Mutual Fund Rebalancing in context to Indian Mutual Funds

Shaurya Rawat

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A portfolio optimization problem

Abstract

This project explores mutual fund rebalancing for a given mutual fund portfolio (or set of portfolios) in the Indian mutual fund market using a hybrid risk model. We will first model risk at qualitative risk (based on determined category-level ratings of funds) and then integrate with quantitative risk (derived from volatility and inverse Sortino ratio). We will use my own portfolio for the analysis (fund level details are added) given we have market data atleast for the last 5 years, needed for deriving quantitative risk and also backtesting our models. We implement and compare three portfolio optimization methods: a) Mean–Variance Optimization (MVO), b) Conditional Value-at-Risk (CVaR), c) Hybrid CVaR model using our hybrid risk model (combining qualitative and quantitative risk) The objective is to design an optimized rebalanced portfolio that maximizes return while controlling downside and category risk. We will also plot efficient frontiers and compare returns and risk on backtested data.

1 Objective and Approach

Mutual fund investors in India often face challenges in maintaining an optimal portfolio allocation as market conditions, fund performances, and category-level risks evolve. Manual rebalancing decisions are frequently based on limited information, such as short-term returns or basic category risk ratings, rather than comprehensive quantitative evaluation. The project explores a data-driven rebalancing framework for Indian mutual funds that integrates both qualitative and quantitative risk factors in very well known frameworks (MVO and CVaR), thereby enabling systematic and informed portfolio adjustments.

The objective of this project is to develop a portfolio optimization and rebalancing model that:

1. **Risk Profiling:** Develops a risk scoring framework using:
    - **Qualitative Risk:** Based on mutual fund category (e.g., large-cap, mid-cap, hybrid, sectoral).
    - **Quantitative Risk:** Derived from historical volatility and downside deviation measures.

- **Hybrid Risk:** A composite measure integrating both qualitative and quantitative risk indicators.

2. **Portfolio Rebalancing (Optimization):** Rebalances the user's existing mutual fund portfolio using:

   - Qualitative only Risk to maximise expected returns keeping category risk constant.
   - Mean-Variance Optimization (MVO) to minimize portfolio variance for a given expected return.
   - Conditional Value-at-Risk (CVaR) Optimization to control for extreme downside risk.
   - Hybrid CVaR to integrate both market-based risk and category-level risk penalties.

   The models are compared through backtesting on five years of fund data, with average trailing returns as expected returns.

2 Risk Profiling and Rebalancing Framework

2.1 A. Qualitative Risk Profiling

From understanding of Indian mutual funds, qualitative risk scores (1–5) are assigned to each fund for it's category based on its inherent volatility and investment strategy. The mapping between derived mutual fund categories and their corresponding qualitative risk levels is defined as follows:

Group Name	Categories Included (DERIVED_CATEGORY)	Risk Score
Debt / Liquid	LIQUID, ULTRA_SHORT_DURATION, MONEY_MARKET, LOW_DURATION	1 (Low)
Large Cap / Index	LARGE_CAP, INDEX_LARGE_CAP, VALUE_AND_CONTRA	2 (Low-Mid)
Mid Cap / Blend	MID_CAP, FLEXI_CAP, MULTI_CAP, LARGE_AND_MID_CAP	3 (Mid)
Small Cap / Aggressive	SMALL_CAP, THEMATIC_TECH, GLOBAL_OTHER, FOCUSED_FUND	4 (High)
Hybrid / Balanced	DYNAMIC_ASSET_ALLOCATION, EQUITY_SAVINGS, AGGRESSIVE_ALLOCATION	3 (High)
Thematic / Sectoral	TECH, INFRA, HEALTHCARE, ESG, PSU, DEFENCE, etc.	5 (Very High)

Figure 1: Qualitative Category to risk-score mapping for Indian mutual funds Categories

This qualitative mapping provides an intuitive, category-driven baseline for risk assessment prior to quantitative modeling.

