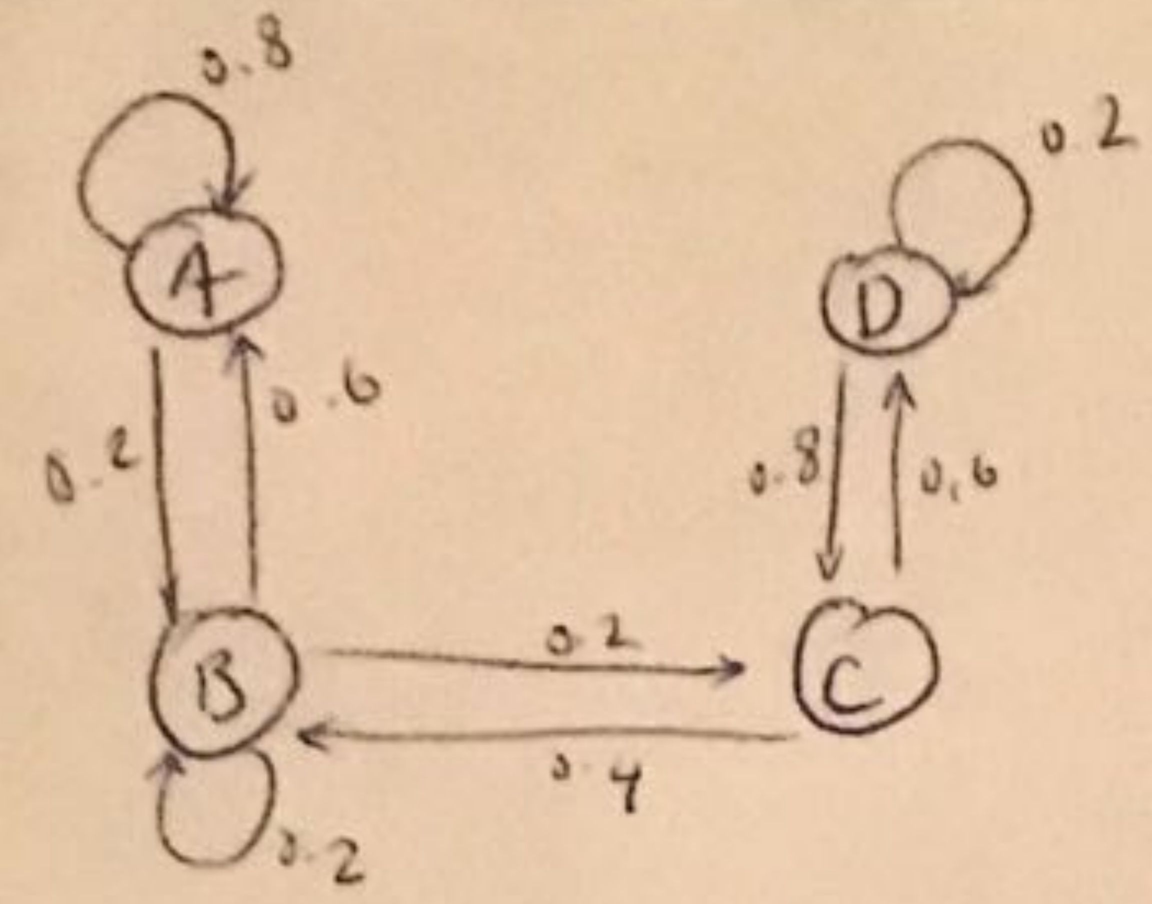

Markov's Chains

1) $P = \begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & 0.8 & 0.2 & 0 & 0 & 1 \\ B & 0.6 & 0.2 & 0.2 & 0 & 1 \\ C & 0 & 0.4 & 0 & 0.6 & 1 \\ D & 0 & 0 & 0.8 & 0.2 & 1 \end{array}$

2) Is it irreducible? ~~No~~, Yes, as it is possible to get from any state to any other state in a finite number of steps. A can reach B, B can reach C, and C can reach D, the chain is connected.

