UNIVERSITATEA TEHNICĂ
DIN CLUJ-NAPOCA

Control Engineering 2

Tuning digital PID controllers using “via s” methods

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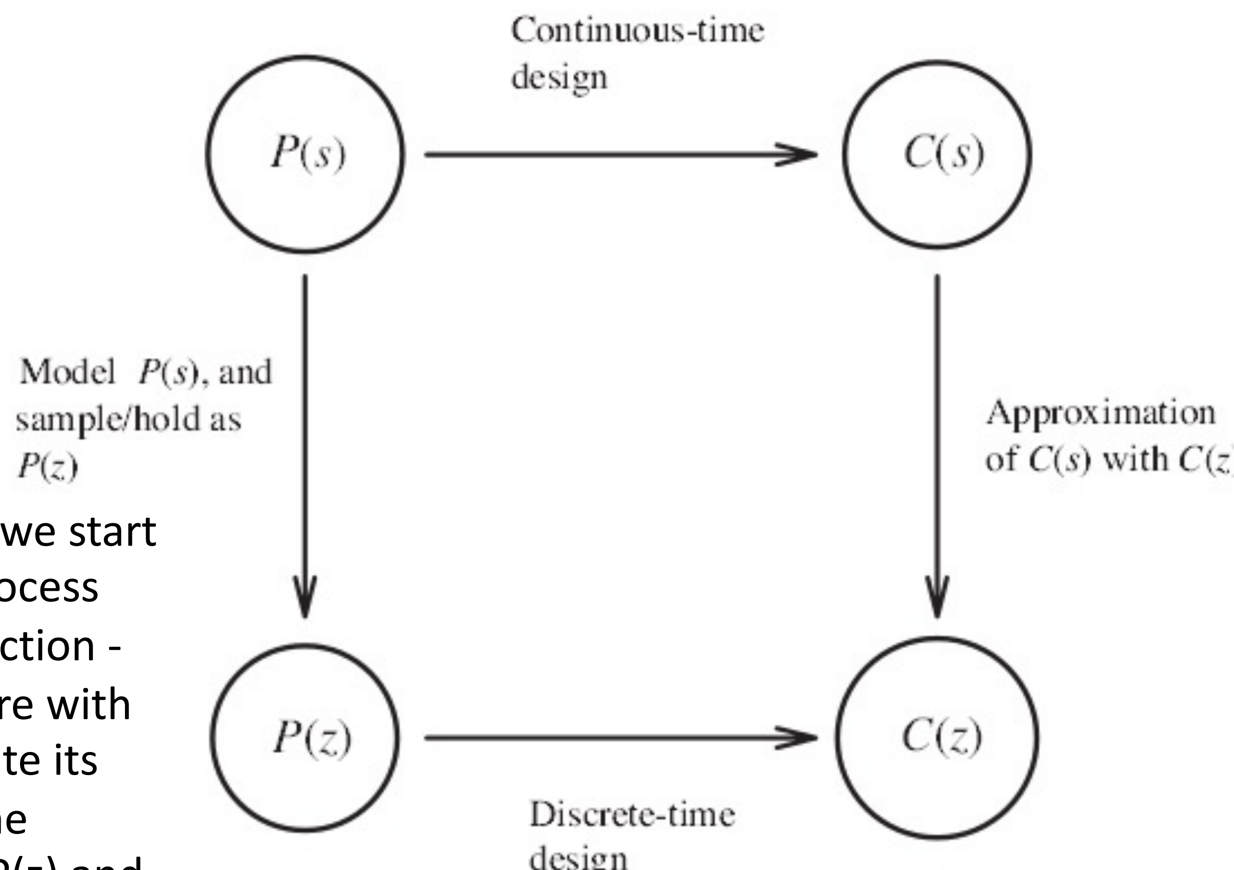
Outline and objectives