2.2 B. Quantitative Risk Estimation

Quantitative risk is modeled using empirical return data over a three-year historical horizon. For each mutual fund, the mean of the daily one-year trailing returns r_t is computed as:

$$\mu = \frac{1}{n} \sum_{t=1}^{n} r_t \tag{1}$$

where μ is the expected annualized return and n represents the total number of trading days across the sample window.

Volatility

Volatility (σ) measures the dispersion of returns around the mean:

$$\sigma = \sqrt{\frac{1}{n} \sum_{t=1}^{n} (r_t - \mu)^2} \tag{2}$$

Higher volatility implies greater uncertainty and potential risk in fund performance.

Sortino Ratio (Downside Risk Adjustment)

To capture return-adjusted risk, the Sortino Ratio is employed:

$$\text{Sortino Ratio} = \frac{\mu - r_f}{\sigma_{DD} + \epsilon} \tag{3}$$

where μ is the expected return, r_f is the risk-free rate (approximated as 7.26% per annum based on the 10-year Indian government bond yield), and σ_{DD} represents downside deviation:

$$\sigma_{DD} = \sqrt{\frac{1}{n} \sum_{r_t < r_f} (r_t - r_f)^2} \tag{4}$$

The Sortino Ratio penalizes only downside volatility, providing a more meaningful risk-adjusted measure than total variance.

2.3 C. Comparative Analysis of Qualitative vs Quantitative Risk

To validate the qualitative scoring system, an empirical comparison is made between qualitative category risk and average quantitative metrics (volatility, return, and Sortino ratio). The summary statistics are shown below:

Category Risk	Fund Count	Average Return (%)	Volatility (%)	Sortino Ratio
1	95	7.11	2.75	0.13
2	77	20.08	12.75	2.63
3	200	22.08	13.53	4.29
4	105	19.81	15.57	1.85
5	90	27.52	18.79	4.56

Table 1: Quantitative risk characteristics by qualitative category.

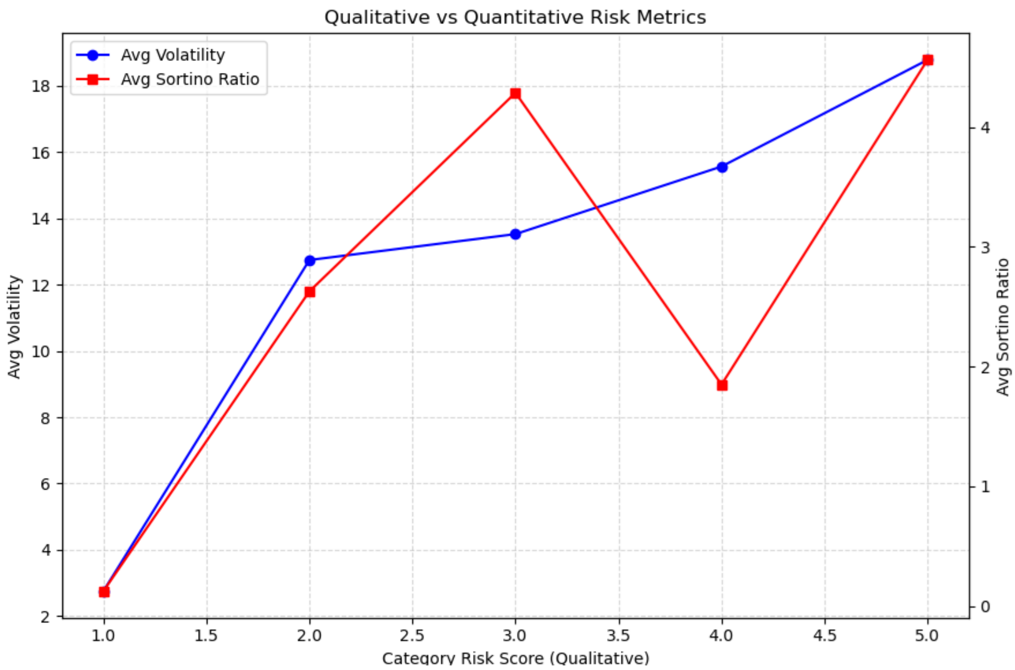

Figure 2: Qualitative against Quantitative Risk (Correlation)

1. Qualitative risk score correlates strongly with both volatility and expected return. As category risk increases, both metrics rise consistently.

