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Bachelor of Science in Applied Physics

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*Learning dynamics in a cellular automata model of classroom  
peer-to-peer interactions*

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## ENDORSEMENT

This is to certify that this thesis entitled **Learning dynamics in a cellular automata model of classroom peer-to-peer interactions**, prepared and submitted by Clarence Ioakim T. Sy in partial fulfillment of the requirements for the degree of Bachelor of Science in Applied Physics, is hereby accepted.

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To thinking complexly

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## ABSTRACT

### LEARNING DYNAMICS IN A CELLULAR AUTOMATA MODEL OF CLASSROOM PEER-TO-PEER INTERACTIONS

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Peer instruction has recently become one of the popular means of classroom instruction in Physics Education. Such educational setup must involve both physical interaction with things and actually doing some procedural steps mentally or physically. In this study, we investigate the effects of different seating arrangements on the students' learning efficiency in peer instruction by modeling the transfer of knowledge within the class as a probabilistic cellular automata model. We compared the efficiency of learning between the traditional learning model and the peer instruction model. We found that in square classrooms with different lengths  $L \in \{32, 48, 64, 96, 128\}$ , the inner corner seating arrangement performed the best among the peer instructions setups in terms of both the time  $t_{max}$  it takes for all the students to learn and the classroom's learning rate  $m$ . This result is different from a previous study, where they found that the outer corner seating arrangement performed the best. The difference stems from the simplifications made in this model that may not reflect real world factors. Our model uses binary values in an isotropic system and does not consider memory or unlearning. However, despite these simplifications, we found that in smaller classrooms with slow learners, peer instruction is more efficient compared to the traditional learning model, just as previous studies have suggested.

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# Chapter 1

## Peer Instruction and Traditional Models of Teaching

something

### 1.1 History of Peer Instruction

Something something Mazur something something

## **1.2 Difference of Peer Instruction from Traditional Models**

### **1.2.1 Details of Peer Instruction**

### **1.2.2 Benefits of Peer Instruction**

### **1.2.3 Drawbacks of Peer Instruction**

## **1.3 Numerical models for learning in the class-room**

# Chapter 2

## The Classroom as a Binary Probabilistic Cellular Automata Model

The classroom is a complex system that can be modeled as a probabilistic cellular automata. This chapter will discuss the classroom as a complex system and the probabilistic cellular automata model. The chapter will also discuss the implications of the model on the classroom and the teaching-learning process. (AI Generated Text)

Probably add objectives here???

### 2.1 Cellular Automata and its use in modelling complex systems

A two-dimensional (2D) rectangular cellular automata can be defined by a five-tuple [1] + Reinier MS:

$$CA = \{\mathcal{S}, \mathcal{C}, \mathcal{L}, \mathcal{N}, \mathcal{R}\} \quad (2.1)$$

where

$\mathcal{S}$  = is the set of possible states that each cell can assume. The state  $s$  can use any kind of representation such as the set of integers  $\{0, \dots, n - 1\}$  with  $n$  as the total number of possible states.

$\mathcal{C} = \{c = i, j \mid i \in \{1, 2, 3, \dots, L_1\}, j \in \{1, 2, 3, \dots, L_2\} \text{ s.t. } L_1 \times L_2 = N\}$  is the set of identifiers for each cell in the automaton where  $N$  is the

total number of cells and  $L_1$  and  $L_2$  are the lengths of each side of the automaton space. The cells can then be identified by their position in the automaton  $(i, j)$ . So, the state of cell  $c \in \mathcal{C}$  can be written as  $s_c = s_{i,j} \in \mathcal{S}$

$\mathcal{L}$  = defines the lattice neighborhood which is generally a mapping  $f : \mathcal{C} \rightarrow \mathcal{C}^M$  where  $M$  is the number of neighbors of a cell  $c \in \mathcal{C}$ . Any given cell  $c$  is mapped to another tuple of cells:  $L_{i,j} = \{(i-1, j-1), (i-1, j), (i-1, j+1), (i, j-1), \dots, (i, j)\}$ . Where  $r$  is the radius of the Moore neighborhood. We then say that  $\mathcal{L}_{i,j}$  contains the set of neighboring cell for  $c_{i,j}$ .

$\mathcal{N} = \mathcal{S}^M$ , the set of neighborhood states. Thus,  $N_c = N_{i,j} \in \mathcal{N}$  such that each  $\mathcal{N}$  is in the form of the  $M$ -tuple  $\{s_{i-1,j-1}, s_{i-1,j}, s_{i-1,j+1}, s_{i,j-1}, \dots, s_{i,j}\}$ .

$\mathcal{R}$  = defines the set of rules implemented in the CA with  $g : s_{i,j} \mid \mathcal{L} \rightarrow \mathcal{S}$  as the mapping of any neighborhood state  $N_c$  to a new state  $s'_{i,j}$  of the cell  $c$ . At the next time step,  $s'_{i,j}$  replaces the original state  $s_{i,j}$ .

$\mathcal{N}$  can vary with the neighborhood structure and the boundary conditions of the automaton. The neighborhood structure dictates the shape the neighborhood in the lattice. Common neighborhood structures include the von Neumann (diamond) and Moore (square) neighborhoods. Boundary conditions dictate how the automaton treats cells at the edge of the lattice when determining the neighborhood. Common boundary conditions include toroidal, spherical, and fixed boundary conditions.