- Introduction: direct methods vs indirect methods
- Guillemin-Truxal example
- Tuning controllers using phase margin criteria and gain crossover frequency
  - Time domain and frequency domain equivalence
- Main Objectives:
  - review of indirect methods
  - importance of the sampling period in control loops
  - tuning of discrete time controllers using “via s” methods

Direct methods vs “via s” methods



Direct way: we start from the process transfer function - denoted here with P(s)- compute its discrete-time equivalent P(z) and then design the controller C(z)

Indirect way: using CE1 methods design first C(s) using P(s) and then compute the discrete-time equivalent of C(z)

Discrete-Time Controller Design Using Indirect Techniques

- Why are indirect tuning methods used?
  - Greater knowledge and experience in designing continuous-time than discrete-time controllers
  - Many practical systems already incorporate a continuous-time controller that we desire to replace with a discrete-time controller

Guillemin-Truxal example

- Given the process transfer function: $H_f(s) = \dfrac{10}{s(5s+1)}$

- Design a digital controller such that the closed loop system will exhibit 15% overshoot and 10 seconds settling time.

- Guillemin-Truxal: $H_0(s) = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$$\sigma = 15\% = 0.15 \Rightarrow \xi = 0.517 \qquad t_s = \frac{4}{\xi\omega_n} \Rightarrow \omega_n = \frac{4}{\xi t_s} = 0.7737$$

$$H_R(s) = \frac{1}{H_f(s)} \frac{H_0(s)}{1 - H_0(s)} = \frac{s(5s+1)}{10} \frac{0.5986}{s(s+0.8)} = 0.05986 \frac{(5s+1)}{(s+0.8)}$$

Controller transfer function

$$T_{min} = \frac{1}{0.8} = 1.25 \Rightarrow T_E \le \frac{T_{min}}{2} = 0.625$$

Sampling period! Shannon’s Theorem

Guillemin-Truxal example

$$H_R(s) = 0.05986\frac{(5s+1)}{(s+0.8)}$$

$$T_E = 0.5$$

$$H_R(z^{-1}) = \frac{0.2993 - 0.2746z^{-1}}{1 - 0.6703z^{-1}}$$

Guillemin-Truxal example

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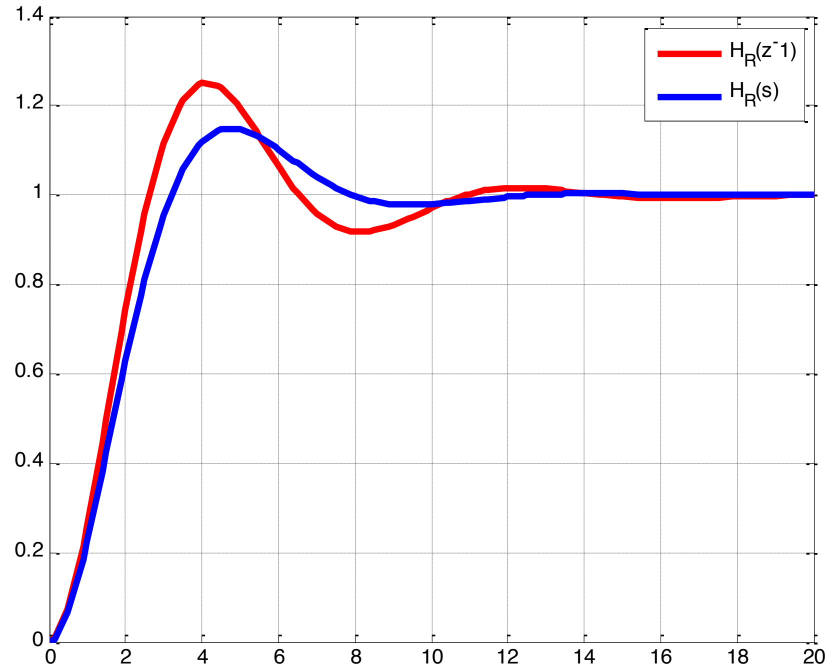