2. Sortino ratios peak at category risk levels 3 and 5, indicating superior return-per-unit of downside risk in medium and very high risk categories.

3. Overall, qualitative risk is positively correlated with average volatility and return, confirming internal consistency between qualitative and quantitative dimensions.

This dual-layer risk framework (qualitative + quantitative) serves as the foundation for one of the subsequent portfolio rebalancing methods.

3 Portfolio Rebalancing

With the risk modeling framework established, the next stage involves portfolio rebalancing. The objective is to enhance portfolio efficiency—either by improving expected returns for a given level of risk. Three rebalancing optimization approaches are explored:

- **Rebalancing via Qualitative Risk Only**
- **Rebalancing via Quantitative Risk (Mean-Variance (MVO) and CVaR)**
- **Rebalancing via Hybrid Risk (CVaR with Hybrid Penalization)**

For 2,3 the optimization is carried out on the user's **existing mutual fund holdings only**, without introducing new funds. Expected returns are estimated as the mean of the last three years of one-year trailing returns.

3.1 Phase 1: Rebalancing via Qualitative Risk

This approach optimizes portfolio returns within each qualitative risk group, restricting reallocation to funds within the same risk category. This preserves the overall qualitative risk exposure while reallocating toward top-performing schemes.

For each category risk score $C \in \{1, 2, 3, 4, 5\}$:

1. Compute the total invested amount in category C.
2. Rank all funds within C by average historical return.
3. Reallocate the total category amount to the top $k = 2$ funds to preserve minimal diversification.

Let $A_{u,i}$ denote the amount invested by user u in fund i, belonging to category j. The optimization preserves the total capital per category:

$$\sum_{i \in C_j} A_{u,i} = S_j \quad \text{(constant for each category)}.$$

The reallocation rule maximizes expected category return under constant total investment:

$$\max_{i \in C_j} \sum_i A_{u,i}\mu_i \quad \text{s.t.} \quad \sum_i A_{u,i} = S_j.$$

	ISIN	FUND_NAME	DERIVED_CATEGORY	category_risk_score	avg_return	total_amount_invested
0	INF204K01XI3	Nippon India Large Cap Direct Growth	LARGE_CAP	2.00	25.27	₹39,998
1	INF109K016L0	ICICI Prudential Bluechip Direct Growth	LARGE_CAP	2.00	22.22	₹21,999
2	INF194KB1CR7	Bandhan Nifty 100 Index Fund Direct Growth	INDEX_LARGE_CAP	2.00	19.10	₹6,000
3	INF247L01445	Motilal Oswal Midcap Direct Growth	MID_CAP	3.00	38.67	₹23,999
4	INF204K01XF9	Nippon India Multi Cap Direct Growth	MULTI_CAP	3.00	30.89	₹5,000
5	INF179K01YS4	HDFC ELSS Tax saver Direct Growth	ELSS_TAX_SAVINGS	3.00	27.23	₹14,999
6	INF966L01986	Quant ELSS Tax Saver Direct Growth	ELSS_TAX_SAVINGS	3.00	26.01	₹11,999
7	INF761K01DM6	Bank of India Mid & Small Cp Eq & Debt Direct Growth	AGGRESSIVE_ALLOCATION	3.00	24.73	₹32,998
8	INF879O01027	Parag Parikh Flexi Cap Fund Direct Growth	FLEXI_CAP	3.00	23.05	₹10,000

