REPORT - PROJECT 1

Ioanna Maria Lazarou
AST3310 - Astrophysical plasma and the interior of the stars
University of Oslo

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Contents

Abstract

The aim of this project is the calculation of the energy production at the center of a star given a temperature and density. A function that is able to accept a temperature and density was written that can calculate the amount of energy produced by each branch of the proton proton chain and the CNO chain of fusion reactions. This paper is structured around five main parts. First, it is introduction 1. Next, in 2 we discuss the involved set of equations and the method for solving the equations. In 3, results are presented, and in the following part 4, we discuss the findings. Finally, in the last part 5, we present a summary and some thoughts and problems regarding the project.

1 Introduction

Stars, for at least a portion of their lives shine due to fusion in their central region. Their stellar engines that can generate energy that can traverse from the interior of the star to the stellar surface, by either electromagnetic radiation or by the movement of hot gas, a procedure called convection, and radiate into outer space. The energy and heat are generated by the nuclear fusion of light elements into heavier ones. Such a process is possible because the energy per nuclear particle needed to bind protons and neutrons together into an atomic nucleus, is lower and lower the heavier the nucleus becomes.

The kind of conditions needed for nuclear fusion to take place is an intense environment of enormous pressures and temperatures that can reach over 15 million Kelvin degrees. Once these conditions are reached in the core of a star, nuclear fusion converts hydrogen atoms into helium atoms through a multiple stage process. Between two atomic nuclei there is a strong repulsion force because of the positive electric charge both nuclei have, and that makes it very hard for those to get close enough to each other for the fusion to begin. Therefore, there is a large dependence on temperature and pressure and this stems from the thermal motions necessary to overcome the repulsion between two nuclei. The larger the nuclei, the higher their charge, and hence, the Coulomb repulsion force becomes larger and higher and higher temperatures are needed for the overcoming of the repulsion and fusion to happen.

Stars initially burn hydrogen into helium and this is most efficient at relatively low temperatures, because it only involves hydrogen with one proton in its nucleus. When energy production begins to decrease due to the depletion of the amount of hydrogen available as fuel, the stellar core begins to contract, heats up and raises the core temperature to then ignite helium that fuses into more heavier and heavier elements. Heavier nuclei only fuse at higher temperatures because they contain more protons in their nuclei, and so have a larger charge. To be able to burn heavier and heavier elements, stars must have really large masses. More specifically, only stars of approximately 20 solar masses can continue cycles of heavier and heavier element fusion up to iron, the final element produced in a star before it collapses.

In general, light stars radiate less energy, while heavy stars radiate more because heavy stars in their centers have higher pressures and temperatures, making the fusion more effective. It becomes so much more effective that the heavy stars can consume much faster their usable hydrogen than light stars. Stars can emit energy in their core as long as they have hydrogen fuel. Once hydrogen has depleted, the fusion reactions will shut down and the star will start to shrink and cool. Some stars will simply turn into white dwarfs, while more massive stars will continue the fusion process using helium and other heavier elements.

There are simple relations between the main parameters of stars, their mass, their radius and their energy output or luminosity, under some quite strict assumptions. The first assumption is that stars are spherically symmetric and the second assumption is that stars are in hydrostatic equilibrium, the pressure gradient at all locations is in balance with the gravitational force.

In order to work for this project, we have the following assumptions. First, the mass fraction of each atomic species is independent of radius, and given. Second, When calculating the electron density we can assume that all elements are fully ionized. In addition, we may combine the first two steps of the PP chain as a single step with the same reaction rate. And last, we do not need to consider changes over time, as you are looking at a snapshot of a star at a particular moment in time.

2 METHOD

In order to be able to calculate the energy production in a star , we need to consider change in number density of the different elements(the evolution of element abundances). The number density , which is the concentration of atoms or molecules per unit volume, is an easier quantity to find when the material density ρ is given.