3) Is the chain aperiodic?

state A: possible transitions: $A \to A$ (period = 1). gcd = 1
state B: $B \to B$ — " ——
state D: —— " —— $D - D$ — " ——
MC is periodic

state C: $C \to D \to C$ (period = 2)^{2,4,6} gcd = 2.

4) Does it have a stationary distribution? ~~No~~ Yes ~~because to have q, need to be~~ if aperiodic and irreducible ⇒ the chain will converge to the stationary distribution in the long run ~~otherwise~~ otherwise won't.

$\pi_A = 0.8\pi_A + 0.6\,\pi_B$
$\pi_B = 0.2\,\pi_A + 0.2\,\pi_B + 0.4\,\pi_C$
$\pi_C = 0.2\,\pi_B + 0.8\pi_D$
$\pi_D = 0.6\,\pi_C + 0.2\,\pi_D$
$\pi_A + \pi_B + \pi_C + \pi_D = 1.$

$\pi_A, \pi_B, \pi_C, \pi_D = \left[\frac{8}{13}, \frac{8}{39}, \frac{4}{39}, \frac{1}{13}\right]$

5) Is it reversible?

$\pi_i P_{ij} = \pi_j P_{ji}$

For $i = A$, $j = B$: $\pi_A \cdot P_{AB} = \frac{8}{13} \cdot 0{,}2 = \frac{8}{65}$, $\pi_B \cdot P_{BA} = \frac{8}{39} \cdot 0{,}6 = \frac{8}{65}$

For $i = B$, $j = C$: $\pi_B \cdot P_{BC} = \frac{8}{39} \cdot 0{,}2 = \frac{8}{195}$, $\pi_C \cdot P_{CB} = \frac{4}{95} \cdot 0{,}4 = \frac{8}{195}$

For $i = C$, $j = D$: $\pi_C \cdot P_{CD} = \frac{4}{39} \cdot 0{,}6 = \frac{8}{65}$, $\pi_D \cdot P_{DC} = \frac{1}{13} \cdot 0{,}8 = \frac{8}{65}$

Since the detailed balance condition is satisfied, for all pairs of states i, j ⇒ MC is reversible.

