Palierne's model for the complex shear modulus $G_b^*(\omega)$ of the blend is:

$$G_b^*(\omega) = G_m^*(\omega)\frac{1 + 3\int_0^\infty \frac{E(\omega,R)}{D(\omega,R)}\nu(R)\,dR}{1 - 2\int_0^\infty \frac{E(\omega,R)}{D(\omega,R)}\nu(R)\,dR} = G_b^{'}(\omega) + iG_b^{''}(\omega), \tag{1}$$

In the Palliena model, which describes the rheological properties of emulsions or composites, the distribution of droplet or particle sizes in the dispersed phase is represented by a volume fraction distribution function $\nu(R)$. This function describes the fraction of the dispersed phase that exists at each droplet radius R.

Under the simplifying assumption that all dispersed droplets have a single, uniform radius, $\nu(R)$ can be represented as a Dirac delta function:

$$\nu(R) = \phi\delta(R - R_v) \tag{2}$$

where:

- ϕ is the total volume fraction of the dispersed phase.
- $\delta(R - R_v)$ is the Dirac delta function centered at $R = R_v$.

This representation implies that all droplets are of size R_v, simplifying the distribution to a single average radius. It also ensures the conservation of the total volume fraction, as shown by the integral:

$$\int_0^\infty \nu(R)\,dR = \phi \tag{3}$$

where the integration over all possible droplet sizes yields the total volume fraction ϕ.

In this way, $G_b^*(\omega)$ could be simplified as:

$$G_b^*(\omega) = G_m^*(\omega)\frac{1 + 3\phi\frac{E(\omega,R_v)}{D(\omega,R_v)}}{1 - 2\phi\frac{E(\omega,R_v)}{D(\omega,R_v)}} \tag{4}$$

Furthermore, in this model, the interfacial properties are characterized by material parameters, such as interfacial moduli β_{10} or β_{20}, and interfacial relaxation times λ_{if1} or λ_{if2}. By setting either $\beta'(\omega) = 0$ or $\beta''(\omega) = 0$, the complexity of the model is reduced without compromising the uniqueness of the parameter-data relationship. The simplified expression for the interfacial modulus under a single Maxwell mode is given by:

$$\beta'(\omega) = \beta_{10(20)}\frac{i\omega\lambda_{\text{if1(2)}}}{1 + i\omega\lambda_{\text{if1(2)}}}. \tag{5}$$

This approach allows the Palierne model to effectively capture the effects of interfacial elasticity and viscosity, ensuring a balance between model complexity and accuracy.

where

$$E(\omega,R) = [G_d^*(\omega) - G_m^*(\omega)]\,[19G_d^*(\omega) + 16G_m^*(\omega)] + 4\frac{\alpha}{R}\,[5G_d^*(\omega) + 2G_m^*(\omega)] + E_{1(2)},$$

$$E_1 = \frac{\beta'(\omega)}{R}\left[24\frac{\alpha}{R} + 23G_d^*(\omega) - 16G_m^*(\omega)\right], \quad \text{for } \beta'' = 0,$$

with

$$E_2 = \frac{2\beta''(\omega)}{R}\left[8\frac{\alpha}{R} + 13G_d^*(\omega) + 8G_m^*(\omega)\right], \quad \text{for } \beta' = 0, \text{ and}$$

$$D(\omega, R) = \left[2G_d^*(\omega) + 3G_m^*(\omega)\right]\left[19G_d^*(\omega) + 16G_m^*(\omega)\right] + 40\frac{\alpha}{R}\left[G_d^*(\omega) + G_m^*(\omega)\right] + D_{1(2)},$$

$$D_1 = \frac{2\beta'(\omega)}{R}\left[24\frac{\alpha}{R} + 23G_d^*(\omega) + 32G_m^*(\omega)\right], \text{ for } \beta'' = 0,$$

with

$$D_2 = \frac{4\beta''(\omega)}{R}\left[8\frac{\alpha}{R} + 13G_d^*(\omega) + 12G_m^*(\omega)\right], \quad \text{for } \beta' = 0.$$

In the Palierne model, the interfacial properties are characterized by material parameters such as interfacial moduli β_{10} or β_{20} and interfacial relaxation times λ_{if1} or λ_{if2}. Analysis reveals that the parameters $\beta'(\omega)$ and $\beta''(\omega)$ may be interchangeable, as they enter terms with similar structures. To simplify the model and avoid parameter ambiguity, they set either $\beta' = 0$ or $\beta'' = 0$, reducing the complexity without compromising the uniqueness of the parameter-data relationship. The resulting single Maxwell mode with interfacial modulus and relaxation time is expressed as follows:

$$\beta'(\omega) = \beta_{10(20)}\frac{i\omega\lambda_{\text{if1(2)}}}{1 + i\omega\lambda_{\text{if1(2)}}} \approx \beta_{10(20)}, \tag{6}$$

where $\lambda_{\text{if1(2)}} = \infty$.

They provide the complex shear modulus formulas $G_m^*(\omega)$ and $G_d^*(\omega)$ for the matrix and dispersed phases:

$$G_m^*(\omega) = \frac{i\omega\eta_m}{1 + i\omega\lambda_m}, \quad G_d^*(\omega) = \frac{i\omega\eta_d}{1 + i\omega\lambda_d}, \tag{7}$$

where

$$G_m'(\omega) = \frac{\omega^2\eta_m\lambda_m}{1 + (\omega\lambda_m)^2}, \quad G_m''(\omega) = \frac{\omega\eta_m}{1 + (\omega\lambda_m)^2}, \tag{8}$$

$$G_d'(\omega) = \frac{\omega^2\eta_d\lambda_d}{1 + (\omega\lambda_d)^2}, \quad G_d''(\omega) = \frac{\omega\eta_d}{1 + (\omega\lambda_d)^2}, \tag{9}$$

Influence of the interfacial tension $\alpha = 2, 0.2, 0.02 mN/m$ on the storage modulus $G_b'(\omega)$ and the loss modulus $G_b''(\omega)$ of the blend for the parameters $\beta_{20} = 0.01$ mN/m, $R_v = 0.1\,\mu$m, $\eta_m = 10^5$ Pa s, $\eta_d = 10^4$ Pa s, $\phi = 0.075$, $\lambda_m = 0.3$ s, and $\lambda_d = 0.1$ s.