The number density of an element is easily defined as:

$$n = \frac{\rho \chi_a}{a m_{\mathrm{u}}}, \tag{1}$$

where χ is the number fraction of an element, and a is the atomic number of the element. We denote X, Y, Z to be the number fractions of hydrogen, helium and heavier metals, respectively. The number densities can be used in order to calculate the reaction rates that we will discuss next about.

To be successful, fusion processes have at least one strong barrier to be overcome. If we try to bring two protons (hydrogen nuclei) together the electrostatic interaction tends to cause them to repel. The repulsion of Coulomb between two positively charged nuclei makes it extremely difficult for them to interact but it must be overcome if the protons are to fuse. In order for the thermal motions to be strong enough to overcome the Coulomb repulsion, the kinetic energy of the incoming atomic nucleus must be larger than the electrostatic potential produced by the other atomic nucleus. So, speed is one consideration that is needed to overcome the Coulomb barrier but it is not enough. The other and very important is the quantum tunneling effect that allows the incoming particle to the pass the Coulomb barrier. The actual process whereby two protons can fuse that involves tunneling in practice requires the protons to have extremely high kinetic energies. It means they have to travel very fast, that is, they have extremely high temperatures.

Nuclear fusion only starts in the cores of stars when the density in the core is great and the temperature reaches about 10 million K. The tunneling effect relies among other things on the principle of non-locality, which means that there is a non-zero probability that the particle is in fact within the barrier at any time. In general, Heisenbergs theory of uncertainty shows that the origin of the particles is never fully known, allowing the particle to appear within the Coulomb barrier. Taking the tunneling into account, the reaction rate is the number of reactions made per second. In stellar interiors, the distributions of the atoms are almost exactly Maxwellian. And that means that velocity, and energy of an ensemble of particles in the nuclear burning centre of a star is distributed according to a Maxwell-Boltzmann distribution. This distribution is applicable when the temperature is high enough or the particle density is low enough to render quantum effects negligible.

One and easy way to calculate the reaction rates is as a function of energy. The other way, that will not be used in this project, is by using the relative velocity. Often the reaction rates (unit is kg^{-1}, s^{-1}) are not given, but instead the function that allows the rate to be calculated. The proportionality function depends upon energy or temperature. Nevertheless, it is independent of the particle density involved in the fusion process. The proportionality function is often denoted by λ in the literature. It is connected to the rate per unit mass through. The reaction rates per unit mass is defined by:

$$r_{ik} = \frac{n_i n_k}{\rho(1+\delta_{ik})}\lambda_{ik}, \tag{2}$$

where n_i, n_k is the number density for an element, δ_{ik} is the Kronecker delta and λ_{ik} is the reaction rate of a fusion.

All lambdas, e.g. the reaction rates, for all reactions of the PP and CNO chain, that are found in table 3.1 of the lecture notes of the course, are used in the function that was written for the project. Measured in units cubic cm per second modified from Caughlan and Fowler (1988). Temperature T_9 is in units of 10^9 K. The electron capture by ${}^7_4Be$ has an upper limit at temperatures below 10^6 K of $N_A\lambda_{e7} \leq 1.5710^7/n_e$. When λ is the reaction rates, the reactions in the center of a star can be written for all the elements that are consumed or produced in the three PP chains and the CNO cycle.

Finally, The total energy generation per unit mass ε, is found by:

$$\varepsilon = \sum Q'_{ik} r_{ik}, \tag{3}$$

where the sum is over all reactions, i,k represents two elements, Q'_{ik} is the energy released from the fusion of two elements, r_{ik} is the reaction rates per unit mass for two elements. The Q energy is the part of the energy released, delivered into the thermal bath in which the reactions occur, neutrinos escape from the star without interactions so their energy Q of neutrinos is lost and does not contribute to the star's energy balance.