$\mathcal{R}$  can also be affected by other factors such as whether the rules are deterministic or probabilistic and whether they are implemented synchronously or asynchronously. An automaton with deterministic rules will always produce the same output given the same input, while an automaton with probabilistic rules will produce different outputs given the same input. In Conway's Game of Life, a cell dies when it has three live neighbors, while a cell is born when it has two or three live neighbors. This is an example of a deterministic rule. An example of a probabilistic rule would be a cell dying with a probability of 0.25 when it has three live neighbors. An automaton with synchronous rules will update all cells simultaneously, while an automaton with asynchronous rules will update cells one at a time. (something explanation something about sync vs async)

Due to the flexibility of cellular automata, they can be used to model a wide variety of complex systems. Cellular automata have been used to model physical

systems such as fluid dynamics, biological systems such as the spread of diseases, and social systems such as traffic flow [3]. Its discreteness and locality make it a good model for systems that are composed of many interacting parts. Thus, we have chosen to use a two-state probability cellular automata to simulate the learning process for students in the classroom

## 2.2 PI as a Discrete Probabilistic CA Model

We used a two-dimensional binary probabilistic cellular automata (PCA) model to simulate the learning process in a classroom. In this PCA model, each cell in the automaton represents a student and the state of each cell represents their aptitude  $S = \{\text{unlearned}, \text{learned}\} = \{0, 1\}$ . We assign the neighborhood to be an outer-totalistic Moore neighborhood of radius  $r = 1$  and define the boundary conditions to be fixed wherein the grid does not wrap around itself and  $s_{i,j} = 0$  for  $i, j \notin [1, L]$ . The rules of the automaton describes how the students learn from their neighbors based on three parameters. First (1), the learning rate  $\lambda_{i,j}$  of student  $c_{i,j}$  which describes how receptive they are to peer instruction. Secondly (2), the positional learning factor  $\rho_{i+\delta i, j+\delta j}$  which describes how likely it is to learn from the neighbor  $c_{i+\delta i, j+\delta j}$  based solely on their relative position with respect to  $c_{i,j}$ . Lastly (3), the aptitude level of the neighbor  $s_{i+\delta i, j+\delta j}$  which dictates whether student  $c_{i,j}$  can learn from them. The probability for a student to learn in each time step is then determined by the following equation:

$$P_{i,j} = 1 - \prod_{\forall \delta i, \delta j} [1 - (\lambda_{i,j})(\rho_{i+\delta i, j+\delta j})(s_{i+\delta i, j+\delta j})] \quad (2.2)$$

where

$P_{i,j} \in [0, 1]$  is the probability of  $c_{i,j}$  to learn in each time step,

$\lambda_{i,j} \in [0, 1]$  is the learning rate of  $c_{i,j}$  with values  $\lambda_{i,j} = \{\lambda_0 \pm \delta\lambda\}$  where  $\lambda_0 = 0.5$  and  $\delta\lambda = \{0.1, 0.2, 0.3, 0.4\}$ ,

$\rho_{i+\delta i, j+\delta j} \in [0, 1]$  is the probability of  $c_{i,j}$  to learn from their neighbors in seats  $\{c_{i+\delta i, j+\delta j} \mid \forall \delta i, \delta j \in \{-1, 0, 1\}\}$  solely based from their relative positions with each other, and

$s_{i+\delta i, j+\delta j} = \{\text{unlearned}, \text{learned}\} = \{0, 1\}$  are the neighbors aptitude level.

The derivation of Equation 2.2 is shown in Appendix A.1.

In the five-tuple form, the PCA model for the classroom can be written as:

$$\mathcal{S} = \{\text{learned}, \text{unlearned}\} = \{0, 1\}$$

$$\mathcal{C} = \{(1, 1), (1, 2), \dots, (1, L), (2, 1), (2, 2), \dots, (2, L), \dots, (L, L)\} \text{ where } L \text{ is the length of the square classroom.}$$

$$\mathcal{L} = f(c) \leftarrow [L_c = \{(i + \delta i, j + \delta j) \mid \forall (\delta i \wedge \delta j), \delta i, \delta j \in \{-1, 0, 1\}\}] \text{ as a mapping for outer-totalistic Moore neighborhood of radius } r = 1 \text{ with a fixed boundary condition.}$$

$$\mathcal{N} = \{00000000, 00000001, \dots, 11111111\} \text{ such that the representation of the neighborhood state } N_c \in \mathcal{N} \text{ is equivalent to } N_c = \{s_{i+\delta i, j+\delta j} \mid \forall \delta i, \delta j \in \{-1, 0, 1\}\}.$$

$$\mathcal{R} = \text{the probabilistic rule defined by equation 2.2.}$$

The numerical procedure is outlined in Figure 2.1. Each simulation starts the classroom with four learned students  $n_0 = 4$  placed in different seats in the classroom. The simulation is considered finished once all the students have learned.

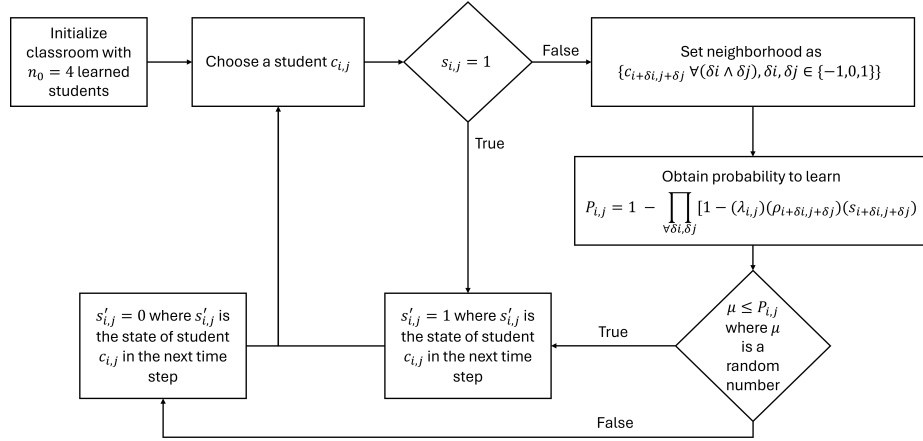


Figure 2.1: Numerical process for simulation of 2D BPCA for PI set-ups.