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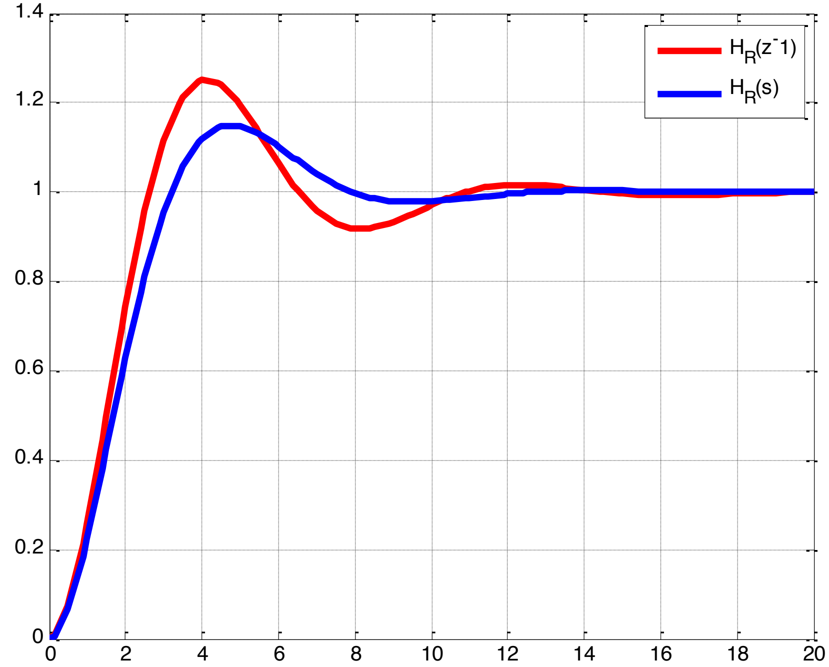

$$T_E = 0.1$$

$$H_R(z^{-1}) = \frac{0.2993 - 0.2935z^{-1}}{1 - 0.9231z^{-1}}$$

Guillemin-Truxal example

$$H_R(s) = 0.05986\frac{(5s+1)}{(s+0.8)}$$

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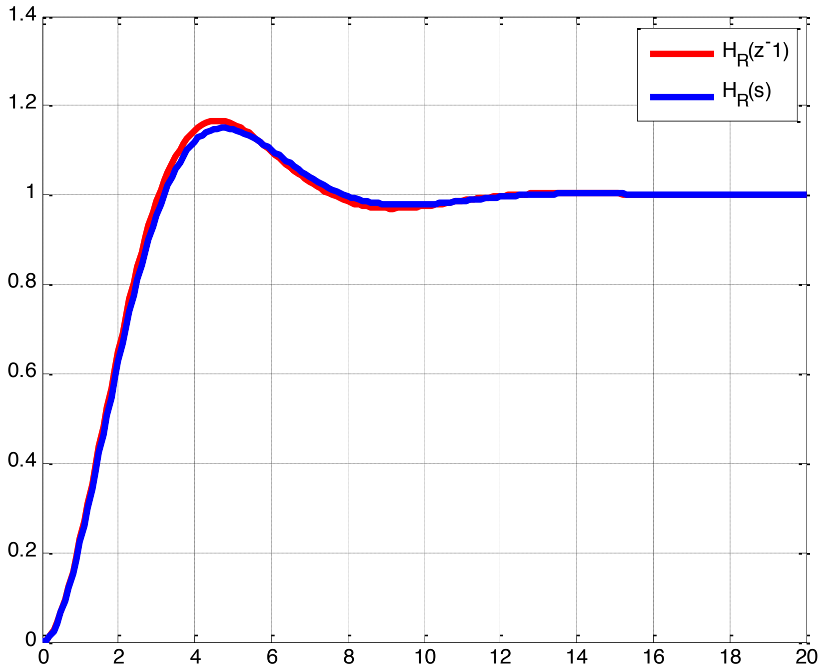

$$T_E = 0.1$$

$$H_R(z^{-1}) = \frac{0.2993 - 0.2935z^{-1}}{1 - 0.9231z^{-1}}$$

Observe the results with a smaller sampling period

Tuning controllers using phase margin criteria and gain crossover frequency

- Time domain performance specifications:
  - Settling time
  - Overshoot
  - Steady state errors
  - Peak time
  - Rise time
- Frequency domain performance specifications:
  - Phase margin
  - Gain crossover frequency