It is exciting how the energy output from the reactions in the PP chain and CNO cycle is generated. There are two main processes by which hydrogen fusion takes place in stars. Those are the proton-proton chain and the CNO(Carbon Nitrogen Oxygen) cycle. The nuclear fusion in the cores of stars involves positive hydrogen nuclei, ionised hydrogen atoms or protons, to slam together, releasing energy in the process. At each stage of the reaction, the combined mass of the products is less than the total mass of the reactants. The mass difference is what releases the energy according to Einstein's famous equation $E = m \cdot c^2$, where E is the energy, m the mass and c the speed of light in a vacuum or $E = \Delta m \cdot c^2$, where Δm is the change in mass.

Proton-Proton (PP) Chain: The PP chain is the primary mechanism of less than 1.5 solar masses in our Sun, and all stars, that is responsible for generating energy on most stars. PP chain has 3 branches and it accounts for 85% of the fusion energy released in the Sun. All three branches are resulting in ${}^4_2He$ as the end product, but two branches are involving the heaver elements lithium, beryllium and boron as intermediate products. The three branches each have their own dependency on temperature. There are two steps in the overall reaction that occur twice and those are common for all three branches to begin with, and all end up producing helium-4. Those are:

${}^1_1H + {}^1_1H \rightarrow {}^2_1D + e^+ + \nu_e + 0.155 MeV$
${}^2_1D + {}^1_1H \rightarrow {}^3_2He + 5.494$ MeV

In the first step, two protons fuse to make a deuterium nucleus(1 proton and 1 neutron) and in the second, a deuterium nucleus and a proton fuse together to make a nucleus of He-3. In branch one, two helium-3 nuclei fuse to make helium-4(two protons, two neutrons), releasing two excess protons during the process.

${}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2{}^1_1H + 12.860$ MeV

The neutral neutrinos that have extremely low rest masses, simply do not interfere with normal matter. They travel straight out of the core and at nearly the speed of light they escape from the star. Such neutrinos hold around 2 per cent of the energy released in the pp chain. When a positron collides with an electron, it annihilates and releases yet more gamma-photons of high energy. The electron neutrino e produced in this reaction is very weakly interacting and passes through the star without interacting. The energy it possesses (0.265 MeV) is lost and cannot contribute to the star's pressure balance. This first reaction in the PP chain is very slow, because it requires a proton's decay to a neutron that takes an average of one billion years for an individual proton. A positron is ideally produced when the proton decays to satisfy the conservation of energy.

The positron e^+ very quickly annihilates with a free electron e and produces two photons γ of energy 511 keV each. This energy can readily be absorbed in the stellar core. So the energy output from this reaction is: Qpp = 0.155 MeV + 1.022 MeV = 1.177 MeV
The energy output can be calculated similarly from each of the following reactions, considering that neutrino energy is lost and knowing that if a positron is produced then the energy of annihilation must be included as well.

Branches two and three contribute about 15% of the energy production in the Sun and in the intermediate steps, they generate heavier elements but they are all consumed during later phases. In the pp2 chain, a He-3 nucleus produced via the first stages of the pp1 chain undergoes fusion with a He-4 nucleus, producing Be-7 and releasing a gamma photon. The Be-7 nucleus then collides with a positron, releasing a neutrino and forming Li-7. This in turn fuses with a proton, splitting to release two He-4 nuclei. The pp2 steps are summarized as follows:

$^3_2He + ^4_2He \rightarrow ^7_4Be + 1.586$ MeV
$^7_4Be + e^- \rightarrow ^7_3Li + \nu_e + \delta E$
$^7_3Li + ^1_1H \rightarrow 2\ ^4_2He + 17.346$ MeV

A more rare event is the pp3 chain whereby a Be-7 nucleus produced as above fuses with a proton to form B-8 and release a gamma photon. B-8 is unstable, undergoing beta positive decay into Be-8, releasing a positron and a neutrino. Be-8 is also unstable and splits into two He-4 nuclei. The PP3 branch is not very effective in the Sun. Unless the temperature is above 20 MK or the abundance of the heavy elements needed is much higher than in the Sun, the PP3 branch produces little energy. This process only contributes 0.02% of the Sun's energy. The steps of pp3 branch:

