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Control Engineering 2

Digital controller design based on Dahlin method

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Outline and Objectives

- Dahlin's algorithm for direct synthesis method
- Illustrative example
- An analysis of ringing:
  - methods for removing the ringing pole
  - Vogel-Edgar algorithm
- Main objectives:
  - Discrete-time controller tuning using Dahlin's algorithm
  - Evaluate the closed loop performance
  - Problems associated with direct tuning methods
  - Comparisons with different direct tuning algorithms

Dahlin's algorithm

- Direct method to compute discrete-time controllers
- The dead-beat controller is very sensitive to plant characteristics, may lead to an oscillatory response or ringing
- Analytical design method that consists in a modification of the dead-beat algorithm
- Advantage of the Dahlin controller: produces an exponential response which is smoother than that of the dead-beat controller
- Disadvantage: less demanding in terms of closed loop performance when compared to the dead-beat algorithm
- First step consists in the discretization of the process transfer function

Dahlin's algorithm

- If the process has a time delay equal to τ_m seconds, then the closed loop tf will have the exact same time delay:

$$H_0(s) = \frac{e^{-\tau_m s}}{\lambda s + 1}$$

- Considering a sampling period of T_s seconds and using ZOH, the discrete form of the closed loop transfer function will be:

$$H_0(z^{-1}) = \frac{(1 - e^{-T_s/\lambda})z^{-N-1}}{1 - e^{-T_s/\lambda} z^{-1}}$$

N is time delay in discrete-time

Dahlin's algorithm

$$H_0(s) = \frac{1}{\lambda s+1} e^{-\tau_m s}$$

$$H_0(z^{-1}) = \mathcal{Z}\left\{\mathcal{L}^{-1}\left\{ \underbrace{\frac{1-e^{-T_s \cdot s}}{s}}_{ZOH} \cdot H_0(s)\right\}\right\}$$

$$H_0(z^{-1}) = (1-z^{-1}) \cdot \mathcal{Z}\left\{\mathcal{L}^{-1}\left\{\frac{1}{s(\lambda s+1)} e^{-\tau_m s}\right\}\right\}$$

Assume $N = \frac{\tau_m}{T_s} \Rightarrow e^{-\tau_m s} = z^{-N}$

$$\frac{1/\lambda}{s(s+\frac{1}{\lambda})} = \frac{A}{s} + \frac{B}{s+\frac{1}{\lambda}} \Rightarrow As + \frac{1}{\lambda}A + Bs = \frac{1}{\lambda}$$

$$s(A+B) + \frac{1}{\lambda} \cdot A = \frac{1}{\lambda} \Rightarrow$$

$$\Rightarrow A \cdot \frac{1}{\lambda} = \frac{1}{\lambda} \Rightarrow A = 1$$

$$A+B=0 \Rightarrow B=-A \Rightarrow B=-1$$

Dahlin's algorithm

$$H_c(z^{-1}) = (1-z^{-1}) \cdot z^{-N} \cdot \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{ \frac{A}{s} + \frac{B}{s+\frac{1}{\lambda}} \right\}\right\}$$

$$= (1-z^{-1}) \cdot z^{-N}\, \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{ \frac{1}{s} - \frac{1}{s+\frac{1}{\lambda}} \right\}\right\}$$

$$= (1-z^{-1}) \cdot z^{-N} \cdot \left\{ \frac{1}{1-z^{-1}} - \frac{1}{1-e^{-\frac{1}{\lambda}\cdot T_s} \cdot z^{-1}} \right\}$$

$$= \cancel{(1-z^{-1})} \cdot z^{-N} \cdot \frac{\cancel{1} - e^{-T_s/\lambda} \cdot z^{-1} - \cancel{1} + z^{-1}}{\cancel{(1-z^{-1})}\left(1-e^{-T_s/\lambda} \cdot z^{-1}\right)}$$

$$H_c(z^{-1}) = \frac{z^{-1}\left(1-e^{-T_s/\lambda}\right)}{\left(1-z^{-1} \cdot e^{-T_s/\lambda}\right)} z^{-N} \Rightarrow \boxed{H_c(z^{-1}) = \frac{\left(1-e^{-T_s/\lambda}\right) z^{-N-1}}{1-e^{-T_s/\lambda}\, z^{-1}}}$$

Dahlin’s algorithm

- Tuning approach:
  - Select the closed loop time delay (based on process time delay)
  - Select the closed loop time constant (according to performance specifications)
  - Obtain the discrete time transfer function of the process
  - Compute the discrete time transfer function for the closed loop system
  - Compute the digital controller transfer function

$$H_0(z^{-1}) = \frac{H_f(z^{-1})H_c(z^{-1})}{1 + H_f(z^{-1})H_c(z^{-1})}$$

$$H_c(z^{-1}) = \frac{1}{H_f(z^{-1})} \frac{H_0(z^{-1})}{1 - H_0(z^{-1})}$$

$$H_0(z^{-1}) = \frac{(1 - e^{-T_s/\lambda})z^{-N-1}}{1 - e^{-T_s/\lambda}z^{-1}}$$

  - For all N, an integrator will be present at the denominator of the controller tf:

$$H_c(z^{-1}) = \frac{1}{H_f(z^{-1})} \frac{(1 - e^{-T_s/\lambda})z^{-N-1}}{1 - e^{-T_s/\lambda}z^{-1} - (1 - e^{-T_s/\lambda})z^{-N-1}}$$

Dahlin's algorithm

- Tuning approach:
  - Select the closed loop time delay (based on process time delay)
  - Select the closed loop time constant (according to performance specifications)

$$H_0(s) = \frac{e^{-\tau_m s}}{\lambda s + 1}$$

  - Remark:
    - Small values for λ lead to small settling time and produce tight control
    - Large values for λ lead to larger settling time (more sluggish control), but loosened control effort
    - For λ->0, Dahlin's algorithm is equivalent to dead-beat

Dahlin's algorithm

- Example: For the process given by the transfer function:

$$H_f(s) = \frac{5}{10s+1} e^{-2s}$$

- Design a controller using Dahlin's algorithm that ensures a settling time of 22 seconds.
- Design a controller using Dahlin's algorithm that ensures a settling time of 12 seconds.

