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# REPORT - PROJECT 2

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## Abstract

In the first project, given a temperature and density, the energy production at the center of a star was calculated. More specifically, the amount of energy produced by each branch of the proton proton chain and the CNO chain of fusion reactions. This second term project involves modelling the central parts of a Sun-like star, including both radiative and convective energy transport. We model the stellar core of a star by solving a set of differential equations which let us further study how the different features of the core changes as we move through the star. The code developed in project 1 will be used for the energy production in your star. This paper is structured around four main parts. First, is introduction 1. Next, in Method 2 we discuss the involved set of equations and the method for solving the equations. Finally, in Results 3, we present results, and in Conclusion 4, we discuss findings and some thoughts and problems regarding the project.

# 1 INTRODUCTION

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In the first project, given a temperature and density, the energy production at the center of a star was calculated. More specifically, the amount of energy produced by each branch of the proton proton chain and the CNO chain of fusion reactions. This second term project involves modelling the central parts of a Sun-like star, including both radiative and convective energy transport. We model the stellar core of a star by solving a set of differential equations which let us further study how the different features of the core changes as we move through the star.

All assumptions from Project 1 hold. Additional assumptions for this project is: ideal gas and only radiation and convection. One way the energy produced by the fusion in the central parts of the star is transported to the stellar surface is electromagnetic radiation or by the movement of hot gas from the interior of the star to the stellar surface. Convection, the transport of energy by moving parcels of gas, is somewhat akin to what can be observed in a pot of hot water on the stove just before it boils. Patterns consisting of upwellings of hot water from below, and thin lanes of water that has cooled at the surface and moves back down to be reheated in the bottom of the pot. In stars the same effect happens, and in the Sun, the effect is almost identical. Stars have a nuclear burning core where the fusion of elements provides the energy that is initially transported by radiation in the first two thirds of the radius, until the gas becomes convective and the gas motions transport the bulk of the energy to the surface where it is radiated into space.

## 2 METHOD

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The following equations govern the internal structure of radiative core of the Sun.

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \quad (1)$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} \quad (2)$$

$$\frac{\partial L}{\partial m} = \epsilon \quad (3)$$

$$P = P_G + P_{rad} \quad (4)$$

$$\frac{\partial T}{\partial m} = -\frac{3\kappa L}{256\pi^2 \sigma r^4 T^3} \quad (5)$$

The first thing to notice is that the differential equation are all differentiated with respect to  $m$  instead of  $r$ . The reason for this is a more stable solution when performed numerically with respect to  $m$  than with respect to  $r$ . Since the density becomes larger as we approach the core of the star the the mass within a radius  $R$  decreases less and less.  $R$  on the other hand goes rather quickly towards the core, hence the more stable solution when differentiating with respect to  $m$ .

Equation 1 is simply stating the relation between taking an infinitesimal step in  $r$ , compared to an infinitesimal step in  $m$ .

Equation 2 is the assumption of hydrostatic equilibrium, and states that if the gas in the star is to be at rest, then the outward pressure must exactly balance the force of gravity acting on the gas.

The  $\epsilon$  in 3 represents the amount of energy produced by nuclear fusion per time and mass. It comes from the definition of luminosity,  $\partial L = \partial m * \epsilon$ , where  $\epsilon$  is nuclear energy produced per mass inside the core. The luminosity of the star is simply the energy produced. We derive the value of  $\epsilon$  by looking at the reactions that produce energy inside a star. It is given by:

$$\epsilon = \sum r_{ik} Q_{ik}, \quad (6)$$

where  $Q_{ik}$  is the energy output corresponding to reaction  $ik$ . In project 1, we were asked to calculate the energy production from each branch, where the reaction rate from the common step was not included. When we calculate the luminosity in project 2, we need the entire energy production.

The differential equations we are considering are all in one way or another dependant of the pressure and density in the star. As a result, we need equations which describe these quantities in order to proceed. The pressure inside our star at a given radial position is a sum of all the different contributing pressure components. The pressure is given as 4. where  $P_G$  is the gas pressure, and  $P_{rad}$  is the radiative pressure.

Equation 5, is the temperature gradient need to transport all the produced energy out of the star. It should be noted that this is only the correct temperature gradient when all the energy is transported by radiation. It is calculated by considering the radiative flux on the surface of the star in terms of luminosity.