$^3_2He + ^4_2He \rightarrow ^7_4be + 1.586$ MeV
$^7_4Be + ^1_1H \rightarrow ^8_5B + 0.137$ MeV
$^8_5B \rightarrow ^8_4Be + e^+ + \nu_e + 7.345 MeV$
$^8_4Be \rightarrow 2\ ^4_2He + 2.995$ MeV

The net effect of the process is that four protons, hydrogen nuclei, undergo a series of fusion reactions to form a helium nucleus-4. Other products include the He-4 nucleus, 2 neutrinos, 2 high-energy gamma photons and 2 positrons. Each of these products carries a part of the energy released from a slight decrease in the system's total mass. All sets of reactions produce an energy of 26.732 MeV per ^4_2He nucleus which is equivalent to the mass deficit between four free protons and one ^4_2He nucleus, an alpha particle.

Reaction	Energy output	Neutrino loss %
PP1	$1.08265105 \cdot 10^{-3}$	1.98
PP2	$4.81367312 \cdot 10^{-3}$	4.04
PP3	$1.564530270 \cdot 10^{-3}$	26.09
CNO	$5.666905884 \cdot 10^{-13}$	6.37

Table 1: The energy output from the reactions of each of the branches in the PP chain and CNO cycle, and a percentage of the total energy that is lost due to neutrinos.

CNO Cycle: Stars about more massive than 1.5 times the solar mass and temperatures above 10^7 K generate the most of their energy through another form of hydrogen fusion, the CNO cycle. Carbon, nitrogen and oxygen nuclei of these elements are involved in the cyclical process. To start the cycle a proton needs to fuse with a C-12 nuclei. The resultant N-13 nucleus is unstable and it must undergo beta positive decay to C-13. Next, this then fuses with another proton to from N-14, which in turn fuses with a proton to give O-15. The unstable O-15 undergoes beta positive decay to form N-15. When this fuses with a proton, the resultant nucleus immediately splits to form a He-4 nucleus and a C-12 nucleus. This carbon nucleus can then start another cycle. Carbon-12 acts like a nuclear catalyst. That means that it is necessary for the process to proceed but ultimately is not used up by it. As with the different forms of the pp chain, during the process gamma photons and positrons are released along with the final helium and carbon nuclei and all of these have energy. The dominant branches of the CNO cycle described are as follows:

$^{12}_{6}C + ^{1}_{1}H \rightarrow ^{13}_{7}N + 1.944\text{MeV}$
$^{13}_{7}N \rightarrow ^{13}_{6}C + \text{e}^{+} + \nu_e + 0.491 MeV$
$^{13}_{6}C + ^{1}_{1}H \rightarrow ^{14}_{7}N + 7.551\text{MeV}$
$^{14}_{7}N + ^{1}_{1}H \rightarrow ^{15}_{8}O + 7.297\text{MeV}$
$^{15}_{8}O \rightarrow ^{15}_{7}N + \text{e}^{+} + \nu_e + 0.735 MeV$
$^{15}_{7}N + ^{1}_{1}H \rightarrow ^{12}_{6}C + ^{4}_{2}He + 4.966\text{MeV}$

The neutrinos produced in steps 2 and 5 are both low energy neutrinos transporting away 0.707 and 0.997 MeV respectively. The other branches of the CNO cycle are only effective at much higher temperatures. The next most energy producing branch of the CNO cycle provides just 0.05 per cent of the total energy produced in the Sun by the CNO cycle.

3 Results

Taking into account the methodology, the Appendix A, B, and also the tables 3.2 and 3.3 from the lecture notes, the energy output from the reactions in the PP chain and CNO cycle was calculated. For each of the branches (PPI, PPII, PPIII and CNO) the percentage of the total energy that is lost due to neutrinos was calculated as well. In table 1, we can see the results. Energy output is measured in $Jkg^{-1}\ s^{-1}$.