- Solution?

$$H_f(s) = \frac{5}{10s+1} e^{-2s}$$

$$T_s = 1 \Rightarrow H_f(z^{-1}) = z^{-2} \frac{0{,}4758 z^{-1}}{1-0{,}9048 z^{-1}} = \frac{0{,}4758 z^{-3}}{1-0{,}9048 z^{-1}}$$

$$H_0(z^{-1}) = \frac{1-e^{-T_s/\lambda}}{1-e^{-T_s/\lambda} z^{-1}} \cdot z^{-N-1}$$

Since $H_0(s) = \frac{1}{\lambda s+1} e^{-\tau_m s} \Rightarrow N = \frac{\tau_m}{T_s} = \frac{2}{1} = 2$

$t_s^* = 22 = t_s + \tau_m = 4\lambda + \tau_m = 4\lambda + 2 \Rightarrow t_s = 4\lambda = 22-2 = 20$

- $t_s = 20$ seconds $\Rightarrow \lambda = \frac{20}{4} = 5$ seconds

$$\lambda = 5 \Rightarrow H_0(s) = \frac{1}{5s+1} e^{-2s} \Rightarrow H_0(z^{-1}) = \frac{1-e^{-1/5}}{1-e^{-1/5} z^{-1}} z^{-2-1} \Rightarrow$$

$$\Rightarrow H_0(z^{-1}) = \frac{0{,}1813 z^{-3}}{1-0{,}8187 z^{-1}}$$

$$H_R(z^{-1}) = \frac{1}{H_f(z^{-1})} \cdot \frac{H_0(z^{-1})}{1-H_0(z^{-1})} = \frac{1-0{,}9048 z^{-1}}{0{,}4758 z^{-3}} \cdot \frac{\frac{0{,}1813 z^{-3}}{1-0{,}8187 z^{-1}}}{1-\frac{0{,}1813 z^{-3}}{1-0{,}8187 z^{-1}}}$$

$$H_R(z^{-1}) = \frac{1-0{,}9048 z^{-1}}{0{,}4758 \cancel{z^{-3}}} \frac{0{,}1813 \cancel{z^{-3}}}{1-0{,}8187 z^{-1} - 0{,}1813 z^{-3}}$$

$$H_R(z^{-1}) = \frac{0{,}3810(1-0{,}9048 z^{-1})}{1-0{,}8187 z^{-1} - 0{,}1813 z^{-3}} \Rightarrow c(0) = 0{,}3810$$

$t_s^* = 12 \Rightarrow 4\lambda = 12 - T_m = 10$

- $t_s = 10$ seconds $\Rightarrow \lambda = \frac{10}{4} = 2{,}5$

$$H_0(z^{-1}) = \frac{1-e^{-1/2{,}5}}{1-e^{-1/2{,}5}z^{-1}} \cdot z^{-3} = \frac{0{,}3297\, z^{-3}}{1-6703\, z^{-1}}$$

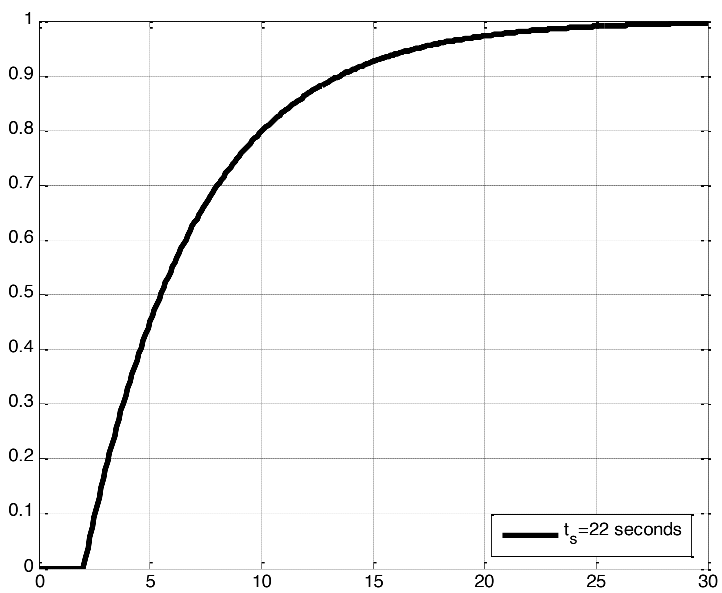
$$H_R(z^{-1}) = \frac{1-0{,}9048\, z^{-1}}{0{,}4758\, \cancel{z^{-3}}} \cdot \frac{0{,}3297\, \cancel{z^{-3}}}{1-0{,}6703\, z^{-1} - 0{,}3297\, z^{-3}} \Rightarrow$$

$$H_R(z^{-1}) = \frac{0{,}6929(1-0{,}9048\, z^{-1})}{1-0{,}6703\, z^{-1} - 0{,}3297\, z^{-3}} \Rightarrow C(0) = 0{,}6929$$

