28. 正20面体，20个面 2组 0~9
检验均匀。800次实验

0	1	2	3	4	5	6	7	8	9
74	92	83	79	80	73	77	75	76	91

问是否均匀？
使用 χ^2 测试
$H_0: P(X=i)=\frac{1}{10},\ i=0,\cdots 9$

$$V=\sum_{i=0}^{9}\left(\nu_i-\frac{n}{10}\right)^2\Big/\frac{n}{10}=5.125$$

H_0下 V 近似服从 9个自由度 χ^2分布
$P(V>16.9)=0.05$.
$5.125<16.9$
认为是均匀的

29. Kolmogorov Test $N=25$ $\alpha=0.05$ 符合 $N(0,1)$？
-0.42 -0.39 -2.21 -2.46 -2.11 -1.23 -0.99
-0.15 -0.10 -0.07 -0.02 0.27 0.40 0.42 0.44
0.7 0.81 0.88 1.07 1.39 1.40 1.47 1.62 1.64 1.76

$H_0: f(x)=f_0(x)$

设 $F_0(x)$ 为 $N(0,1)$ 的 CDF

$$D_n=\max_{1\le k\le 25}\max\left(\frac{k}{25}-F_0(x_k),\ F_0(x_k)-\frac{k-1}{25}\right)$$

计算得 $D_n=0.17542$
查表 $n=25$ $\alpha=0.05$ $\lambda=0.264$
$D_n<\lambda$
认为符合.

1. 定理 2.1

$$\hat{b} \overset{①}{=} \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2} \overset{②}{=} \frac{\sum_{i=1}^{n}(x_i-\bar{x})y_i}{\sum_{i=1}^{n}(x_i-\bar{x})^2} \overset{③}{=} \frac{\sum_{i=1}^{n} x_i(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

证：定理中 $\hat{b} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})y_i - \left(\sum_{i=1}^{n}(x_i-\bar{x})\right)\bar{y}}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$

$= \frac{\sum_{i=1}^{n}(x_i-\bar{x})y_i}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$ (②) $\quad \hat{b} = \frac{\sum_{i=1}^{n} x_i(y_i-\bar{y}) - \bar{x}\sum_{i=1}^{n}(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{\sum_{i=1}^{n} x_i(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$ (③)

$\hat{b} = \frac{\sum_{i=1}^{n} x_i y_i - \bar{x}\sum_{i=1}^{n} y_i - \bar{y}\sum_{i=1}^{n} x_i + n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i + n\bar{x}^2} = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}$ (①)

(数据 $(x_1,y_1) \cdots (x_n,y_n)$)

2. $\hat{a}, \hat{b}$ 一元线性回归， $\hat{y}_i = \hat{a} + \hat{b}x_i$

则 $\sum_{i=1}^{n}(y_i - \hat{y}_i) = 0$, $\quad \sum_{i=1}^{n} x_i(y_i - \hat{y}_i) = 0$

证：$\sum_{i=1}^{n}\hat{y}_i = n\hat{a} + \hat{b}\sum_{i=1}^{n} x_i = n(\bar{y} - \hat{b}\bar{x}) + n\hat{b}\bar{x}$

$= n\bar{y} = \sum_{i=1}^{n} y_i$

$$\begin{aligned}
\sum_{i=1}^{n} x_i(y_i - \hat{y}_i) &= \sum_{i=1}^{n} x_i(y_i - \hat{a} - \hat{b}x_i) \\
&= \sum_{i=1}^{n} x_i y_i - \hat{a}n\bar{x} - \hat{b}\sum_{i=1}^{n} x_i^2 \\
&= \sum_{i=1}^{n} x_i y_i - n\bar{x}(\bar{y} - \hat{b}\bar{x}) - \hat{b}\sum_{i=1}^{n} x_i^2 \\
&= \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} + \hat{b}\left(n\bar{x}^2 - \sum_{i=1}^{n} x_i^2\right) \\
&= 0 \quad (\text{由上一题})
\end{aligned}$$

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