Rate Symbol	Energy	Sanity	Relative Error
λ_{pp},λ_{pd}	$4.048 \cdot 10^2$	$4.04 \cdot 10^2$	0.001971
λ_{33}	$8.608 \cdot 10^{-9}$	$8.68 \cdot 10^{-9}$	0.008270
λ_{34}	$4.840 \cdot 10^{-5}$	$4.86 \cdot 10^{-5}$	0.004074
λ_{e7}	$1.423 \cdot 10^{-6}$	$1.49 \cdot 10^{-6}$	0.044863
$\lambda_{'17}$	$5.266 \cdot 10^{-4}$	$5.29 \cdot 10^{-4}$	0.004618
$\lambda_{17}, decay$	$1.619 \cdot 10^{-6}$	$1.63 \cdot 10^{-6}$	0.006671
λ_{p14}	$9.180 \cdot 10^{-8}$	$9.18 \cdot 10^{-8}$	0.000042

Table 2: The energy production from each of the PP branches and the CNO cycle.

We notice that the energy output for the PP reactions is less than the CNO cycle. CNO generates higher amounts of energy and also PP2 is stronger than PP3. PP3 would begin to make much larger relative contributions in the energy output but, however, at higher temperatures, the CNO cycle would quickly become more important than the PP chains in stellar energy production. In PP2 and PP3 branch, the neutrino percentage loss is high but between those two PP3 has the most neutrino lost. The neutrinos in the pp1, pp2, pp3 chains and CNO cycle carry away 1.3%, 11.5%, 26.7%, and 6.4% of the energy in those reactions, respectively. These high energy neutrinos have been the source of the solar neutrino problem. The problem was that far fewer neutrinos were observed than the number predicted by solar models. However, it is now recognized that on their way from the Sun to Earth, the neutrinos of the PP III chain must go through a conversion.

Table 2, shows the energy production results, measured in $Jkg^{-1}\ s^{-1}$. In general, The relative frequency of the different branches depends on the chemical composition, the temperature and the density. The energy production from each of the PP branches and the CNO cycle was calculated, using the temperature and density of the solar core. The calculation were made by taking into account that neutrino energy is lost and including the annihilation energy when a positron is created. The rate symbols of the reactions are shown in the first column just to avoid confusion about which reactions energy refers to. Reactions involving nuclei with the smallest charges are favoured, due to smaller Coulomb barrier. This is the reason why in the Sun, pp is the most dominant energy production mechanism, accounting for the most part of the total energy production. Reactions that involve nuclei with larger and larger charges are less likely, since more massive particles are involved in the tunneling. In the third column are the values of the sanity for each reaction in $Jm^{-3}\ s^{-1}$ and in the fourth, the relative errors of the calculated energy production levels. All relative errors are almost zero, fact, that verifies calculations inside the function should be correct.

Figure 1 shows the relative energy output (ε) of proton–proton chain (PP) and CNO cycle as a function of temperature, where $T \in [10^4,\ 10^9]$. In this plot, logarithmic scales have been used in order to respond to skewness towards large values (cases in which one or a few points are much larger than the bulk of the data).

