Classroom learning dynamics using a cellular automata spatiotemporal model comparing peer instruction and traditional instruction

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Peer instruction (PI) has recently become one of the popular means of classroom instruction in Physics Education. Such instruction method is vastly different from how classes are traditionally handled where the instructor conducts a lecture for the entire duration of the class. In this study, we model the transfer of knowledge within classes of students as a probabilistic cellular automata model and investigate the effects of different factors such as seating arrangements, class size N, average learning rate λ, and heterogeneity $\delta\lambda$ on the class overall learning efficiency. The learning efficiency of the class is measured in terms of the overall class rate using the initial rate trend and the total learning time $t_{\max}$ for the whole class. PI mode appears to be more applicable with heterogenous classes (in-class high learning rate max difference $\delta\lambda$). As commonly known TI mode is shown to be more advantageous for large N and homogenous class aptitude (low $\delta\lambda$). We show that the learning curves of highly heterogenous classes with TI mode have two stages of learning: a fast initial stage and a slow final stage. The slow final state class learning rate is due to the late learners with lower λ-values and have general shape dependent on other model parameters as well. Such curves are absent with PI which consistently shifts depending on the other factors. Differing from a previous empirical data for the PI mode, classes perform best when the best in the class are situated within the inner corner. The model we used for this study currently ignores the actual non-isotropic seating orientation with which students do not share knowledge readily in all neighboring seating directions. Our model simplified the student capabilities to binary values and does not consider the effect of aptitude similarity during peer interactions as described in previous studies. We therefore find that PI performs at better than, of not similar to, traditional instruction. A mix of TI and PI would be the optimal mode of instruction taking advantage of TI for fast learning in the initial stage and the robust learning curve of PI in the later stages of class learning. Despite these simplifications, our model is able to provide general insights that are in agreement with previous studies and existing practices.

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I. INTRODUCTION

Peer instruction (PI)–an increasingly popular method of instruction–attempts to solve a disconnect between students' ability to answer quantitative and conceptual questions, particularly in physics subject matters [1, 2]. Such disconnect is said to be attributed to the typical approach of memorizing physics problem patterns and applying the same solution patterns regardless of their potential non-applicability. The PI method forces students to think critically and to engage with peers guided by the learning material as opposed to simply absorbing the information from lectures given by the instructor as in the traditional mode of instruction (TI). The method's effectivity has been shown not only in high school physics settings, but also in other disciplines and higher education settings [3–5].

The PI method as outlined by Mazur and Somers [1] typically involves an instructor giving a short lecture on the topic, followed by a multiple-choice question (MCQ) that the student answer individually. PI emphasizes on the next phases of instruction which involve repeated cycle of students sharing their answers to the MCQ and discussing them with peers, usually until they get the concepts right. This part makes PI method student centric requiring the instructor to intervene with slightly modified steps to adjust to the needs of the class. As long as the same learning objectives are tackled, the instructor can choose variations in the method [6]. The instructor may often decide on: increasing the multiplicity of the peer sharing and discussion phase; having different kinds of requirements and/or levels of evaluation [7]; providing different MCQ sets; or giving another short lecture to ensure clarity of concepts involved [8]. Modifications to the PI method have also been reported to minimize some undesired behavioral patterns such as just copying off each other [6]. The PI therefore contrasts significantly against the traditional method (TI) with which the instructor simply gives a lecture for the entire duration, typically with minimal peer interactions among the students during class time.

Despite the unique approach of de-emphasizing actual problem-solving during the PI lecture phase, quantitative problem-solving skills have been consistently shown to remain not compromised, and in few cases even improved, relative to TI. Several advantages were pointed out in favor of the PI method in relation to the following dimensions: decrease in the number of students with extremely low scores [7–9], improved conceptual understanding [7], as well as being independent of the background knowledge [8, 10].

II. MATHEMATICAL MODELS

Although there are mathematical models for learning and PI, we have not found a model that compares PI and TI using mathematical models.

