Benchmark 1. Balanced Item Placement

Given I items to be placed in B bins using R types of resources, let s_{ir} be the amount of fixed resource type r requred by item i, C_{br} the fixed capacity of bin b for resource type r. Place all items in bins while, minimizing the imbalance of the resources used across all bins.

Parameters:

s_{ir}: amount of fixed resource typoe r required by item i; $i = 1, \ldots, I$ and $r = 1, \ldots, R$

C_{br}: amount of fixed capacity allocated for resource type r in bin b; $b = 1, \ldots, B$ and $r = 1, \ldots, R$

Decision Variables:

Explicit:
x_{ib}: $\in \{0, 1\}$ 1 if item i is placed in bin b, 0 otherwise ; $i = 1, \ldots, I$ and $b = 1, \ldots, B$.

Implicit:
d_{br}: $\in [0, 1]$ normalized imbalance of resource r in bin b; $b = 1, \ldots, B$ and $r = 1, \ldots, R$.
m_r: $\in [0, 1]$ maximum normalized imbalance of resource r across all bins; $r = 1, \ldots, R$.

Model:

$$\min\ 10 \times B \times R \times \sum_{r=1}^{I} m_r + \sum_{b=1}^{B} \sum_{r=1}^{R} d_{br} \qquad \text{(total imbalance)}$$

$$\text{s.t.} \quad \sum_{b=1}^{B} x_{ib} = 1 \qquad i = 1, \ldots, I. \qquad \text{(assignment)}$$

$$\sum_{i=1}^{I} s_{ir} x_{ir} \leq C_{br} \qquad b = 1, \ldots, B, r = 1, \ldots, R. \qquad \text{(bin capacities)}$$

$$1 - \frac{B}{\sum_{i=1}^{I} s_{ir}} \sum_{i=1}^{I} s_{ir} x_{ir} \qquad b = 1, \ldots, B, r = 1, \ldots, R.$$

(nomarlized imbalance resources)

$$d_{br} \leq md_r \qquad b = 1, \ldots, B, r = 1, \ldots, R.$$

(max normalized imbalance resources)

$$x_{ib} \in \{0, 1\}, d_{br}, md_r \in [0, 1], \qquad i = 1, \ldots I, b = 1, \ldots, B, r = 1, \ldots, R..$$

(binary, nonnegativity)

Benchmark 2. Workload Apportionment

Given J workloads to be processed by I workers, let A_j be the set of workers allowed to process workload j, L_j the amount of work required to process workload j, c_j and C_j, the activation cost and capacity of worker i, repectively. Minimize the total cost of processing all workloads, under the constraint that any one worker is allowed to fail (robust apportionment).

Sets:

A_j: set of workers allowed to process workload j; $j = 1, \ldots, J$

Parameters:

c_i: activation cost of worker i; $i = 1, \ldots, I$

C_i: capacity of worker i; $i = 1, \ldots, I$

Lj: amount of work required to process workload j; $j = 1, \ldots, J$

Decision Variables:

x_{ij}: amount of work reserved on worker i for workload j; $i = 1, \ldots I$ and $j = 1, \ldots, J$.
y_i: $\in \{0, 1\}$ 1 if worker i is activate, 0 otherwise ; $i = 1, \ldots, I$.

Model:

$$
\begin{aligned}
\min \quad & \sum_{i}^{I} c_i y_i && && \text{(total cost)} \\
\text{s.t.} \quad & \sum_{j=1}^{J} x_{ij} \leq C_i && i = 1, \ldots, I. && \text{(worker capacity)} \\
& \sum_{i'=i}^{I} x_{i'j} \geq L_j && \forall\ i \in A_j, j = 1, \ldots, J. && \text{(robust apportionment)} \\
& \sum_{j=1}^{J} x_{ij} \leq \max\{C_i, L_j\} y_i && i = 1, \ldots, I, j = 1, \ldots, J. && \text{(worker activation)} \\
& x_{ij} \leq 0 && \forall\ i \notin A_j, j = 1, \ldots, J. && \text{(only allowed workers)} \\
& x_{ij} \geq 0, y_i \in \{0, 1\} && i = 1, \ldots I, j = 1, \ldots, J. && \text{(nonnegativity, binary)}
\end{aligned}
$$

Benchmark 3. Anonymous Problem

The third problem benchmark is anonymous, and thus the description of the problem instances is not provided.