

Lecture 5

topic: Regression Analysis

material: Chapters 6 (book "Data Mining for Business Intelligence")



Data Mining:

 Provides results useful for decision making

Summary from previous lectures

- Process involves multiple steps:
 - Business understanding → Data preparation →
 - Model building → Testing & Evaluation → Deployment
- Various options for the model:
 - Regression
 - Clustering
 - Decision trees
 - Association rules
 - etc.

Summary of previous lectures / Agenda

Date		Lecture contents	Lecturer	Lab topics	Test
Jan-27	1	Intro. to BI+ Data Management	Caron		
Jan-30				SQL-1	1
Jan-28	2	Data warehousing	Caron		
Feb-06				SQL-2	2
Feb-03	3	OLAP business databases & dashboard	Caron		
Feb-13				SQL-3 & OLAP	3a & 3b
Feb-10	4	Data mining introduction	loannou		
	5	Regression models	Ioannou		
Feb-17	6	Naïve Bayes	Ioannou		
	7	k nearest neighbors	loannou		
Feb-20				Bayes & neighbors	4
Feb-27	8	Performance measures	loannou		
Mar-02	9	Decision trees	loannou		
Mar-05				Dec. trees	5
Mar-09	10	Association rules	Ioannou		
Mar- 11,12&13				Ass. Rules	6
Mar-16	11	Clustering (+20 mins exam preparation)	loannou		
Mar-19				Clustering	7

Data Mining



Motivation

Data Mining:

Provides results useful for decision making

Examples:

- Store X wants to know the amount of water bottles to place in the fridge during June
- The tax office wants to detect if the family income relates to the overall monthly spending of families
- The university wants to find out if reducing the lab sessions will result in lower course grades
- The university wants to find out if the color of the student hair affects the final exam score

Variables

- Store X wants to know the amount of water bottles to place in the fridge each week
- Variables are any measurement on the records
- Dependent variables (denoted as Y):
 - The ones we want to explain / predict
 - Their value depend on something else
 - Example? amount of water bottles to place for June
- Independent variables (denoted as X):
 - The variables that explain / justify the dependent ones
 - Example? mean temperature from past years with the expected temperature, i.e., forecast; stock control information for the stock of a new store; etc.

Relationships

- Store X wants to know the amount of water bottles to place in the fridge each week
- Variables are any measurement on the records
- A relationship shows the variable association
- Examples?
 - Place more water bottles in the fridge during Summer
 - Special events near the store don't affect something
 - Etc.



Regression Analysis: fit a relationship between a numerical outcome variable and a set of predictors

- Numerical outcome variable Y
 also called response, target, or dependent variable
- Set of predictors X₁, X₂, ..., X_n also referred to as independent variables, input variables, regressors, or covariates

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Linear Regression Model

→ arranged in, or extending, along a straight or nearly straight line



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Single Linear Regression Model

→ one independent predictor, i.e., single variable X



Regression Analysis: fit a relationship between a numerical outcome variable and a set of predictors

- <u>Numerical outcome</u> variable Y also called response, target, or dependent variable
- Set of <u>numeric predictors</u> $X_1, X_2, ..., X_n$ also referred to as <u>independent variables</u>, input variables, regressors, or covariates

Single Linear Regression Model Multiple Linear Regression Model

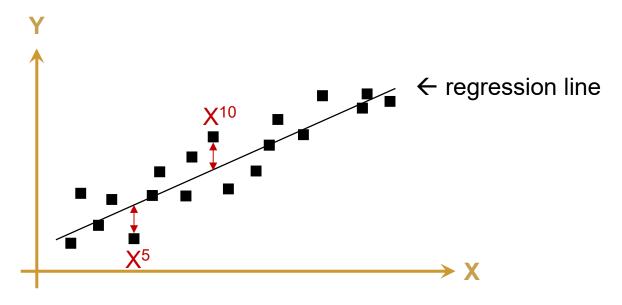
→ two or more predictors, i.e., X1, X2, ...



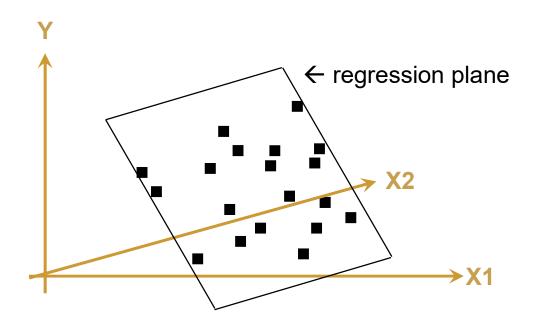
Single / Multiple Linear Regression

Intuition for Single Linear Regression

- Fitting a linear relationship (i.e., equation) between:
 - a numerical outcome (i.e., variable Y) and
 - one predictor (i.e., variable X)
- Not all points go through the line
 - → Error, i.e., deviation of data from line



Intuition for Multiple Linear Regression



- Fitting a linear relationship (i.e., equation) between:
 - o a numerical outcome (i.e., variable Y) and
 - two or more predictors (i.e., variables X1, X2, ...)