Nitta [11] gives us a few existing models that model PI. Nitta presents a model by Pritchard et al. [12] where PI is modeled as a set of differential equations that is dependent on the probability of students learning to stick (memory model, Equation 1) and the ability for students to associate new learnings from old knowledge via logistic differential equation (connectedness model, Equation 2.) The Memory model requires that

$$\frac{dU_T(t)}{dt} = -\alpha_m U_T(t) \tag{1}$$

while the Connectedness model requires

$$\frac{dU_T(t)}{dt} = -\alpha_c U_T(t) K_T(t), \tag{2}$$

where knowledge is taken to grow at a uniform rate, as in the Tutoring model:

$$K_T(t) = \alpha_{tu} t + K_T(0) \tag{3}$$

$$U(t) + K(T) = 1 \tag{4}$$

In these equations $U(T)$ and $K(T)$ are the unknown and known knowledge domains respectively. The parameters α_m, α_c, and α_{tu} are the corresponding rates for the memory model, connectedness model, and tutoring model

In deriving their own model of PI, Nitta arrived at analytic equations similar to the Hake gain (Equation 7) to evaluate the effectiveness of PI for a concept test question (Equation 5) and Pritchard's connectedness model (Equation 2) to model students' learning after each MCQ (Equation 6).

$$\eta(q) \equiv \frac{\rho_2(q;c) - \rho_1(q;c)}{1 - \rho_1(q;c)} \tag{5}$$

$$\rho_2 = \rho_1 + \rho_1(1 - \rho_1) \tag{6}$$

Comparing their equations to data, they concluded that these metrics and equations roughly agree with the data and could give us insights on the learning dynamics of the classroom.

Another model that was presented is a generalized Ising Model by Bordogna and Albano [13, 14] where they consider three sources of information for the student to learn from: teacher instruction, peer interaction, and bibliographic materials (books, lecture notes, etc.) Their model shows that students learn more when they engage discussions with their peers than those who only listen to lectures. They also show that group structure affects student learning, and that low aptitude students may learn at the expense of high aptitude peers - a transmissionist view of PI.

Roxas et al. [15] used actual assessment results to train a neural network to map student interactions in PI classrooms. Using this neural network, they were able to characterize information transfer and investigate the effects of group homogeneity. Their study also investigated the optimal seating arrangement for students under PI methods based on their aptitude. In their paper, the measure of students' improvement was calculated via the Hake gain as shown in Equation 7 [16]. They also used the output/input ratio (O/I), which was the ratio of second assessment scores vs first assessment scores, to gauge student improvement. However, it should be noted that O/I values tend to be biased towards low-scoring students.

$$\langle g \rangle = \frac{\langle 2^{\text{nd}} \text{ assessment} - 1^{\text{st}} \text{ assessment} \rangle}{\langle 1 - 1^{\text{st}} \text{ assessment} \rangle} \tag{7}$$

The results of their study show that the outer corner seating arrangement (SA) performed the best, followed by inner corner, then random, then center (see Figure 1 for SA visualizations). In simulated classrooms, each with 64 students and 10 classrooms in total, they found that homogenous classrooms with low aptitudes have significantly higher O/I values. This means that low aptitude students benefit the most from being grouped together.

The process of PI is complex, with many interacting components. Existing models are either predictive, as in the case of the neural network modeling of Roxas et al. [15] or lack the spatial aspect of the process as with the differential equations of Pritchard [12] and Nitta [11]. While Bordogna et al. [13, 14] present to us a dynamical model in their generalized Ising model, it lacks some aspects of PI we'd like to consider like seating arrangements, students' learning rate, and heterogeneity.

