Image Processing HW #2

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CH3

(6)

Histogram equalization determines the transformation function to produce an output image. This output image has uniform of flat histogram. Histogram requires pixel intensities to form the flat histogram.

For obtaining uniform or flat histogram, all the pixel intensities have to be distributed again so as to get a resulting image with intensity levels having equal number of pixels. This means if there are N # of pixels and k histogram bins or components, then each bin should have the size of N/k.

This task is not done in case of discrete histogram equalization. In case of discrete histogram equalization, the mapping of histogram is redone onto the intensity scale.

(9) (a) $T(r)$ is a monotonically increasing function in the interval $0 \le r \le L-1$;

(b) $0 \le T(r) \le L-1$ for $0 \le r \le L-1$

Consider a distribution f

$$p_r(r) = \begin{cases} 0 , & 0 \le r < \frac{L}{2}-1 \\ 2/L , & \frac{L}{2}-1 \le r \le L-1 \end{cases}$$

Since $p_r(r)$ is positive, $T(r)$ is a monotonically increasing function in the interval $0 \le r \le L-1$

To find the minimum value of S,

$$S = (L-1)\int_0^0 P_r(w)\,dw = 0$$

The maximum value of S would be

$$S = (L-1)\int_0^{L-1} P_r(w)\,dw = (L-1)\cdot\frac{2}{L}\int_{L/2-1}^{L-1} dw$$

$$= (L-1)\frac{2}{L}\left((L-1)-(L/2-1)\right)$$

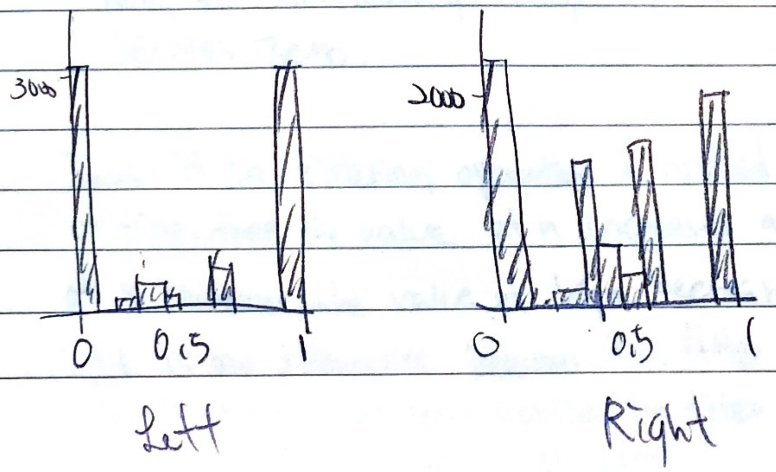
$$= (L-1)\cdot\frac{2}{L}\left(\frac{L}{2}\right) = \underline{(L-1)}$$

$\therefore$ $0 \le T(r) \le L-1$ in the interval of $0 \le r \le L-1$.

(14) First of all, the given problem list does not include the two images in the textbook, so I found those on the internet. So the answer would be little bit different if the pictures I saw is different from the original.

(a) Based on the picture I saw, my answer is no. Because when the images are blurred, boundary points will give rise to a larger number of different values for images on the right as the number of boundary points between black & white regions is much larger in the image on the right. So the histogram of the two blurred images will be different.

(b) If we consider the image of 80x80 and black as 0 pixel and white as 1, the histogram would be like below.



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The general form of the result of filtering an image with a 3X3 low pass spatial filter is the equation below.

$$R = \frac{1}{9}\sum_{i=1}^{9} \underbrace{w_i}_{\text{mask pixels}} \overbrace{f_i}^{\text{Image pixels.}}$$

each time the mask is applied, the image pixels are scaled down by a factor of 9 along with blurring of the image.

∴ the filtering operation if applied for n times then the resulting image pixels will consist of a term $1/9n$.

∴ For sufficiently large value of n (limiting value), the resulting image will consist of all zeros.

This is the limiting effect of repeatedly filtering an image with a 3X3 low pass spatial filter.

Here is an example.

consider the image & mask as shown below,

$$f = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad w = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

the result of convolution will be $f_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 9 & 9 & 9 & 0 \\ 0 & 9 & 9 & 9 & 0 \\ 0 & 9 & 9 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

If you repeat this convolution, it is clear that the pixel values are reducing at each iteration. Thus, for sufficiently large number of n, the value becomes zero.

Thus, if the filtering operation is applied for more number of times, then the value of n increases and as the value of n increases, the value of $1/9n$ decreases.