The seating arrangement (SA) were chosen from a previous study that showed that the SA can affect the learning process [4]. These SA's are namely: inner corner, outer corner, center, and random. The SA configurations are shown in Figure 2.2.

Figure 2.3 shows an example of classroom's evolution over time. The sample classroom is set with  $L = 64$ ,  $\lambda_0 = 1$ , and  $\rho_0 = 0.5$  with the inner corner SA.

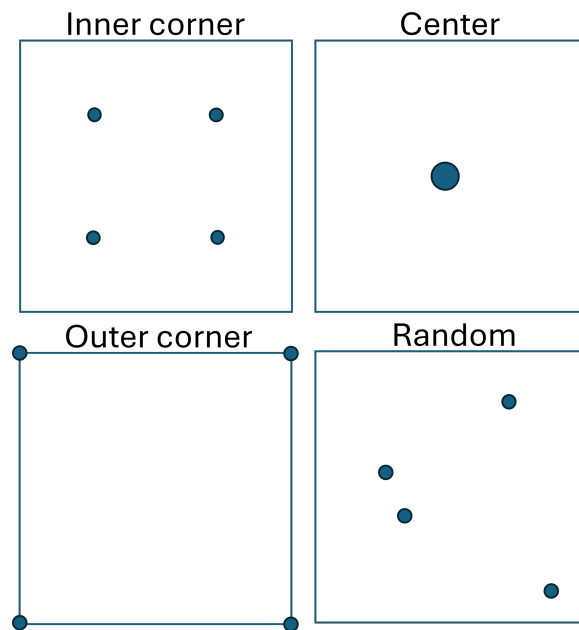


Figure 2.2: Peer instruction seating arrangements. Circles denote high aptitude students. The inner corner configuration places high aptitude students halfway between the center and the corner of the classroom. The outer corner configuration places high aptitude students at the corner of the classroom. The center configuration places high aptitude students in the center of the classroom. The random configuration places high aptitude students randomly throughout the classroom.