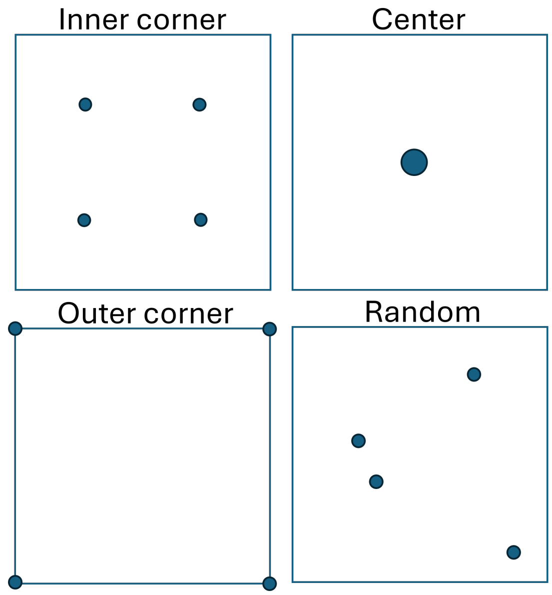


FIG. 1. Seating arrangements considered for PI. Dots represent position of high aptitude students.

III. A PROBABILISTIC CELLULAR AUTOMATA MODEL

Here, we show that a probabilistic cellular automata (PCA) model can be used to study the spatiotemporal dynamics of both PI and traditional instruction when incorporating these different aspects into the model. We previously used a probabilistic cellular automata to model the classroom [17], the results of which were presented in the 42$^{\text{nd}}$ Samahang Pisika ng Pilipinas Conference in July 2024. The findings presented in the paper are also presented in this study as part of the results.

For the PCA model, each cell in the automaton represents a student where each student is assigned a learning rate $\lambda_{i,j}$ to describe how fast they learn as an individual. The state of each cell represents their aptitude $s_{i,j} = \{\text{unlearned}, \text{learned}\} = \{0, 1\}$. We assign the neighborhood to be an outer totalistic Moore neighborhood of radius $r = 1$ and define the boundary conditions to be fixed wherein the grid does not wrap around itself and $s_{i,j} = 0$ for $i, j \notin [1, L]$. For each time step, we find the probability of a student to learn and update the state of the student based on this probability. The probability of a student to learn is calculated differently for traditional instruction and peer instruction.

III.1. Update Rules for Traditional Instruction (TI)

For traditional instruction, the probability of a student to learn in each time step ($P_{i,j}$) is given by:

$$P_{i,j} = \lambda_{i,j}\rho_0 \tag{8}$$

where $P_{i,j} \in [0, 1]$ is the probability of student $c_{i,j}$ to learn in each time step, $\lambda_{i,j} \in [0, 1]$ is the learning rate of student $c_{i,j}$, and $\rho_0 \in [0, 1]$ is the probability of $c_{i,j}$ to learn from the teacher based on their relative position from the teacher.

We consider the case that students have heterogeneous learning rates $\lambda_{i,j} = \lambda_0 \pm \delta\lambda$ where $\lambda_0 = 0.5$. This allows us to investigate the effects of student heterogeneity on the classroom's learning dynamics.

Having a positional learning coefficient $\rho_{i,j}$ allows us to model a classroom where students closer to the teacher in front learn faster than those farther away or vice versa. However, for this research, we only consider the case such that $\rho_{i,j} = \rho_0 \forall i, j$.

The numerical procedure is outlined in Figure 2. Each simulation for the TI model starts with all students unlearned.

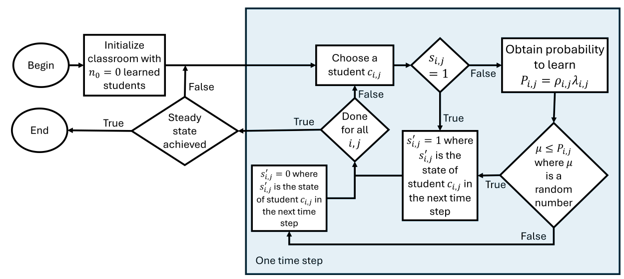


FIG. 2. Numerical process for simulation of 2D BPCA for traditional instruction.