The equations contain seven unknowns,  $(r, \rho, P, L, P_G, P_{rad}, T)$  so the next assignment was to find two more equations which would let us solve them; an equation of state and an equation for the radiative pressure. When assuming an ideal gas inside the star we can use the equation of state for an ideal gas to give us an equation connecting  $\rho$ ,  $P$  and  $T$ . This equation states:

$$PV = N\kappa_B T \quad (7)$$

where  $P$  is the gas pressure,  $V$  is the volume,  $N$ ,  $N = \frac{m}{m_\mu \mu}$ , is the number of particles and  $T$  is the temperature of the gas.  $\mu$  is the average atomic weight and  $m_\mu$  is the atomic mass unit. To find the average atomic weight we must first find the total number of particles in the star. The number densities of each element was given by equation 2 so the mean molecular weight must therefore be:

$$\mu_0 = \sum_i \frac{\frac{\rho}{n_i}}{m_\mu} = \sum_i \frac{\rho m_\mu}{\frac{X_i}{A_i} \rho m_\mu} \quad (8)$$

In our case:  $\mu_0 = \frac{1}{2X + 3/4Y + Z/2}$ . The second equation we need is related to the radiative pressure  $P_{rad}$ . The gas pressure is:  $P_G = \frac{4\sigma}{3c} T^4$ , where  $\sigma$  is the Stefan-Boltzmann constant and  $c$  is the speed of light. In addition to the gas pressure, we have radiation pressure  $P_{rad}$ , so that the total pressure is  $P = P_g + P_{rad}$ . The expression for radiation pressure  $P_{rad} = \frac{a}{3} T^4$  depends only on temperature, where  $a = 4\sigma/c$  is a constant. This gives us finally the additional equation of state we need:

$$P = \frac{\rho}{\mu m_u k_B T + \frac{a}{3} T^4} \Leftrightarrow \rho = (P - \frac{a}{3} T^4) \frac{\mu m_u}{k_B T} \quad (9)$$

The equation is given also with respect to density. This relation can be used to find  $P$  given  $\rho$  and  $T$ , and vice versa.

To create our model we need to include some additional important equations. The temperature gradient of adiabatic rising parcel:

$$\nabla_{ad} = \left( \frac{\partial \ln(T)}{\partial \ln(P)} \right)_s = \frac{2}{5} \quad (10)$$

is equal to a constant since we assume ideal gas. This more obvious if one considers the alternate form of the adiabatic gradient:

$$\nabla_{ad} = \frac{P\delta}{T\rho C_p} \quad (11)$$

This is due to the assumption of ideal gas and the non energy exchange between the rising convective gas bubbles.  $C_p$  then becomes:

$$C_p = \frac{5}{2} \frac{k_B}{\mu m_u} \quad (12)$$

Thus, the expression for the adiabatic gradient will dictate the convection criteria at later stages. For the other gradients we need to consider more equations. Starting with:

$$F_{conv} = \rho C_p T \sqrt{g\delta} H_p^{3/2} \left( \ln \frac{l_m}{2} \right)^{1/2} (\nabla^* - \nabla_p)^{3/2} \quad \text{and} \quad F_{rad} = \frac{16\sigma T^4}{3\kappa\rho H_p} \nabla^* \quad (13)$$

These two equations complete the set of equations for  $(\nabla^* - \nabla_p)$  and we can use them in:

$$F_{con} + F_{rad} = \frac{16\sigma T^4}{3\kappa\rho H_p} \nabla_{stable} \quad (14)$$

where  $F_{rad} = L/(4\pi r^2)$  is the radiative flux given as the luminosity over the area of the spherical mass shell, and  $F_{con}$  is the convective flux, which is 0 in this case.  $\nabla_{stable}$  is the temperature gradient of the non convective star, and lastly,  $H_p$  is the pressure scale height defined as the height where the pressure has dropped by a factor  $e$ . This height is:

$$H_p = -PTr \quad (15)$$

Solving equation (14) for the temperature gradient, and inserting the radiative flux  $F_{rad} = L/(4\pi r^2)$  results in:

$$\nabla_{stable} = \frac{3L\kappa\rho H_p}{64\pi r^2 \sigma T^4} \quad (16)$$

Thus, when we do not have convection, the temperature gradient of the star is  $\nabla^* = \nabla_{stable}$ . The criteria for whether or not our star is stable is based on the inequality of the adiabatic and stable temperature gradient. Convection occurs when the star is unstable, that is if  $\nabla_{stable} > \nabla_{ad}$ .

When a star is not stable, the expression for the gradient is more complicated. The convective flux of a star is given as:

$$F_C = \rho C_p T \sqrt{g\delta} H_p^{3/2} \left( \frac{l_m}{2} \right)^2 (\nabla^* - \nabla_p)^{3/2}, \quad (17)$$

where  $g$  is the gravitational acceleration,  $l_m = 2\delta r$  is the mixing length (which is 2 times the distance  $\delta r$  a parcel of gas has travelled), and  $\nabla_p$  is the temperature gradient of the parcel. This is derived using the previously mentioned model where a parcel of gas moves up the unstable star. When we have no

convection, equation (14) can be written in terms of  $\nabla^*$  as

$$F_R = \frac{16\sigma T^4}{3\kappa\rho H_P} \nabla^* \quad (18)$$

Inserting equations (17) and (18) into (14), we get

$$\frac{16\sigma T^4}{3\kappa\rho H_P} \nabla^* + \rho c_P T \sqrt{g\delta H_P}^{-3/2} \left(\frac{l_m}{2}\right)^2 (\nabla^* - \nabla_p)^{3/2} = \frac{16\sigma T^4}{3\kappa\rho H_P} \nabla_{\text{stable}}$$

