Uniqueness and Stability Results in Some Inverse Spectral Problems

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When $\Omega \subset \mathbb{R}^N$ is a bounded Lipschitz domain and $a, \rho \in L^\infty(\Omega)$ are three functions satisfying $\min(a, \rho) \geq \varepsilon_0$, for some fixed $\varepsilon_0 > 0$ and $q \in L^\infty(\Omega)$, one considers the linear elliptic operator

$$u \mapsto Lu := -\mathrm{div}(a\nabla u) + qu$$

under various boundary conditions (for instance Neumann boundary condition). We associate to this operator its *boundary spectral data*, that is the set

$$\mathrm{BSD}(a, \rho, q) := \{(\lambda_k, \gamma_0(\varphi_k))\ ;\ k \geq 1\}$$

where the eigenvalues λ_k and the eigenfunctions φ_k are given by

$$L\varphi_k = \lambda_k \rho \varphi_k, \quad (a\nabla\varphi_k) \cdot \mathbf{n} = 0 \ \text{ on } \ \partial\Omega, \quad \int_\Omega \varphi_k(x)\varphi_j(x)\, \rho(x)dx = \delta_{kj},$$

and $\varphi \mapsto \gamma_0(\varphi) := \varphi_{|\partial\Omega}$ denotes the trace operator on the boundary $\partial\Omega$.

We shall give a short review of results pertaining to the question which consists in the determination of either of the coefficients a, ρ, a through the knowledge of the boundary spectral data $\mathrm{BSD}(a, \rho, q)$.

The case of a waveguide, where $\Omega := \omega \times \mathbb{R}$ with $\omega \subset \mathbb{R}^2$ a bounded Lipschitz domain, while $a \equiv \rho \equiv 1$ and q is periodic in the direction x_3, will be the main subject of our presentation.