III.2. Update Rules for Peer Instruction (PI)

In contrast to TI, the probability of a student to learn in each time step for PI is additionally dependent on the student's neighbors and their states. The probability of a student to learn in each time step ($P_{i,j}$) is given by:

$$P_{i,j} = 1 - \prod_{\forall \delta i, \delta j} [1 - (\lambda_{i,j})(\rho_{\delta i, \delta j})(s_{i+\delta i, j+\delta j})] \tag{9}$$

where $P_{i,j} \in [0, 1]$ is the probability of student $c_{i,j}$ to learn in each time step, and $\lambda_{i,j} \in [0, 1]$ is the learning rate of student $c_{i,j}$. We consider the case that students have heterogeneous learning rates $\lambda_{i,j} = \lambda_0 \pm \delta\lambda$ where $\lambda_0 = 0.5$. The variables $\rho_{\delta i, \delta j} \in [0, 1]$ is the probability of $c_{i,j}$ to learn from their neighbors in seats $\{c_{i+\delta i, j+\delta j} \forall \delta i, \delta j \in \{-1, 0, 1\}\}$ solely based from their relative positions with each other, and $s_{i+\delta i, j+\delta j} = \{\text{unlearned, learned}\} = \{0, 1\}$ are the neighbors' aptitude level.

The numerical procedure is outlined in Figure 3. Each simulation for the PI model starts with only four learned students $n_0 = 4$.

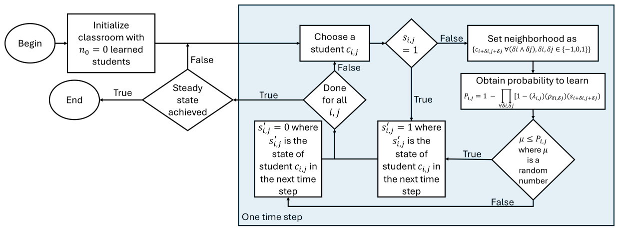


FIG. 3. Numerical process for simulation of 2D BPCA for peer instruction.

III.3. Model parameters

The values of the model input parameters we considered for the class size $N = L^2$, the positional learning coefficient ρ_0, and the learning rate heterogeneity $\delta\lambda$ are as follows:

- $L = \{32, 48, 64, 96, 128\}$
- $\rho_0 = \{0.1, 0.2, 0.3, \ldots, 0.8, 0.9, 1.0\}$
- $\delta\lambda = \{0.0, 0.1, 0.2, 0.3, 0.4\}$

To assess how well the class performed, we used the time it took for all the students to learn $t_{\max}$. For both TI and PI, the simulation ends when all students have learned. Each set of parameters was run 5 independent times to get the average and standard deviation of $t_{\max}$ for each set of parameters.

IV. RESULTS AND DISCUSSION

IV.1. Different Dynamics of Traditional and Peer Instruction

For TI, varying the different model parameters yield different dynamics. As shown in Figure 4, the progression of the fraction of learned students was not affected with increasing class size N. The time to learn $t_{\max}$, however, increased with class size N. This is expected as TI is set up such that the students can all learn at the same time from the teacher — so the dynamics are independent of class size N. Positional learning factor ρ_0 changes the initial number of learned students and the rate at which students learn throughout the simulation (Figure 5). Upon initial inspection of Figure 6 for the effects of learning rate heterogeneous, we see that it has a similar effect to that of positional learning factor ρ_0 without changing the initial number of learned students. However, we notice that the effect is on the rate of which students learn is not linear. The difference between $\delta\lambda = 0.4$ and $\delta\lambda = 0.2$ is greater than that of $\delta\lambda = 0.2$ and $\delta\lambda = 0.0$. This is consistent with the nonlinear effect of ρ_0 on the learning rate. Upon further inspection, we notice that TI classes with high learning rate heterogeneity $\delta\lambda$ have a two-stage learning process. The first stage is characterized by a sharp increase in the number of learned students caused by the majority of fast students ($\lambda = \lambda_0 + \delta\lambda$) learning in the first few time steps. The second stage is characterized by a much slower increase in the number of learned students caused by waiting for the slower students ($\lambda = \lambda_0 - \delta\lambda$) to learn. An example case where this phenomenon is observable is shown in Figure 7. In this case, we see at $t = 2$ that majority of the students that are learned are fast students and at $t = 13$ the slower students have started to learn, but the rate of learning has slowed down significantly. Despite the promising-looking start to this simulation, it will spend up to $t = 232$ waiting for all the students to learn.

