Inverse Dirichlet-to-Neumann problem for compact surfaces in $\mathbb{R}^3$

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Let X be a compact or bordered connected oriented two dimensional surface in $\mathbb{R}^3$ equipped with a constant scalar conductivity or equivalently with conformal structure induced by the standard euclidean metric of $\mathbb{R}^3$. Let $\bar{\partial}$ be corresponding Cauchy-Riemann operator, $d^c = i(\bar{\partial} - \partial)$ and $d = \partial + \bar{\partial}$.

Solution of the Dirichlet problem on such surface has the following electrostatic formulation.

Theorem (Riemann, 1851, Klein, 1881).

Let u be an "electrical" potential on bX (this assumption is empty, when X is compact surface) and X has "electrical" real charges $\pm c_j$ concentrated at points $a_j^{\pm}$, $1 \leq j \leq \nu$ ($\sum c_j = 0$ when X is compact). Then there exists unique "electrical" potential U on X, extending (when X is non compact) u to X, such that $dd^cU = 0$ on $X \backslash \{a_j^{\pm}, 1 \leq j \leq \nu\}$ and the residues $Res_{a_j^{\pm}}(d^cU)$ of d^cU at $a_j^{\pm}$ are $\pm c_j$, $1 \leq j \leq \nu$.

Gelfand, 1962, has formulated the following inverse problem: whether the conformal structure of compact surface $X \subset \mathbb{R}^3$ can be reconstructed from spectre of laplacian dd^c on X?

Related inverse problem was formulated recently by R.Wentworth, 2010: how to reconstruct the conformal structure of compact Riemann surface X from inverse Dirichlet-to-Neumann type data on X collected in some small domain $D \subset X$?

The theorem below gives for compact surfaces in $\mathbb{R}^3$ the result inverse to the formulated Riemann-Klein result and gives also the constructive answer to the question of R.Wentworth.

The following theorem is development of the constructive inverse Dirichlet-to-Neumann result for the bordered surfaces (Henkin, Michel, 2007).

Theorem (Henkin, Michel, 2012).

Let X and X' be compact connected oriented two dimensional surfaces in $\mathbb{R}^3$ equipped with the conformal structures, induced by the euclidean metric of $\mathbb{R}^3$. Assume that $X \cap X'$ contains smoothly bordered domain D, in which three pairs of mutually distinct points $a_l^{\pm}$ are given with fixed "electrical" non zero charges $\pm c_l$, $l = 0, 1, 2$. Let $U_l (resp. U_l')$ be harmonic functions on $X \backslash \{a_l^{\pm}\} (resp. X' \backslash \{a_l^{\pm}\})$ generated by these data such that $U_l|_D = U_l'|_D$ and forms $\partial U_l (resp. \partial U_l')$ have simple poles at $a_l^{\pm}$ with residues $\pm c_l$, $l = 0, 1, 2$. Then there is biholomorphism between X and X'. Moreover, X can be explicitly reconstructed from non degenerated meromorphic mapping

$$F = \left(\frac{\partial U_1}{\partial U_0}, \frac{\partial U_2}{\partial U_0}\right) : D \to \mathbb{C}^2.$$