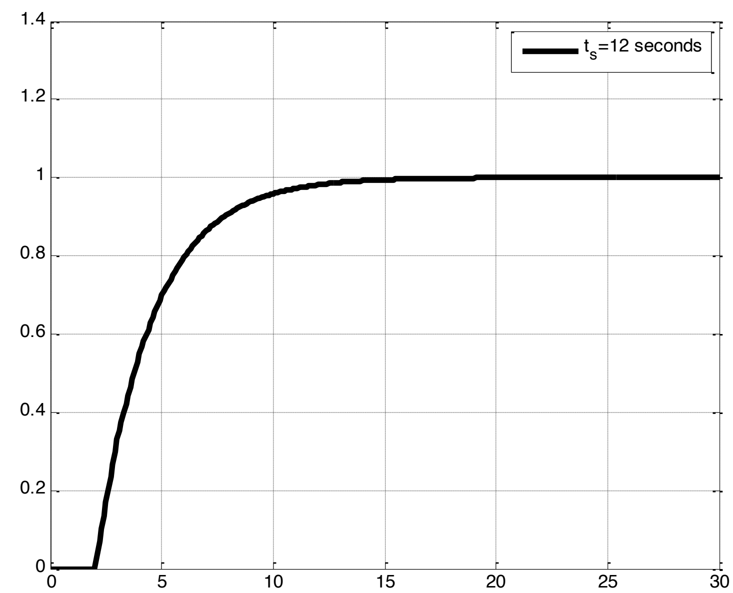
Dahlin's algorithm

Closed loop performance comparison

Settling time of 22 seconds



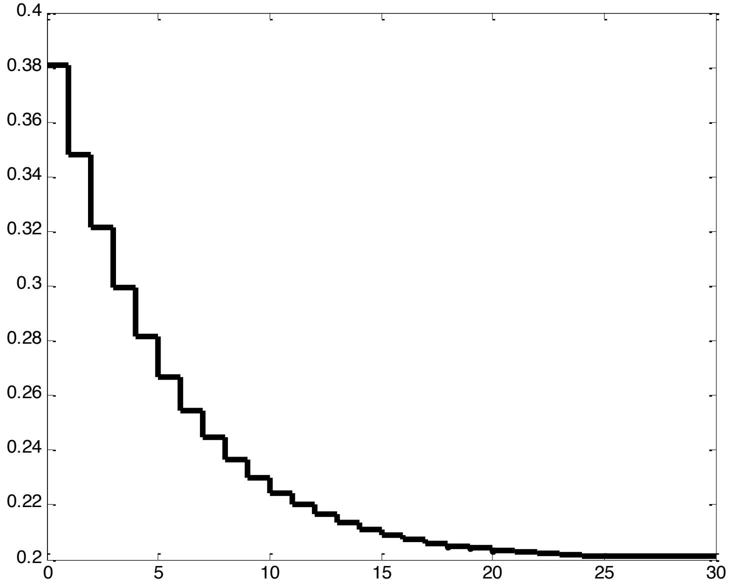
Settling time of 12 seconds



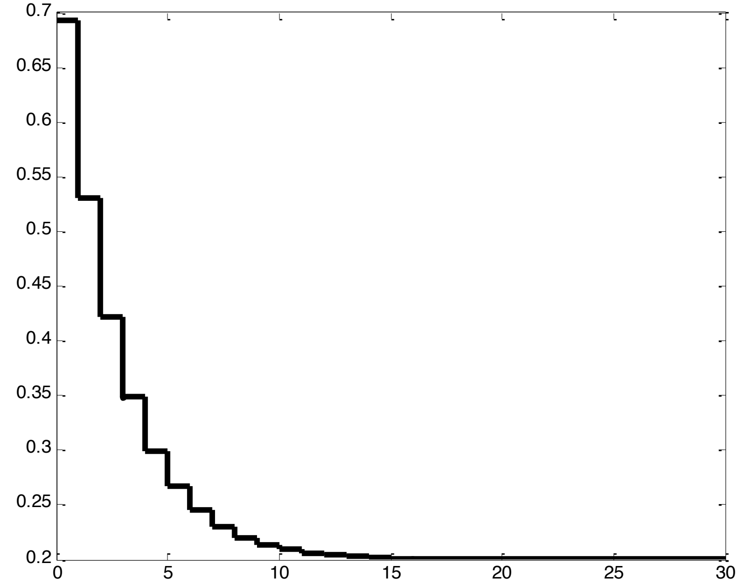
Dahlin's algorithm

Closed loop performance comparison

Settling time of 22 seconds



Settling time of 12 seconds



An analysis of ringing

- For the process given by the transfer function:

$$H_f(s) = \frac{1}{(5s+1)(3s+1)}$$

- Design a controller using Dahlin's algorithm that ensures a settling time equal to 8 seconds
- Compute the first 3 values of the control signal

- The controller transfer function is...?

An analysis of ringing

$$H_f(s) = \frac{1}{(5s+1)(3s+1)}$$

$$H_0(s) = \frac{1}{\lambda s+1} e^{-\tau_m s}$$

$$\left.\begin{array}{l} \tau_m = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} H_0(s) = \dfrac{1}{\lambda s+1} \\ t_s = 8 \text{ seconds} \end{array}\right\} \Rightarrow H_0(s) = \frac{1}{2s+1}$$

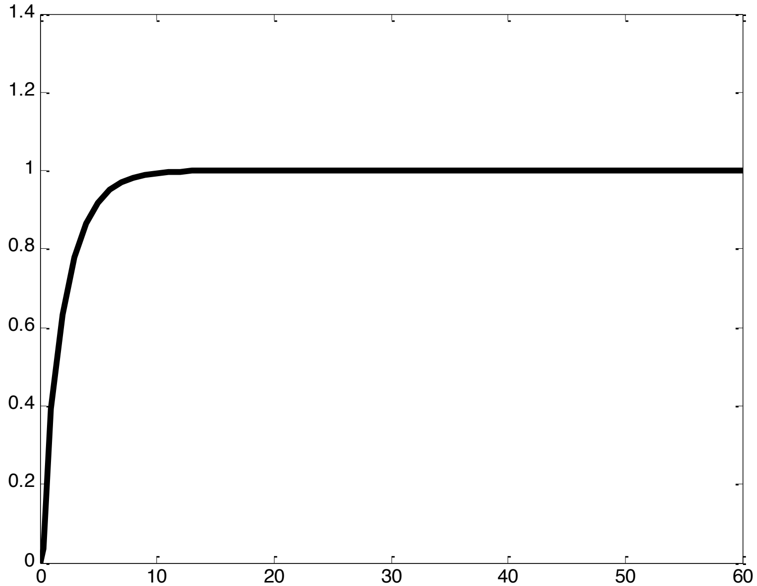
$$T_s = 1$$

$$H_f(z^{-1}) = \frac{0{,}02797\, z^{-1} + 0{,}02341\, z^{-2}}{1 - 1{,}535\, z^{-1} + 0{,}5866\, z^{-2}} \qquad H_0(z^{-1}) = \frac{0{,}3935 \cdot z^{-1}}{1 - 0{,}6065\, z^{-1}}$$

$$H_R(z^{-1}) = \frac{1 - 1{,}535\, z^{-1} + 0{,}5866\, z^{-2}}{0{,}02797\, z^{-1} + 0{,}02341\, z^{-2}} \cdot \frac{0{,}3935\, z^{-1}}{1 - 0{,}6065\, z^{-1} - 0{,}3935\, z^{-1}}$$

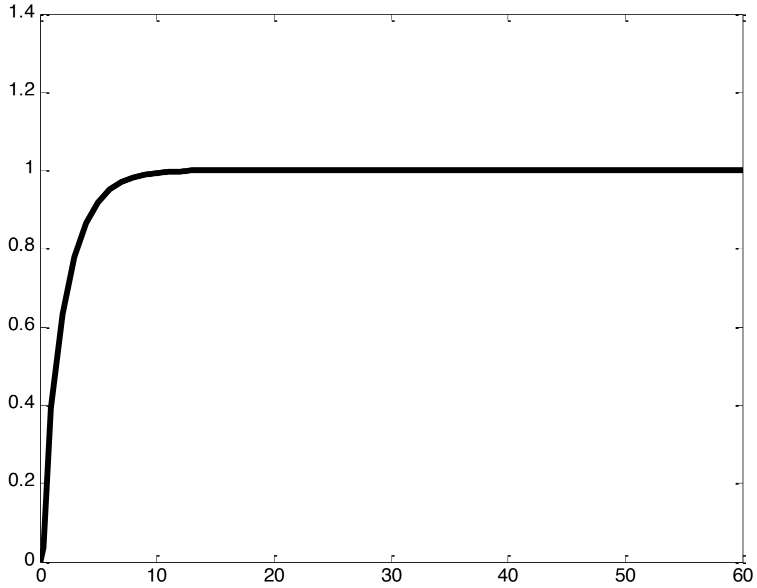
$$\boxed{H_R(z^{-1}) = \frac{0{,}3935\,(1 - 1{,}535\, z^{-1} + 0{,}5866\, z^{-2})}{(0{,}02797 + 0{,}02341\, z^{-1})(1 - z^{-1})}} = \frac{0{,}3935\,(1 - 1{,}535\, z^{-1} + 0{,}5866\, z^{-2})}{0{,}02797 - 0{,}00456\, z^{-1} - 0{,}02341\, z^{-2}}$$

