8.2

1 (1) $(x+2y)\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$

特征方程 $\frac{dx}{x+2y} = \frac{dy}{-y} = \frac{2dy}{-2y} = \frac{dx+2dy}{x} = \frac{d(x+y)}{x+y}$

$x\,dx = (x+2y)\,d(x+2y)$

$x^2 = (x+2y)^2 + C$

$\psi_1 = xy + y^2$ $\boxed{u = \varphi(xy+y^2),\ \varphi \text{ 连续可微}}$

(3) $x(y^2+z^2)\frac{\partial u}{\partial x} + y(z^2+x^2)\frac{\partial u}{\partial y} + z(y^2-x^2)\frac{\partial u}{\partial z} = 0$

特征方程 $\frac{dx}{x(y^2+z^2)} = \frac{dy}{y(z^2+x^2)} = \frac{dz}{z(y^2-x^2)}$

$\| \quad \frac{x\,dx}{x^2y^2+x^2z^2} = \frac{z\,dz}{z^2y^2-z^2x^2} = \frac{\frac{1}{2}d(x^2+z^2)}{y^2(x^2+z^2)} = \frac{y\,dy}{y^2z^2+y^2x^2}$

$2dy = \frac{d(x^2+z^2)}{y}$ $\psi_1 = x^2+z^2-y^2$

寻找 φ_2.

$dx = kx(y^2+z^2)$

$dy = ky(z^2+x^2)$

$dz = kz(y^2-x^2)$

$d\left(\frac{yz}{x}\right) = \frac{x(y\,dz+z\,dy) - yz\,dx}{x^2}$

$= \frac{kxyz(y^2-x^2) + kxyz(z^2+x^2) - kxyz(y^2+z^2)}{x^2} = 0$

$\psi_2 = \frac{yz}{x}$ $\boxed{u = \varphi\left(x^2+z^2-y^2,\ \frac{yz}{x}\right),\ \varphi \text{ 连续可微}}$

2(3)

$\frac{\partial z}{\partial x} + (2e^x - y)\frac{\partial z}{\partial y} = 0,\quad x=0 \Rightarrow z=y$

特征方程 $\frac{dx}{1} = \frac{dy}{2e^x-y}$ $2e^x\,dx = y\,dx + dy$

$2e^{2x}\,dx = d(e^x y)$

$e^x y = e^{2x} + C$ $\psi_1 = e^x y - e^{2x}$

$z = \varphi(e^x y - e^{2x})$ 代入初值条件 $y = \varphi(y-1)$ $\varphi(t) = t+1$

$\boxed{z = e^x y - e^{2x} + 1}$

8.3 1(1) $xy\frac{\partial z}{\partial x}+(x-2z)\frac{\partial z}{\partial y}=yz$

特征方程 $\frac{dx}{xy}=\frac{dy}{x-2z}=\frac{dz}{yz}=\frac{y\,dy}{xy-2yz}=\frac{2dz}{2yz}=\frac{y\,dy+2dz}{xy}$

$\frac{dx}{x}=\frac{dz}{z}$ $\quad \psi_1=\frac{z}{x}$

$dx=y\,dy+2dz$, $d(x-\frac{1}{2}y^2-2z)=0$

$\psi_2=x-\frac{1}{2}y^2-2z$

通解 $\boxed{V(\frac{z}{x},\ x-\frac{1}{2}y^2-2z)=0\ ,\text{且 } V_z'\neq 0}$ #.

2(3)

$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=2xy$, $y=x$ 时 $z=x^2$

特征方程 $\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{2xy}$

$\psi_1=\frac{y}{x}$

$dz-y\,dx-x\,dy=0$

$dz-d(xy)=0$

$\psi_2=z-xy$

通解 $V(\frac{y}{x},\ z-xy)=0$,且 $V_2'\neq 0$

设解为 $z-xy=f(\frac{y}{x})$,则 $f(1)=0$

故 $\boxed{z=xy+f(\frac{y}{x}),\quad f(1)=0.\quad f\in C^1.}$

3 求一曲面，使一点处切平面在x轴截距为切点x坐标的一半.

设曲面 $z=z(x,y)$ 在 (x_0,y_0,z_0) 处切平面为

$$\left.\frac{\partial z}{\partial x}\right|_{x=x_0}(x-x_0)+\left.\frac{\partial z}{\partial y}\right|_{y=y_0}(y-y_0)+z_0-z=0$$

与x轴交点为 x_1，则，$x_1=\frac{x_0}{2}$，且 $\left.\frac{\partial z}{\partial x}\right|_{x=x_0}\neq 0$.

$$\left.\frac{\partial z}{\partial x}\right|_{x=x_0}\left(-\frac{x_0}{2}\right)+\left.\frac{\partial y}{\partial y}\right|_{y=y_0}(-y_0)+z(x_0,y_0)=0$$

PDE为 $\left(\frac{x}{2}\right)\frac{\partial z}{\partial x}+(y)\frac{\partial z}{\partial y}=z$

特征方程为 $\frac{2dx}{x}=\frac{dy}{y}=\frac{dz}{z}$

$$\varphi_1=\frac{x^2}{y}\qquad \varphi_2=\frac{z}{y}$$

曲面为 $V\left(\frac{x^2}{y},\frac{z}{y}\right)=0$，$V_2'\neq 0$，$\frac{\partial z}{\partial x}\neq 0$.

$\boxed{\text{例如}\quad z=x^2}$