

## Lab 9

### Quadrature formulas (2)

The rectangle (midpoint) quadrature formula is

$$\int_a^b f(x)dx = (b-a)f\left(\frac{a+b}{2}\right) + R_1(f). \quad (1)$$

The repeated rectangle (midpoint) quadrature formula is

$$\int_a^b f(x)dx = \frac{b-a}{n} \sum_{i=1}^n f(x_i) + R_n(f), \quad (2)$$

with  $x_1 = a + \frac{b-a}{2n}$ ,  $x_i = x_1 + (i-1)\frac{b-a}{n}$ ,  $i = 2, \dots, n$ .

#### Problems:

1. a) Use the rectangle formula (1) to evaluate the integral

$$\int_1^{1.5} e^{-x^2} dx.$$

b) Plot the graph of the function  $f$  and the graph of the rectangle which area approximates the integral by formula (1).

c) Use the repeated rectangle formula (2), for  $n = 150$  and  $500$ , to evaluate the integral

$$\int_1^{1.5} e^{-x^2} dx.$$

(Result: 0.1094)

2. Use two Romberg schemes (see [Course 8, formulas (2) and (5)]) for approximating the integral

$$\int_0^1 \frac{2}{1+x^2} dx,$$

with precision  $\varepsilon = 10^{-4}$ .

3. Plot the graph of  $f : [1, 3] \rightarrow \mathbb{R}$ ,  $f(x) = \frac{100}{x^2} \sin \frac{10}{x}$ . Use an adaptive quadrature algorithm for Simpson formula to approximate the integral

$$\int_1^3 f(x) dx,$$

with precision  $\varepsilon = 10^{-4}$ . Compare the obtained result with the one obtained applying repeated Simpson formula for  $n = 50$  and  $100$ . (The exact value is  $-1.4260247818$ .)