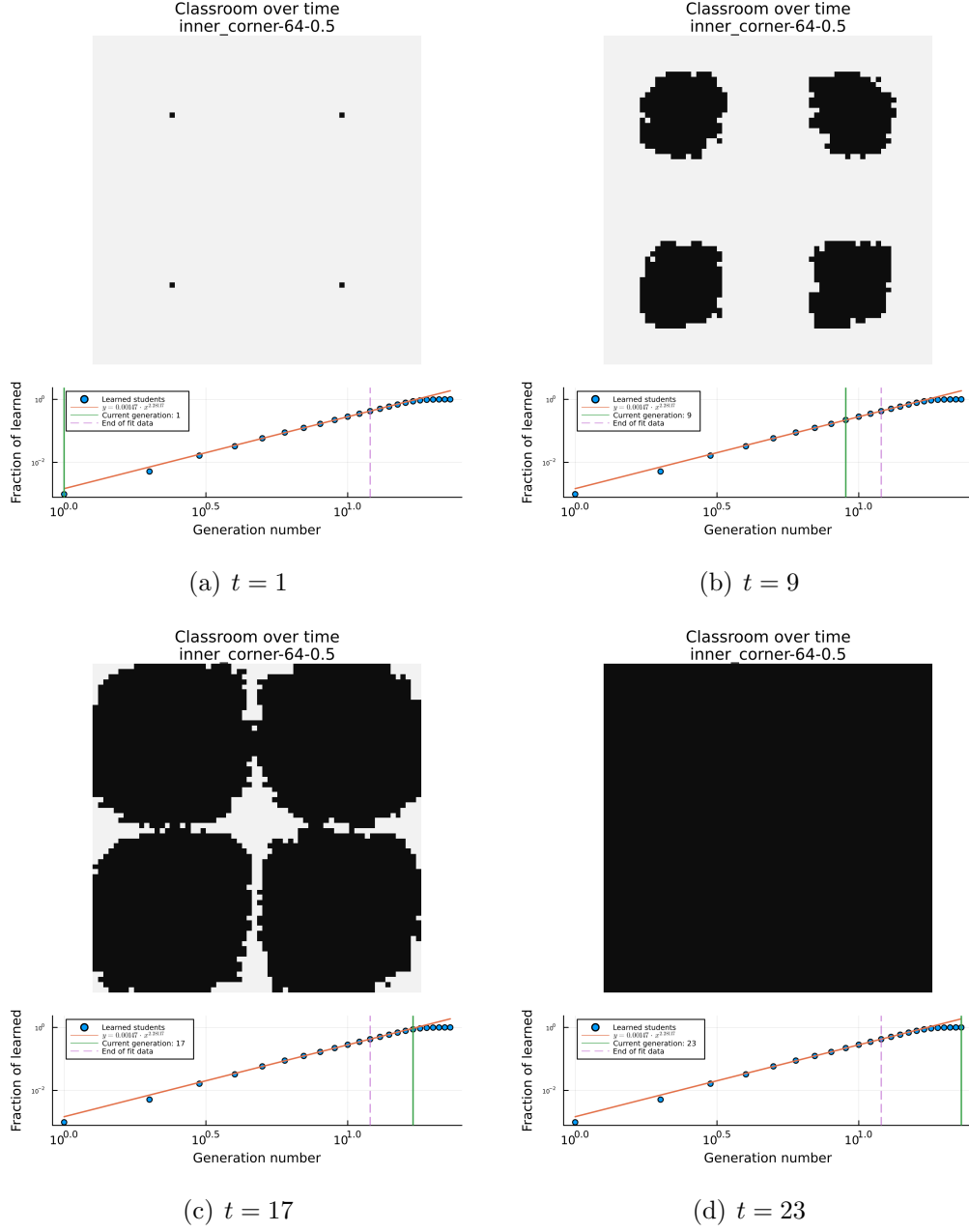


Figure 2.3: Sample classroom evolution for  $L = 64$ ,  $\lambda_0 = 1$ , and  $\rho_0 = 0.5$  with the inner corner SA. Black squares represent learned students while white squares represent unlearned students. The accompanying graphs show the fraction of learned students as a function of time. The blue circle represents the data points while the orange line shows the power law fit. The broken pink line shows the where we truncate the data for fitting the power law and the green line shows the current time step in the plot.

## 2.3 The binary probabilistic cellular automata model for a traditional classroom set up

The main difference between the PI model and the traditional instructional model is that the latter only relies on one source of information - the teacher. To simulate this, for each time step, we set the probability of each student to learn  $P_{ij}$  to be the product of their individual learning rate  $\lambda_{i,j}$  and probability of learning from the teacher  $\rho_0$ .

$$P_{ij} = \lambda_{ij}\rho_0 \quad (2.3)$$

The five-tuple of the traditional instructional model then only differs from the PI model in the update rule  $\mathcal{R}$ .

The numerical procedure is outlined in Figure 2.4. Each simulation for the traditional model starts with all students unlearned. Similar with the PI model, the simulation is considered finished once all the students have learned.

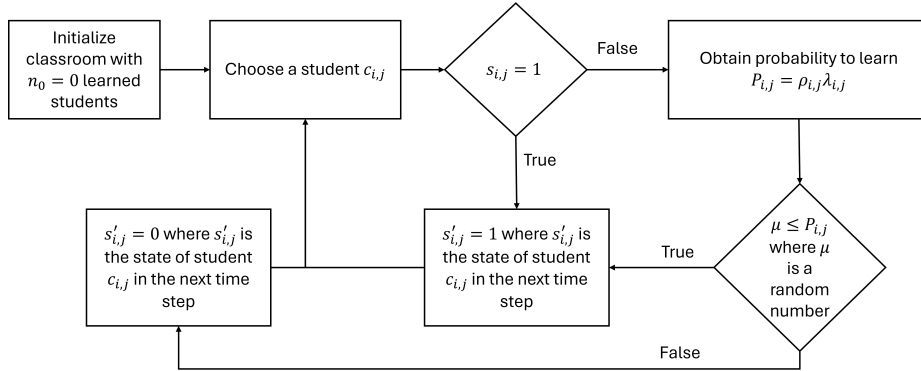


Figure 2.4: Numerical process for simulation of 2D BPCA for traditional set-ups.

## 2.4 Results: Homogenous PI vs Traditional

In this chapter, we only consider the case where all students have the same learning rate  $\lambda_{i,j} = \lambda_0 = 1$  and an isotropic positional learning factor  $\rho_{i,j} = \rho_0 \forall (\delta i \wedge \delta j), \delta i, \delta j \in \{-1, 0, 1\}$ . From the simulations, we compared both the average number time steps  $\langle t_{max} \rangle$  it takes for all the students in the classroom to learn and the average learning rate  $\langle m \rangle$  across different configurations over 5 independent runs. The learning rate  $m$  for each trial was obtained by using a Levenberg-Marquardt algorithm to fit a power law ( $y = ax^m$ ) to the fraction of learned students as a

function of the generation number. We only considered the first 50% of the data for the PI model or the first 25% of the data for the traditional model. This truncation was done so that we only fit the part of the data before the finite size effect starts to affect the simulation.

#### **2.4.1 Time to learn $t_{max}$ vs positional learning factor $\rho_0$**

The data in Figure 2.5 shows that the time to learn  $t_{max}$  decreases with increasing positional learning factor  $\rho_0$  for all classroom sizes for both traditional models and PI models. Among PI models, the inner corner configuration has the lowest  $t_{max}$  for all classroom sizes. The outer corner and center SAs performed similarly, while the random SA has the highest  $t_{max}$ . The traditional model performs situationally better than even the inner corner PI model, something we will investigate in Section 2.4.3.

