:

- Proveedores
- Transbordo
- Almacenamiento

- Tanto la oferta como la demanda son fijas. Además, los nodos de transbordo, no pueden almacenar stock, todo lo que ingresa debe salir hacia otro nodo.
- Existe la posibilidad de usar o no los nodos de transbordo, ya que es posible el envío directo de las planta a los almacenes.
- Los nodos pueden enviar y recibir mientras el grafo lo permita.
- Se pide minimizar el costo de transporte de productos, cumpliendo con la demanda. El quiebre de stock no es una opción.

Datos

Matriz de costos de envío

origen / destino	Planta 1	Planta 2	Transbordo 1	Transbordo 2	Almacén 1	Almacén 2
Planta 1	X	X	12	22	65	31
Planta 2	X	X	8	22	30	30
Transbordo 1	X	X	X	4	20	18
Transbordo 2	X	X	X	X	15	20
Almacén 1	X	X	X	X	X	14
Almacén 2	X	X	X	X	X	X

Oferta y demanda de cada nodo

origen / destino	Planta 1	Planta 2	Transbordo 1	Transbordo 2	Almacén 1	Almacén 2	Total
Oferta	160	220	X	X	X	X	380
Demanda	X	X	X	X	200	180	380

Consignas

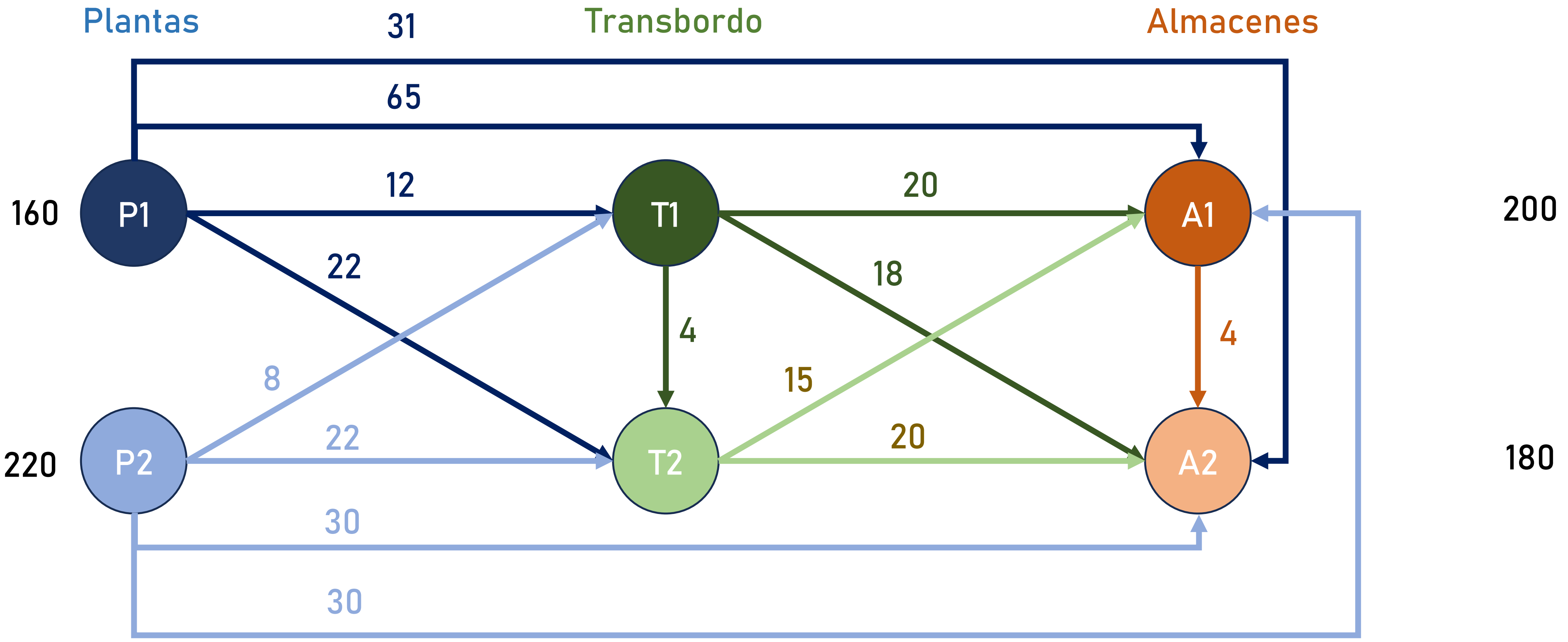
1- Construir el grafo de la cadena logística.