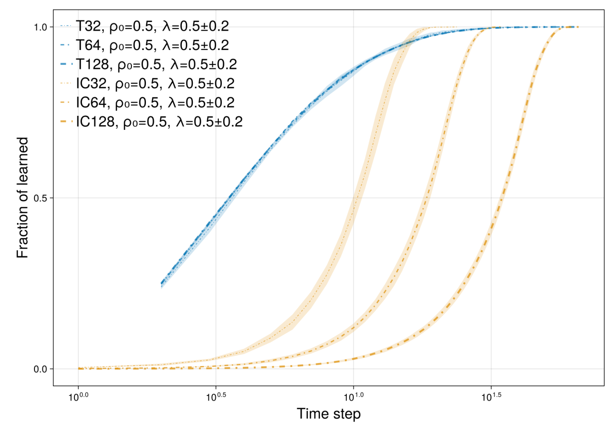


FIG. 4. Comparison of time to learn $t_{\max}$ and fraction of learned students for class size N. Each SA corresponds to a different color — blue for traditional, orange for inner corner. Different class sizes correspond to different line widths where bigger classroom sizes are represented by thicker lines. The bands around each line show the standard deviation of the data over 5 trials. Higher fraction of learned students indicate better learning.

As for PI, varying the different parameters do not lead to as varied dynamics as TI. The progression of the fraction of learned students are affected the same way when varying class size N, positional learning factor ρ_0, and learning rate heterogeneity $\delta\lambda$. Generally, there is a time delay before the learning starts to speed up. As shown in Figure 4, Figure 5, and Figure 6, increasing class size N and learning rate heterogeneity $\delta\lambda$, and decreasing positional learning factor ρ_0 all increase this time delay. A combination of these factors may affect to the slope of classes' progression over time once learning speeds up, but these factors individually do not noticeably affect the progression. Additionally, we see in Figure 8 that different SAs perform differently even with the same set of parameters with the inner corner SA performed the best. The random SA may outperform the center and outer corner SAs in the middle of the simulation but still ends up having the highest time to learn $t_{\max}$.

The plots in this section show that classes under TI have more students learning in the earlier time steps, but has lower time to learn $t_{\max}$ when compared to classes under PI with the same set of parameters. These find-

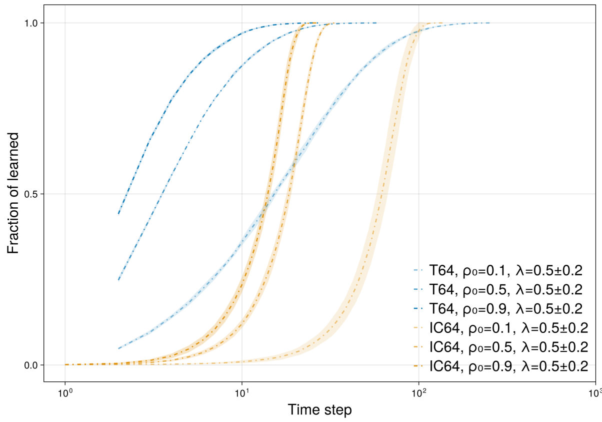


FIG. 5. Comparison of time to learn $t_{\max}$ and fraction of learned students for positional learning factor ρ_0. Each SA corresponds to a different color — blue for traditional, orange for inner corner. Different ρ_0 values correspond to different line transparencies where darker lines represent higher ρ_0 values. The bands around each line show the standard deviation of the daya over 5 trials. Higher fraction of learned students indicate better learning.