Overall Portfolio Summary

total_investment	avg_return
₹166,992	26.35

Figure 3: Current Portfolio

Recommended Portfolio After Qualitative Rebalancing

	ISIN	FUND_NAME	category_risk_score	avg_return	score_weight	new_amount
0	INF204K01XI3	Nippon India Large Cap Direct Growth	2.00	25.27	53.21%	₹36,181
1	INF109K016L0	ICICI Prudential Bluechip Direct Growth	2.00	22.22	46.79%	₹31,815
2	INF247L01445	Motilal Oswal Midcap Direct Growth	3.00	38.67	55.59%	₹55,036
3	INF204K01XF9	Nippon India Multi Cap Direct Growth	3.00	30.89	44.41%	₹43,959

New Portfolio Summary (Post-Rebalance)

total_investment	avg_return
₹166,992	29.26

Figure 4: Rebalance Portfolio : Qualitative Risk only

Result Summary: Backtesting over three years indicated approximately **4% gain improvement** (e.g., portfolio return improved from 25% to 29%) by reallocating within each risk group to the top two funds. This demonstrates that, even without cross-category switching, significant gains can be achieved through efficient rebalancing within fixed risk categories.

3.2 Phase 2A: Mean-Variance Optimization (MVO)

The classical Markowitz Mean-Variance Optimization (MVO) framework is applied to adjust portfolio weights across existing holdings, to maximize expected returns for a given risk aversion parameter λ.

The optimization problem is formulated as:

$$\max_{\mathbf{w}} \; \mu^\top \mathbf{w} - \lambda \, \mathbf{w}^\top \Sigma \mathbf{w}$$

subject to:

$$\sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0 \quad \forall i,$$

where:

- $\mathbf{w}$: Portfolio weights vector $(n \times 1)$
- μ: Expected returns vector
- Σ: Covariance matrix of returns
- λ: Risk aversion parameter

Covariance matrix elements are estimated empirically as:

$$\Sigma_{ij} = \mathrm{Cov}(R_i^{(1Y)}, R_j^{(1Y)}).$$

This quadratic program balances expected return and risk (variance). Additional constraints are applied to prevent negligible allocations (e.g., $w_i \geq 0.25\, w_i^{(\text{current})}$).

3.3 Phase 2B: Conditional Value-at-Risk (CVaR) Optimization

While the Mean-Variance Optimization (MVO) model minimizes overall variance, it does not adequately capture extreme downside losses. The Conditional Value-at-Risk (CVaR) framework addresses this limitation by focusing on the expected loss in the tail of the return distribution, beyond a specified confidence level α.

$$\mathrm{CVaR}_\alpha = \mathbb{E}[L \mid L \geq \mathrm{VaR}_\alpha],$$

where L denotes the portfolio loss, and VaR_α represents the Value-at-Risk at confidence level α.

The optimization problem can be formulated as:

$$\max_{\mathbf{w},\,\mathrm{VaR},\,\mathbf{z}} \quad \mu^\top \mathbf{w} - \lambda_{\mathrm{CVaR}} \cdot \frac{1}{T} \sum_{t=1}^{T} z_t$$

subject to the following constraints:

$$\sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0,$$

$$z_t \geq 0, \quad z_t \geq -(r_t^\top \mathbf{w} - \mathrm{VaR}), \quad \forall t = 1, \ldots, T.$$

This linear programming formulation (based on the Rockafellar–Uryasev approach) maximizes expected portfolio returns while penalizing exposure to extreme downside risk. It is particularly suitable for modeling non-normal and asymmetric return distributions, which are common in mutual fund performance data.

Phase 2C: Conditional Value-at-Risk (CVaR) with Hybrid Risk Penalization

This phase extends the pure CVaR optimization model by incorporating a **hybrid risk penalty**, which combines both *quantitative* and *qualitative* risk components. The **Conditional Value-at-Risk (CVaR)**, or *Expected Shortfall*, measures the expected loss beyond a specified Value-at-Risk (VaR) threshold — capturing tail risk more effectively than variance.

$$\mathrm{CVaR}_\alpha = \mathbb{E}[X \mid X \leq \mathrm{VaR}_\alpha]$$

where:

- X denotes the portfolio loss (negative return),
- VaR_α is the Value-at-Risk at confidence level α (e.g., 95% or 99%).