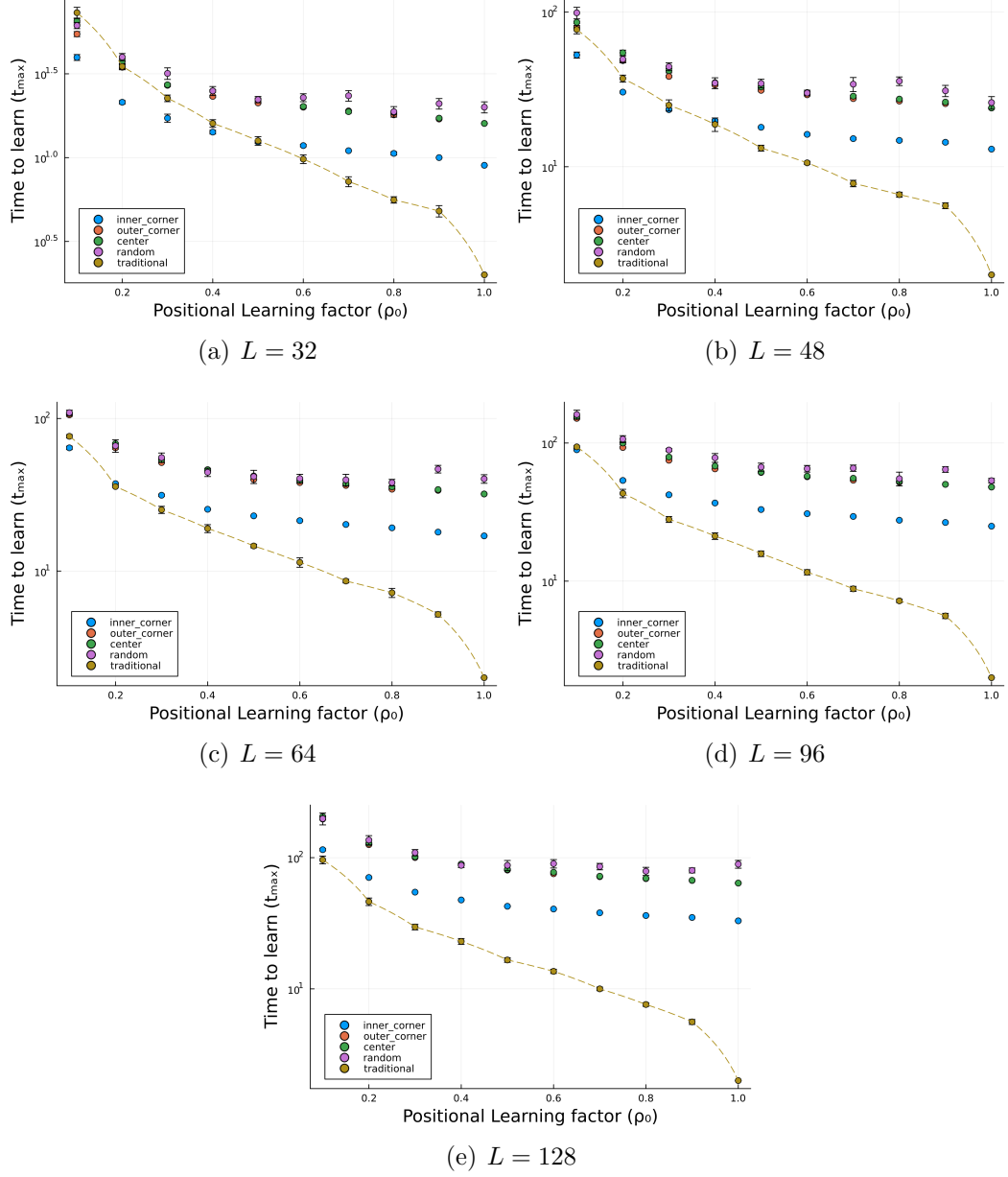


Figure 2.5: Time to learn  $t_{max}$  as a function of positional learning factor  $\rho_0$  for different classroom sizes  $L \in \{32, 48, 64, 96, 128\}$ . Lower time to learn  $t_{max}$  indicate better performance.

### 2.4.2 Learning rate $m$ vs positional learning factor $\rho_0$

The data in Figure 2.6 shows that the learning rate  $m$  does not generally change for different positional learning factors  $\rho_0$  when comparing against similar SAs for the PI model. The traditional model, however, shows a increase in learning rate  $m$  with increasing  $\rho_0$  for all classroom sizes. Similar to the findings in section 2.4.1, the inner corner configuration has the highest learning rate  $m$  for all classroom sizes. The outer corner and center SAs performed similarly, while the random SA has the lowest learning rate  $m$ . We still find that the traditional model performs situationally better than even the inner corner PI model.

The sudden decrease in learning rate  $m$  for the traditional model at  $\rho_0 = 1.0$  is due to the improper truncation and fitting of the data. In the traditional model where  $\rho_0 = 1.0$ , all the students will transition from unlearned to learned in one time step, so fitting the power law to the first 25% of the data would yield a learning rate  $m$  that does not represent the results of the simulations well.

We notice that these results differ slightly from the ones presented in section 2.4.1. The results in this section show that the traditional model performs better than the PI model in terms of learning rate  $m$  for all classroom sizes at low  $\rho_0$ . This is in contrast to the results in section 2.4.1 where the PI model performed better only in small classrooms with low positional learning factor  $\rho_0$ . This discrepancy is likely due to the different metrics used to evaluate the performance of the models. The learning rate  $m$  is a measure of how quickly the students learn, while the time to learn  $t_{max}$  is a measure of how long it takes for all the students to learn. In cases where the classroom is large and positional learning factor  $\rho_0$  is small, the traditional model is able to teach all the students in a shorter amount of time despite the PI model being able to teach the students more effectively. *This could be what other researches find when they claim that PI is a more effective method without accounting for the classroom room size or the students' learning rates. citation needed*