2- Armar modelo algebraico de FMC.

3- Armar el modelo matricial de FMC.

4- Resolver el modelo algebraico con python.

1-Grafo de la cadena logística



2-Modelo algebraico FMC

La cadena logística pertenece al caso balanceado, oferta igual a demanda.

$$Min \sum_{i} \sum_{j} X_{ij} d_{ij}$$

st:

$$b_i = \sum_{j,\ ij \subset A} X_{ij} - \sum_{j,\ ji \subset A} X_{ji} \quad ; \forall i$$

$$\text{cota inferior} \leq X \leq cota\ superior$$

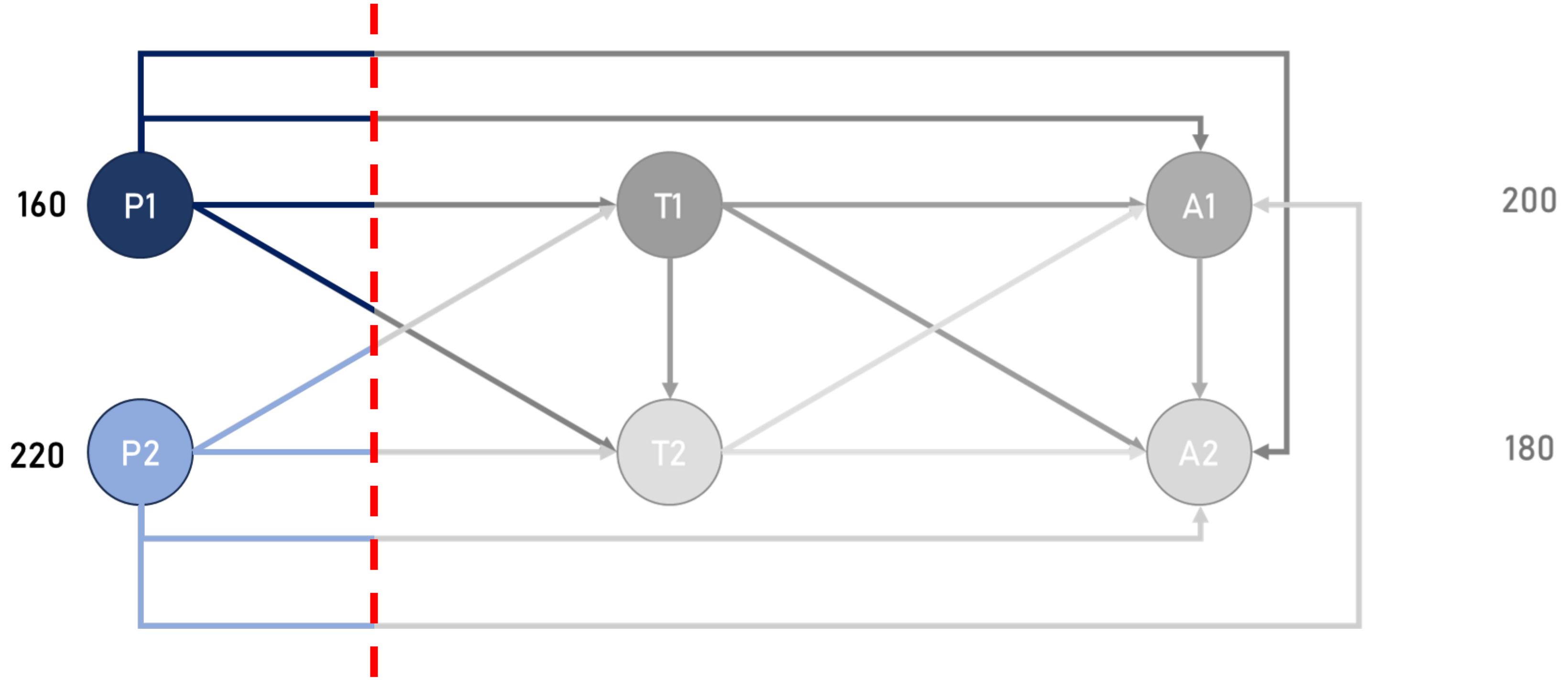
2-Función objetivo

origen / destino	Planta 1	Planta 2	Transbordo 1	Transbordo 2	Almacén 1	Almacén 2
Planta 1	X	X	12	22	65	31
Planta 2	X	X	8	22	30	30
Transbordo 1	X	X	X	4	20	18
Transbordo 2	X	X	X	X	15	20
Almacén 1	X	X	X	X	X	14
Almacén 2	X	X	X	X	X	X

$$Min \sum_{i} \sum_{j} X_{ij} d_{ij}$$

$$Min \quad 12\, X_{P1,T1} + 22\, X_{P1,T2} + 65\, X_{P1,A1} + 31\, X_{P1,A2} +$$
$$8\, X_{P2,T1} + 22\, X_{P2,T2} + 30\, X_{P2,A1} + 30\, X_{P2,A2} +$$
$$4\, X_{T1,T2} + 20\, X_{T1,A1} + 18\, X_{T1,A2} +$$
$$15\, X_{T2,A1} + 20\, X_{T2,A2} +$$
$$14\, X_{A1,A2}$$

