

Palierne's model for the complex shear modulus $G_b^*(\omega)$ of the blend is:

$$G_b^*(\omega) = G_m^*(\omega) \frac{1 + 3 \int_0^\infty \frac{E(\omega, R)}{D(\omega, R)} \nu(R) dR}{1 - 2 \int_0^\infty \frac{E(\omega, R)}{D(\omega, R)} \nu(R) dR} = G_b'(\omega) + iG_b''(\omega), \quad (1)$$

In the Palliena model, which describes the rheological properties of emulsions or composites, the distribution of droplet or particle sizes in the dispersed phase is represented by a volume fraction distribution function $\nu(R)$. This function describes the fraction of the dispersed phase that exists at each droplet radius R .

Under the simplifying assumption that all dispersed droplets have a single, uniform radius, $\nu(R)$ can be represented as a Dirac delta function:

$$\nu(R) = \phi \delta(R - R_v) \quad (2)$$

where:

- ϕ is the total volume fraction of the dispersed phase.
- $\delta(R - R_v)$ is the Dirac delta function centered at $R = R_v$.

This representation implies that all droplets are of size R_v , simplifying the distribution to a single average radius. It also ensures the conservation of the total volume fraction, as shown by the integral:

$$\int_0^\infty \nu(R) dR = \phi \quad (3)$$

where the integration over all possible droplet sizes yields the total volume fraction ϕ .

In this way, $G_b^*(\omega)$ could be simplified as:

$$G_b^*(\omega) = G_m^*(\omega) \frac{1 + 3\phi \frac{E(\omega, R_v)}{D(\omega, R_v)}}{1 - 2\phi \frac{E(\omega, R_v)}{D(\omega, R_v)}} \quad (4)$$

Furthermore, in this model, the interfacial properties are characterized by material parameters, such as interfacial moduli β_{10} or β_{20} , and interfacial relaxation times λ_{if1} or λ_{if2} . By setting either $\beta'(\omega) = 0$ or $\beta''(\omega) = 0$, the complexity of the model is reduced without compromising the uniqueness of the parameter-data relationship. The simplified expression for the interfacial modulus under a single Maxwell mode is given by:

$$\beta'(\omega) = \beta_{10(20)} \frac{i\omega \lambda_{if1(2)}}{1 + i\omega \lambda_{if1(2)}}. \quad (5)$$

This approach allows the Palierne model to effectively capture the effects of interfacial elasticity and viscosity, ensuring a balance between model complexity and accuracy.

where

$$E(\omega, R) = [G_d^*(\omega) - G_m^*(\omega)] [19G_d^*(\omega) + 16G_m^*(\omega)] + 4\frac{\alpha}{R} [5G_d^*(\omega) + 2G_m^*(\omega)] + E_{1(2)},$$

$$E_1 = \frac{\beta'(\omega)}{R} \left[24 \frac{\alpha}{R} + 23G_d^*(\omega) - 16G_m^*(\omega) \right], \quad \text{for } \beta'' = 0,$$

with

$$E_2 = \frac{2\beta''(\omega)}{R} \left[8 \frac{\alpha}{R} + 13G_d^*(\omega) + 8G_m^*(\omega) \right], \quad \text{for } \beta' = 0, \text{ and}$$

$$D(\omega, R) = [2G_d^*(\omega) + 3G_m^*(\omega)] [19G_d^*(\omega) + 16G_m^*(\omega)] + 40 \frac{\alpha}{R} [G_d^*(\omega) + G_m^*(\omega)] + D_{1(2)},$$

$$D_1 = \frac{2\beta'(\omega)}{R} \left[24 \frac{\alpha}{R} + 23G_d^*(\omega) + 32G_m^*(\omega) \right], \quad \text{for } \beta'' = 0,$$

with

$$D_2 = \frac{4\beta''(\omega)}{R} \left[8 \frac{\alpha}{R} + 13G_d^*(\omega) + 12G_m^*(\omega) \right], \quad \text{for } \beta' = 0.$$

In the Palierne model, the interfacial properties are characterized by material parameters such as interfacial moduli β_{10} or β_{20} and interfacial relaxation times λ_{if1} or λ_{if2} . Analysis reveals that the parameters $\beta'(\omega)$ and $\beta''(\omega)$ may be interchangeable, as they enter terms with similar structures. To simplify the model and avoid parameter ambiguity, they set either $\beta' = 0$ or $\beta'' = 0$, reducing the complexity without compromising the uniqueness of the parameter-data relationship. The resulting single Maxwell mode with interfacial modulus and relaxation time is expressed as follows:

$$\beta'(\omega) = \beta_{10(20)} \frac{i\omega\lambda_{\text{if1}(2)}}{1 + i\omega\lambda_{\text{if1}(2)}} \approx \beta_{10(20)}, \quad (6)$$

where $\lambda_{\text{if1}(2)} = \infty$.

They provide the complex shear modulus formulas $G_m^*(\omega)$ and $G_d^*(\omega)$ for the matrix and dispersed phases:

$$G_m^*(\omega) = \frac{i\omega\eta_m}{1 + i\omega\lambda_m}, \quad G_d^*(\omega) = \frac{i\omega\eta_d}{1 + i\omega\lambda_d}, \quad (7)$$

where

$$G_m'(\omega) = \frac{\omega^2\eta_m\lambda_m}{1 + (\omega\lambda_m)^2}, \quad G_m''(\omega) = \frac{\omega\eta_m}{1 + (\omega\lambda_m)^2}, \quad (8)$$

$$G_d'(\omega) = \frac{\omega^2\eta_d\lambda_d}{1 + (\omega\lambda_d)^2}, \quad G_d''(\omega) = \frac{\omega\eta_d}{1 + (\omega\lambda_d)^2}, \quad (9)$$

Influence of the interfacial tension $\alpha = 2, 0.2, 0.02 \text{ mN/m}$ on the storage modulus $G_b'(\omega)$ and the loss modulus $G_b''(\omega)$ of the blend for the parameters $\beta_{20} = 0.01 \text{ mN/m}$, $R_v = 0.1 \text{ }\mu\text{m}$, $\eta_m = 10^5 \text{ Pa s}$, $\eta_d = 10^4 \text{ Pa s}$, $\phi = 0.075$, $\lambda_m = 0.3 \text{ s}$, and $\lambda_d = 0.1 \text{ s}$.