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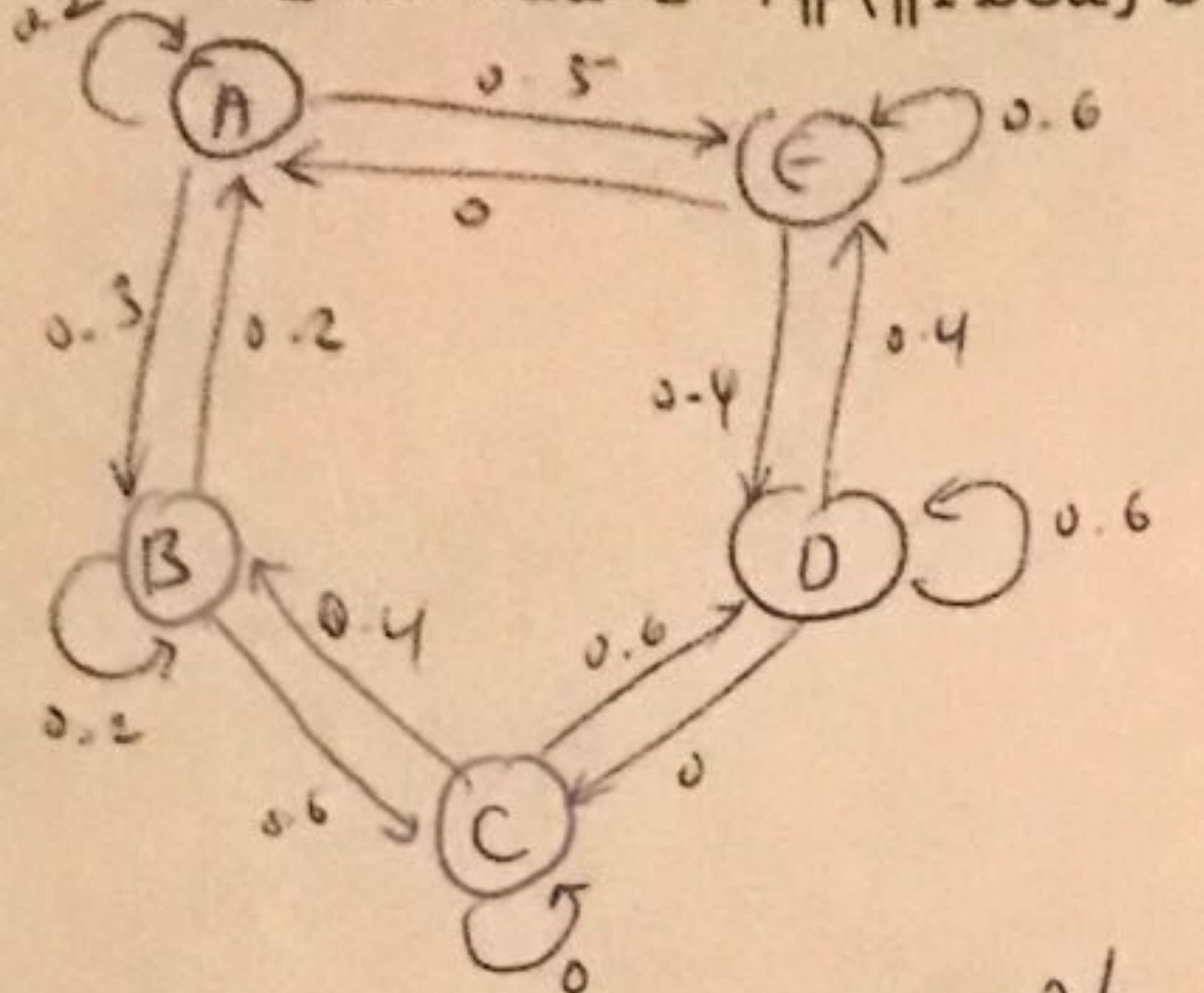

$$
P = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\left[\begin{array}{ccccc}
0.2 & 0.3 & 0 & 0 & 0.5 \\
0.2 & 0.2 & 0.6 & 0 & 0 \\
0 & 0.4 & 0 & 0.6 & 0 \\
0 & 0 & 0 & 0.6 & 0.4 \\
0 & 0 & 0 & 0.4 & 0.6
\end{array}\right]
\end{array}
\begin{array}{c} \\ =1 \\ =1 \\ =1 \\ =1 \\ =1 \end{array}
$$

2, Is it irreducible? No because the state space is divided into 2 disconnected components: $\{A, B, C\}$, $\{D, E\}$.

$d(i) = 1$: aperiodic

$d(i) > 1$: periodic

$d(i) = GCD\{n : P^n(i,i) > 0\}$

STATE A: $A \to A$ → period=1.

STATE B: $B \to B$ — // ——

STATE C: $C \to B \to C$ —//— $=2$

STATE D: $D \to D$ period = 1

STATE E: $E \to E$ period = 1

$\Bigg\}$ Chain - is periodic

3. ~~Is~~ Does it have a stationary distribution?

* Stationary Distribution $\pi = [0, 0, 0, 0.5, 0.5]$

states A, B, C : have $\pi = 0 \Rightarrow$ Transient states.

The chain will concentrate to states D and E in the long term.

$\pi_i P_{ij} = \pi_j P_{ji}$

for $i = D$ $j = E$: $\pi_D \cdot P_{DE} = \pi_E \cdot P_{ED} \iff 0.5 \cdot 0.4 = 0.5 \cdot 0.4$ ✓

It is reversible.