An analysis of ringing

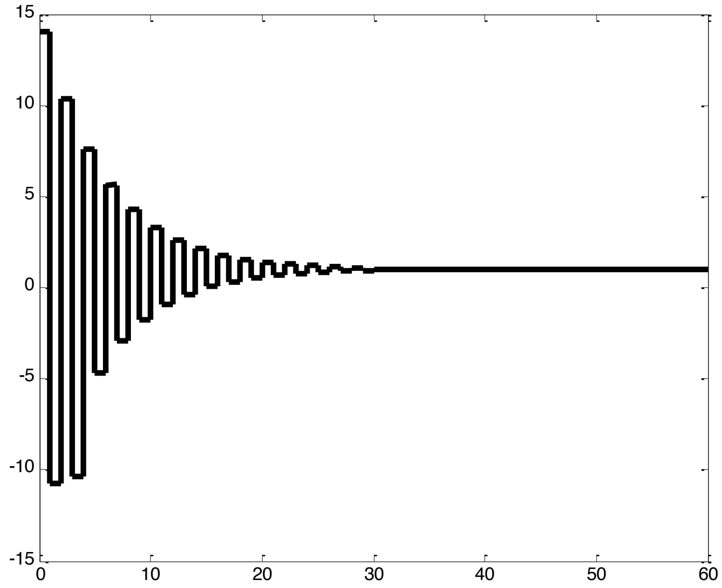


Expected closed loop behaviour

An analysis of ringing

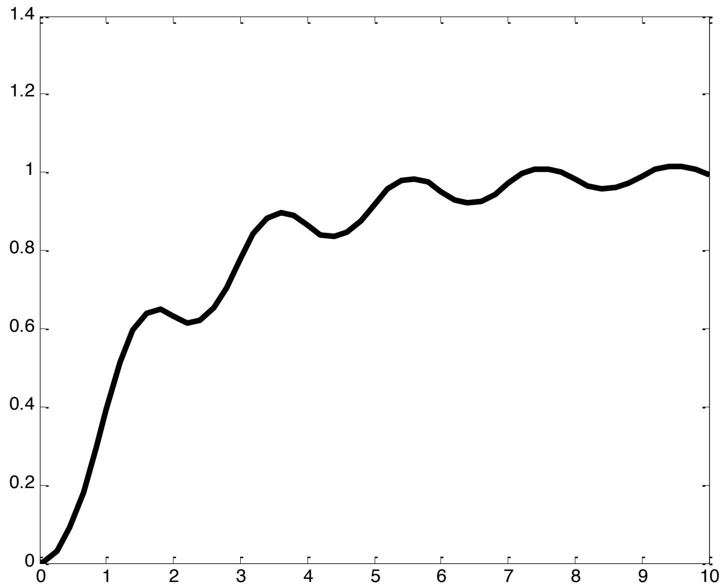


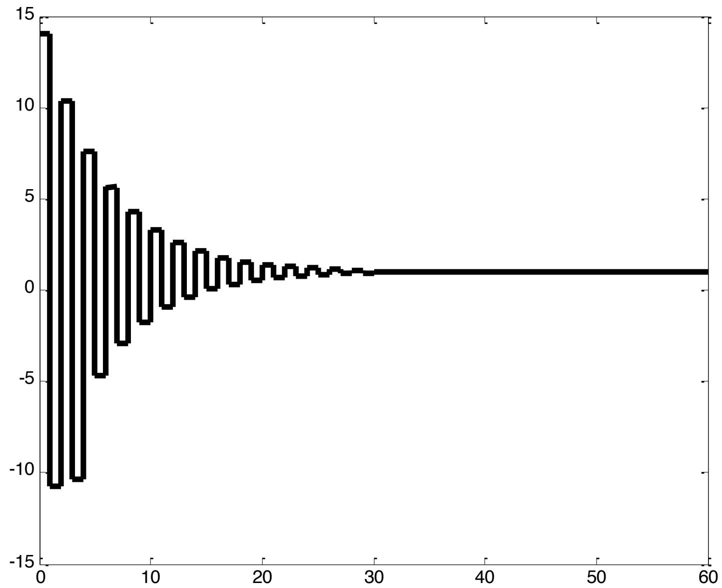
Expected closed loop behaviour



Control effort

An analysis of ringing





Expected closed loop behaviour
Actual closed loop behaviour

Control effort

An analysis of ringing

- Ringing is unique to discrete-time direct synthesis methods
- Produces excessive actuator movement and wear
- Should be avoided…
- Ringing occurs when:

$$H_R(z^{-1}) = \frac{1}{1 - p_1 z^{-1}} H'_R(z^{-1}) \qquad z = p_1$$

Negative real pole, stable but close to the unit circle

Why is this responsible for ringing?
In the time domain $(p_1)^n$ – n is the time step

- Methods to avoid ringing…

An analysis of ringing

- Avoiding ringing: analysis of the transfer function between the controller output (control signal, process input - c) and the reference input (w)

$$\frac{C(z^{-1})}{W(z^{-1})} = \frac{H_R(z^{-1})}{1 + H_R(z^{-1}) H_f(z^{-1})}$$

$$\frac{C(z^{-1})}{W(z^{-1})} = \frac{H_R(z^{-1}) H_f(z^{-1})}{1 + H_R(z^{-1}) H_f(z^{-1})} \frac{1}{H_f(z^{-1})}$$

$$\frac{C(z^{-1})}{W(z^{-1})} = \frac{1}{H_f(z^{-1})} H_0(z^{-1})$$

Check the poles!

- Assume a second order continuous-time process approximated by a discrete-time second order transfer function

$$H_f(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

An analysis of ringing

- Assume the process may be approximated by a second order transfer function (no time delay!)

$$H_f(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$\cancel{H_0(z^{-1}) = \frac{(1 - e^{-T_s/\lambda}) z^{-N-1}}{1 - e^{-T_s/\lambda} z^{-1}}}$$

- No time delay, thus N=0:

$$H_0(s) = \frac{e^{-\tau_m s}}{\lambda s + 1} = \frac{1}{\lambda s + 1} \longrightarrow H_0(z^{-1}) = \frac{(1 - e^{-T_s/\lambda}) z^{-1}}{1 - e^{-T_s/\lambda} z^{-1}}$$

- Then:

$$\frac{C(z^{-1})}{W(z^{-1})} = H_0(z^{-1}) \frac{1}{H_f(z^{-1})}$$

$$\frac{C(z^{-1})}{W(z^{-1})} = \frac{1 + a_1 z^{-1} + a_2 z^{-2}}{\boxed{(b_1 + b_2 z^{-1})} z^{-1}} \frac{(1 - e^{-T_s/\lambda}) z^{-1}}{\boxed{1 - e^{-T_s/\lambda} z^{-1}}}$$

May cause ringing

Never causes ringing!

