Dynamical Systems
Spring Semester 2020
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Exercise 1: Consider the non-linear dynamical system $\dot{x} = f(x) = \lambda + x - x^3, \lambda \in \mathbb{R}$.

i. Determine the fixed points of the system and identify their stability. If this procedure cannot be done analytically, it may be conducted computationally. Take into account the fact that in many cases graphical solutions provide additional insight even when a closed analytical solution exists.
ii. Provide the phase-space plot of the function $f(x)$ along with the associated potential function $V(x)$. Validate the results you obtained for the previous question by utilizing the acquired potential function.
iii. Identify the "interesting" values of the parameter λ where bifurcations occur.
iv. Implement a computational procedure that graphically represents the phase-space plot of the dynamical system for the various values of the parameter λ. You may use the available code samples in MATLAB that we covered during the lectures.
v. Implement a computational procedure that constructs the bifurcation diagram for the given dynamical system.

Exercise 2: Romeo is in love with Juliet, but in our version of this story, Juliet is a fickle lover. The more Romeo loves her, the more Juliet wants to run away and hide. But when Romeo gets discouraged and backs off, Juliet begins to find him strangely attractive. Romeo, on the other hand, tends to echo her: he warms up when she loves him, and grows cold when she hates him. Let

$$R(t) = Romeo's\ love\ or\ hate\ for\ Juliet\ at\ time\ t$$

$$J(t) = Juliet's\ love\ or\ hate\ for\ Romeo\ at\ time\ t$$

Positive values of $R(t)$ and $J(t)$ signify love while negative values signify hate. Then a model for their star-crossed romance may be given by the following linear system of first-order differential equations as:

$$\begin{aligned} \dot{R}(t) &= a \cdot J(t), \qquad a > 0 \\ \dot{J}(t) &= -b \cdot R(t), \qquad b > 0 \end{aligned}$$

i. Identify the fixed point of the dynamical system and characterize its stability.
ii. Given that at time $t = 0, R(0) = R_0$ and $J(0) = J_0$, predict the outcome of their love affair by determining the analytical solution for the system of differential equations.
iii. Can you specify the percentage of time for which they manage to achieve simultaneous love, that is the percentage of time where $R(t)$ and $J(t)$ are both positive?