Solving this equation for  $(\nabla^* - \nabla_p)^{3/2}$ :

$$(\nabla^* - \nabla_p)^{3/2} = \frac{U}{l_m^2} (\nabla_{\text{stable}} - \nabla^*), \quad (19)$$

where we have substituted  $U = \frac{64\sigma T^3}{3\kappa\rho^2 c_P} \sqrt{\frac{H_P}{g\delta}}$  for simplification. Next is to consider the quantity  $(\nabla_p - \nabla_{\text{ad}})$ :

$$(\nabla_p - \nabla_{\text{ad}}) = \frac{32\sigma T^3}{3\kappa\rho^2 c_P v} \frac{S}{Qd} (\nabla^* - \nabla_p), \quad (20)$$

where  $v$  is the velocity of the parcel, which is given as:  $v = \sqrt{\frac{g\delta l_m^2}{4H_P}} (\nabla^* - \nabla_p)^{1/2}$ , and  $\frac{S}{Qd}$  is a geometrical factor for the parcel. Assuming a spherical parcel the surface of the sphere is  $S = 4\pi r_p^2$ ,  $d = 2r_p$  is the diameter, and  $Q = \pi r_p^2$  is the area of a circle within the sphere where the flux is computed. Combined gives:  $S/Qd = 2/r_p$ , where  $r_p$  is the parcel radius.

To determine the parcel radius, we could either make an estimate using the speed of sound within the star and the mean free path of the photons or simply make an assumption that the radius is somewhat of the same order as the distance the parcel travel,  $\partial r$  through one mass step. Usually the assumption for this distance is:  $\partial r = l_m/2$ . We then assume that the radius of the parcel is some factor  $\beta$  times this distance. we use  $\beta = 1$ , so that  $r_p = \frac{2}{l_m}$ . Inserting for this radius into the geometrical factor, one finds  $\frac{S}{Qd} = \frac{4}{l_m}$ . We substitute for the velocity and this geometrical factor into equation (20):

$$(\nabla_p - \nabla_{\text{ad}}) = 4 \left( \frac{U}{l_m^2} \right) (\nabla^* - \nabla_p)^{1/2}$$

We use  $A = \frac{U}{l_m^2}$ , so that the final expression becomes:  $(\nabla_p - \nabla_{\text{ad}}) = 4A(\nabla^* - \nabla_p)^{1/2}$

By adding and subtracting  $\nabla^*$ , we rewrite:  $(\nabla_p - \nabla_{\text{ad}}) = (\nabla^* - \nabla_{\text{ad}}) - (\nabla^* - \nabla_p)$

$$4A(\nabla^* - \nabla_p)^{1/2} = (\nabla^* - \nabla_{\text{ad}}) - (\nabla^* - \nabla_p) \quad (21)$$

We next substitute  $\xi = (\nabla^* - \nabla_p)^{1/2}$  and rewrite equation (21):

$$4A\xi = (\nabla^* - \nabla_{\text{ad}}) - \xi^2 + 0 \quad (22)$$

We solve for  $\nabla^*$  and we get the temperature gradient for a unstable star:

$$\nabla^* = \xi^2 + 4A\xi + \nabla_{\text{ad}} \quad (23)$$

Inserting for  $\nabla^*$  from equation (23) into equation (19) in addition to rewriting it in terms of  $\xi$ :

$$\xi^3 = A(\nabla_{\text{stable}} - \xi^2 - 4A\xi - \nabla_{\text{ad}})$$

We can numerically solve this equation for  $\xi$  by expressing this as a cubic equation:

$$\frac{1}{A}\xi^3 + \xi^2 + 4A\xi - (\nabla_{\text{stable}} - \nabla_{\text{ad}}) = 0 \quad (24)$$

We now have all the expressions for the temperature gradients for both the stable and unstable case in our star. The temperature gradient of the star  $\nabla^*$  can be expressed in terms of the height pressure scale on the form:

$$\begin{aligned} \nabla^* &= -\frac{H_P}{T} \frac{\partial T}{\partial r} \\ \frac{\partial T}{\partial r} &= -\frac{T}{H_P} \nabla^* \end{aligned} \quad (25)$$

We use the chain rule and rewrite this expression in terms of  $\partial m$  where  $\partial m = 4\pi\rho r^2\partial r$ . In addition, we insert for the definition of  $H_P$ :

$$\frac{\partial T}{\partial m} = -\frac{Gm}{4\pi r^4} \frac{\mu m_u}{k_B \rho} \nabla^*$$

The first fraction in the expression is the definition of  $\partial P/\partial m$ , while the second fraction is  $T/P$  from the ideal gas law equation of state. Our final expression then for the change in temperature with respect to mass becomes:

$$\frac{\partial T}{\partial m} = \frac{T}{P} \frac{\partial P}{\partial m} \nabla^* \quad (26)$$

We make all the appropriate implementations to our model by defining all the new quantities in our program, followed up by implementing an if statement which checks whether or not the criteria for convection is fulfilled. If  $\nabla_{\text{ad}} < \nabla_{\text{stable}}$ , we let  $\nabla^* = \nabla_{\text{stable}}$ . Otherwise, the expression for the temperature changes (26) to  $\nabla^* = \xi^2 + 4A\xi + \nabla_{\text{ad}}$  (23). We also can study the radiation and convection zones in our star which are calculated through the flux calculations from equations (4), (17) and (18).