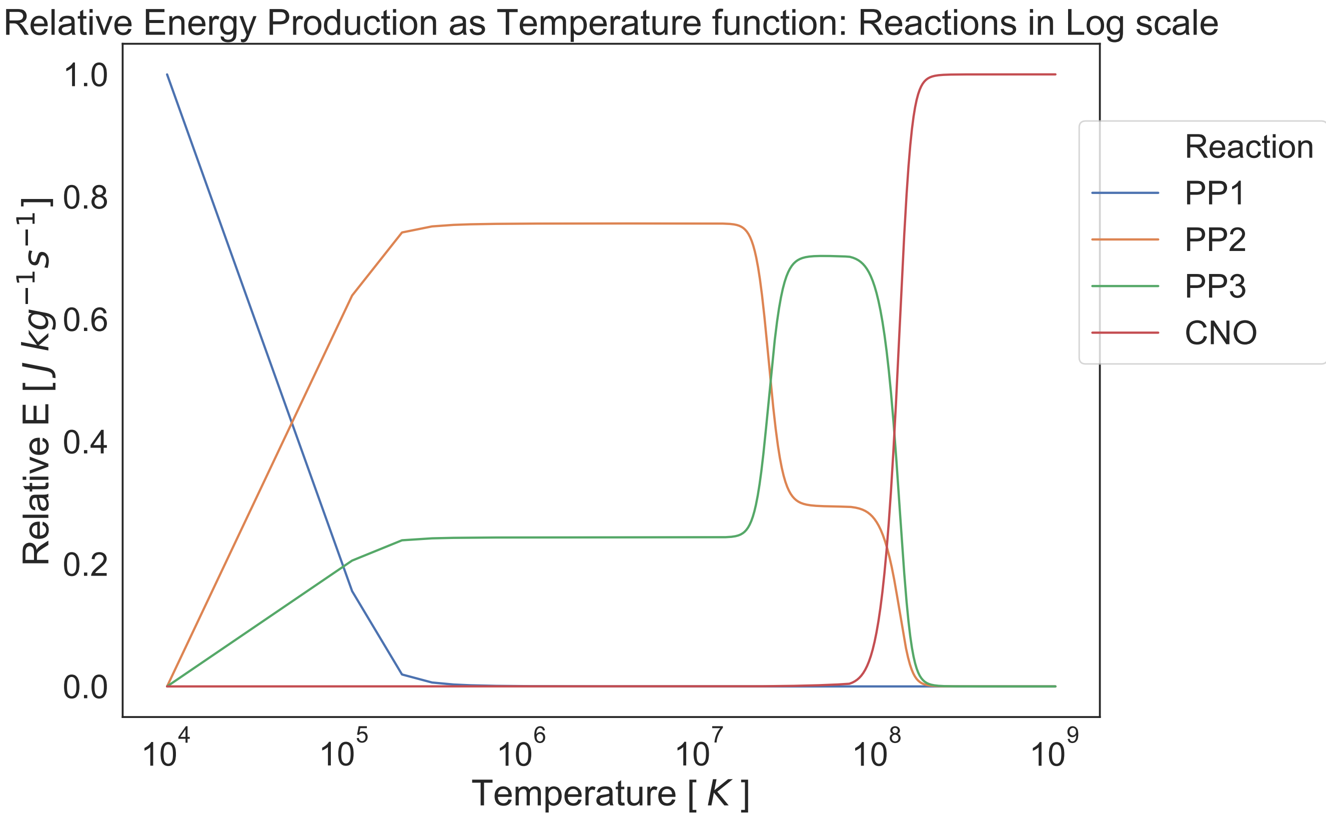

Figure 1: Relative energy production from each of the PP branches and the CNO cycle as functions of temperature. Logarithmic scales have been used on X,Y axes, Temperature is expressed in K and Energy in $Jkg^{-1}s^{-1}$.

Due to the difficulty in distinguishing details, at the top left corner of the graph, we can see a zoomed part of the graph of a specific range of energy output and temperature. At the Sun's core temperature, we can see, generally, that the PP process is more efficient. We can notice that approximately under 10^8 K degrees, the energy output of the pp chain is higher than the CNO cycle whereas in higher temperatures the CNO cycle generates higher amount of energy production than the PP chain. At a certain temperature (shown by the intersection of the 4 curves) the two reactions are equal. We can include that any further slight increase in core temperature leads to a greater increase in energy output.

The CNO cycle becomes the primary energy source in stars with 1.5 or higher solar masses. In these stars the core temperatures are 18 million K or greater. As the core temperature of the Sun is around 16 million K, the CNO cycle represents just one minute fraction of the total energy released. The first stage of the pp chain involves two protons fusing together whereas in the CNO cycle, a proton has to fuse with a carbon-12 nucleus. As carbon has six protons the Coulomb repulsion is greater for the first step of the CNO cycle than in the pp chain. The nuclei thus require greater kinetic energy to overcome the stronger repulsion. This means they have to have a higher temperature to initiate a CNO fusion.