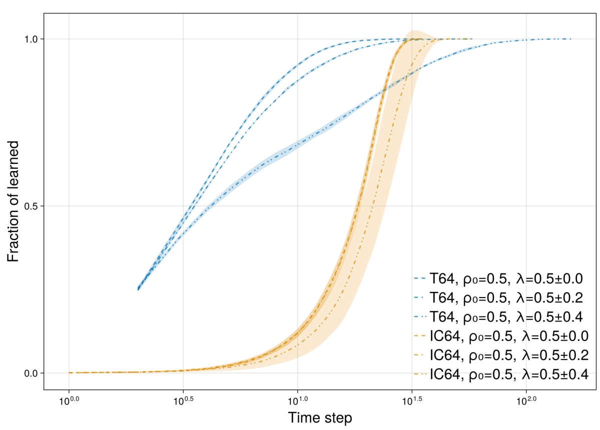
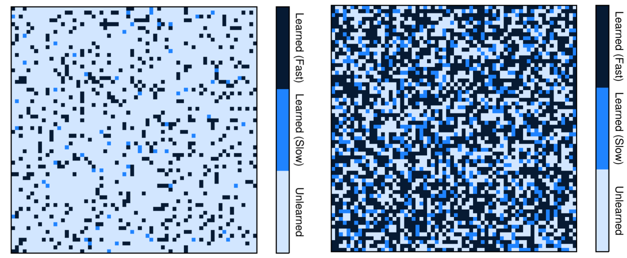


FIG. 6. Comparison of time to learn $t_{\max}$ and fraction of learned students for different learning rate heterogeneity $\delta\lambda$. Each SA corresponds to a different color — blue for traditional, orange for inner corner. Different $\delta\lambda$ values correspond to different line styles where $\delta\lambda = 0, 0.2, 0.4$ are represented by dashed lines, alternating dots and dashes, and two dots and a dash respectively. The bands around each line show the standard deviation of the data over 5 trials. Higher fraction of learned students indicate better learning.



(a) $t = 2$ (b) $t = 13$

FIG. 7. Example classroom evolution for TI with $L = 64$, $\rho_0 = 0.3$, and $\delta\lambda = 0.4$ at different time steps t showing a two-stage learning process. Dark blue cells represent learned students with learning rate $\lambda = \lambda_0 + \delta\lambda$, blue cells represent learned students with learning rate $\lambda = \lambda_0 - \delta\lambda$, and light blue cells represent unlearned students.

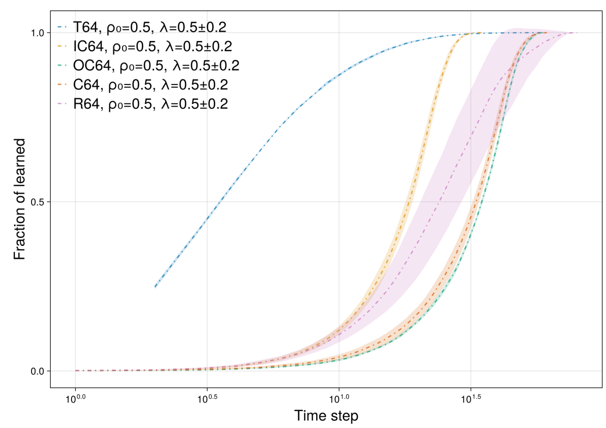


FIG. 8. Comparison of time to learn $t_{\max}$ and fraction of learned students for different SAs. Each SA corresponds to a different color — blue for traditional, orange for inner corner, green for outer corner, orange for center, and pink for random. The bands around each line show the standard deviation of the data over 5 trials. Higher fraction of learned students indicate better learning.

ings suggest that to optimize the learning in the classroom, it would be best to perform TI at the start of the class to take advantage of the fast initial learning stage, then switch to PI where having learned students spread throughout the classroom benefit those who have yet to learn. This is consistent with current practices [1, 6, 8, 15], where the students are expected to have read up on the material in advance and the instructor gives a short lecture before the students engage in peer discussion.