An analysis of ringing

Dead-beat analogy

- The term $b_1 + b_2 z^{-1}$ will cause ringing if both coefficients have the same sign
- For dead-beat control , then $H_0(z^{-1}) = z^{-1}$, then ringing occurs depending on the signs of b_1 and b_2

$$\frac{C(z^{-1})}{W(z^{-1})} = H_0(z^{-1})\frac{1}{H_f(z^{-1})}$$

- The Dahlin controller for:

$$H_f(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \qquad H_0(z^{-1}) = \frac{(1 - e^{-T_s/\lambda})z^{-1}}{1 - e^{-T_s/\lambda} z^{-1}}$$

Is

$$H_R(z^{-1}) = \frac{1 + a_1 z^{-1} + a_2 z^{-2}}{\boxed{(b_1 + b_2 z^{-1})}} \frac{(1 - e^{-T_s/\lambda})}{1 - e^{-T_s/\lambda} z^{-1} - (1 - e^{-T_s/\lambda})z^{-1}}$$

- Avoiding ringing:

$$H_R(z^{-1}) = \frac{1 + a_1 z^{-1} + a_2 z^{-2}}{\boxed{(b_1 + b_2)}} \frac{(1 - e^{-T_s/\lambda})}{1 - e^{-T_s/\lambda} z^{-1} - (1 - e^{-T_s/\lambda})z^{-1}}$$

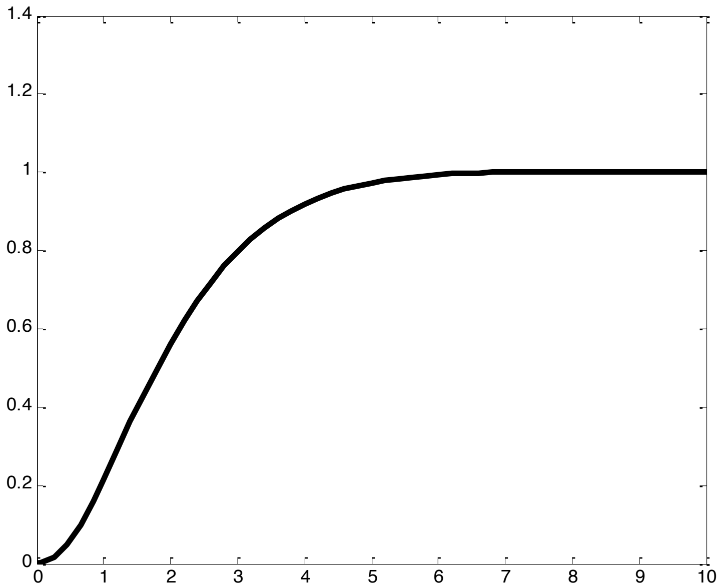
Preserve steady state, remove dynamic component! Make $z^{-1}=1$!

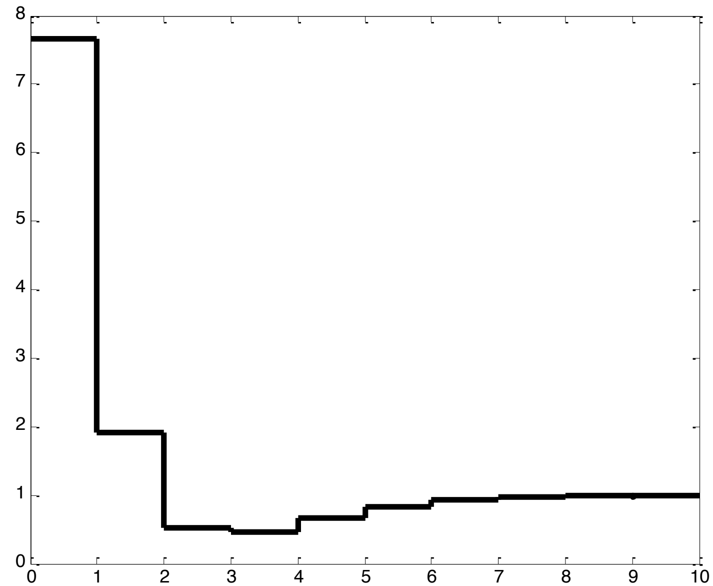
Modified Dahlin controller with ringing pole removed

- Recall our previous example:

$$H_f(s) = \frac{1}{(5s+1)(3s+1)}$$

- What is the non-ringing Dahlin controller?





Non-ringing Dahlin controller

Initial controller – caused ringing

$$H_R(z^{-1}) = \frac{0{,}3935\,(1-1{,}535\,z^{-1}+0{,}5866\,z^{-2})}{(0{,}02797+0{,}02341\,z^{-1})(1-z^{-1})} = \frac{0{,}3935\,(1-1{,}535\,z^{-1}+0{,}5866\,z^{-2})}{0{,}02797-0{,}00456\,z^{-1}-0{,}02341\,z^{-2}}$$

poles $z_1 = 1$ (integrator effect)

$z_2 = -0{,}836$ (close to the unit circle) – ringing pole

Dahlin suggested to replace the ringing pole with the sum of its coefficients (setting $z=1$ in the ringing pole)

Ringing removed:

$$H_R(z^{-1}) = \frac{0{,}3935\,(1-1{,}535\,z^{-1}+0{,}5866\,z^{-2})}{0{,}05138\,(1-z^{-1})}$$

Generalizing the method for removing the ringing pole

- Assume the process may be approximated by a second order transfer function with time delay:

$$H_f(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} z^{-N} \qquad N = \frac{\tau_m}{T_s}$$

- Closed loop transfer function, with equal time delay as the process is:

$$H_0(z^{-1}) = \frac{(1 - e^{-T_s/\lambda}) z^{-1}}{1 - e^{-T_s/\lambda} z^{-1}} z^{-N}$$

- Then:

$$\frac{C(z^{-1})}{W(z^{-1})} = \frac{1 + a_1 z^{-1} + a_2 z^{-2}}{\boxed{(b_1 + b_2 z^{-1})}} \frac{(1 - e^{-T_s/\lambda})}{\boxed{1 - e^{-T_s/\lambda} z^{-1}}}$$