Higher-mass stars have a stronger gravitational pull in their cores which leads to higher core temperatures. Stars evolve because of changes in their abundance of their constituent elements over their lifespans. First by burning hydrogen, then helium, and progressively burning higher elements resulting in different levels of energy production, according to different temperatures.

4 Discussion

Some more things that we need to discuss is the reason Why we only need to use the reaction rate from ${}^{14}_{7}N +^{1}_{1} H \rightarrow^{15}_{8}O + 7.30MeV$ for the entire CNO cycle and where does the ${}^{12}_{6}C$ in the first CNO reaction (Eq. 3.13 of the lecture notes) come from. When pp and CNO cycles are in equilibrium, the rate of production equals that of consumption, all reactions proceed at the same rate. We can then compute the total energy generation rate by taking the reaction rate of one reaction and multiplying it by the energy generation of one whole reaction chain. The slowest reaction within the cycle is that witch converts ${}^{14}_{7}N$ into ${}^{15}_{8}O$. Nitrogen is the most numerous of the three. Nearly all of the initially present C, N and O nuclei will therefore be found as ${}^{14}_{7}N$, waiting to be transformed to ${}^{15}_{8}O$. CNO cycle that absorbs protons at each step but we end up getting an alpha particle out of this whole cycle. Carbon, nitrogen and oxygen, basically undergo transformations but they end up behaving like a catalyst. They do not transform into something else. They enable the reaction to take place, but are not themselves consumed or created by it. Regardless of the amount of carbon which goes into the cycle, we end up getting the same amount coming out of it. Carbon, nitrogen, and oxygen act as catalysts in this kind of a process where hydrogen combines together to create alpha particle. Thus, the ${}^{12}_{6}C$ nucleus used in the initial reaction is regenerated in the final one and hence acts as a catalyst for the whole cycle.

Moreover we have to make clear how we make sure that no step consumes more of an element than the previous steps are able to produce. The binding energy is the energy required to split the nuclei into free protons and neutrons. Naturally, hydrogen has zero binding energy because it consists of a single proton. Since it takes energy to split helium, that energy must then be available when fusing hydrogen into helium. That energy is converted into free energy and ends up as heat. Fusion of elements releases energy as long as the nuclei entering the process have lower binding energy than the nuclei that are produced. The binding energy per nucleon. Binding energy is the amount of energy needed to split the nucleus apart into isolated protons and neutrons. One important thing to mention when calculating ϵ is that no reaction should happen more often than the reaction(s) that produced the reactant(s) of the first reaction. For instance, in the PP3 we have the two steps:

${}^{3}_{2}He + {}^{4}_{2}He \rightarrow {}^{7}_{4}be + \gamma$

${}^{7}_{4}Be \rightarrow \mathrm{e}^{-}\ {}^{7}_{3}Li + \nu_e + \gamma$

Here, the second step can happen no more often than the first step. If this was not the case, the Beryllium would quickly disappear and we end up using Beryllium that does not exist. A similar considerations have to be made to all dependencies of each step in the cycles.

5 Conclusion

During working on this project, I learned a lot about the processes that occur inside the stellar cores and the importance of temperature and pressure. Moreover, I learned about how energy is generated inside the core of the sun and in stars that are more massive or less massive than our sun. Exploring all the physics behind energy production-studying the fusion reactions of the proton-proton chain and the CNO cycle was really interesting as well as plotting the relative energy production from each of the PP branches and the CNO cycle as functions of temperature. The project was enjoyable to work on, and I feel quite satisfied with how it turned out. Much extra work related to this project arose when trying to debug various problems. Getting the energy production correct, initially, caused some trouble but eventually, everything turned out smooth. I am having this course as a single course, and as I am not a physics student yet, I faced several difficulties throughout the whole working project period. However, learning interest about stars and dedication have proved keys for some improvement.

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