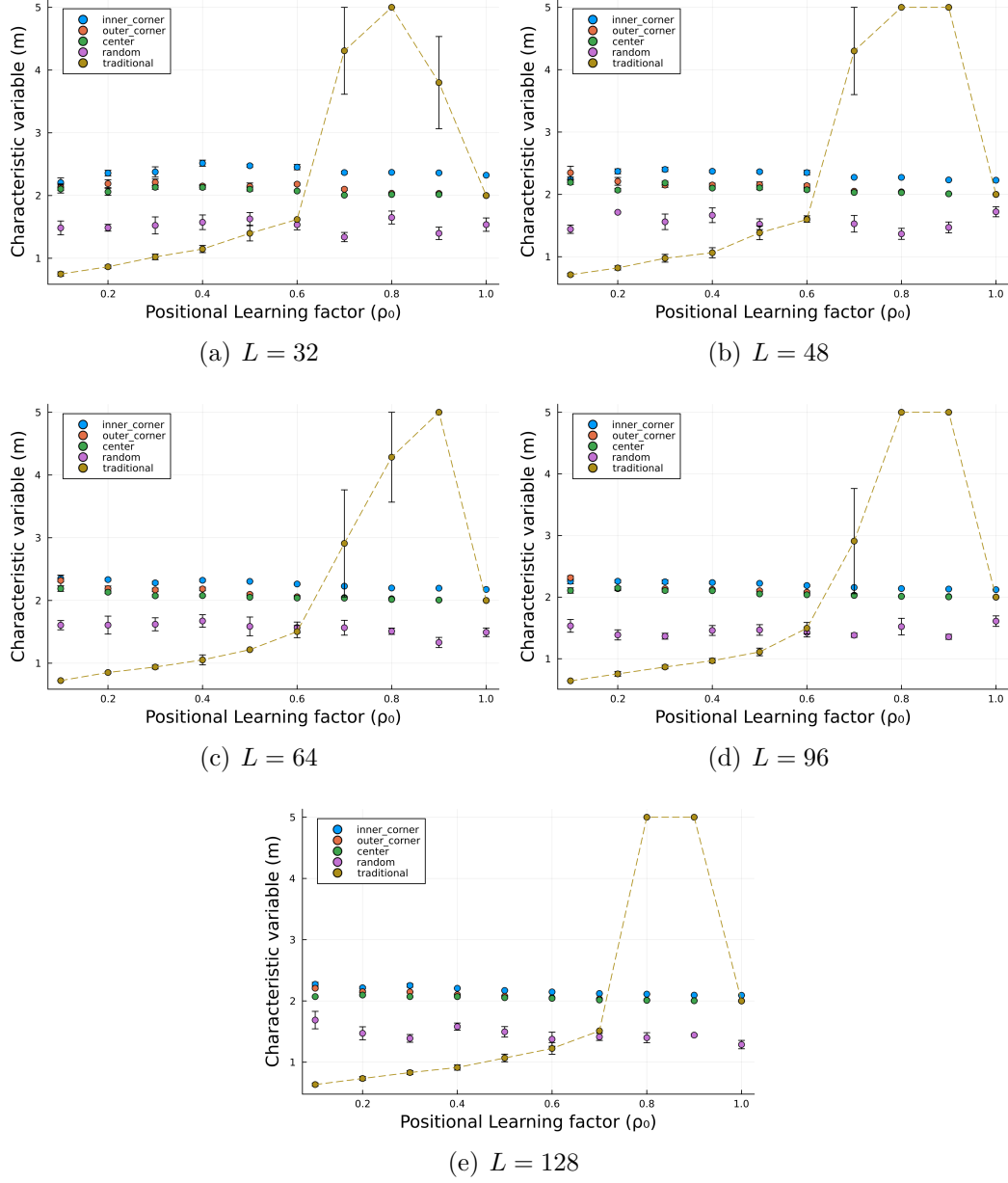


Figure 2.6: Learning rate  $m$  as a function of positional learning factor  $\rho_0$  for different classroom sizes  $L \in \{32, 48, 64, 96, 128\}$ . Higher learning rate  $m$  values indicate better performance.

### 2.4.3 Time to learn $t_{max}$ vs class size $N$

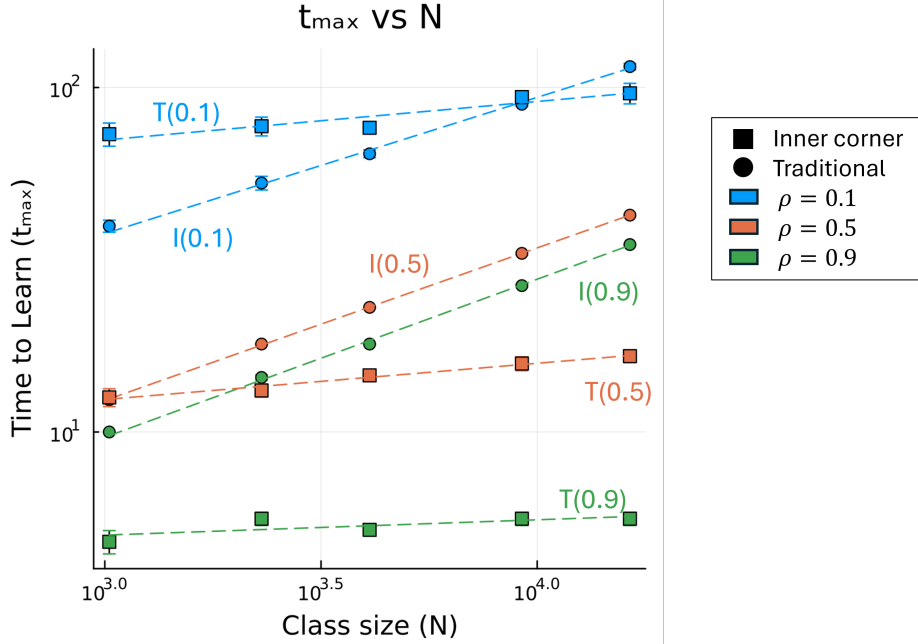


Figure 2.7: Maximum time  $t_{max}$  as a function of class size  $N$  for P2P and traditional models. Circles and squares denote data points for the inner corner P2P configuration and traditional learning model, respectively. The dash lines are the corresponding power law fit of the form  $t_{max} = a \cdot N^b$  using  $\rho \in \{0.1, 0.5, 0.9\}$ . The fitted parameters are shown in the table with an average of  $\langle b \rangle = 0.4532 \pm 0.019$  for the inner corner and  $\langle b \rangle = 0.087 \pm 0.021$  for the traditional model.