### 3 RESULTS

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We were given two sanity checks that include an interpolation sanity check found in table 1 and a sanity gradient test shown in table 2 for this project. The relative errors produced were negligible. The temperature inside a star is amongst other variables proportional to the opacity( $\kappa$ ), the resistance to have energy transported throughout the gas. The opacity of a gas depends on the ionization degree and the atomic species present in the gas. We need this value to be able to continue with solving the differential equations. The values provided were the values achieved by the interpolation function were which as we can see match relatively well with the "sanity check" values given as shown in table 1. A series of gradient values was and the interpolated data points were compared with known values. A set tolerance was used for verification.

The initial parameter plays an important role in the outcome of our star. In table 3 we see an overview of the initial parameter values that were used to create the sanity cross section plot of the star (1). For initial parameters equal to that of the sun, our model generated the following results that resemble known solar parameters (2). In figure 1 we see the cross section of the star. Both of these figures seems to be equivalent with the given sanity check figures, which suggests that the program also works as intended



*Interpolation Sanity check results*

$\log_{10} T$	$\log_{10} R$ (cgs)	$\log_{10} \kappa$ (SI)	$\kappa$ Sanity Values	Relative error
3.750	-6.00	2.84e-03	2.84e-03	0.000004
3.755	-5.95	3.11e-03	3.11e-03	0.000001
3.755	-5.80	2.69e-03	2.68e-03	0.000012
3.755	-5.70	2.45e-03	2.46e-03	0.000015
3.755	-5.55	2.12e-03	2.12e-03	0.000004
3.770	-5.95	4.72e-03	4.70e-03	0.000018
3.780	-5.95	6.23e-03	6.25e-03	0.000023
3.795	-5.95	9.44e-03	9.45e-03	0.000007
3.770	-5.80	4.12e-03	4.05e-03	0.000074
3.775	-5.75	4.55e-03	4.43e-03	0.000120
3.780	-5.70	5.03e-03	4.94e-03	0.000091
3.795	-5.55	6.89e-03	6.86e-03	0.000034
3.800	-5.50	7.69e-03	7.69e-03	0.000001

**Table 1:** Table of achieved  $\kappa$  values as shown from the interpolation sanity check.

*Gradient Sanity check results*

Variable	Calculated	Expected	Relative error
$\nabla_{stable}$	3.146	3.260	0.03486
$H_P$ [Mm]	3.152e+07	3.250e+07	0.03028
$U$	6.041e+05	5.940e+05	0.01696
$\xi$	1.203e-03	1.175e-03	0.02351
$\nabla^*$	4.000e-01	4.000e-01	0.00000
$u$ [ $ms^{-1}$ ]	6.616e+01	6.562e+01	0.00827
Convective flux	8.729e-01	8.800e-01	0.00810
Radiative flux	1.271e-01	1.200e-01	0.05943

**Table 2:** Table of achieved gradient values as shown from the gradient sanity check.

for full simulations. This is not an optimal model but though verifies its validity, more specifically the radiative and convective zones in the interior verify the validity of the model. We also look at the evolution of other stellar parameters found in figure 2.

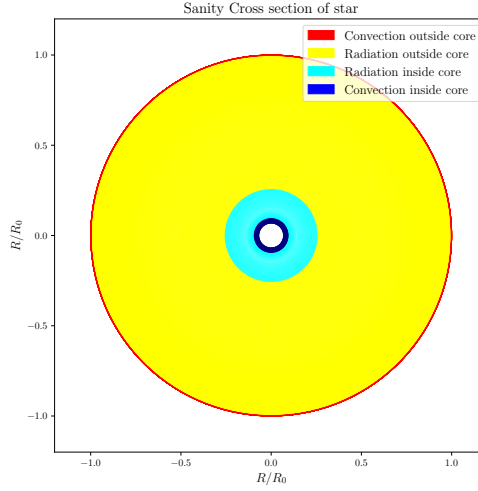
### 3.1 Tweaking parameters

In order to determine which parameters to tweak in order to produce a model that best describes our criteria eight stars were modelled - one for a high and low value for each of the four initial parameters.

*Solar Model Parameters-Cross section of star*

Initial Parameter	Symbol	Value
Density	$\rho_{\odot}$	1.9994e-04 [ $kg/m^3$ ]
Luminosity	$L_{\odot}$	3.8460e+26 [ $W/m^2$ ]
Mass	$M_{\odot}$	1.9891e+30 [kg]
Radius	$R_{\odot}$	6.9600e+08 [m]
Temperature	$T_{\odot}$	5.7700e+03 [K]

**Table 3:** Initial parameters for creating the cross section sanity of a star.



**Figure 1:** Sanity plot-Cross section of a star.

Particularly, we investigate what happens to our star when we Change the value of  $R_0$ ,  $T_0$ ,  $\rho_0$ ,  $P_0$  (keeping all other initial parameters unchanged). Plots of the results from the experiments for each of these are depicted in figure 3.