2-Restricciones: plantas

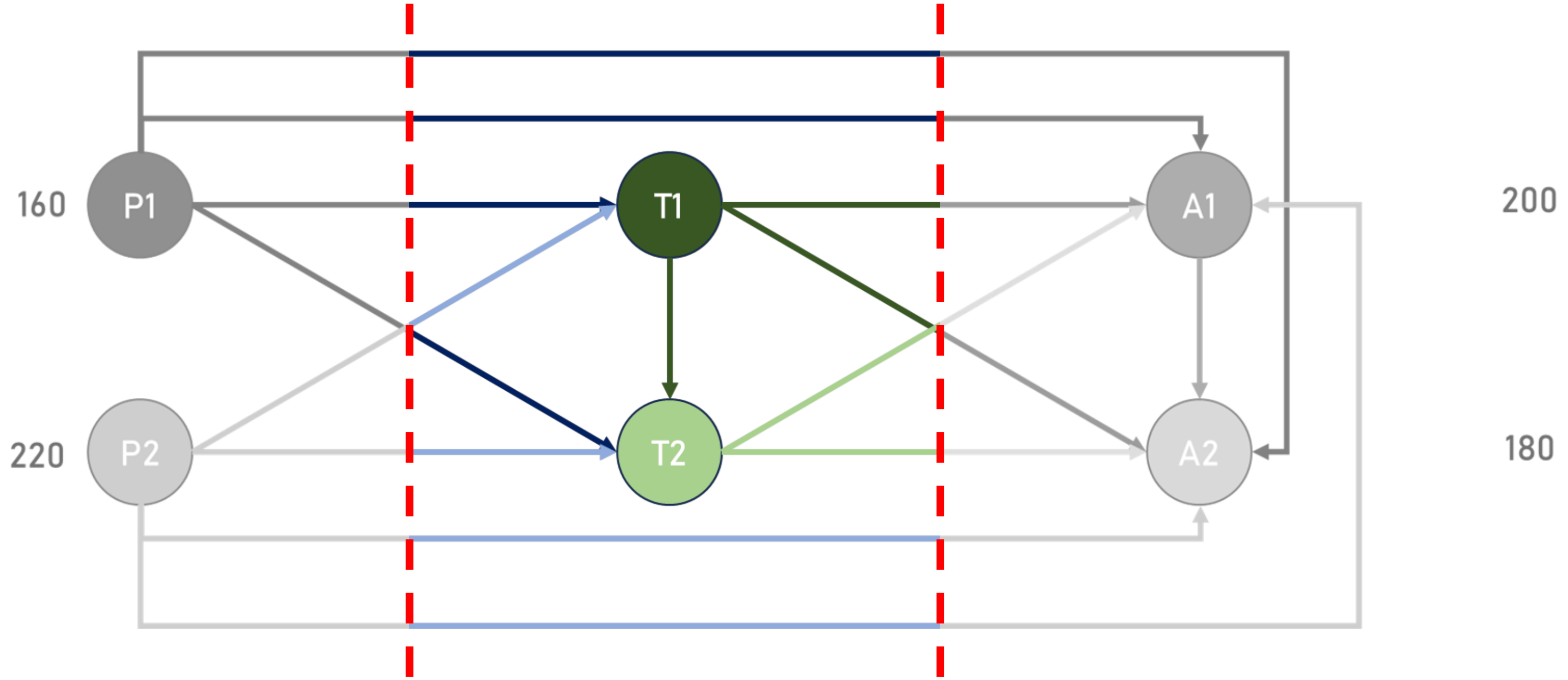


$$b_i = \sum_{j,\ ij\subset A} X_{ij} - \sum_{j,\ ji\subset A} X_{ji} \quad ; \forall i$$