IV.2. Summary of effects of class size N, positional learning factor ρ_0, and learning rate heterogeneity $\delta\lambda$

As we vary the different parameters, as shown in Figure 9, we are able to identify which cases are more favorable for TI or PI. We find that PI suffers more than TI when handling larger class sizes N, and that TI suffers more from increased learning rate heterogeneity $\delta\lambda$ compared to PI. However, low positional learning factor ρ_0 affects both TI and PI similarly.

The same figure also shows us the magnitude of the advantage of one method of instruction over the other. We find that when TI is advantageous, like in cases of big classrooms and low learning rate heterogeneity, the advantage over PI is not that large. However, in cases where PI is advantageous, like in cases of small classrooms and high learning rate heterogeneity, the advantage over TI is very large. This suggests that PI can be a safe option —

never performing much worse than TI, but can perform much better in certain cases.

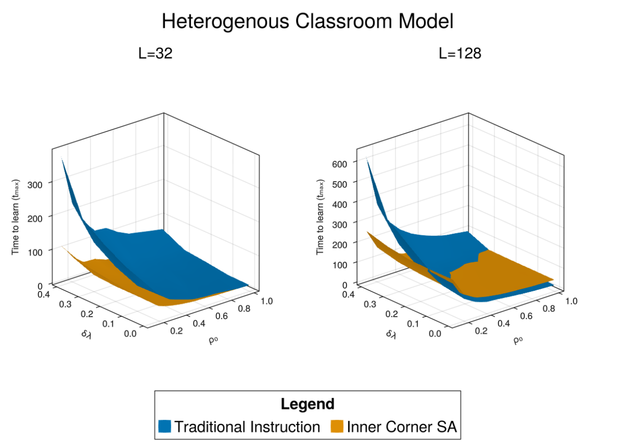
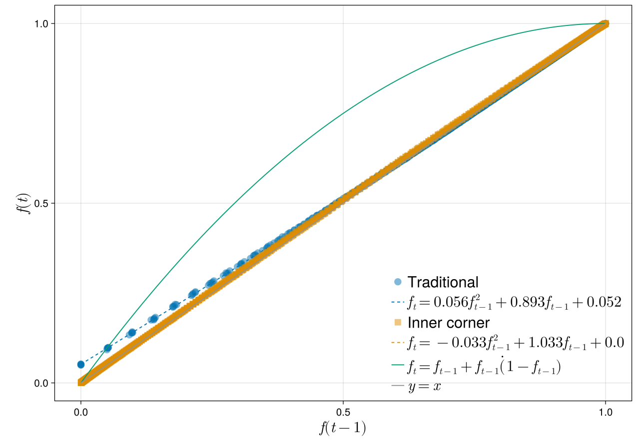


FIG. 9. Time to learn $t_{\max}$ as a function of positional learning factor ρ_0 and heterogeneity $\delta\lambda$ of PI (inner corner SA) and TI for different class sizes $N = L^2$. The blue surface represent TI and the orange surface represents PI. Lower time to learn indicates better learning.

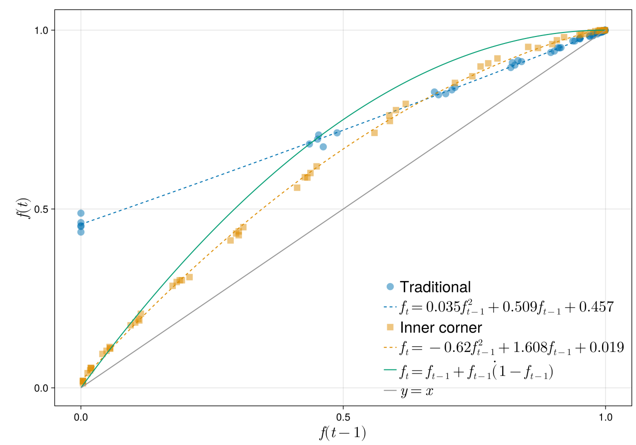
IV.3. Comparison with existing research/data