We only allow convection when  $\nabla_{stable} > \nabla_{ad}$ .  $\nabla_{stable}$  is proportional to the density while  $\nabla_{ad}$  is inverse proportional to it. This should mean that the higher the initial density, the larger the convection zone outside the core should be. After applying different initial densities, results suggested that with larger initial density, the convection values outside the core increase; the convection zones are getting larger. Since  $\nabla_{stable}$  inverse proportional to  $T^4$  and  $\nabla_{ad}$  is inverse proportional to  $T$  we would expect that for high temperatures the convection zone outside the core would get smaller since high temperatures would give us a small  $\nabla_{stable}$  which means that the convection test  $\nabla_{stable} > \nabla_{ad}$  will not get fulfilled.

If we would start with a relatively small radius, we would expect a large temperature gradient as we move in closer to the star's core. If the star's temperature gradient is large then we can expect a large convection zone for these initial radius values, specially when considering that  $\nabla_{ad}$  is not directly dependent of radius.

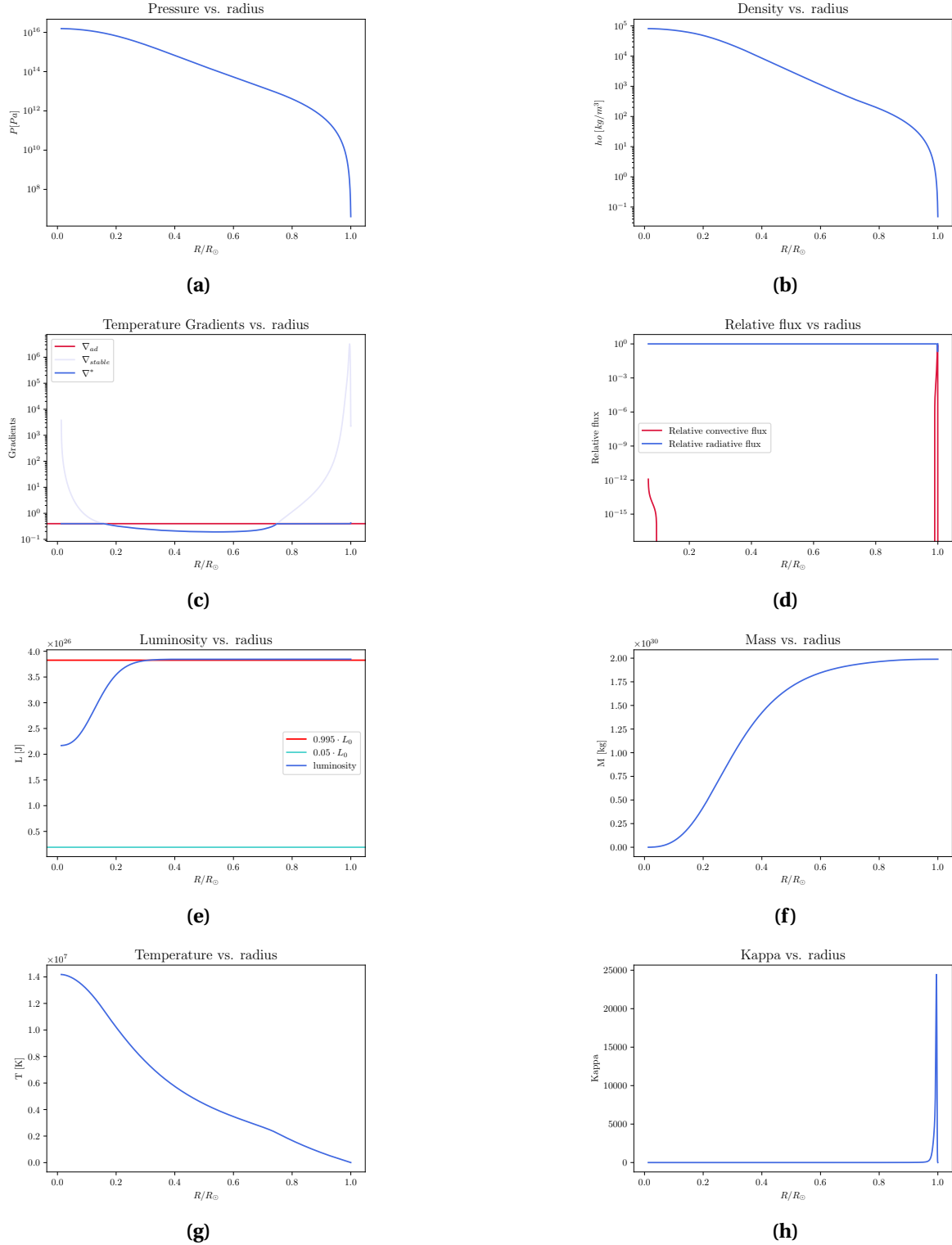
For a a wider convection zone in the star, we require lower temperatures as then, after experimenting, we notice deep convective envelopes. Great temperatures in the star give thinner convection zones and wide radiative zones. If we wanted to create a large outer convection zone in our final star model, we could either increase the initial density or decrease the initial temperature or increase the initial radius, or we could also have a combination of what has been said.