Objective Function

The optimization seeks to maximize expected return while penalizing both **tail risk (CVaR)** and **hybrid risk** associated with individual assets:

$$\max_{\mathbf{w}} \quad \mu^\top \mathbf{w} - \lambda_{\mathrm{CVaR}} \cdot \mathbb{E}[\mathrm{CVaR}] - \lambda_{\mathrm{hybrid}} \cdot \sum_i w_i \cdot R_i$$

where:

- $\mu^\top \mathbf{w}$ — expected portfolio return,
- $\mathbb{E}[\text{CVaR}]$ — expected tail loss (downside risk),
- R_i — hybrid risk score (combining qualitative and quantitative risk) for asset i,
- λ_{CVaR} — penalty coefficient controlling sensitivity to tail risk,
- λ_{hybrid} — penalty coefficient controlling exposure to hybrid risk.

Interpretation

- The first term encourages high-return portfolios.
- The second term penalizes **extreme downside losses**, improving resilience against adverse market conditions.
- The third term biases allocations away from assets with high hybrid risk scores, aligning portfolio composition with the investor's risk preference.

This formulation results in a linear program that can be efficiently solved using convex optimization tools such as `cvxpy`. It ensures scalability and interpretability within the mutual fund rebalancing framework, particularly when integrating category-level risk modeling.

3.4 Results Comparison and Plotting Efficient Frontier

The results from all optimization phases are summarized below: The MVO solution lies on the efficient frontier — the set of portfolios that yield the maximum expected return for a given level of risk. The frontier is plotted to visualize risk–return trade-offs among optimized, current, and equally weighted portfolios.

	portfolio_name	avg_return	volatility	avg_risk_score
0	Current Portfolio	26.729991	14.537198	1.050680
1	MVO Portfolio	26.046321	12.340860	1.012022
2	CVaR Portfolio	37.559386	19.128181	1.191425
3	MVO Hybrid Portfolio	37.474049	19.074173	1.187997
4	CVaR + Hybrid Portfolio	37.474052	19.074167	1.187998
5	Equal Weight Portfolio	26.351295	14.522726	1.071892