	Inner corner		Traditional	
	$a$	$b$	$a$	$b$
$\lambda = 0.1$	2.4515	0.3955	32.4884	0.1119
$\lambda = 0.5$	0.5795	0.4428	6.0079	0.1052
$\lambda = 0.9$	0.4055	0.4589	3.6914	0.0445

Table 2.1: Power law fit parameters  $a$  and  $b$  for  $t_{max}$  vs  $N$ , where  $t_{max} = a \cdot N^b$

When we analyze the dependence of the time to learn  $t_{max}$  on the class size  $N$ , we find that the trend for both models follow the power law  $t_{max} = a \cdot N^b$ . In Figure 2.7 we observe break-even points where the traditional model becomes more efficient than the PI model. These points happen at lower class sizes and lower  $\rho$  values. We also find two major groups of power laws based on their  $b$  values, as summarized in Table 2.1. The first group has an average  $b$  value of  $\langle b \rangle = 0.4532 \pm 0.019$  for the inner corner configuration, while the second group has an average  $b$  value of  $\langle b \rangle = 0.087 \pm 0.021$  for the traditional model. The inner corner configuration has higher  $b$  values than

the traditional model, indicating that the traditional model is less affected by class size  $N$  and so is more scalable than the PI model.

## 2.5 Discussion/conclusions?

In this chapter, we have shown that between different seating arrangements for the PI model, the inner corner SA is the most efficient in terms of time to learn  $t_{max}$  and learning rate  $m$ . The outer corner and center SAs performed similarly worse than the inner corner SA, while the random configuration performed the worst. This is different from what existing literature has shown, where the outer corner SA is the most efficient [4]. This is likely because of the simplifications made in our model. Our model does not incorporate factors such as the similarity effect mentioned in previous studies [4, 5]. This effect is the phenomenon where in students of similar aptitude levels tend to learn better when seated together and are able to learn from each other regardless their actual aptitude. Our model also does not consider anisotropic positional learning factors  $\rho_{i,j}$ , where the probability of learning from a neighbor varies with the neighbors' relative positions. Besides being binary, which introduces granularity, our model also does not consider that not all students are equally receptive to peer instruction or have the same learning rate. Introducing these factors, which we will explore in the next chapters, may make our model reflect reality better and provide us with better understanding of the learning dynamics in the classroom.

The nature of our model lends itself to be heavily influenced by geometric factors. The most impactful factor in this case is then the distance  $d_{max}$  of any student to a high aptitude or learned student as shown in figure 2.8. The inner corner SA minimizes  $d_{max}$ , which is why it is the most efficient. The outer corner and center SAs have equal  $d_{max}$  higher than the inner corner SA, which explains why they performed very similarly albeit worse than the inner corner SA.

Our analysis on the dependence of  $t_{max}$  on  $N$  gives us an explanation on why the traditional model performed better than the PI model in most cases. With lower  $b$ -values, the traditional model is less affected by class size than the PI models are. Because of this, the traditional model performed better in cases with larger classrooms.

With regards to the performance of PI methods with traditional teaching methods, we found that classrooms with higher  $\rho_0$  values performed better in PI models, while



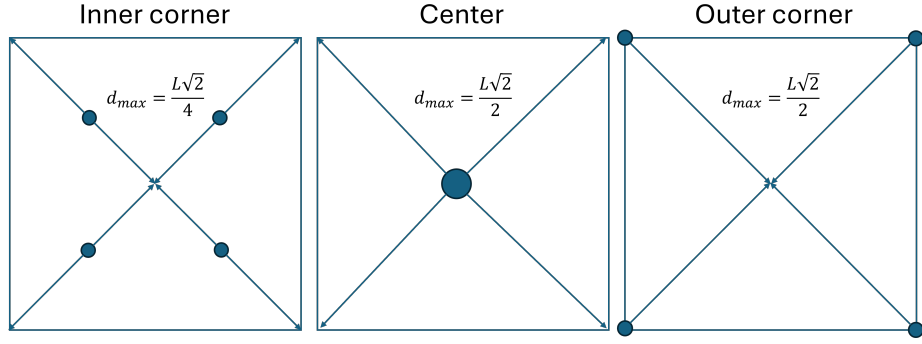


Figure 2.8: Distance  $d_{max}$  of any student to a high aptitude or learned student for different seating arrangements.

classrooms with lower  $\rho_0$  values performed better in traditional models. A previous similar study [4] found that classes with lower aptitude levels were the ones who benefited the most from the PI methods. Although we cannot directly conclude this from our model, we were able to show that students with a lower probability of learning or students who learn slower, can benefit as much from PI as in traditional set ups. This reinforces other studies' findings [2] that show that PI methods can be effective even regardless of the students' actual aptitude levels.

# Chapter 3

## Modifications on the Binary PCA Classroom model

### 3.1 Heterogeneous Learning Rates

To introduce learning rate heterogeneity in the classroom, we revisit equation 2.2.

$$P_{ij} = 1 - \prod_{\forall \delta i, \delta j} [1 - (\lambda_{ij})(\rho_{i+\delta i, j+\delta j})(s_{i+\delta i, j+\delta j})] \quad (2.2 \text{ revisted})$$

We can adjust the parameter  $\lambda_{ij}$  to introduce differences in each student's learning rate. We set a student's learning rate as  $\lambda_{ij} = \lambda_0 \pm \delta\lambda$  where  $\delta\lambda \in \{0.0, 0.1, 0.2, 0.3, 0.4\}$  and  $\lambda_0 = 0.5$ . Each student has a 50% chance of having a learning rate that is either faster ( $\lambda_0 + \delta\lambda$ ) or slower ( $\lambda_0 - \delta\lambda$ ) than the average learning rate  $\lambda_0$ .