∴ It is no difference between 3X3 filter or 5X5 filter in the sense that you applied the filter repeatedly. It will gradually reach all zeros.

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The response of the mask is the average of the pixels that it covers. If you calculate the pixels, the vertical bars are 5 pixels wide and their separation is 20 pixels according to the figure 3.33. in textbook.

∴ when the mask moves one pixel to the right, it loses one column on the left and pick up another on the right. When a 25X25 pixel mask is used, the mask moves one pixel to the right, it does lose one value of the vertical bar on the left, but it picks up an identical one on the right. So the response doesn't change. Thus, the # of pixels belonging to the vertical bars covered by the mask doesn't change.

∴ the bars have merged in case of 25X25 pixel mask. However, this is not the case with 23X23 and 45X45 pixel masks and hence, a clear separation exists between the bars.

Ch4

③ use translation property of fourier transform,

$f(x)e^{j2\pi\left(\frac{u_0 x}{M}\right)} \xleftrightarrow[F \cdot T]{} F(u-u_0).$

Put) $f(x) = \cos(2\pi n t)$

$\because \cos(jt) = \frac{e^{jt}+e^{-jt}}{2}$

$\therefore f(x) = \frac{e^{j(2\pi n t)}+e^{-j(2\pi n t)}}{2}$

$= \frac{1}{2}\left[e^{j(2\pi n t)}+e^{-j(2\pi n t)}\right]$

$= \frac{1}{2}\cdot\left[1\cdot e^{j(2\pi n t)}+1\cdot e^{-j(2\pi n t)}\right]$ for easier fourier transform.

We know that

$$\left[\begin{array}{l} 1 \xleftrightarrow{F.T} \delta(t) \\ e^{j2\pi n t} \longleftrightarrow \delta(\mu-n) \\ e^{-j2\pi n t} \longleftrightarrow \delta(\mu+n) \end{array}\right]$$

$$\boxed{\therefore F(\mu) = \frac{1}{2}\left[\delta(\mu-n)+\delta(\mu+n)\right]} = \frac{1}{2}\left[\delta(\mu+n)+\delta(\mu-n)\right]$$

$\xleftrightarrow[F \cdot T]{} f(t) = \cos(2\pi n t)$ //

④ $f(t) = \cos(2\pi n t)$

(a) The period of $\cos$ is 2π.

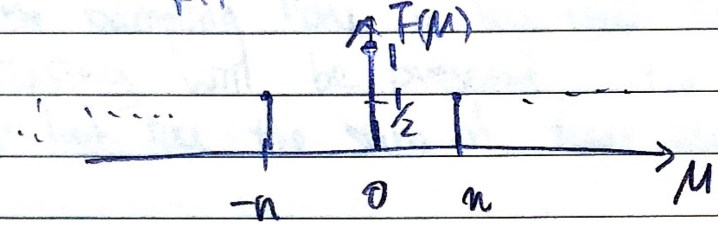
$\therefore 2\pi = 2\pi n t \rightarrow \boxed{t = \frac{1}{n}}$

(b) $f = \frac{1}{T} \rightarrow \therefore$ frequency is n.

$\boxed{F = n}$

$f(t) \xleftrightarrow{F.T} F(\mu)$

$\cos(2\pi n t) \underset{F.T}{\longleftrightarrow} \frac{1}{2}[\delta(\mu+n) + \delta(\mu-n)]$