### 3.2 Best Fitted Model

With temperature and radius to be as close as possible to that of the sun, as our best model we choose the initial parameters:  $T_0 = 1.1T_\odot$   $R_0 = 0.85R_\odot$   $\rho = 3.38e^{-5}\rho_\odot$ . In figure 4 we can see the results of the star's cross section and evolution of stellar parameters that represent our best chosen model. Any of the parameters do not present any asymptotic behaviour.

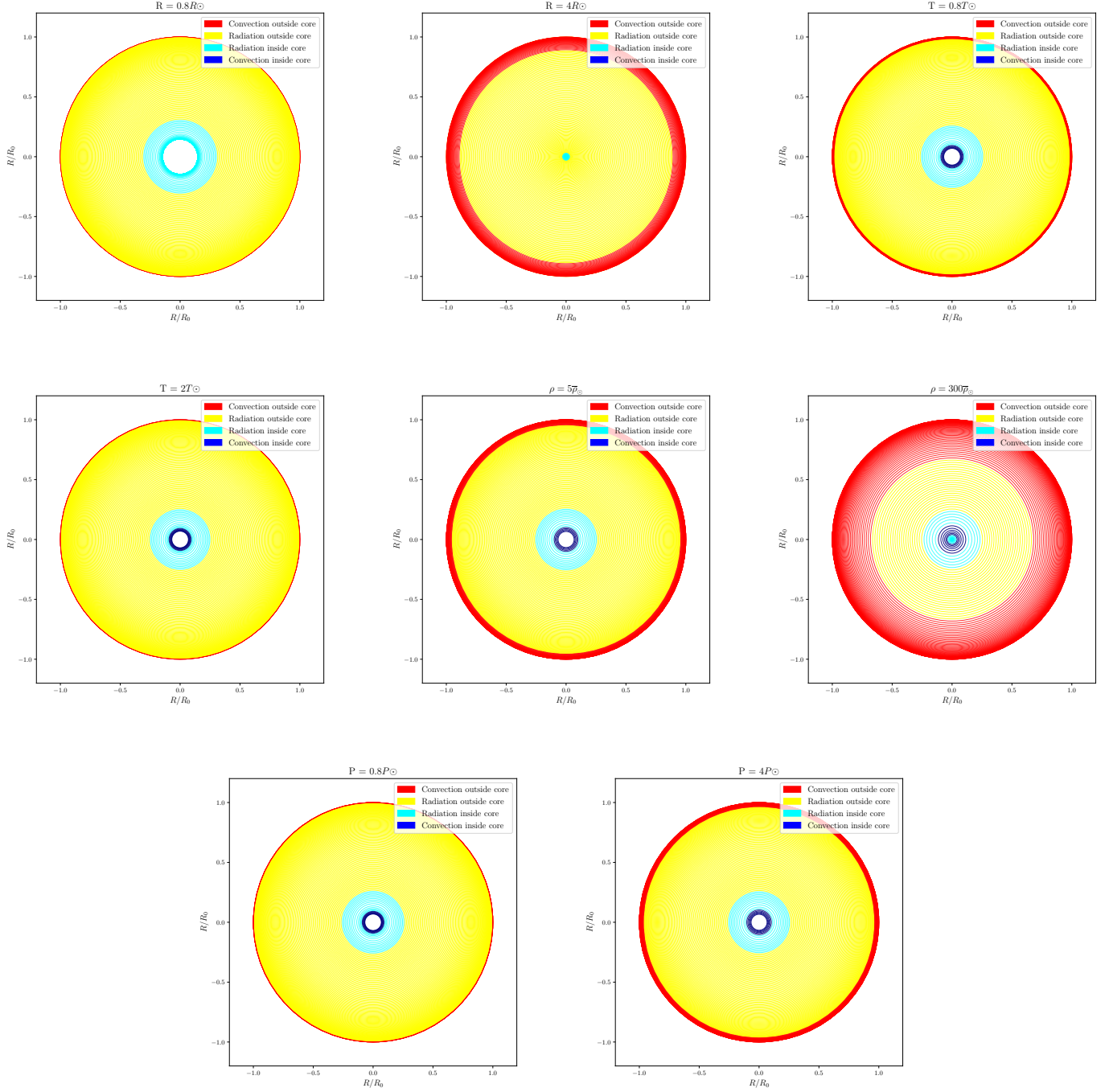
From the cross section's fractions of convective and radiative flux we conclude an outer convective zone