#### 3.1.1 $m$ vs $\rho$

#### 3.1.2 $t$ vs $\rho$

#### 3.1.3 $m$ vs $\delta\lambda$

#### 3.1.4 $t$ vs $\delta\lambda$

### 3.2 Anisotropic Interactions

# Chapter 4

## Another Customized Chapter

In this chapter, a sample for making equations and sub-equations are demonstrated.

### 4.1 Equations and sub-equations

In the following, a set of equations is shown.

$$\vec{\nabla} \cdot \vec{D} = \rho \tag{4.1a}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{4.1b}$$

$$\vec{\nabla} \times \vec{E} = -\partial_t \vec{B} \tag{4.1c}$$

The last one being

$$\vec{\nabla} \times \vec{H} = \vec{J} + \partial_t \vec{D} \tag{4.1d}$$

Note that text can still be placed between sub-equations within the `subequations` environment.

When using a solitary equations, you may use the usual equation syntax in L<sup>A</sup>T<sub>E</sub>X.

$$E = mc^2 \tag{4.2}$$

# Chapter 5

## Summary and Conclusions

A short sample thesis/dissertation is presented. Although not complete, it will be useful for newbies in L<sup>A</sup>T<sub>E</sub>X. Any questions? email me at the following address: [johnrob.bantang@gmail.com](mailto:johnrob.bantang@gmail.com).

# Appendix A

## Sample appendix

This gives an example of an appendix chapter. Note that this file has been included **after** the line `\appendix` in `main.tex`.

### A.1 Derivation of Equation 2.2

For any event  $e$ , the desired outcome occurs with probability  $p$  or not with probability  $q$  where

$$p + q = 1. \tag{A.1a}$$

For  $n$  events, each being event  $e$ :

$$\prod_{\forall e} (p_e + q_e) = 1. \tag{A.1b}$$

Expanding equation A.1b, we get

$$\prod_{\forall e} p_e + \dots + \prod_{\forall e} q_e = 1. \tag{A.1c}$$

where the sum of the first  $n - 1$  terms is the probability of the desired outcome occurring at least once over  $n$  events and the last term is the of the probability of the desired outcome not occurring at all. Thus, we can rewrite the probability of the desired outcome occurring at least once as

$$P = 1 - \prod_{\forall e} q_e. \tag{A.1d}$$

substituting equation A.1a into equation A.1d, we get

$$P = 1 - \prod_{\forall e} (1 - p_e). \tag{A.1e}$$

## A.2 Equations in appendix

You don't need to worry about equations within the appendix since L<sup>A</sup>T<sub>E</sub>X automatically formats the equation numbers for you. For example,

$$c^2 = a^2 + b^2 \tag{A.2}$$

becomes the Pythagorean theorem where  $c$  is the length of the longest side of any right triangle.

## A.3 Codes as appendix

Include your codes when necessary to your thesis/dissertation. To do this, you may use `verbatim` environment as follows. **WARNING:** All `verbatim` and `verbatiminput` environments should always be treated as a separate paragraph. When included in a text paragraph, it sometimes happen to reduce the 1.5 spacing to the usual single-spaced text.

```
#include <iostream>
using std::cout;
using std::endl;

int main( void )
{
    cout << "Hello world!" << endl;
    return 0;
}
```

The `{\small }` bracketed region is used to lower the font size of the entire `verbatim` text. This will save you much space and give a more aesthetical look in your manuscript.

On the other hand, when very long codes are wished to be included automatically without the tedious cut and paste procedure, you may include them using the `\verbatiminput` command as follows. You may want to include a short description of the code of course.

```
//Johnrob Y. Bantang, Natinal Institute of Physics
//Created: 03 October 2002
// Makes new C files
// usage: newC filename
//Modifications:
```

```

// >> 21 Jan 2003, Johnrob
//    included the constant AUTHOR and AFFILIATION for portability

#include <stdlib.h>
#include <iostream.h>
#include <fstream.h>
#include <sstream.h>
#include <time.h>
#include <string.h>

const char *const AUTHOR= "Johnrob Y. Bantang";
const char *const AFFILIATION= "National Institute of Physics";
const char *const EXTENSION= ".cpp";

int main(int argc,char **argv){
if(argc!=2){
cout<<"usage: newC filename"<<endl;
exit(0);
}
char *fname= new char[strlen(argv[1])+5];
ostringstream out(fname,strlen(argv[1])+5);
out<<argv[1]<<EXTENSION;
time_t date=time(NULL);

ofstream file(fname,ios::nocreate,0);
//opens normal file that **already exists**;
if(file){
for(int i=1;i<5;i++)
cout<<"WARNING! file already exists!"<<endl;
cout<<endl<<"you can type"<<endl<<endl;
cout<<"\t\"head "<<fname<<"\"<<endl;
cout<<"in command line to *view version*"<<endl<<endl;
cout<<"please enter 1 to OVERWRITE this file"<<endl;
cout<<"type anything to cancel"<<endl;
for(int i=1;i<5;i++)
cout<<"WARNING! file already exists!"<<endl;
int n;
cin>>n;
if(n!=1){
cout<<"*no* file is created... exiting..."<<endl<<endl;
exit(0);
}
file.close();
file.open(fname);
if(!file)
cout<<"**cannot create new file!**"<<endl;
cout<<"OLD FILE: "<<fname<<" *overwritten!*"<<endl;
}
if(!file){

```





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