How the damping term ensures the uniqueness in the final data inverse source problems related to vibration of the Euler-Bernoulli beam?

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We discuss the unique determination of unknown spatial load $F(x)$ in a damped Euler-Bernoulli beam equation $\rho(x)u_{tt} + \mu u_t + (r(x)u_{xx})_{xx} = F(x)G(t)$ from final time displacement, $u_T(x) := u(x,T)$ or velocity, $\nu_{t,T}(x) := u_t(x,T)$ measurement. It is shown in [A. Hasanov Hasanoglu and V.G. Romanov, *Introduction to Inverse Problems for Differential Equations*, Springer, New York, 2017] that the unique determination of the spatial load $F(x)$ in the undamped wave equation $u_{tt} - (k(x)u_x)_x = F(x)G(t)$ from final data *is not possible.* This result is also valid for the undamped beam equation $u_{tt} + (r(x)u_{xx})_{xx} = F(x)G(t)$. We prove that in the presence of damping term μu_t, the solution of the final data inverse source problem possesses the unique and convergent singular value decomposition, that is the spatial load can be uniquely determined by the final time output, under some acceptable conditions with respect to the final time $T > 0$ and the damping coefficient $\mu > 0$. The results not only clearly demonstrate the key role of the damping factor μu_t, but also allows determine first the admissible values of the final time, and then, to reconstruct the unknown spatial load $F(x)$ in the damped Euler-Bernoulli beam equation.