Planta 1: $X_{P1,T1} + X_{P1,T2} + X_{P1,A1} + X_{P1,A2} = 160$

Planta 2: $X_{P2,T1} + X_{P2,T2} + X_{P2,A1} + X_{P2,A2} = 220$

2-Restricciones: transbordo

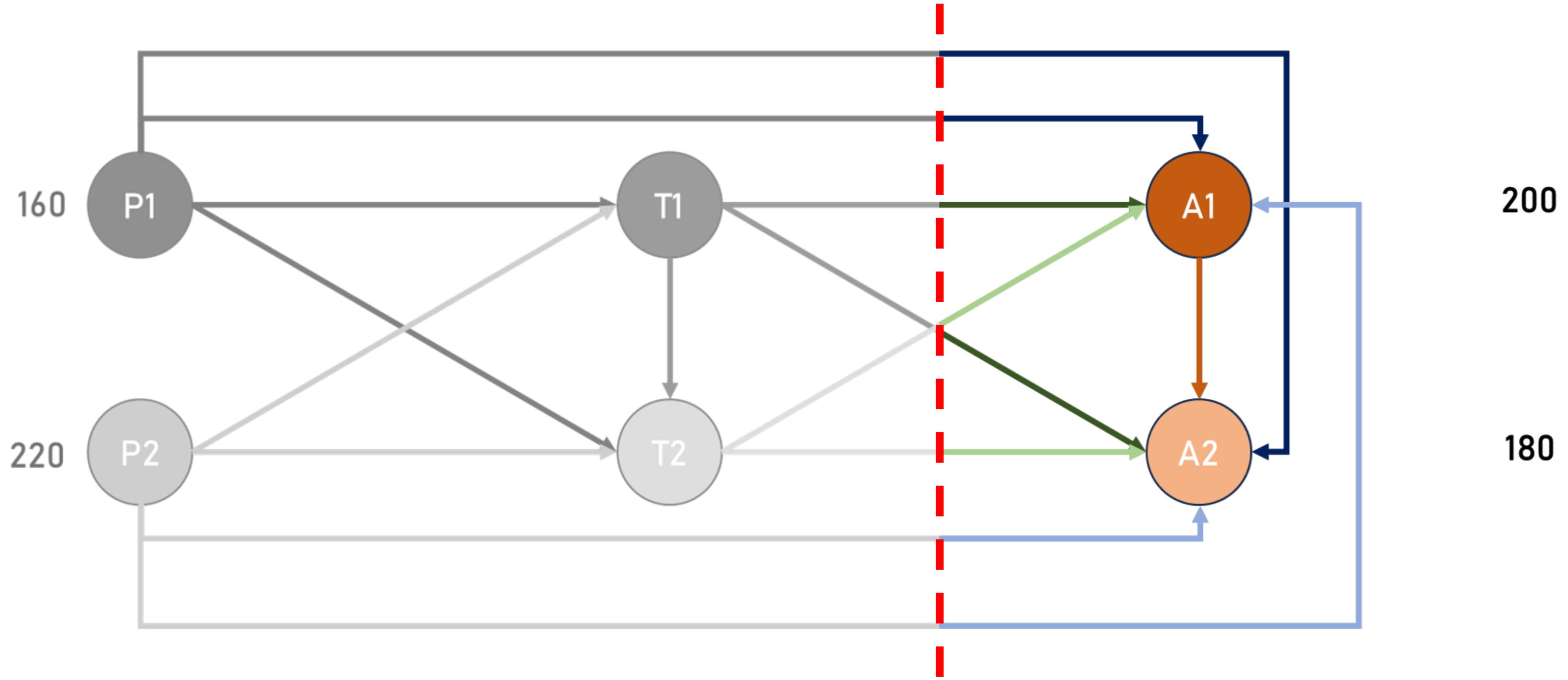


$$b_i = \sum_{j,\ ij \subset A} X_{ij} - \sum_{j,\ ji \subset A} X_{ji} \quad ; \forall i$$

Transbordo 1: $X_{T1,A1} + X_{T1,A2} + X_{T1,T2} - X_{P1,T1} - X_{P2,T1} = 0$

Transbordo 2: $X_{T2,A1} + X_{T2,A2} - X_{P1,T2} - X_{P2,T2} = 0$

2-Restricciones: almacenes



$$b_i = \sum_{j,\ \ ij \subset A} X_{ij} - \sum_{j,\ \ ji \subset A} X_{ji} \quad ; \forall i$$

Almacén 1: $X_{A1,A2} - X_{T1,A1} - X_{T2,A1} - X_{P1,A1} - X_{P2,A1} = 200$

Almacén 2: $-X_{T1,A2} - X_{T2,A2} - X_{P1,A2} - X_{P2,A2} = 180$

2-Modelo algebraico FMC

$$Min \quad 12\,X_{P1,T1} + 22\,X_{P1,T2} + 65\,X_{P1,A1} + 31\,X_{P1,A2} + 8\,X_{P2,T1} + 22\,X_{P2,T2} + 30\,X_{P2,A1} + 30\,X_{P2,A2} + 4\,X_{T1,T2} + 20\,X_{T1,A1} + 18\,X_{T1,A2} + 15\,X_{T2,A1} + 20\,X_{T2,A2} + 14\,X_{A1,A2}$$

