Customer Selection and Profit Maximization in Vehicle Routing Problems

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2 authors:

Deniz Aksen
Koc University

Necati Aras
Bogazici University

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Deniz Aksen[1], Necati Aras[2]

[1] Koç University, College of Administrative Sciences and Economics, Rumelifeneri Yolu, Sarıyer, 34450 İstanbul, Turkey

[2] Boğaziçi University, Department of Industrial Engineering, Bebek, 34342 İstanbul, Turkey

1 Introduction

The capacitated vehicle routing problem (CVRP or simply VRP) is one of the most studied combinatorial optimization problems in the literature of operations research. The main reason for this much attention is the abundance of its real-life applications in distribution logistics and transportation. In this study we focus on the single-depot capacitated VRP with profits and time deadlines (VRPP-TD).VRPP-TD is a generalization of the VRP where visiting each customer incurs a fixed revenue, and it is not necessary to visit all customers. The objective is to find the number and routes of vehicles under time deadline restrictions so as to maximize the total profit, which is equal to the total revenue collected from the visited customers less the traveling cost. For this problem we propose an efficient iterative marginal profit analysis method called iMPA applied in a two-phase framework. The first phase involves solving a time-deadline constrained vehicle routing problem (VRP-TD) using simulated annealing given a set of customers. The second phase is the implementation of iMPA where each customer's marginal profit value is calculated with respect to the set of routes found in the first phase. It is decided upon which customers to retain and which ones to discard. With the remaining set of customers determined in phase 2, phase 1 is repeated. A final correction is performed on the final solution in order to make sure that all routes have positive total profit values. In our numerical experiments we report the results obtained with this framework and assess the solution quality of our approach.

2 Literature Review

A recent paper by Feillet et al. (2005) elaborates on the traveling salesman problem with profits (TSPP) which is a generalization of the traveling salesman problem (TSP) where it is not necessary to visit all vertices of the given graph. With each customer is associated a profit that is known a priori. TSPP can be formulated as a discrete bicriteria optimization problem where the two goals are maximizing the profit and minimizing the traveling cost. It is also possible to define one of the goals as the objective function and the other as a satisfiability constraint. In one version, which is known as orienteering problem (OP), selective TSP (STSP), or maximum collection problem (MCP) in the literature, the objective is the maximization of the collected profit such that the total traveling cost (distance) does not exceed an upper bound. The other version, named as the prize collecting TSP (PCTSP), is concerned with determining a tour with minimum total traveling cost where the collected profit is greater than a lower bound. Feillet et al. (2005) provide an excellent survey of the existing literature on TSPP. Their survey lists various modeling approaches to TSSP and exact as well as heuristic solution techniques.

The extension of TSPP to multiple vehicles is referred to as the VRP with profits (VRPP). The multi-vehicle version of the OP is called the team orienteering problem (TOP) which is studied by Chao et al. (1996). The authors propose a 5-step metaheuristic based on deterministic annealing for its solution. A very recent paper on TOP is due Tang and Miller-Hooks (2005) who develop a tabu search heuristic for the problem. Butt and Cavalier (1994) address the multiple tour maximum collection problem (MTMCP) in the context of recruiting football players from high schools. They propose a greedy tour construction heuristic to solve this problem. Later on, Butt and Ryan (1999) develop an exact algorithm for the MTMCP based on branch and price solution procedure. Gueguen (1999) also proposes branch and price solution procedures for the so-called selective VRP and for the prize collecting VRP both of which are further constrained by time windows.

In this paper we focus on the VRPP-TD and propose a new heuristic called iMPA for its solution. Given a complete graph G=(N, E), where $N = \{0,1,2,\ldots,n\}$ is the set of (n+1) vertices (customers and one depot) and $E = \{(i, j) : i, j \in N \wedge i \neq j\}$ is the edge set, the objective of the VRPP-TD is to find the best routes for vehicles which depart from the depot (vertex 0), visit a set of customers, and then return to the depot so as to maximize the total profit. Profit equals the total revenue collected from the visited customers less the traveling cost. We assume that demand and profit of each customer, customer locations and the location of the depot are known with certainty. In our formulation we take into account customer-specific temporal constraints referred to as time deadlines, maximum route duration/length constraints and a uniform (homogeneous) vehicle capacity. Capacity and time deadline constraints in addition to arbitrary customer demands differentiate our problem from the TOP studied by Chao et al. (1996) and Tang and Miller-Hooks (2005).