Tuning controllers using phase margin criteria and gain crossover frequency

- Imposing phase margin = ensuring a certain overshoot

$$\gamma_k = \arccos\left(\frac{1}{2\xi^2 + \sqrt{1 + 4\xi^4}}\right) \Rightarrow \xi \Rightarrow \sigma = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$$

- Imposing gain crossover frequency = ensuring a certain settling time

$$\omega_c \Rightarrow t_s = \frac{2}{\xi^2 \omega_c}$$

Tuning controllers using phase margin criteria and gain crossover frequency

- Designing a PI controller such that the open loop system has a certain phase margin and cutting frequency:
- $H_c(s) = k_p\left(1 + \frac{1}{T_i s}\right)$
- $H_{OL}(s) = H_c(s)H_p(s)$

- Phase equation: $\angle H_{OL}(j\omega_c) = -180^o + \gamma_k$

Imposed phase margin

- Modulus equation: $|H_{OL}(j\omega_c)| = 1$

Imposed cutting frequency

Tuning controllers using phase margin criteria and gain crossover frequency

- Phase equation: $\angle H_{OL}(j\omega_c) = -180^o + \gamma_k$
- $\angle H_c(j\omega_c) + \angle H_p(j\omega_c) = -180^o + \gamma_k$
- $\angle H_c(j\omega_c) = -180^o + \gamma_k - \angle H_p(j\omega_c)$

In this case the cutting frequency is imposed. The process phase can be easily computed, either analitically or using Matlab Bode plots. Then, the right hand side of the phase equation above can be easily determined.

- Controller phase:
- $\angle H_c(j\omega_c) = \angle k_p + \angle(1 + jT_i\omega_c) - \angle jT_i\omega_c = \tan^{-1}\left(\frac{T_i\omega_c}{1}\right) - 90^o$
- $\angle H_c(j\omega_c) = \tan^{-1}(T_i\omega_c) - 90^o$

Tuning controllers using phase margin criteria and gain crossover frequency

- Then replace the general phase equation of the controller:

$$\angle H_c(j\omega_c) = \tan^{-1}(T_i\omega_c) - 90^o$$

into

$$\angle H_c(j\omega_c) = -180^o + \gamma_k - \angle H_p(j\omega_c)$$

- It follows that:

$$\tan^{-1}(T_i\omega_c) - 90^o = -180^o + \gamma_k - \angle H_p(j\omega_c)$$

- Apply the tangent function:

$$T_i\omega_c = \tan\left(-90^o + \gamma_k - \angle H_p(j\omega_c)\right)$$

- The integral time constant is then:

$$T_i = \frac{\tan\left(-90^o + \gamma_k - \angle H_p(j\omega_c)\right)}{\omega_c}$$

Tuning controllers using phase margin criteria and gain crossover frequency

- Use now the modulus equation to determine the proportional gain:

$$|H_{OL}(j\omega_c)| = 1$$

or

$$|H_c(j\omega_c)|\left|H_p(j\omega_c)\right|=1$$

In this case the cutting frequency is imposed (so known!). The process modulus can be easily computed, either analitically or using Matlab Bode plots.