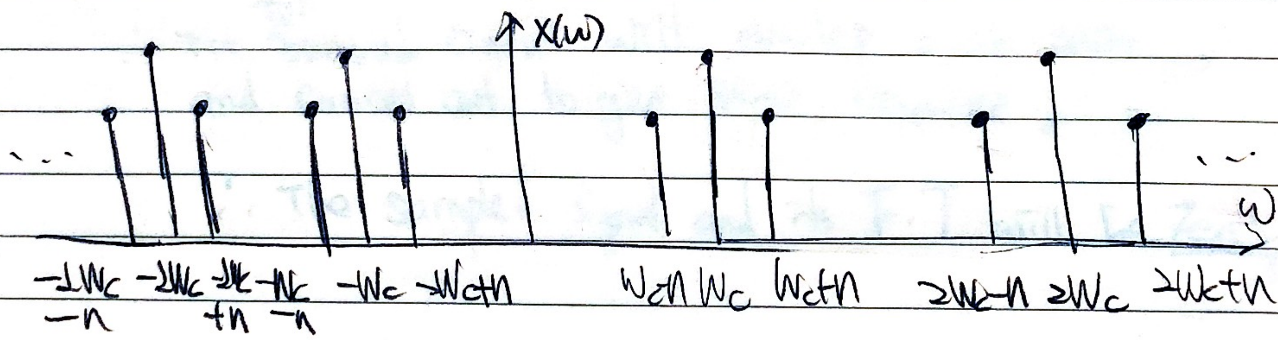


(c) If we sample the above function, It will look like

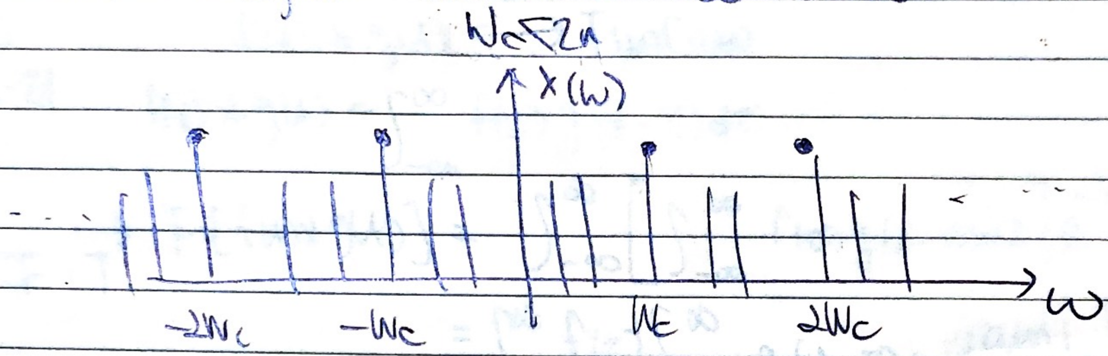
$X(W_c t)\cos(2\pi n t) = \frac{1}{2}[X(W_c+n) - X(W_c-n)]$

the sampled rate is higher than the Nyquist rate, It you sample it lower than the Nyquist rate, the original signal would be distorted.

$\therefore W_c \geq 2n$.



(d) Like I said above, if you sample signal at a rate lower than Nyquist rate, the signal would be overlapped. And the signal would be like below.



As the sampling rate is less than Nyquist rate, aliasing will be present ∴ the sampled signal will look like the sum of two sampled cosine waves.

(e) Nyquist sampling rate $= 2 \times$ Maximum Frequency
$= 2\Omega$.

If $f(t)$ is sampled at the Nyquist rate,

$\boxed{f_s = 2\Omega}$

Also, sampling Period ΔT

$\therefore \Delta T \Rightarrow \frac{1}{\Delta T} = 2\Omega \rightarrow \boxed{\Delta T = \frac{1}{2\Omega}}$

∴ The two sampled signal will overlap each other and cancel out to give zero response.

∴ The sampled signal and its F.T will be zero.

5)

the expression for the 1-D continuous convolution theorem for time domain is this:

(a) $f(t) * g(t) \Longleftrightarrow F(\mu)G(\mu)$

4.2-21 $f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau) g(t-\tau) d\tau$

F.T $\rightarrow F[f(t) * g(t)] = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(\tau) g(t-\tau) d\tau \right] e^{-j2\pi\mu t} dt$

$= \int_{-\infty}^{\infty} f(\tau) \left[\int_{-\infty}^{\infty} g(t-\tau) e^{-j2\pi\mu t} d\tau \right] dt$

use

$\boxed{F(g(t-\tau)) = G(\mu) \cdot e^{-j2\pi\mu t}}$

$= \int_{-\infty}^{\infty} f(\tau) \left[G(\mu) e^{-j2\pi\mu\tau} \right] dt$

$= G(\mu) \int_{-\infty}^{\infty} f(\tau) e^{-j2\pi\tau\mu} dt$

$\boxed{\therefore F[f(t) * g(t)] = G(\mu) F(\mu)}$

(b) The 1-D continuous convolution theorem for convolution in frequency domain is this: $f(t) g(t) \Longleftrightarrow F(\mu) * G(\mu)$

4.2-22

$\rightarrow F[f(t) g(t)] = \int_{-\infty}^{\infty} f(t) g(t) e^{-j2\pi\mu t} dt$

$= \int_{-\infty}^{\infty} f(t) \left[\int_{-\infty}^{\infty} G(\tau) e^{j2\pi\tau t} d\tau \right] e^{-j2\pi\mu t} dt$

$= \int_{-\infty}^{\infty} G(\tau) \left[\int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} \cdot e^{j2\pi\tau t} dt \right] d\tau$

$= \int_{-\infty}^{\infty} G(\tau) \cdot F(\mu-\tau) d\tau$

$\boxed{\therefore F(f(t) g(t)) = G(\mu) * F(\mu)}$