Figure 5: Comparison of Portfolio Optimization Results

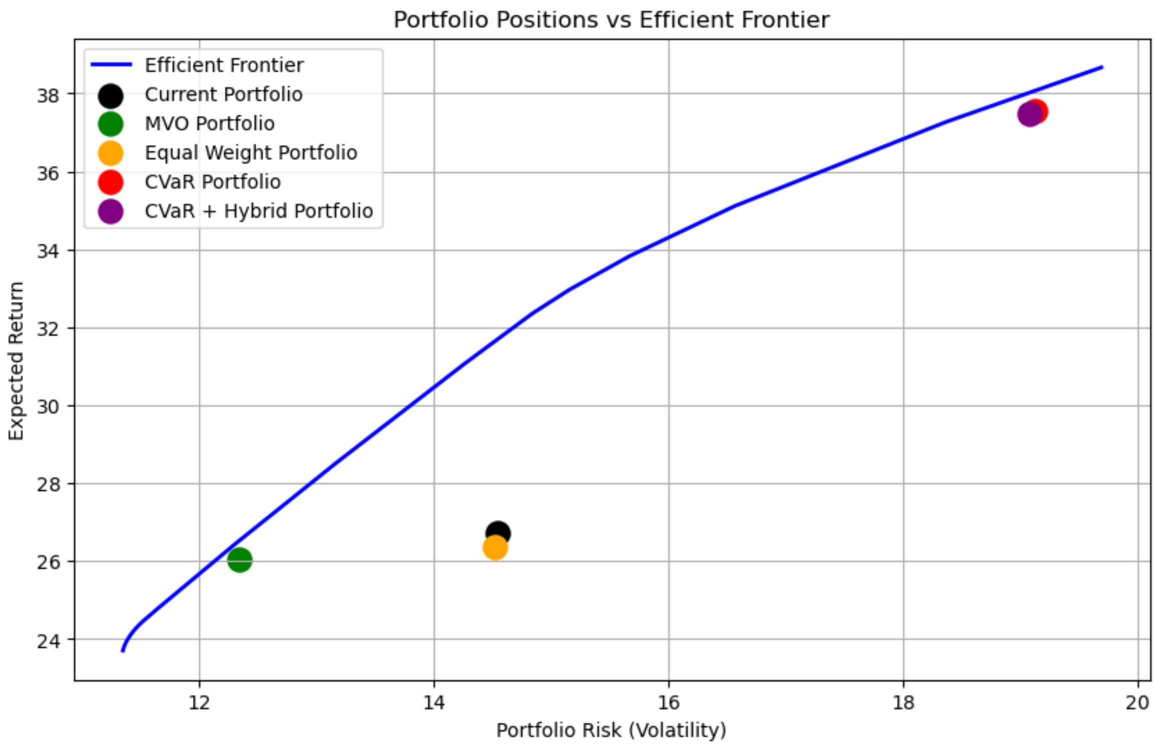

Figure 6: Qualitative against Quantitative Risk (Correlation)

Insights:

- The classical MVO reduces portfolio volatility but slightly lowers expected returns.
- CVaR-based portfolios achieve substantially higher returns (11% improvement), at the cost of higher volatility.
- Hybrid CVaR and MVO models effectively balance tail-risk protection with higher expected returns.
- The efficient frontier plots confirm that hybrid portfolios achieve superior risk-adjusted positioning compared to the current portfolio.

4 Conclusion

Portfolio Type	Avg. Return (%)	Volatility (%)	Hybrid Risk Score
Current Portfolio	26.73	14.54	1.05
MVO Portfolio	26.05	12.34	1.01
CVaR Portfolio	37.56	19.13	1.19
CVaR + Hybrid Portfolio	37.47	19.07	1.19
Equal Weight Portfolio	26.35	14.52	1.07

Table 2: Comparison of portfolio optimization outcomes.

The study demonstrates that systematic, data-driven rebalancing of mutual fund portfolios can yield measurable improvements in performance. Qualitative-only rebalancing delivers immediate efficiency gains without altering the risk profile, while quantitative and hybrid optimization frameworks provide additional enhancements in return at manageable risk levels.

Overall, the hybrid CVaR approach emerges as the most balanced model, effectively integrating downside protection and higher expected returns within the Indian mutual fund context.

5 Future Work: Incorporating Fund Switching Logic -Phase 3

All prior phases restricted optimization within the user's current holdings. Incorporating *fund switching* would allow inclusion of new funds within similar or adjacent categories, guided by a user-defined risk appetite. We have already demonstrated a fund-switching logic in the Qualitative Risk Model (Phase 1), where selection was made for the top-performing funds within each category.

Extending Phase 2 to Fund Switching : To fully incorporate fund switching into the quantitative optimisation (MVO or CVaR), we would need to extend the portfolio universe beyond current holdings. Conceptually, this also expands the optimization problem beyond the current efficient frontier. In mathematical terms, this enlarges the feasible region $\mathcal{W}$ of portfolio weights to include additional ISINs, resulting in a shift and extension of the efficient frontier:

This extension expands the feasible portfolio set:

$$\mathcal{W}_{\text{extended}} = \{\mathbf{w} \in \mathbb{R}^m \mid \sum_{i=1}^{m} w_i = 1,\, w_i \geq 0\}, \quad m > n.$$

Such an approach can potentially improve returns but would require additional considerations such as:

- Transaction costs and exit load implications (¡1 Year holdings)
- Long-term and short-term capital gains (LTCG/STCG) taxation in Indian Market.
- Modeling of user-specific risk tolerance (conservative, balanced, aggressive) so we can optimize return basis on given risk.

While this is beyond the current scope, it provides a natural next step for extending this work toward active portfolio optimization.