- What is the controller modulus equation, for the general case?

$$|H_c(j\omega_c)| = k_p \frac{|1 + jT_i\omega_c|}{|jT_i\omega_c|} = k_p \frac{\sqrt{T_i^2\omega_c^2 + 1}}{T_i\omega_c}$$

Tuning controllers using phase margin criteria and gain crossover frequency

- Replace then the general equation for the PI controller modulus into the modulus condition:

$$|H_c(j\omega_c)|\left|H_p(j\omega_c)\right|=1$$

yielding

$$k_p \frac{\sqrt{T_i^2\omega_c^2+1}}{T_i\omega_c}\left|H_p(j\omega_c)\right| = 1$$

$$k_p = \frac{T_i\omega_c}{\left|H_p(j\omega_c)\right|\sqrt{T_i^2\omega_c^2+1}}$$

Tuning controllers using phase margin criteria and gain crossover frequency

- Tuning example:
  - For the process described by the following transfer function

$$H_f(s) = \frac{10}{(5s+1)(3s+1)}$$

design a PI controller such that the open loop system has a cutting frequency $\omega_c = 0.1$ rad/s and a phase margin $\gamma_k = 60^\circ$. Compute the discrete time equivalent of the previously designed PI controller.

Tuning example PI controller

- Process modulus at the cutting frequency:

$$\left|H_p(j\omega_c)\right| = \frac{10}{\sqrt{25\omega_c^2+1}\sqrt{9\omega_c^2+1}} = 8.57$$

- Process phase at the cutting frequency:

$$\angle H_p(j\omega_c) = -\tan^{-1}(5\omega_c) - \tan^{-1}(3\omega_c)$$
$$\angle H_p(j\omega_c) = -43.26^o$$

- The integral time constant is then:

$$T_i = \frac{\tan\left(-90^o + \gamma_k - \angle H_p(j\omega_c)\right)}{\omega_c}$$
$$T_i = \frac{\tan(-90^o + 60^o + 43.26^o)}{0.1} = 2.35$$

- The proportional gain is:

$$k_p = \frac{T_i\omega_c}{\left|H_p(j\omega_c)\right|\sqrt{T_i^2\omega_c^2+1}}$$
$$k_p = 0.0267$$

Tuning example PI controller

- The resulting PI controller transfer function is:

$$H_c(s) = 0.0267\left(1 + \frac{1}{2.35s}\right)$$

- Determining the discrete-time PI controller:
  - We will use Tustin $s = \frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}}$
  - PI controller time constant: 2.35 (just one! The integral time constant!)
  - Process time constants: 5 and 3
  - Shannon: $T_s \leq \frac{2.35}{2}$(sampling time). Select T_s=0.5
  - Discrete-time transfer function obtained as:

$$H_c(z^{-1}) = 0.0267\left(1 + \frac{1}{2.35 \cdot \frac{2}{0.5}\frac{1-z^{-1}}{1+z^{-1}}}\right)$$

$$H_c(z^{-1}) = \frac{0.02954 - 0.02386z^{-1}}{1-z^{-1}}$$

Tuning example PI controller

- Recurrent relation for the control signal

$$H_c(z^{-1}) = \frac{0.02954 - 0.02386z^{-1}}{1 - z^{-1}} = \frac{u(z^{-1})}{\varepsilon(z^{-1})}$$

$$\mathrm{u}(z^{-1})\,(1 - z^{-1}) = \varepsilon(z^{-1})(0.02954 - 0.02386z^{-1})$$

$$\mathrm{u}(k) - u\,(k-1) = 0.02954\varepsilon(k) - 0.02368\varepsilon(k-1)$$

$$\mathrm{u}(k) = u\,(k-1) + 0.02954\varepsilon(k) - 0.02368\varepsilon(k-1)$$

Tuning controllers using phase margin criteria and gain crossover frequency

- Designing a PD controller such that the open loop system has a certain phase margin and cutting frequency:
- $H_c(s) = k_p\left(\frac{1+T_d s}{1+\beta T_d s}\right)$
- $H_{OL}(s) = H_c(s)H_p(s)$

- Phase equation: $\angle H_{OL}(j\omega_c) = -180^o + \gamma_k$

Imposed phase margin

- Modulus equation: $|H_{OL}(j\omega_c)| = 1$

Imposed cutting frequency

Tuning controllers using phase margin criteria and gain crossover frequency

- Phase equation: $$\angle H_{OL}(j\omega_c) = -180^o + \gamma_k$$
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In this case the cutting frequency is imposed. The process phase can be easily computed, either analitically or using Matlab Bode plots. Then, the right hand side of the phase equation above can be easily determined.