st:

$$X_{P1,T1} + X_{P1,T2} + X_{P1,A1} + X_{P1,A2} = 160$$

$$X_{P2,T1} + X_{P2,T2} + X_{P2,A1} + X_{P2,A2} = 220$$

$$X_{T1,A1} + X_{T1,A2} + X_{T1,T2} - X_{P1,T1} - X_{P2,T1} = 0$$

$$X_{T2,A1} + X_{T2,A2} - X_{P1,T2} - X_{P2,T2} = 0$$

$$X_{A1,A2} - X_{T1,A1} - X_{T2,A1} - X_{P1,A1} - X_{P2,A1} = -200$$

$$-X_{T1,A2} - X_{T2,A2} - X_{P1,A2} - X_{P2,A2} = -180$$

$$0 \leq X, X \in \mathbb{R}$$

3-Modelo matricial FMC

La cadena logística pertenece al caso balanceado, oferta igual a demanda.

$$Min\ C^T X$$

st:

$$AX = b$$

$$\text{cota inferior} \leq X \leq cota\ superior$$

3-Variables de decisión y costos

Variables de decisión:

$$X = \begin{bmatrix} x_{P1,T1} \\ x_{P1,T2} \\ x_{P1,A1} \\ x_{P1,A2} \\ x_{P2,T1} \\ x_{P2,T2} \\ x_{P2,A1} \\ x_{P2,A2} \\ x_{T1,T2} \\ x_{T1,A1} \\ x_{T1,A2} \\ x_{T2,A1} \\ x_{T2,A2} \\ x_{A1,A2} \end{bmatrix}$$