May cause ringing

Never causes ringing!

$$H_R(z^{-1}) = \frac{(1 + a_1 z^{-1} + a_2 z^{-2})(1 - e^{-T_s/\lambda})}{\boxed{(b_1 + b_2 z^{-1})}(1 - e^{-T_s/\lambda} z^{-1} - (1 - e^{-T_s/\lambda}) z^{-N-1})}$$

Generalizing the method for removing the ringing pole

$$H_R(z^{-1}) = \frac{(1+a_1 z^{-1} + a_2 z^{-2})(1-e^{-T_s/\lambda})}{(b_1 + b_2 z^{-1})(1-e^{-T_s/\lambda} z^{-1} - (1-e^{-T_s/\lambda})z^{-N-1})}$$

with ringing

$$H_R(z^{-1}) = \frac{(1+a_1 z^{-1} + a_2 z^{-2})(1-e^{-T_s/\lambda})}{(b_1 + b_2)(1-e^{-T_s/\lambda} z^{-1} - (1-e^{-T_s/\lambda})z^{-N-1})}$$

without ringing

- Transfer function from reference to control signal:

$$\frac{C(z^{-1})}{W(z^{-1})} = \frac{H_R(z^{-1})}{1+H_R(z^{-1})H_f(z^{-1})} = \frac{(1+a_1 z^{-1} + a_2 z^{-2})(1-e^{-T_s/\lambda})}{(b_1+b_2)(1-e^{-T_s/\lambda} z^{-1} - (1-e^{-T_s/\lambda} z^{-1})z^{-N-1}) + (1-e^{-T_s/\lambda})(b_1 + b_2 z^{-1})z^{-N-1}}$$

Additional ringing poles could appear

Vogel-Edgar algorithm

- Algorithm that eliminates the ringing pole for second order processes with time delay:

$$H_f(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} z^{-N} \qquad N = \frac{\tau_m}{T_s}$$

- Closed loop transfer function, similar to Dahlin:

$$H_0(z^{-1}) = \frac{(1 - e^{-T_s/\lambda})(b_1 + b_2 z^{-1})}{(1 - e^{-T_s/\lambda} z^{-1})(b_1 + b_2)} z^{-N-1}$$

$$H_0(z^{-1}) = \frac{(1 - e^{-T_s/\lambda})}{1 - e^{-T_s/\lambda} z^{-1}} z^{-N-1}$$

- Controller transfer function:

$$H_R(z^{-1}) = \frac{(1 + a_1 z^{-1} + a_2 z^{-2})(1 - e^{-T_s/\lambda})}{(b_1 + b_2)(1 - e^{-T_s/\lambda} z^{-1}) - (1 - e^{-T_s/\lambda})(b_1 + b_2 z^{-1}) z^{-N-1}}$$

- What is the transfer function from reference to control signal?

$$H_c(z^{-1}) = \frac{\left(1-e^{-TE/\lambda}\right) z^{-N-1}}{1-e^{-TE/\lambda} z^{-1}} \cdot \frac{(b_1+b_2 z^{-1})}{b_1+b_2}$$

$$\frac{c(z^{-1})}{w(z^{-1})} = \frac{1}{H_f(z^{-1})} \cdot H_o(z^{-1})$$

$$\frac{c(z^{-1})}{w(z^{-1})} = \frac{a_0+a_1 z^{-1}+a_2 z^{-2}}{\cancel{(b_1+b_2 z^{-1})}\,\cancel{z^{-N-1}}} \cdot \frac{\left(1-e^{-TE/\lambda}\right)\cancel{z^{-N-1}}}{\underbrace{1-e^{-TE/\lambda} z^{-1}}_{\text{non ringing pole}}} \cdot \frac{\cancel{(b_1+b_2 z^{-1})}}{b_1+b_2}$$

Controller transfer function

$$H_{R_{VE}}(z^{-1}) = \frac{a_0+a_1 z^{-1}+a_2 z^{-2}}{\cancel{(b_1+b_2 z^{-1})}\,\cancel{z^{-N-1}}} \cdot \frac{\left(1-e^{-TE/\lambda}\right)\cancel{(b_1+b_2 z^{-1})} \cdot \cancel{z^{-N-1}}}{\left(1-e^{-TE/\lambda} z^{-1}\right)(b_1+b_2)-(b_1+b_2 z^{-1}) \cdot \left(1-e^{-TE/\lambda}\right) z^{-N-1}}$$

$$H_{R_{VE}}(z^{-1}) = \frac{\left(a_0+a_1 z^{-1}+a_2 z^{-2}\right)\left(1-e^{-TE/\lambda}\right)}{\left(1-e^{-TE/\lambda} z^{-1}\right)(b_1+b_2)-(b_1+b_2 z^{-1})\left(1-e^{-TE/\lambda}\right) z^{-N-1}}$$

Dahlin/Dahlin without ringing/Vogel-Edgar comparison

- For the process described by the following transfer function, design a Dahlin/Dahlin without ringing and a Vogel-Edgar controller that ensures a settling time for the closed loop system of 18 seconds:

$$H_f(s) = \frac{5}{(5s+1)(10s+1)} e^{-2s}$$

- What are the three controllers transfer functions?

$$H_f(s) = \frac{5}{(5s+1)(10s+1)} e^{-2s}$$

$$\left.\begin{array}{l} t_s = 18\,\text{sec} \\ \tau_m = 2 \end{array}\right\} \Rightarrow t_s = 4\lambda + \tau_m \Rightarrow \lambda = 4 \Rightarrow H_0(s) = \frac{1}{4s+1} e^{-2s}$$

$$T_E = 1 \Rightarrow H_f(z^{-1}) = \frac{(0{,}04528 + 0{,}04097 z^{-1})\, z^{-3}}{1 - 1{,}724 z^{-1} + 0{,}7408 z^{-2}}$$

$$H_R(z^{-1}) = \frac{1}{H_f(z^{-1})} \cdot \frac{H_0(z^{-1})}{1 - H_0(z^{-1})}$$

$$H_0(z^{-1}) = \frac{0{,}2212\, z^{-3}}{1 - 0{,}7788 z^{-1}}$$

Dahlin ctrl.

$$H_R(z^{-1}) = \frac{1-1,724z^{-1}+0,7408z^{-2}}{\cancel{z^{-3}}(0,04528+0,04097z^{-1})} \cdot \frac{0,2212\cancel{z^{-3}}}{1-0,7788z^{-1}-0,2212z^{-3}}$$

$$\frac{C(z^{-1})}{W(z^{-1})} = \frac{1-1,724z^{-1}+0,7408z^{-2}}{\underbrace{(0,04528+0,04097z^{-1})}_{\substack{z=-0,9048 \\ \text{ringing pole}}}\cancel{z^{-3}}} \cdot \frac{0,2212\cancel{z^{-3}}}{\underbrace{1-0,7788z^{-1}}_{z=0,7788}}$$

