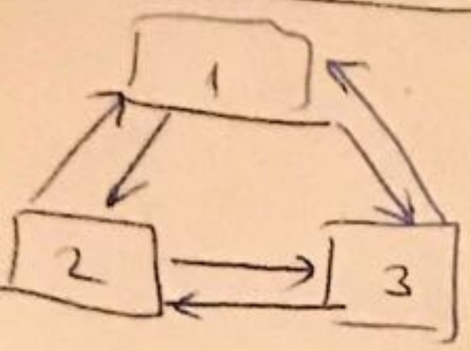
Markov Chain 1



Irreducible as we can access any state from any other state

$1 \to 2 \to 1$ cycle length period = 2.

$1 \to 2 \to 3 \to 1$ period = 3

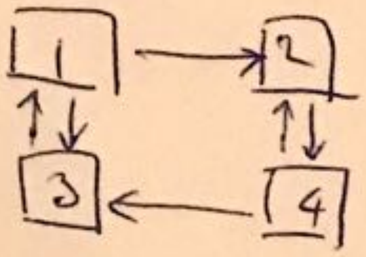
} Aperiodic (since different multiple for returning).

$\gcd(2, 3) = 1$

* A state in a discrete-time MC is PERIODIC if the chain can return to the state only at multiples of some specific integer larger than 1. 1 specific integer

eg. ABCA period = 3 $k = 3, 6, 9, 12 \ldots$ multiple of 3

MC 2



Irreducible – access any state from any other state

$1-2-3-4-1$ cycle length period = 4

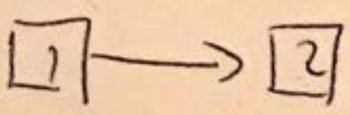
$1-2-3-1$ period = 2

Check all states

} Periodic with $p = 2$

$\gcd(2, 4) = 2$

MC 3



~~No~~ Reducible

Aperiodic

MC

Reducible

X

Irreducible & Finite → The MC has a unique stationary distribution π

& Fraction of time spent at a state $j \to \pi(j)$ as $N \to \infty$

Periodic

X

Aperiodic

$\pi_n \to \pi$, π is stationary distribution as $N \to \infty$

if $\pi \cdot P = \pi$ [converges]. π: unique

Stationary Distribution: $x = \{s_0, \ldots s_n\}$. $P = (P(x, y))$ has a stationary distribution if $\pi = \{\pi(s_0), \pi \ldots, \pi(s_n)\}$

if 1) $\pi(x) > 0$ for any x, $\Sigma \pi(x) = 1$

2) $\pi \cdot P = \pi$ (fixed point equation).