Comparing our data to Nitta's model (Equation 6) [11], as in Figure 10, only our data for PI approaches their model — this is expected since their model is for PI and not TI. Furthermore, PI only approaches their model when the conditions are favorable for PI. Considering that their model has been shown to agree with experimental data, it suggests that real world students' behavior and learning dynamics are better represented by the values of high positional learning factor ρ_0 and small class sizes N. It should also be noted that Nitta's model and data tracks individual students' learning while our model tracks the class's learning as a whole. We chose to track learning this way because there is not much insight to be gained from tracking individual students' learning in a binary-state model. Nevertheless, the agreement of our model to theirs and to experimental data suggests that our model can be a good approximation of the real world with further improvements.

Compared to Roxas' work [15] which found the outer corner SA to perform the best, we found the inner corner SA to perform the best. This difference can be attributed to the simplifications made in our model. We were not able to consider the aptitude similarity effect that happens where students of similar aptitude learn better when grouped together regardless of their actual aptitude level [6]. The implementation of this phenomenon is better suited for a continuous-state model rather than a binary-state model. Additionally, we did not consider the orientation of the students in the classroom, resulting in an isotropic system. In addition to the findings regarding seating arrangements, our model also agrees with their findings that homogenous classes find better improvements compared to heterogeneous classes.



(a) $L = 128, \rho_0 = 1, \lambda = 0.5 \pm 0.4$



(b) $L = 32, \rho_0 = 9, \lambda = 0.5 \pm 0.0$

FIG. 10. Return map for selected parameter sets for both TI and PI. Blue circles represent the data points for TI and orange squares represent the data points for PI. The corresponding best-fit quadratic function obtained using the Levenberg-Marquardt algorithm for each set of data points are shown in blue and orange dashed lines respectively. The green line represents Nitta's theoretical model (Equation 6) and the gray line represents $y = x$.

V. SUMMARY AND CONCLUSION

We propose a probabilistic cellular automata model as a new way of investigating learning dynamics in the classroom. Through this model, we can investigate how different factors can affect students' learning in the classroom for both traditional instruction (TI) and peer instruction (PI).

We found that TI performs better in larger classes and when the students have low learning rate heterogeneity. On the other hand, PI performs better in smaller classes and when the students have high learning rate heterogeneity. To be more precise, class size, heterogeneity, and low positional learning factor affect both TI and PI negatively, but TI is not as affected by class size compared to PI and PI is not as affected by heterogeneity compared to TI.

We also found that when a highly heterogeneous class is under TI, the class exhibits a two-stage learning process where majority of the fast students learn in the first few time steps and the rest of the time is spent waiting for slower students to learn. Our findings regarding class heterogeneity are in line with those of Roxas, et al. (2010) [15]. However, our findings regarding seating arrangement differ from theirs where they found that the outer corner seating arrangement perform best while we found that the inner corner seating arrangement perform best. This difference is likely because of the simplifications made in our model — namely learning isotropy and non-implementation of an aptitude similarity effect. These simplifications are from the limitations of having a binary-state model.

Another limitation of having a binary-state model is that there are not much insight to be gained from tracking individual students' learning. Hence, we chose to track the class's learning as a whole. Despite having a different way to assess the methods of instruction effectiveness, our findings are consistent with the existing works like that of Nitta, et al. [11] which is backed by real-world data.

While our model can be a good approximation of the real world, there are still improvements that can be made. The current model only considers the peer discussion part of a whole PI session. A mixed model that considers the whole session with a mix of TI and TI would be a better representation of the real world. Additionally, transitioning to a continuous-state model would allow us to incorporate more real-world phenomena like aptitude similarity and orientation effects.

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