Image Processing HW #4

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CH4

37 (a).

Sol) $h(t,z) = A \cdot 2\pi\sigma^2 \cdot e^{-2\pi^2\sigma^2(t^2+z^2)}$; Gaussian low pass filter transfer function.

Take 2-D DFT,

$H(u,v) = e^{-D^2(u,v)/2D_0^2}$.

(where D_0 = Cut-off freq. of Gaussian low pass filter)

$\therefore G(u,v) = H(u,v)F(u,v)$

$= e^{-D^2(u,v)/2D_0^2} F(u,v)$

let k be the # of applications of the filter.

$$\boxed{G_k(u,v) = e^{-kD^2(u,v)/2D_0^2} F(u,v)}$$

This equation shows passing k times Gaussian filter gives $g_k(x,y)$. After passing several times, it will pass only one frequency centered at $F(0,0)$ and this is the average value of the image. Thus, it will simply produce a constant image.

$$F(0,0) = \sum_{x=0}^{M-1}\sum_{y=0}^{M-1} f(x,y) = \underbrace{MN\bar{f}(x,y)}_{\text{average value of the image}}$$

CH5

5.1 (a), (b), (c)

Sol) 7 pixels wide x 210 pixels high.

Consider the result of arithmetic mean filtering for a set of coordinates in sub image S_{xy} of $m \times n$ centered at point (x, y).

$$\boxed{\hat{f}(x,y) = \frac{1}{mn} \cdot \sum_{(s,t) \in S_{xy}} g(s,t)}$$

It smoothes the local variations in an image and blurs the image.
Also, the smoothening of the corners will increase as the size of arithmetic mean filter is increased.
The results of applying $n \times n$ arithmetic mean filter are shown in matlab code attached.

5.2

(a), (b), (c)

Sol)

the result of geometric mean filtering for a set of coordinates in sub image S_{xy} of $m \times n$ centered at point (x, y). is

$$\boxed{\hat{f}(x,y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t)\right]^{\frac{1}{mn}}}$$

It smoothes the local variations in an image but tend to lose less image detail compared to arithmetic mean filters.
The results of applying $m \times m$ geometric mean filter are shown in matlab code attached.

5.10

(a) The geometric mean filter is associated with less blurring effect as compared to arithmetic mean filter. This is due to the fact that whenever at least one pixel in the mask neighborhood is black, the geometric mean will be zero.
However, this is not the case with arithmetic mean filter as arithmetic mean is zero only if the entire neighborhood consists of pixels with zero intensity. Therefore, intermediate intensity levels will be generated in case of arithmetic mean filter and thus, more blurring is observed.

(b) whenever at least one digit in a sequence of numbers is zero, the geometric mean will be zero.

Hence, in case of geometric mean filtering, whenever at least one pixel in the mask neighborhood is black, the geometric mean and hence the new value for the center pixel will be zero.

However, this is not the case with arithmetic mean filter as arithmetic mean is zero only if the entire neighborhood consists of pixels with zero intensity. Thus, the black components in the right image are thicker.

5.16

Sol) The output image $g(x,y)$?

$h(x-\alpha, y-\beta) = e^{-[(x-\alpha)^2+(y-\beta)^2]}$; The impulse response

The input image $f(x,y) = \delta(x-a)$.

It is clear that the output of a system is obtained by performing convolution of input and impulse response.

$$\therefore g(x,y) = f(x,y) * h(x,y)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\, h(x-\alpha, y-\beta)\, d\alpha\, d\beta = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(\alpha-a)\, e^{-[(x-\alpha)^2+(y-\beta)^2]}\, d\alpha\, d\beta$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(\alpha-a) e^{-(x-\alpha)^2} e^{-(y-\beta)^2}\, d\alpha\, d\beta$$

$$\therefore g(x,y) = e^{-(x-a)^2} \int_{-\infty}^{\infty} e^{-(y-\beta)^2} d\beta = \underline{\sqrt{\pi}\, e^{-(x-a)^2} \int_{-\infty}^{\infty} \frac{\sqrt{2}}{\sqrt{2\pi}} e^{-(\beta-y)^2} d\beta}$$

This is a form of Gaussian distribution with mean 'y' and $\boxed{\sigma = \frac{1}{\sqrt{2}}}$

Also the area under the Gaussian distribution curve is always 1. Thus the value of the integral will be 1 too.

$$\therefore g(x,y) = \sqrt{\pi} \cdot e^{-(x-a)^2} \underbrace{\int_{-\infty}^{\infty} \frac{\sqrt{2}}{\sqrt{2\pi}} \cdot e^{-(\beta-y)^2} d\beta}_{=1} = \sqrt{\pi} \cdot e^{-(x-a)^2}$$

$$\boxed{\therefore g(x,y) = \sqrt{\pi} \cdot e^{-(x-a)^2}}$$

$\boxed{5.17}$

Sol) Assume the motion in vertical direction is denoted by $x_0(t)$ and the motion in horizontal direction is denoted by $y_0(t)$ as they are function of time.

$$\therefore x_0(t) = \begin{cases} \frac{at}{T_1}, & 0 \le t \le T_1 \\ a, & T_1 < t \le T_1 + T_2 \end{cases}$$

(where a is the displacement in x direction)

$$y_0(t) = \begin{cases} 0, & 0 \le t \le T_1 \\ \frac{b(t-T_1)}{T_1}, & T_1 < t \le T_1 + T_2 . \end{cases}$$

(where b is the displacement in y direction)

$$\therefore H(u,v) = \int e^{-j2\pi(ux_0(t)+vy_0(t))} dt$$

$$= \int_0^{T_1} e^{-j2\pi\left(u\frac{at}{T_1}\right)} dt + \int_{T_1}^{T_1+T_2} e^{-j2\pi\left(ua + v\frac{b(t-T_1)}{T_1}\right)} dt.$$

$$= \frac{T_1}{\pi ua} \sin(\pi ua) e^{-j\pi ua} + e^{-j2\pi ua} \cdot \frac{T_1}{\pi vb} \sin(\pi vb) e^{-j\pi vb}$$

$$\boxed{\therefore H(u,v) = \frac{T_1}{\pi ua} \sin(\pi ua) \cdot e^{-j\pi ua} + e^{-j2\pi ua} \cdot \frac{T_1}{\pi vb} \sin(\pi vb) e^{-j\pi vb}.}$$