3 Model Formulation and Solution Methodology

In the mixed-integer linear program (*MIP*) below, p_i is the revenue collected by visiting customer i; q_i is the demand of customer i; d_{ij} is distance between customers i and j; β denotes the unit traveling cost, and Q is the vehicle capacity. Decision variable y_i is 1 if customer i is visited by some vehicle, 0 otherwise; x_{ij} is 1 if customer j is visited after customer i by some vehicle, 0 otherwise; u_i is a weight associated with each customer i bounded between the demand of the customer and the vehicle capacity.

$$\text{maximize} \quad \sum_{i \in N \setminus \{0\}} p_i y_i - \beta \sum_{i \in N} \sum_{\substack{j \in N \\ i \neq j}} d_{ij} x_{ij} \tag{1}$$

$$\text{s.t.} \quad \sum_{\substack{j \in N \\ j \neq i}} x_{ij} = \sum_{\substack{j \in N \\ j \neq i}} x_{ji} \qquad i \in N \tag{2}$$

$$\sum_{\substack{j \in N \\ j \neq i}} x_{ij} = y_i \qquad i \in N \setminus \{0\} \tag{3}$$

$$Q \sum_{i \in N \setminus \{0\}} x_{0i} \geq \sum_{i \in N \setminus \{0\}} q_i y_i \tag{4}$$

$$u_i - u_j + Q x_{ij} + (Q - q_i - q_j) x_{ji} \leq Q - q_j \qquad i, j \in N \setminus \{0\} \wedge i \neq j \tag{5}$$

$$q_i \leq u_i \leq Q \qquad i \in N \setminus \{0\} \tag{6}$$

$$y_i, x_{ij} \in \{0,1\} \qquad i, j \in N \tag{7}$$

In this formulation, the first constraint is a degree balance constraint for all vertices in N. The second constraint ensures that a customer has no incoming and outgoing arcs unless it is visited. The third constraint imposes a minimum number of vehicle routes according to visits to customers. The next two constraints are lifted Miller-Tucker-Zemlin subtour elimination constraints for the VRP first proposed by Desrochers and Laporte (1991), corrected later by Kara et al. (2005). Finally, the last constraint pertains to the integrality of y_i and x_{ij}'s. Note that Q must be chosen larger than the maximum customer demand such that every customer is eligible to be visited by a vehicle.

The *MIP* formulation in (1)-(7) needs to be supplemented by time deadline constraints and consequently arrival time variables for each customer node if we want to solve a VRPP-TD instance with it. However, after the incorporation of those extra constraints and variables, a commercial solver such as CPLEX cannot solve problems with more than 20 customers. Therefore, we need efficient heuristics to tackle large size instances of VRPP-TD. To this end, we propose a new heuristic referred to as iterative marginal profit analysis (iMPA) which is based on the idea of the marginal profit of a customer. We employ iMPA in the following solution framework.

1. Solve the given problem instance as a VRP-TD with the current set of visited customers using simulated annealing (SA).

2. For the current set of routes apply iMPA until the marginal profit of each and every remaining customer is positive.
3. If iMPA does not modify the set of visited customers (i.e., no customers are dropped from the current routes), then go to step 4. Otherwise (i.e., some customers are dropped), go to step 1.
4. Check the profit of all routes. If they are positive, then stop. Otherwise, for each route with a nonpositive total profit drop the customer with the lowest marginal profit until the total profit of the route becomes positive *and* the removal of this customer does not improve the total profit of the route.

Step 4 of the above solution framework is necessary since it is possible that the total profit associated with a route can be negative even if the marginal profits of customers on that route are all positive. Therefore, we make a final check to detect such routes, and try to modify them by removing some customers so as to make the profits of these routes positive. It is clear that at the very beginning of step 1 the initial set of visited customers is taken as the set of all customers $N\backslash\{0\}$.

In step 1 of the above procedure we solve a VRP-TD using SA. Given a combinatorial optimization problem with a finite set of solutions and an objective function, the SA algorithm is characterized by a rule to randomly generate a new feasible solution in the neighborhood of the current solution. The new solution is accepted if there is an improvement in the objective value. In order to escape local minima, new solutions with worse objective values are also accepted with a certain probability that depends both on the magnitude of the deterioration Δ and the annealing temperature T. The acceptance probability is taken as $e^{-\Delta/T}$. A certain number of iterations (L_k) are performed at a fixed temperature (T_k), then the temperature is reduced every L_k iterations by the cooling rate α. The most important characteristic of an SA-based heuristic is the definition of the neighborhood structure that determines how new solutions are generated from the current solution. We use three different neighborhood structures, which are 1-0 move, 1-1 exchange, and 2-Opt. In 1-0 move, a customer selected arbitrarily is removed from its current position and inserted in another position on the same or a different route. 1-1 exchange swaps the positions of two customers that are either on the same route or on two different routes. Finally, 2-Opt removes two arcs, which are either in the same route or in two different routes, and replaces them with two new arcs. For a detailed explanation and pictorial description of these local improvement heuristics we refer the reader to Tarantilis et al. (2005).

