Production model in the conditions of unstable demand

Nataliya Obrosova[1], **Alexander Shananin**[2]

[1] *CC RAS, Moscow, Russia;* nobrosova@ya.ru
[2] *MIPT, Moscow, Russia, CC RAS, Moscow, Russia;* alexshan@ya.ru

Russia's accession to the World Trade Organization aggravated problems of product competitiveness of the Russian production as within the country, and in the world market. Thus industries of the Russian economy appear in various conditions. Domestic raw materials corporations were included long ago into the world energy market. Products of an oil and gas complex since the Soviet period are competitive. More difficult is a situation in processing industries. Historically production capacities of this sector were created in the conditions of the closed economy and absence of the competition to the import goods. Up to now there is a technological backwardness of processing sector remained. Products of sector lose in the competition to better import analogs. As a result the producer has delays with sales of products and current assets deficit which becomes covered or at the expense of bank loans, or at the expense of the state grants. The inefficiency of processing sector essentially influences economic indicators of production, and also economy indicators as a whole. The program implementation of upgrade of processing sector shall be carried out after the detailed analysis of their economic consequences. Such analysis shall be carried out taking into account feedback and potential influence of change of indicators of an industry on state of the economy as a whole. The adequate tool for carrying out such researches are the mathematical models of economy constructed on the basis of a system approach [1] to the analysis of economic events and allowing analyzing a consequence of large economic decisions taking into account their indirect consequences.

In 2005-2007 based on a system approach the model of economy of Russia intended for the purposes of short-term and mid-term forecasting [2] was developed. In the model the description of activities of processing sector taking into account the current assets deficit developed in [3] for the first time was used. The calculations for model showed [2] that without an inefficiency of processing sector growth rate of economy appears overestimated by 2-3 %. Thus features of processing sector need additional detailed research on the basis of adequate mathematical models. Results of this research are provided in the report.

Let's assume that demand for a made product is unstable. Sale of a product occurs during the random moments of time forming a Poisson process with parameter λ. As a result the producer is forced to accumulate some amount of products in a warehouse in hope of sale. If sale doesn't come the producer has a current assets deficit which it is possible to cover at the expense of the short-term credit line $K(t)$ under percent r.

Let's designate τ the maximum time which is profitable to the producer to use the credit line $K(t)$ in the conditions of absence of sale. Let Y^*- restriction of trade infrastructure which is understood as the greatest possible consignments. Let's designate: Y_0 - the current product stock in a warehouse of the producer, η - production capacity, y - cost value of a product, p - the product price. Then

$$K(t) = \begin{cases} y\eta, & 0 \le t \le \tau \\ 0, & t > \tau \end{cases}, \tag{1}$$

the loan debt $L(t)$ changes owing to the equation

$$\begin{cases} \frac{dL(t)}{dt} = K(t) + rL(t) \\ L(0) = 0 \end{cases}, \text{ i.e. } L(t) = \frac{y\eta}{r}\left(e^{rt} - e^{r(t-\tau)_+}\right). \tag{2}$$

The product output by the time t is

$$Y(t) = \begin{cases} Y_0 + \eta t, & 0 \le t \le \tau \\ Y_0 + \eta\tau, & t > \tau \end{cases}. \tag{3}$$

The purpose of the owner of manufacture is maximization of the income $W(Y_0)$ discounted with coefficient $\Delta \ge r$ on the unrestricted horizon choosing time τ_0 of credit using. The discounted income of the owner of manufacture $W(Y_0)$ is the solution of the following Bellman equation

$$W(Y_0) = \max_{\tau \ge 0} \int_0^\infty \lambda e^{-(\lambda+\Delta)t}\left[p\min(Y(t), Y^*) - L(t) + W\left((Y(t) - Y^*)_+\right)\right] dt. \tag{4}$$

From the economic point of view $W(Y_0)$ characterizes firm cost in case of Y_0 a product stock.

Theorem 1. *The solution $W(Y_0)$ of the equation (4) exists and is unique in a class $G[0,+\infty)$ of the continuous, non-negative, not decreasing, concave functions limited together with the derivative on a semi-interval $[0,+\infty)$, i.e.*

$$G[0,+\infty) = \left\{ \begin{array}{l} w(x) |\ x \in [0,+\infty),\ w \in C[0,+\infty),\ 0 \le w(x) \le \frac{\lambda}{\Delta} pY^*, \\ 0 \le \frac{dw}{dx} \le p,\ \ w\left(\alpha x + (1-\alpha)y\right) \ge \alpha w(x) + (1-\alpha) w(y) \\ \forall x, y \in [0,+\infty),\ \alpha \in [0,1] \end{array} \right\}.$$

If the first derivative in some point for function w isn't determined (owing to monotony the first derivative can have only discontinuity of the first kind and a set of discontinuity points is not more than countable), as a derivative we will understand a function derivative on the left in all points, except 0, and in 0 - a function derivative on the right.

It is proved that the solution of the equation (4) satisfies the condition

$$W(0) \le \frac{\eta}{\Delta} \frac{\lambda}{\lambda+\Delta} \left(p - \frac{y(\lambda+\Delta)}{\lambda+\Delta-r} \right)_+ .$$

We suppose everywhere further that the condition of profitability of production $p - \frac{y(\lambda+\Delta)}{\lambda+\Delta-r} > 0$ is fulfilled.

