25. $F(x+b) = \int_{-\infty}^{t+b} f(t)dt \qquad F(x+a) = \int_{-\infty}^{t+a} f(t)dt$

$\therefore F(x+b) - F(x+a) = \int_{x+a}^{x+b} f(t)dt.$

$\therefore \int_{-\infty}^{+\infty}\int_{x+a}^{x+b} f(t)\,dt\,dx = \int_{-\infty}^{+\infty}\int_{t-b}^{t-a} f(t)\,dx\,dt$

$= \int_{-\infty}^{+\infty} f(t)dt \int_{t-b}^{t-a} dx = b-a$

27. (1) $\int_{-\infty}^{+\infty} f(x)dx = \int_{-1}^{1} \frac{a}{\sqrt{1-x^2}}dx = a(\arcsin 1 - \arcsin(-1)) = a\pi = 1$

$\therefore a = \frac{1}{\pi}$

(2) $P(|X|<\frac{1}{2}) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\pi\sqrt{1-x^2}}dx = \frac{1}{\pi}\arcsin\frac{1}{2} - \frac{1}{\pi}\arcsin(-\frac{1}{2}) = \frac{1}{3}$

(3) 当$x<-1$时. $f(x)=0 \quad \therefore F(x)=0$

当$-1\le x\le 1$时. $F(x) = \int_{-1}^{x} \frac{1}{\pi\sqrt{1-t^2}}dt = \frac{1}{2} + \frac{1}{\pi}\arcsin x$

当$x>1$时. $f(x)=0 \quad F(x)=1$

$$\therefore F(x) = \begin{cases} 0, & x<-1 \\ \frac{1}{2}+\frac{1}{\pi}\arcsin x, & -1\le x\le 1 \\ 1, & x>1. \end{cases}$$

32. (1) $P(X\le 2) = F(2) = 1-e^{-\frac{1}{50}\times 2} = 1-e^{-0.04}$

$\therefore$ 厂家免费为其更换显示器的概率为 $1-e^{-0.04} \approx 0.0392$

(2) $P(X\ge 10) = 1-P(X<10) = 1-F(10) = e^{-\frac{1}{50}\times 10} = e^{-0.2}$

$\therefore$ 显示器至少可以用10000小时的概率为 $e^{-0.2} \approx 0.8187$

(3) $P(X \geq 20 \mid X \geq 10) = \dfrac{P(X \geq 20, X \geq 10)}{P(X \geq 10)} = \dfrac{P(X \geq 20)}{P(X \geq 10)} = \dfrac{1-F(20)}{1-F(10)} = \dfrac{e^{-0.4}}{e^{-0.2}} = e^{-0.2}$

∴ 其至少还能再使用 10000h 的概率为 $e^{-0.2} \approx 0.8187$.

33 由 $T \sim E(0.5)$ 则 $F(t) = \begin{cases} 1-e^{-0.5t}, & t > 0 \\ 0, & t \leq 0 \end{cases}$

∵ $P(T > 10) = 1 - F(10) = e^{-0.5 \times 10} = e^{-5}$

设打电话超过10分钟的人次为 X 则 $X \sim B(282, e^{-5})$

$P(X \geq 2) = 1 - P(X=0) - P(X=1) = 1-(1-e^{-5})^{282} - C_{282}^{1} e^{-5}(1-e^{-5})^{281} \approx 0.5671$

∴ 在282人次所打的电话中，有两次或两次以上超过 10 min 的概率为 0.5671

35. $P(X < C) = 1 - P(X > C) = P(X > C)$ ∴ $P(X > C) = \frac{1}{2}$. $P(X < C) = \frac{1}{2}$

由正态分布的对称性可得 $C = 2$.

补充习题：设得糖尿病为事件 A，检查中血糖值高于 7 为事件 B

$P(A) = P(X \geq 7) = \Phi\ 1 - (\frac{7-5}{1.56}) \approx 0.1$

$P(B \mid A) = 0.85$ $P(B \mid \bar{A}) = 0.06$

$P(A \mid B) = \dfrac{P(AB)}{P(B)} = \dfrac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid \bar{A})P(\bar{A})} \approx 0.6376$