Costos:

$$C = \begin{bmatrix} 12 \\ 22 \\ 65 \\ 31 \\ 8 \\ 22 \\ 30 \\ 30 \\ 4 \\ 20 \\ 18 \\ 15 \\ 20 \\ 14 \end{bmatrix}$$

3-Matriz nodo-arco y vector b

Matriz nodo-arco:

	x_{P1T1}	x_{P1T2}	x_{P1A1}	x_{P1A2}	x_{P2T1}	x_{P2T2}	x_{P2A1}	x_{P2A2}	x_{T1T2}	x_{T1A1}	x_{T1A2}	x_{T2A1}	x_{T2A2}	x_{A1A2}		b
Plantas	1	1	1	1												160
Plantas					1	1	1	1								220
Transb.	-1				-1				1	1	1				=	0
Transb.		-1				-1			-1			1	1			0
Almacén			-1				-1			-1		-1		1		-200
Almacén				-1				-1			-1		-1	-1		-180

4-Solución con python PuLP

```python
import pulp

lp01 = pulp.LpProblem("ejercicio-supply-chain", pulp.LpMinimize)

# Arcos:
arcos = [
    'p1t1', 'p1t2', 'p1a1', 'p1a2', 'p2t1', 'p2t2', 'p2a1', 'p2a2',
    't1t2', 't1a1', 't1a2', 't2a1', 't2a2', 'a1a2'
]

# Variables:
X = pulp.LpVariable.dicts('x', arcos, 0, None, cat='Continuous')

# Función objetivo:
lp01 += 12*X['p1t1'] + 22*X['p1t2'] + 65*X['p1a1'] + 31*X['p1a2'] + 8*X['p2t1'] +\
        22*X['p2t2'] + 30*X['p2a2'] + 4*X['t1t2'] + 20*X['t1a1'] + 18*X['t1a2'] +\
        15*X['t2a1'] + 20*X['t2a2'] + 14*X['a1a2'], "Z"

# # Restricciones:
lp01 +=  X['p1t1'] + X['p1t2'] + X['p1a1'] + X['p1a2'] == 160
lp01 +=  X['p2t1'] + X['p2t2'] + X['p2a1'] + X['p2a2'] == 220
lp01 +=  X['t1a1'] + X['t1a2'] + X['t1t2'] - X['p1t1'] - X['p2t1'] == 0
lp01 +=  X['t2a1'] + X['t2a2'] - X['p1t2'] - X['p2t2'] == 0
lp01 +=  X['a1a2'] - X['t1a1'] - X['t2a1'] - X['p1a1'] - X['p2a1'] == -200
lp01 +=  - X['t1a2'] - X['t2a2'] - X['p1a2'] - X['p2a2'] == -180
```

```python
# Resolucion:
lp01.solve()

# Imprimimos el status del problema:
print(pulp.LpStatus[lp01.status])

# Imprimimos las variables en su valor óptimo:
for variable in lp01.variables():
    print("%s = %.2f" % (variable.name, variable.varValue))

# Imprimimos el funcional óptimo:
print(f'Función objetivo: {pulp.value(lp01.objective)}')
```

4-Solución con python PuLP

```
>>Optimal
>>x_a1a2 = 0.00
>>x_p1a1 = 0.00
>>x_p1a2 = 0.00
>>x_p1t1 = 160.00
>>x_p1t2 = 0.00
>>x_p2a1 = 200.00
>>x_p2a2 = 0.00
>>x_p2t1 = 20.00
>>x_p2t2 = 0.00
>>x_t1a1 = 0.00
>>x_t1a2 = 180.00
>>x_t1t2 = 0.00
>>x_t2a1 = 0.00
>>x_t2a2 = 0.00
>>Función objetivo: 5320.0
```