23. (1) 设每箱中装入 i 件正品为事件 A_i $(i=0.1.2.3.4)$.

取出的一件为正品为事件 B.

$\therefore P(A_i)=\frac{1}{5}$ $(i=0.1.2.3.4)$

$P(B|A_0)=0$ $P(B|A_1)=\frac{1}{4}$ $P(B|A_2)=\frac{1}{2}$

$P(B|A_3)=\frac{3}{4}$ $P(B|A_4)=1$

$P(B)=\sum_{i=0}^{4}P(A_i)P(B|A_i)=\frac{1}{5}\times(0+\frac{1}{4}+\frac{1}{2}+\frac{3}{4}+1)=\frac{1}{2}$

(2) $P(A_0|B)=\frac{P(A_0B)}{P(B)}=\frac{\frac{1}{5}}{\frac{1}{2}}=\frac{2}{5}$

26. 设甲.乙.丙进球分别为事件 $A_1.A_2.A_3$

恰有 i 人进球为事件 B_i $(i=0.1.2.3)$. 至少有一人进球为事件 C

$P(A_1)=0.5$ $P(A_2)=0.7$ $P(A_3)=0.6$ 且 $A_1.A_2.A_3$ 相互独立.

(1) $P(B_1)=P(A_1)P(\bar{A_2})P(\bar{A_3})+P(\bar{A_1})P(A_2)P(\bar{A_3})+P(\bar{A_1})P(\bar{A_2})P(A_3)$

$=0.5\times0.3\times0.4+0.5\times0.7\times0.4+0.5\times0.3\times0.6=0.29$

(2) $P(B_2)=P(A_1)P(A_2)P(\bar{A_3})+P(A_1)P(\bar{A_2})P(A_3)+P(\bar{A_1})P(A_2)P(A_3)$

$=0.5\times0.7\times0.4+0.5\times0.3\times0.6+0.5\times0.7\times0.6=0.44$

(3) $P(B_0)=P(\bar{A_1})P(\bar{A_2})P(\bar{A_3})=0.06$

$P(C)=P(B_1\cup B_2\cup B_3)=P(\bar{B_0})=1-P(B_0)=0.94$

习题二 2.

① $0 \le G(x) \le 1$.　$0 \le H(x) \le 1$. ∴ $0 \le aG(x) \le a$.　$0 \le bH(x) \le b$.

$0 \le F(x) \le a+b=1$.

$G(-\infty)=H(-\infty)=0$.　$G(+\infty)=H(+\infty)=1$

$F(-\infty)=aG(-\infty)+bH(-\infty)=0+0=0$

$F(+\infty)=aG(+\infty)+bH(+\infty)=a+b=1$

② 当 $x_1<x_2$ 时.　$G(x_1) \le G(x_2)$　$H(x_1) \le H(x_2)$

∴ $F(x_1)=aG(x_1)+bH(x_1) \le aG(x_2)+bH(x_2)=F(x_2)$

③ 由 $G(x)$ 与 $H(x)$ 有右连续性.　∴ $G(x+0)=G(x)$　$H(x+0)=H(x)$

∴ $F(x+0)=aG(x+0)+bH(x+0)=aG(x)+bH(x)=F(x)$

∴ $F(x)$ 也是分布函数.

补充: 1. $P(AC|AB\cup C)=\dfrac{P(AC\cap(AB\cup C))}{P(AB\cup C)}=\dfrac{P(ABC\cup AC)}{P(AB)+P(C)-P(ABC)}$

∵ $BC=\phi$　∴ $ABC=\phi$　且 A 与 B. A 与 C 相互独立.

∴ 原式: $\dfrac{P(AC)}{P(AB)+P(C)}=\dfrac{P(A)P(C)}{P(A)P(B)+P(C)}=\dfrac{1}{4}$　$P(A)=\frac{1}{2}$　$P(B)=\frac{1}{2}$

解得 $P(C)=\dfrac{1}{4}$

2. (1) 设血液检测为阳性为事件A. 患上这种疾病为事件B.

$P(B)=0.001 \quad P(A|B)=0.99 \quad P(A|\bar{B})=0.01$

$$P(A)=P(AB)+P(A\bar{B})=P(B)P(A|B)+P(\bar{B})P(A|\bar{B})$$

$$=0.001\times0.99+0.999\times0.01=0.01098$$

$$P(B|A)=\frac{P(AB)}{P(A)}=\frac{0.00099}{0.01098}\approx0.09016\approx9.2\%$$

(2) 设尿液检查为阳性为事件C.

$P(C|B)=0.95 \quad P(C|\bar{B})=0.05$

$$P(B|A\cup C)=\frac{P(A|B)P(C|B)P(B)}{P(A|B)P(C|B)P(\bar{B})+P(A|\bar{B})P(C|\bar{B})P(\bar{B})}$$

$$\approx0.6527\approx65.3\%$$

(3) 将 $P(B)$ 替换为 0.01. 重新计算可得 $P(B|A\cup C)=95\%$