Linear Regression Model

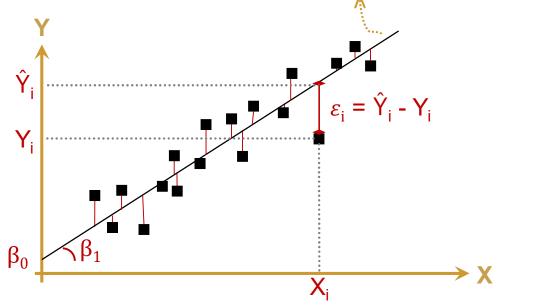
$$Y = \beta_0 + \beta_1 \cdot X_1 + \beta_2 \cdot X_2 + ... + \epsilon$$

- ε is a term that represents the errors associated with the model
- β_0 , β_1 , ... are the regression coefficients (parameters)
- Values for attribute X_1 are $X_{11}, X_{12}, ..., X_{1i}, ...$ where i is a "counter" representing the ith observation for the particular data collection



Graphical Visualization

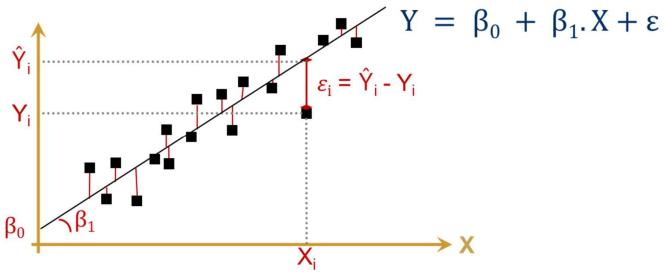
• Single Linear Regression: $Y = \beta_0 + \beta_1 \cdot X + \epsilon$



- β_0 the value of our dependent variable when the independent one is equal to zero, i.e., value of Y when X=0
- β_1 the slope of the regression line
- ε_i corresponds the difference between the observations and the predictions of our model, i.e., distance from the line

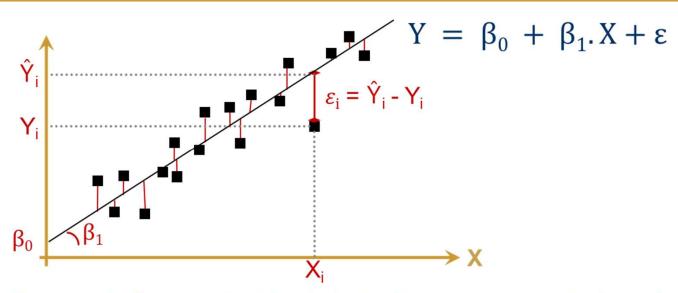


Ordinary Least Squares (OLS)



- Method for estimating the unknown parameters in a linear regression model
- It minimizes the errors associated with predicting values for the dependent variable Y
- It uses a least squares criterion because without square we would allow positive and negative deviations from the model to cancel each other out

Ordinary Least Squares (OLS)



- Find β_0 and β_1 such that total error is minimal
- This criterion is very common and for example also used in Neural Network models
- Error for Y_i is ε_i and the value of ε_i is equal to \hat{Y}_i Y_i

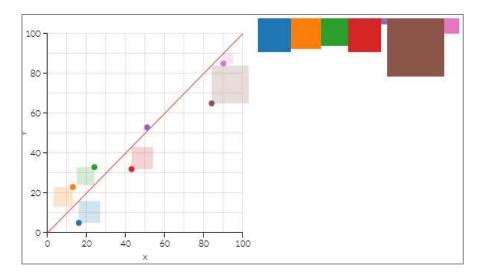
$$\rightarrow \min \sum_{i=1}^{n} (\varepsilon_i)^2 = \min \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

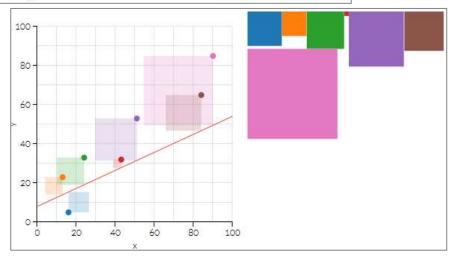
$$= \min \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 Xi)^2$$

Graphical Visualization

OLS: choose betas so that the total area of all the squares is as small as possible

$$\sum_{i=1}^{n} (\varepsilon_i)^2 = \min \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 X_i)^2$$





Plots taken from here:

http://setosa.io/ev/ordinary-least-squares-regression/

the site contains interactive plots to understand the concepts!

Explanatory vs. Predictive



Objectives for single/multiple regression

Two popular but different objectives behind fitting a regression model:

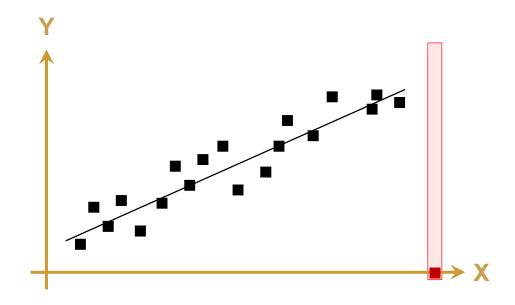
1. Predictive

 Detect the outcome value for new records, given their input values

2. Explanatory or descriptive

- Quantifying / explaining the average effect of inputs on an outcome
- Data are treated as a random sample from a larger population of interest

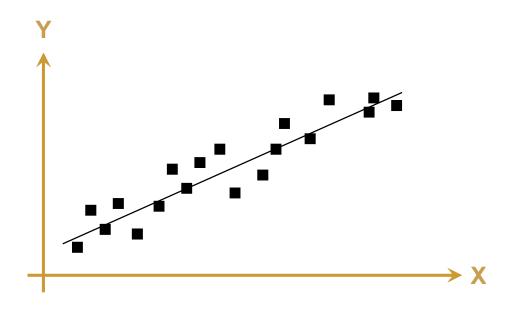
Predictive



• Given a new value for X, estimate the value for Y



Explanatory



Generate statements useful for decision making
 E.g., a unit increase in X is associated with
 an average increase of 2 points in Y

