**%FontSize=32
%TeXFontSize=32
\documentclass{article}
\pagestyle{empty}
\def\Nspaces{\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ }
\begin{document}
\begin{tabular}{lll}
6&\Nspaces\Nspaces\Nspaces&\ \\\hline\hline
\end{tabular}
\end{document}![%FontSize=8
%TeXFontSize=8
\documentclass{article}
\pagestyle{empty}
\begin{document}
%\[
\hrule
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\end{document}](data:image/png;base64,iVBORw0KGgoAAAANSUhEUgAAAAEAAAABAQMAAAAl21bKAAAAA1BMVEX///+nxBvIAAAAAXRSTlMAQObYZgAAAAlwSFlzAAAOxAAADsQBlSsOGwAAAAlwSFlzAABcRgAAXEYBFJRDQQAAAApJREFUCB1jYAAAAAIAAc/INeUAAAAASUVORK5CYII=)Duality Theory and Sensitivity Analysis**

One of the most important discoveries in the early development of linear programming was the concept of duality and its many important ramifications. This discovery revealed that every linear programming problem has associated with it another linear programming problem called the **dual.** The relationships between the dual problem and the original problem (called the **primal**) prove to be extremely useful in a variety of ways. For example, you soon will see that the shadow prices described in Sec. 4.7 actually are provided by the optimal solution for the dual problem. We shall describe many other valuable applications of duality theory in this chapter as well.

One of the key uses of duality theory lies in the interpretation and implementation of *sensitivity analysis.* As we already mentioned in Secs. 2.3, 3.3, and 4.7, sensitivity analysis is a very important part of almost every linear programming study. Because most of the parameter values used in the original model are just *estimates*

of future conditions, the effect on the optimal solution if other conditions prevail instead needs to be investigated. Furthermore, certain parameter values (such as resource amounts) may represent *managerial decisions,* in which case the choice of the parameter values may be the main issue to be studied, which can be done through sensitivity analysis.

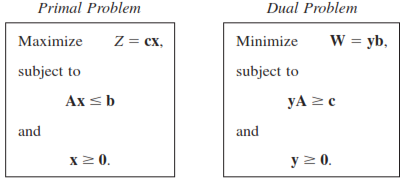
For greater clarity, the first three sections discuss duality theory under the assumption that the *primal* linear programming problem is in *our standard form* (but with no restriction that the *b* values need to be positive). Other forms are then discussed in Sec. 6.4. We begin the chapter by introducing the essence of duality theory and its applications. We then describe the economic interpretation of the dual problem (Sec. 6.2) and delve deeper into the relationships between the primal and dual problems (Sec. 6.3). Section 6.5 focuses on the role of duality theory in sensitivity analysis. The basic procedure for sensitivity analysis (which is based on the fundamental insight of Sec. 5.3) is summarized in Sec. 6.6 and illustrated in Sec. 6.7.

**6.1 THE ESSENCE OF DUALITY THEORY**

Given our standard form for the *primal problem* at the left (perhaps after conversion from another form), its *dual problem* has the form shown to the right.

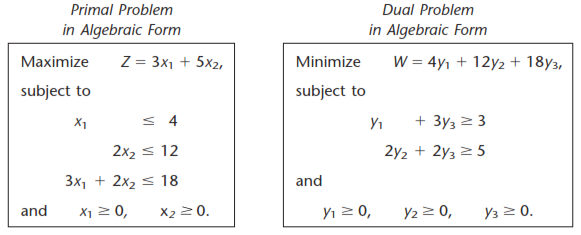
|  |  |
| --- | --- |
| *Primal Problem* | *Dual Problem* |
|  |  |

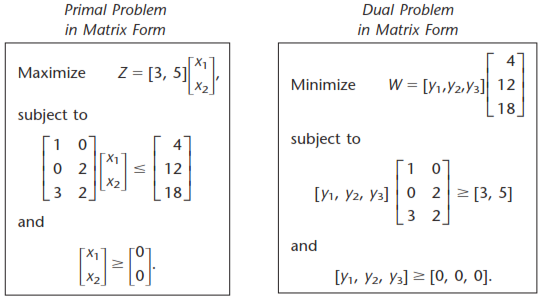
Thus, the dual problem uses exactly the same *parameters* as the primal problem, but in different locations. To highlight the comparison, now look at these same two problems in matrix notation. Thus, the dual problem uses exactly the same *parameters* as the primal problem, but in different locations. To highlight the comparison, now look at these same two problems in matrix notation.



To illustrate, the primal and dual problems for the Wyndor Glass Co. example of Sec. 3.1 are shown in Table 6.1 in both algebraic and matrix form.

**TABLE 6.1 Primal and dual problems for the Wyndor Glass Co. Example**





The **primal-dual table** for linear programming (Table 6.2) also helps to highlight the correspondence between the two problems. It shows all the linear programming parameters %FontSize=9
%TeXFontSize=9
\documentclass{article}
\pagestyle{empty}
\begin{document}
\[
\left(a_{ij},\,b_{j},\,\mbox{\rm\ y\ }\,c_{}\right)
\]
\end{document} and how they are used to construct the two problems. All the headings for the primal problem are horizontal, whereas the headings for the dual problem areread by turning the book sideways. For the primal problem, each *column* (except the Right

Side column) gives the coefficients of a single variable in the respective constraints and then in the objective function, whereas each *row* (except the bottom one) gives the parameters for a single contraint. For the dual problem, each *row* (except the Right Side row) gives the coefficients of a single variable in the respective constraints and then in the objective function, whereas each *column* (except the rightmost one) gives the parameters for a single constraint. In addition, the Right Side column gives the right-hand sides for the primal problem and the objective function coefficients for the dual problem, whereas thebottom row gives the objective function coefficients for the primal problem and the righth and sides for the dual problem.