## Evolution of solar parameters



**Figure 2:** Plots of pressure (a) and density (b) in SI units. Plotted logarithmically as a function of radius of the star. We see that the radius does not reach zero and both pressure and temperature follow the same pattern and reach to a halt. Given on the star's parameters, there is a limit in how dense and how high the pressure of a star can get and therefore both parameters converge towards a value. The plot of the three temperature gradients (c) and the relative convective flux (d) as a function of radius. We observe an synchronized onset and decline of convection in both plots. In e we see the evolution of luminosity and the evolution of mass in f as a function of radius. The luminosity is almost constant before  $R/R_0 \approx 0.25$ , something that is in accordance with the temperature in this region -  $T$  is way too low for fusion to take place on a significant scale. We can observe that as the step-time grows mass decreases a lot faster. Since mass and density are dependent, as density increases substantially in the core so does mass decreases. We see the temperature in g and the opacity (kappa) in h as a function of radius in SI units. Towards the core of the star, the temperature increases rapidly. On the other hand, the opacity has a major spike while in the middle of the convective zone, there where the radiative flux drops. It resembles in shape that of the sun but the spike value is much higher.

### Stellar Cross-Sections



**Figure 3:** Cross sections of star models with modified initial parameters as a function of the relative radius. Experimentation with initial radii, temperature, density and pressure.

reaching down to approximately 65% of the initial radius, while the convective zone of the sun reaches down to about 70% of solar radius. But our value makes sense since our star has the same mass with the sun and about 12% bigger initial radius. From the experimentation we saw that a wider radius along with a higher density, like our case gives a little more wide convective zone. From the relative energy production plot, we see the total energy production divided by its maximum value and the relative energy production for each of the the PP branches for the entire temperature range. We know that PP chains are heavily temperature depended, with PP3 even more that PP2. Density and pressure follow the same pattern as

*Best Model-Cross section of the star*

Parameter	Set Parameter	Initial Value
Density	$240 \cdot \bar{\rho}_{\odot}$	4.7985e-02 [kg/m <sup>3</sup> ]
Luminosity	$1 \cdot L_{\odot}$	3.8460e+26 [W/m <sup>2</sup> ]
Mass	$1 \cdot M_{\odot}$	1.9891e+30 [kg]
Radius	$0.85 \cdot R_{\odot}$	5.9160e+08[m]
Temperature	$1.1 \cdot T_{\odot}$	6.3470e+03 [K]
Parameter	Final Value	
Density	8.152733e+04	
Luminosity	4.361633e+01	
Mass	9.999953e+01	
Radius	9.870551e+01	
Temperature	1.417976e+07	

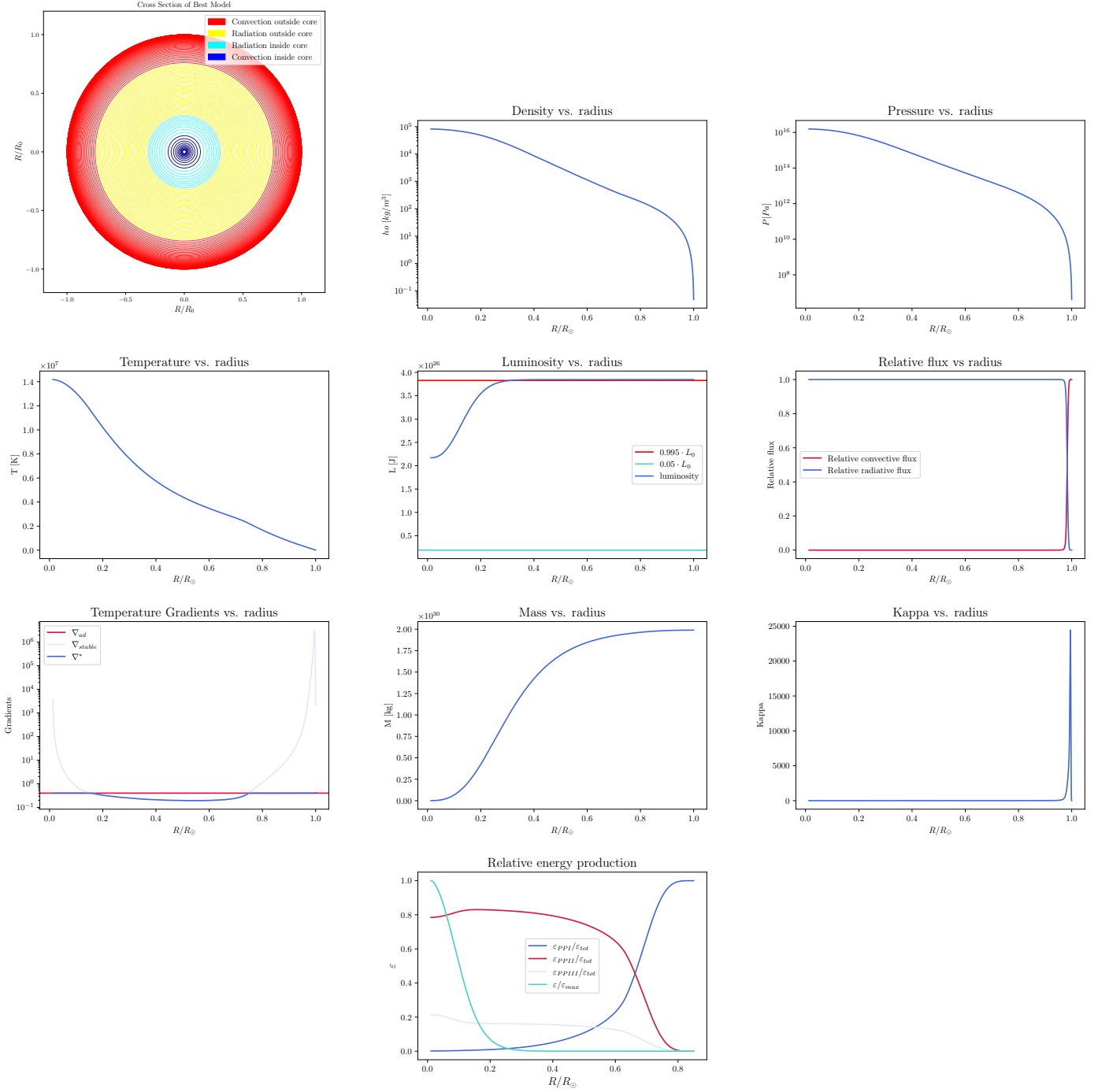
**Table 4:** Initial and final parameters of our chosen best model of the star.

