TEST 1 LINEAR ALGEBRA A (DM 4209) | Thursday, 11:30 - 13:30 | Lecturer: Iwan Njoto Sandjaja

Problem 1. Find the pivots and the solution for these four equations:

$$\begin{aligned} 2x + y &= 0 \\ x + 2y + z &= 0 \\ y + 2z + t &= 0 \\ z + 2t &= 5 \end{aligned}$$

Problem 2. Decide whether the following systems are singular or nonsingular, and whether they have no solution, one solution, or infinitely many solution:

$$\begin{aligned} v - w &= 2 \qquad & v - w &= 0 \qquad & v + w &= 1 \\ u - v &= 2 \qquad & u - v &= 0 \qquad & u + v &= 1 \\ u - w &= 2 \qquad & u - w &= 0 \qquad & u + w &= 2 \end{aligned}$$

Problem 3. Which number a, b, c lead to row exchanges? Which make the matrix singular?

$$A = \begin{bmatrix} 1 & 2 & 0 \\ a & 8 & 3 \\ 0 & b & 5 \end{bmatrix} \ and \ A = \begin{bmatrix} c & 2 \\ 6 & 4 \end{bmatrix}$$

Problem 4. Tridiagonal matrices have zeros entries except on the main diagonal and the two adjacent diagonals. Factor these into $A = LU$ and $A = LDV$:

$$A = \begin{bmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{bmatrix}$$

Problem 5. Compute L and U for the symmetric matrix

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

Problem 6. Find the inverses (in any legal way) of

$$A_1=\begin{bmatrix}0&0&0&1\\0&0&2&0\\0&3&0&0\\4&0&0&0\end{bmatrix},\ A_2=\begin{bmatrix}1&0&0&0\\-\frac{1}{2}&1&0&0\\0&-\frac{2}{3}&1&0\\0&0&-\frac{3}{4}&1\end{bmatrix},\ A_3=\begin{bmatrix}a&b&0&0\\c&d&0&0\\0&0&a&b\\0&0&c&d\end{bmatrix}.$$

Problem 7. The matrix has a remarkable inverse. Find A^{-1} by elimination on $[AI]$. Extend to a 5 by 5 "alternating matrix" and guess its inverse:

$$A=\begin{bmatrix}1&-1&1&-1\\0&1&-1&1\\0&0&1&-1\\0&0&0&1\end{bmatrix}$$

Problem 8. Find and check the inverses (assuming they exist) of these block matrices:

$$\begin{bmatrix}I&0\\C&I\end{bmatrix}\begin{bmatrix}A&0\\C&D\end{bmatrix}\begin{bmatrix}0&I\\I&D\end{bmatrix}$$

Problem 9. By experiment or the Gauss-Jordan method compute

$$\begin{bmatrix}1&0&0\\1&1&0\\m&0&1\end{bmatrix}^n\quad\begin{bmatrix}1&0&0\\1&1&0\\m&0&1\end{bmatrix}^{-1}\quad\begin{bmatrix}1&0&0\\1&1&0\\0&m&1\end{bmatrix}^{-1}$$

Problem 10. Write down the 2 by 2 matrices that

- Reverse the direction of every vector
- Project every vector onto the $\overrightarrow{x_2}$ axis
- Turn every vector counterclockwise through 90°
- Reflect every vector through 45° line $\overrightarrow{x_1}=\overrightarrow{x_2}$

BONUS: Light Outs Games

Below is all of possible Light Outs moves in 4x4:

$$\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 1\end{bmatrix}$$

And the solution is a linear combination of all possible moves in Galois Field. The question is for nxn Light Outs, if we treat all possible moves as basis, do they span all the area? Explain in details! Light Outs game use Von Neumann neighborhood concept.

$$\begin{bmatrix}0 & 1 & 0\\1 & 1 & 1\\0 & 1 & 0\end{bmatrix}$$

If we change the neighborhood concept into Moore neighborhood or X neighborhood, do the linear combination of all possible moves in Galois Field span all the area of nxn Light Outs games?

$$\begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\0 & 1 & 0\\1 & 0 & 1\end{bmatrix}$$