The equation (4) was reduced to the integral equation that allowed to prove the following theorem.

Theorem 2. *The optimal period for using credit* $\tau_0 = \left(\tau_2^0 + \frac{Y^*-Y_0}{\eta} \right)_+$, *where* $\tau_2^0 = \arg\max\limits_{\tau \ge 0} \int\limits_0^{\tau} \left[W'(\eta t) - \frac{y(\lambda+\Delta)}{\lambda+\Delta-r} \right]_+ e^{-(\lambda+\Delta)t} dt.$

Denote

$$\varsigma_0 = \frac{\tau_2^0 \eta}{Y^*},\ \ Y_0 = \varsigma Y^*,\ \ \chi = (\lambda+\Delta)\frac{Y^*}{\eta},\ \ \beta = \frac{\frac{\lambda}{\lambda+\Delta} p - \frac{y(\lambda+\Delta)}{\lambda+\Delta-r}}{\frac{\lambda}{\lambda+\Delta}\left(p - \frac{y(\lambda+\Delta)}{\lambda+\Delta-r}\right)},$$
$$\Psi_0 = \left(\frac{\Delta}{\eta} W(0) - \frac{\lambda}{\lambda+\Delta}\left(p - \frac{y(\lambda+\Delta)}{\lambda+\Delta-r}\right)\right) \left(\frac{\lambda}{\lambda+\Delta}\left(p - \frac{y(\lambda+\Delta)}{\lambda+\Delta-r}\right)\right)^{-1}.$$

Parameter $0 < \beta \leq 1$ is connected to the profitability of production. In force of restriction on $W(0)$ the value $\Psi_0 \in [-1, 0]$.

Theorem 3. *The function $W(Y_0) \in G[0, +\infty)$ is the solution of the equation (4) if and only if the function*

$$\hat{\Phi}(\varsigma) = \left(\frac{\lambda}{\lambda+\Delta}\left(p - \frac{y(\lambda+\Delta)}{\lambda+\Delta-r}\right)\right)^{-1} e^{-(\lambda+\Delta)\varsigma Y^*/\eta}\left[W'(\varsigma Y^*) - \frac{y(\lambda+\Delta)}{(\lambda+\Delta-r)}\right] \tag{5}$$

is the solution of the system

$$\hat{\Phi}(\varsigma) = \begin{cases} \beta e^{-\chi\varsigma} + \Psi_0, \ \ \text{if } 0 \leq \varsigma \leq 1, \\ \Psi_0 + e^{-\chi} - (1-\beta)\, e^{-\chi\varsigma} - \frac{\lambda}{\lambda+\Delta}\chi e^{-\chi} \int\limits_0^{\varsigma-1} \hat{\Phi}_+(\xi)\, d\xi + \\ \frac{\lambda}{\lambda+\Delta}\hat{\Phi}_-(\varsigma-1)\,, \ \text{if } \varsigma > 1, \end{cases} \tag{6}$$

where ς_0, Ψ_0 is the solution of the system

$$\begin{cases} \hat{\Phi}(\varsigma_0) = 0, \\ \Psi_0 + e^{-\chi} = \frac{\lambda}{\lambda+\Delta}\chi e^{-\chi} \int\limits_0^{\varsigma_0} \hat{\Phi}(\xi)\, d\xi. \end{cases} \tag{7}$$

If the system (7) doesn't have solution, then $\varsigma_0 = 0$, $\Psi_0 = -e^{-\chi}$.

Corollary 1. *Let $W(Y_0) \in G[0, +\infty)$ is the solution of (4). Then the following statements are true.*

1. *If profitability condition taking into account sales volume restriction $\beta > e^{-\chi}$ is true, then*

$$\tau_2^0 = \arg\max_{\tau \geq 0} \int\limits_0^{\tau} \left[W'(\eta t) - \frac{y(\lambda+\Delta)}{\lambda+\Delta-r}\right]_+ e^{-(\lambda+\Delta)t} dt > 0.$$

2. *If $\beta \leq e^{-\chi}$, then $\tau_2^0 = 0$, i.e. manufacture works only before achievement of a stock Y^*.*

By the help of steps method the solution of the system (6) is obtained in an explicit form [4].

Corollary 2. *For* $0 < \varsigma_0 \leq 1$ *it is necessary and sufficient that* $e^{-\chi} < \beta \leq \beta_1$*, where* $\beta_1 = \left(1 + \frac{\lambda}{\lambda+\Delta} - \frac{\lambda}{\lambda+\Delta}(1+\chi)\, e^{-\chi}\right)^{-1}$.

The stock of product in the warehouse at the moment of time t is $\varsigma(t)Y^*$. Change of value $\varsigma(t)$ is a random process. The analysis of the process allowed finding average loading of manufacture.

Theorem 4. *In the case of* $\varsigma_0 \in [0, 1)$ *average loading of manufacture is* $u_0 = 1 - \exp(-\lambda\varsigma_0 Y^*/\eta^*)$.

The developed model of production can be used for the description of features of functioning of processing sector in model of Russia economy.

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