IMP

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1 IMP

IMP is a small lanuage of while programs, which called "imperative" lanuage. In the *programming paradigms*, *imperative lanuage* means program execution involves carrying out series of explicit commands to change state.

syntactic sets

Firstly, we give the syntactic sets associated with IMP:

- numbers **N:** the set of signed decimal numerals.
- truth value **T**
- location **Loc:** non-empty strings of letters or such strings follwed by digits.
- arithmetic expressions **Aexp**
- boolean expressions **Bexp**
- commands **Com**

We define the *formation rules* for **Aexp** by:

$$a ::= n|X|a_0 + a_1|a_0 - a_1|a_0 \times a_1.$$

The symbol "::=" should be read as "can be" (p.s. BNF isn't it?)
And for **Bexp**:

$$b ::= \mathbf{true}|\mathbf{false}|a_0 = a_1|a_0 \leq a_1|\neg b|b_0 \wedge b_1|b_0 \vee b_1$$

we define the syntactic of commands:

$$c ::= \mathbf{skip} | X ::- a | c_0; c_1 | \mathbf{if}\ b\ \mathbf{then}\ c_0\ \mathbf{else}\ c_1 | \mathbf{while}\ b\ \mathbf{do}\ c$$

From a *set-theory* point of view, this notaion provides an *inductive definition* of the syntactic set of **IMP**

For the moment, this notation should be viewed as simply telling us how to construct elements of the syntactic sets.

We need some notation to express when two elements e_0, e_1. We use $e_0 \equiv e_1$ to mean e_0 is identical to e_1. The set of *states* Σ consists of functions $\sigma : \mathbf{Loc} \to \mathbf{N}$. Thuse $\sigma(X)$ is the value, or contents, of location **X** in state σ.

Consider the evaluation of an arithmetic expression a in a state σ. We can represent the situation of expression a waiting to be evaluated in state σ by the pair $< a, \sigma >$ We shall **define** an evaluation relation between such pairs and numbers:

$$< a, \sigma > \to n$$

This is meaning: expression a in state σ evaluates to n. Call pairs $< a, \sigma >$, where a is an arithmetic expression and σ is a state.

Evaluation of numbers:

$$< n, \sigma > \to n$$

Evaluation of subtractions:

$$< x, \sigma > \to \sigma(X)$$

Evaluation of sums:

$$\frac{< a_0, \sigma > \to n_0 < a_1, \sigma > \to n_1}{< a_0 * a_1, \sigma > \to n}$$

Evaluation of productions:

$$\frac{< a_0, \sigma > \to n_0 < a_1, \sigma > \to n_1}{< a_0 * a_1, \sigma > \to n}$$

n is the product of n_0 and n_1.

You can read it as: If $\langle a_0, \sigma \rangle \to n_0$ and $\langle a_1, \sigma \rangle \to n_1$ then $\langle a_0 * a_1, \sigma \rangle \to n$