Dahlin with ringing pole removed

$$\underset{NR}{H_R}(z^{-1}) = \frac{1-1,724z^{-1}+0,7408z^{-2}}{(0,04528+0,04097)\cancel{z^{-3}}} \cdot \frac{0,2212\cancel{z^{-3}}}{1-0,7788z^{-1}-0,2212z^{-3}}$$

$$H_{R_{NR}}(z^{-1}) = \frac{0,2212(1-1,724z^{-1}+0,7408z^{-2})}{0,0862(1-0,7788z^{-1}-0,2212z^{-3})}$$

Vogel-Edgar ctrl.

$$H_0(z^{-1}) = \frac{0{,}2212\, z^{-3}}{1-0{,}7788\, z^{-1}} \cdot \frac{(0{,}04528 + 0{,}04097\, z^{-1})}{(0{,}04528 + 0{,}04097)}$$

$$H_{RVE}(z^{-1}) = \frac{1-1{,}724\, z^{-1} + 0{,}7408\, z^{-2}}{\cancel{(0{,}04528+0{,}04097\, z^{-1})}\,\cancel{z^{-3}}} \cdot$$

$$\cdot \frac{\cancel{0{,}2212\, z^{-3}}\,\cancel{(0{,}04528+0{,}04097\, z^{-1})}}{(1-0{,}7788\, z^{-1})\, 0{,}0862 - (0{,}04528+0{,}04097\, z^{-1})(1-0{,}7788)\, z^{-3}}$$

$$H_{RVE}(z^{-1}) = \frac{0{,}2212\,(1-1{,}724\, z^{-1} + 0{,}7408\, z^{-2})}{0{,}0862\,(1-0{,}7788\, z^{-1}) - (0{,}04528+0{,}04097\, z^{-1}) \cdot 0{,}2212\, z^{-3}}$$

check the first $c(k)$ values for $k=0,1$

$$H_{R_{VE}}(z^{-1}) = \frac{c(z^{-1})}{\varepsilon(z^{-1})} \quad \text{or} \quad \frac{c(z^{-1})}{w(z^{-1})} \Rightarrow$$

$$\Rightarrow \frac{c(z^{-1})}{w(z^{-1})} = \frac{1-1{,}724 z^{-1} + 0{,}7408 z^{-2}}{\cancel{(0{,}04528 + 0{,}04097 z^{-1}) z^{-3}}} \cdot \frac{0{,}2212 \cancel{z^{-3}(0{,}04528 + 0{,}04097 z^{-1})}}{(1-0{,}7788 z^{-1})\, 0{,}0862}$$

$$\frac{c(z^{-1})}{w(z^{-1})} = \frac{2{,}5661(1-1{,}724 z^{-1} + 0{,}7408 z^{-2})}{1-0{,}7788 z^{-1}}$$

$$c(z^{-1}) - 0{,}7788 z^{-1} c(z^{-1}) = 2{,}5661 w(z^{-1}) - 4{,}424 z^{-1} w(z^{-1}) + $$
$$+ 1{,}901 z^{-2} w(z^{-1})$$

$$c(k) = 0{,}7788\, c(k-1) + 2{,}5661\, w(k) - 4{,}424\, w(k-1) +$$
$$+ 1{,}901\, w(k-2)$$

$$K=0 \Rightarrow c(0) = +2{,}5661\,w(0) + 0 + 0$$

$$c(0) = 2{,}5661$$

$$K=1 \Rightarrow c(1) = 97788\,c(0) + 2{,}5661\,w(1) - 4{,}424\,w(0) + 0$$

$$c(1) = 97788 \cdot 2{,}5661 + 2{,}5661 - 4{,}424$$

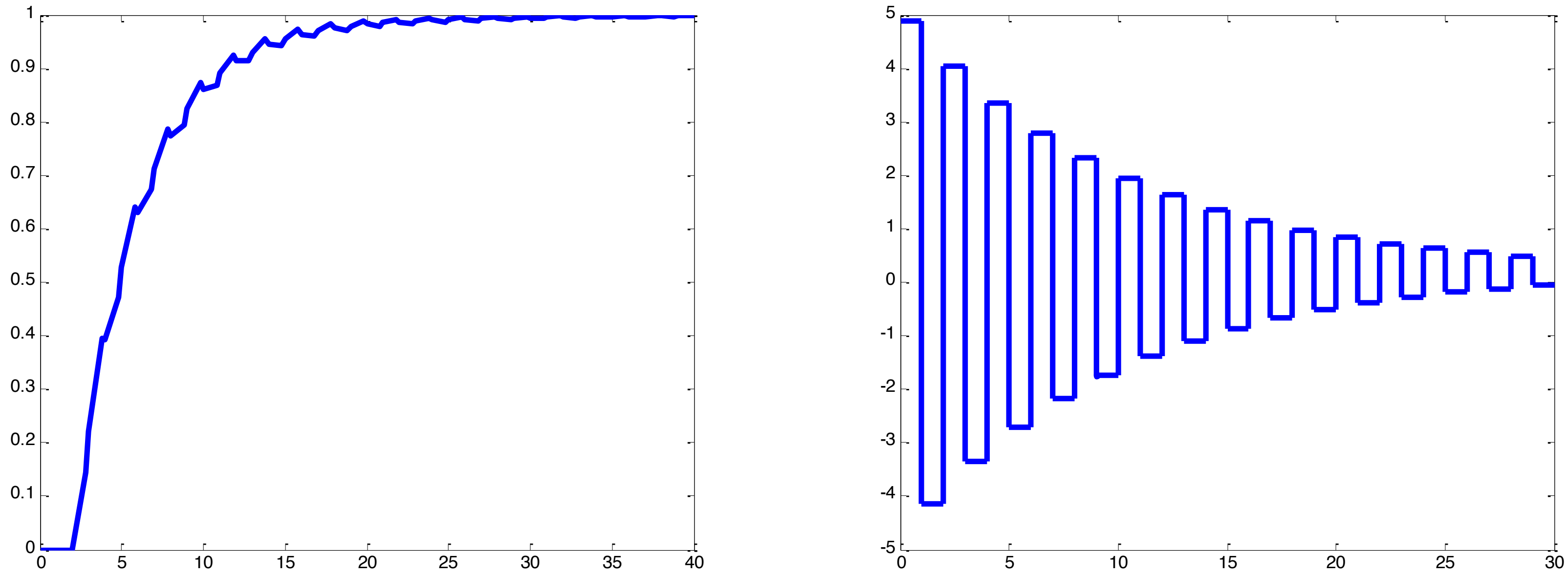
$$c(1) = 0{,}1406$$

$$K=2 \Rightarrow c(2) = 97788\,c(1) + 2{,}5661\,w(2) - 4{,}424\,w(1) + 1{,}901\,w(0)$$