- Controller phase:

$$\angle H_c(j\omega_c) = \angle k_p + \angle(1 + jT_d\omega_c) - \angle 1 + j\beta T_d\omega_c$$
$$\angle H_c(j\omega_c) = \tan^{-1}(T_d\omega_c) - \tan^{-1}(\beta T_d\omega_c)$$
$$\angle H_c(j\omega_c) = \tan^{-1}\left(\frac{T_d\omega_c - \beta T_d\omega_c}{1 + \beta T_d^2\omega_c^2}\right)$$

Tuning controllers using phase margin criteria and gain crossover frequency

- Then replace the general phase equation of the controller:

$$\angle H_c(j\omega_c) = \tan^{-1}\left(\frac{T_d\omega_c - \beta T_d\omega_c}{1 + \beta T_d^2\omega_c^2}\right)$$

into

$$\angle H_c(j\omega_c) = -180^o + \gamma_k - \angle H_p(j\omega_c)$$

- It follows that:

$$\tan^{-1}\left(\frac{T_d\omega_c - \beta T_d\omega_c}{1 + \beta T_d^2\omega_c^2}\right) = -180^o + \gamma_k - \angle H_p(j\omega_c)$$

- Apply the tangent function:

$$\frac{T_d\omega_c - \beta T_d\omega_c}{1 + \beta T_d^2\omega_c^2} = \tan\left(-180^o + \gamma_k - \angle H_p(j\omega_c)\right)$$

- Let's rewrite this:

$$\frac{T_d\omega_c - \beta T_d\omega_c}{1 + \beta T_d^2\omega_c^2} = x$$

Tuning controllers using phase margin criteria and gain crossover frequency

- Solve this equation with T_d the unknown parameter

$$\frac{T_d\omega_c - \beta T_d\omega_c}{1 + \beta T_d^2\omega_c^2} = x$$

Then:

$$\beta T_d^2\omega_c^2 x + T_d(\omega_c - \beta\omega_c) + x = 0$$

- The derivative time constant is:

$$T_{d1,2} = \frac{1 - \beta \pm \sqrt{(1-\beta)^2 - 4\beta x^2}}{2\beta\omega_c x}$$

Select $\beta \in [0.1 - 0.125]$

Tuning controllers using phase margin criteria and gain crossover frequency

- Use now the modulus equation to determine the proportional gain:

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or

$$|H_c(j\omega_c)|\left|H_p(j\omega_c)\right|=1$$

In this case the cutting frequency is imposed (so known!). The process modulus can be easily computed, either analitically or using Matlab Bode plots.

- What is the controller modulus equation, for the general case?

$$|H_c(j\omega_c)| = k_p \frac{|1 + jT_d\omega_c|}{|1 + j\beta T_d\omega_c|} = k_p \frac{\sqrt{T_d^2\omega_c^2 + 1}}{\sqrt{\beta^2 T_d^2\omega_c^2 + 1}}$$

Tuning controllers using phase margin criteria and gain crossover frequency

- Replace then the general equation for the PI controller modulus into the modulus condition:

$$|H_c(j\omega_c)|\left|H_p(j\omega_c)\right|=1$$

yielding

$$k_p \frac{\sqrt{T_d^2\omega_c^2+1}}{\sqrt{\beta^2 T_d^2\omega_c^2+1}}\left|H_p(j\omega_c)\right| = 1$$

$$k_p = \frac{\sqrt{\beta^2 T_d^2\omega_c^2+1}}{\left|H_p(j\omega_c)\right|\sqrt{T_d^2\omega_c^2+1}}$$

At the end of the lecture

- You should be able to…
  - Design discrete-time controllers using “via s” methods
    - Time domain tuning
    - Frequency domain tuning
  - Choose the sampling period in order to meet performance specifications