expected, are both increasing rapidly at first and at the end near the center of the core. As expected, the temperature is exponentially increasing from the surface into the core. The luminosity seems consistent with that of the sun, it remains approximately constant until the outer limits of the core. As about mass, it decreases slowly at first and when approaching to the core it then decreases even more rapidly. We can see the effects of the instability criterion by looking at the relative flux and temperature as functions of relative radius. The lines should appear smoother but due to a numerical error they instead seem more steep before and after the intersection. Before  $\nabla_{stable}$  is greater than  $\nabla_{ad}$ , the convective flux is dominant. As approaching the end of the convective zone all energy transportation is made in radiative way. In reality, as seen in the cross-section convection and radiation will blend and especially at the borders between two zones. In our model we do not have a perfect situation, thus, we make it either convective or radiative in a given region. More specifically, in our model the convective flux becomes zero when the instability criterion does not hold anymore. The convective zone is defined where the flux is different than zero and therefore, this results to a discontinuity in the flux.

## 4 DISCUSSION AND CONCLUSION

In this project, we tried to model the central parts of a Sun-like star, including both radiative and convective energy transport. This required solving a set of differential equations in order to study how the different features of the core changes as we move through the star. Initially, we performed the linear 2D interpolation of the opacity values and implemented a sanity check for certain given values of  $\log T$  and  $\log R$  in order to check if Kappa values are correct. After some calculations in the code, we computed the temperature gradient for when the star is in a stable state. We also computed the temperature gradient for an adiabatic star. Unnecessary as it is, analytically, it found to be equal to 2/5 because of ideal gas, but it was a good check to see if the star behaves like the ideal case (which it does). Next, we calculated all required gradients( $dPdm$ ,  $dTdm$ ,  $drdm$ ,  $dLdm$ ). We used Euler's Method in our code. We created a subroutine that integrates the variables of our interest in a convective star. Next, we experimented with the cross section of the star and had some sanity checks. Varying the initial parameters was interesting.

## Stellar Model results



**Figure 4:** Cross sections of our best star model as a function of the relative radius.

Higher temperatures in the star give thinner convection zones and wide radiative zones. To create a large outer convection zone in our final star model, we could increase the initial density, decrease the initial temperature, increase the initial radius, or apply a combination of those. The project was pleasant to work on and I am rather satisfied with how it turned out. Our model is a decent representation of the energy transportation with some inconsistencies near the core. Those are result of our simplified model and numerical instability. In our model we have assumed that our star is governed by the equation of state of an ideal gas. Our star is one dimensional as we have assumed that our star evolves similarly towards the core from all directions but this is however insufficient as a model. In reality we would need three

dimensions to fully describe the star.

During working on this project, I learned a lot about how energy is transported from the central regions of a star towards its outer regions, about the radiative and convective transport. The temperature of the stellar interior controls the efficiency of the transportation of the energy and the energy production in the core. It is important to understand how the amount of energy is transported away from the stellar core in order to get a realistic model of a star. The efficiency of the transportation of the energy controls the temperature of the stellar interior and the efficiency of the energy production in the core. If the energy transport is extremely efficient, this would allow more energy from the core to be transferred towards the outer regions than the amount of energy produced and, thus, the core would eventually cool down, producing less energy. Both radiation and convection are effective, but it is the local temperature, pressure and chemical composition that decides which of the two dominates.

Combining all pieces of the information to solve the puzzle of modelling a star was challenging, as well as the in depth-understanding of the physics behind. Much extra work related to this project arose when trying to debug various problems. However, learning about how stars generate and transfer their energy was very enjoyable.

## **REFERENCES**

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