$$c(2) = 0{,}1095 + 2{,}5661 - 4{,}424 + 1{,}901$$

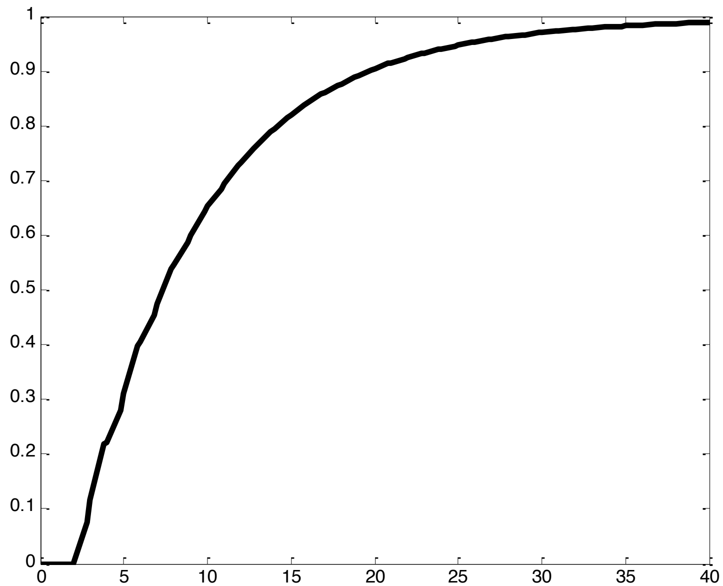
$$c(2) = 0{,}1526$$

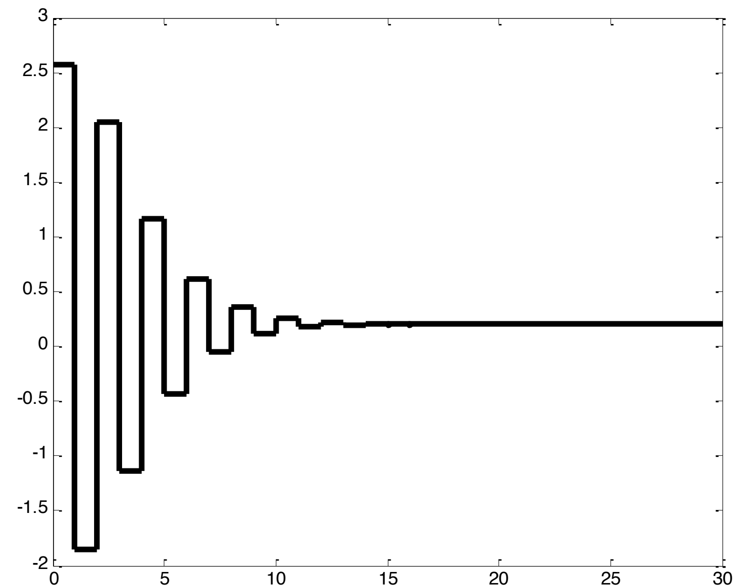
Dahlin/Dahlin without ringing/Vogel-Edgar comparison



Dahlin

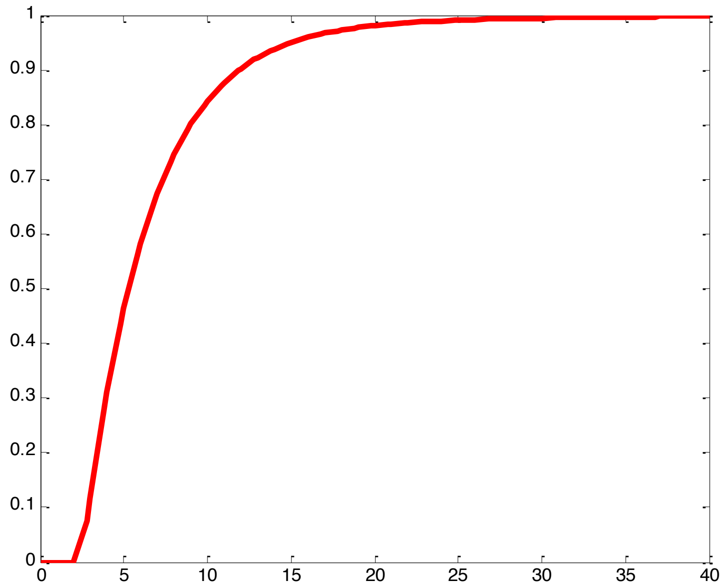
Dahlin/Dahlin without ringing/Vogel-Edgar comparison

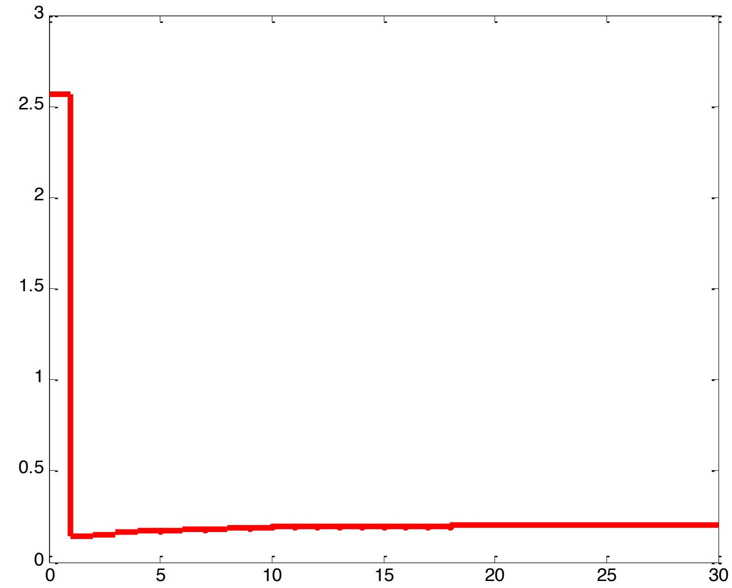




Dahlin without ringing

Dahlin/Dahlin without ringing/Vogel-Edgar comparison





Vogel-Edgar

At the end of the lecture

- You should be able to:
  - Design discrete-time controllers using Dahlin's algorithm
  - Compare Dahlin's algorithm with dead-beat and Kalman algorithms
  - Design discrete-time controllers using Vogel-Edgar algorithm
  - Remove ringing poles