In step 1 of the proposed procedure where VRP-TD is solved by SA with the current set of visited customers, SA has to be provided with an initial solution. For this purpose we use the parallel savings heuristic of Clarke and Wright (1964). At each iteration of the SA heuristic we randomly choose one of the local improvement heuristics to generate a new feasible solution from the current solution. Note that it is possible that one move of the selected local improvement heuristic may result in an infeasible solution to the VRP-TD because this solution may violate vehicle capacity or time deadline constraints. If this happens, we try all possible moves that can be performed by the local improvement heuristic until a feasible solution is found. This implies that one of the feasible solutions in the neighborhood of the current solution is found with certainty if there exists one. In case there are no feasible solutions with respect to the selected local improvement heuristic, then we randomly choose another one. If we cannot generate a feasible solution, the procedure is stopped, and we report the best feasible solution found so far. In step 2 of our solution framework, we apply iMPA for the current set of routes found in step 1. Given a customer *i*, its marginal profit is defined as $\pi_i = p_i - \beta\left(d_{ki} + d_{il} - d_{kl}\right)$ where *k* and *l* are those nodes that, respectively, precede and succeed customer *i*. If $\pi_i \le 0$, then customer *i* is not worth visiting. It can be dropped from its route. Otherwise, it is profitable to visit customer *i*; thus it is kept between nodes *k* and *l*. The formal definition of iMPA is given below.

1. For the current set of routes compute each customer's marginal profit π_i. Sort $\pi_i, i \in N \setminus \{0\}$ values in nondecreasing order and obtain a sorted stack $\pi_{[i]}$.
2. If $\pi_{[1]}$ of customer $i_{[1]}$ (i.e, customer with the lowest marginal profit) is positive, then exit iMPA. Otherwise, go to step 3.
3. Let $succ_{i[1]}$ and $pred_{i[1]}$ be the successor and predecessor nodes, respectively, of customer $i_{[1]}$ on its current route *r*. Delete $i_{[1]}$ from the route. Update the π values of $succ_{i[1]}$ and $pred_{i[1]}$ if they are customer nodes. Cancel route *r* if $i_{[1]}$ was the only customer on it.
4. Set $\pi_{[1]}$ to infinity such that it is put at the end of the stack. Restore the nondecreasing order of π_i values in the stack and go to step 2.

4 Computational Results

In order to test the proposed heuristic we generate random VRPP-TD instances with up to 20 customers. These instances are solved by the proposed heuristic involving iMPA, and the objective values are compared to those found optimally by the solver CPLEX 8.1 operating under GAMS. The heuristic is coded and compiled in Visual C++ 6.0 and run on a 3.20GHz Pentium 4/HT PC with 1 GB RAM.

In the implementation of SA, we adopt the following choices. The number of iterations, L_k, which have to be performed at temperature T_k is set to $250n^2$ where *n* is the number of customers. The parameter α which determines the cooling rate is assigned a value of 0.95. SA algorithm is terminated when the improvement in the objective value of the best solution during the last five temperature updates is less than 1%. The routes of the best solution found by the SA are an input to our heuristic involving iMPA. Results are shown in Table 1 where we report the percent gaps from the best objective values found by CPLEX. In some instances, as can be seen in the table, the maximum allowable CPU time (3 hours) is not sufficient for CPLEX to arrive at a proven optimal solution.

One possible improvement to the method proposed in this paper is in step 2 where iMPA is carried out on a given solution with a set of visited customers. On the basis of marginal profits, iMPA identifies which customers should be dropped from this set. It does not take into account the possibility of adding any currently unvisited customer to this set. We are currently working on the extension of our method that will account also for the repatriation of discarded customers.

Table 4.1. Accuracy of the results obtained by iMPA

Prob. No.	No. of Customers.	Best Value	Proven Optimal	Heuristic	Gap %
1	10	312.81	yes	312.81	0.00
2	10	100.41	yes	91.81	8.56
3	10	108.53	yes	97.99	9.71
4	10	191.06	yes	181.92	4.78
5	10	213.14	yes	194.55	8.72
6	10	194.67	no	252.07	-29.39
7	10	25.54	yes	25.54	0.00
8	15	26.97	no	24.92	7.60
9	15	17.23	yes	11.56	32.91
10	20	90.72	no	90.72	0.00
11	20	247.23	no	230.45	6.78
12	20	186.61	no	156.71	19.08

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