Assignment 3 FMAN95

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February 2024

1 Exercises

Exercise 1. $P_1 = [I \ 0]$ and $P_2 = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$$\underbrace{}_{t}$$

$[t]_\times = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}$

$$F = [t]_\times A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & -2 & 0 \end{pmatrix}$$ ← Fundamental M

$$X = \begin{bmatrix} Y \end{bmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Epipolar line of x in P_1: $\lambda A_1 x + t$

$$\underbrace{\lambda \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{Ax} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} = \text{Epipolar line}$$

$$l = e_2 \times Ax = t \times Ax = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$$

$$l^T\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = 0 \qquad l^T\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 0 \qquad l^T\begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} = 4$$

Points $(2,0)$ & $(2,1)$

Exercise 2.

$P_1 \longrightarrow C_1 = (0,0,0,1)$

P_2: Nullspace:
$$\left.\begin{aligned} x + y + z + 2w &= 0 \\ 2y + 2w &= 0 \\ z &= 0 \end{aligned}\right\} \quad \left.\begin{aligned} x &= -w \\ y &= -w \\ z &= 0 \end{aligned}\right\} \quad \begin{aligned} x &= -t \\ y &= -t \\ z &= 0 \\ w &= t \end{aligned}$$

$C_2 = (-1,-1,0,1)$

$$P_1 C_2 = [I \ 0]\begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = e_1$$ ← epipole

$$P_2 C_1 = \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = e_2$$

$$F = [t]_\times A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix}$$

Exercise 2.

$\det(F) = 0$

$$F e_1 = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$e_2^T F = \begin{pmatrix} 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

Exercise 3: $\tilde{x}_1 \sim N_1 x_1$ & $\tilde{x}_2 \sim N_2 x_2$

$N_2^T x_2^T \tilde{F} N_1 x_1 \longrightarrow F = N_2^T \tilde{F} N_1$

Exercise 4: $F = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

Get e_2 by computing nullspace of $F^T = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$

$$\left.\begin{array}{l} y = 0 \\ x + z = 0 \\ x + z = 0 \end{array}\right\} \begin{array}{l} x = t \\ y = 0 \\ z = -t \end{array} \rightarrow e_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \rightarrow [e_2]_x = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$[e_2]_x F = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & -2 \\ 1 & 0 & 0 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

Add 1 to row 4 to make it homogeneous

$$x_{11} = P_1 X_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad x_{12} = P_2 X_1 = \begin{pmatrix} 2 \\ -10 \\ 0 \end{pmatrix}$$

$$x_{12}^T F x_{11} = (2 \ -10 \ 0) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (-10 \ 2 \ 2) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 0$$

$$x_{21} = P_1 X_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \qquad x_{22} = P_2 X_2 = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$$

$$x_{22}^T F x_{21} = (4 \ -6 \ 2) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = (-6 \ 6 \ 6) \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = 0$$

$$x_{31} = P_1 X_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \qquad x_{32} = P_2 X_3 = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

$$x_{32}^T F x_{31} = (2 \ -2 \ 0) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = (-2 \ 2 \ 2) \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 0$$

Confirmed the epipolar constraint for all of the 3D scene points

Camera center of P_2 is a point at infinity.

From: $0 = \left[[e_2]_x F \quad e_2 \right] \begin{bmatrix} e_1 \\ 0 \end{bmatrix}$

Exercise 6.

$$UV^T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1 & 0 \end{pmatrix} \begin{matrix} 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 \\ 0 & -1 \end{matrix} \rightarrow \det(UV^T) = 1$$

$$E = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 0 & 0 \end{pmatrix}$$

Check plausible by checking the epipolar constraint $x_2^T E x_1 = 0$

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = -\sqrt{2}/2 + \sqrt{2}/2 + 0 = 0$$

$$P_1 X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \\ s \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \longrightarrow \text{Homogenous coordinate of } x_1 = (0,0).$$

$$[UWV^T \; u_3] = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} = L_1$$

$$[UWV^T \; -u_3] = \begin{pmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} = L_2 \qquad [UW^TV^T \; u_3] = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} = L_3$$

$$[UW^TV^T \; -u_3] = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} = L_4$$

$$\underbrace{L_1 X(s) = \begin{pmatrix} -1/\sqrt{2} \\ +1/\sqrt{2} \\ s \end{pmatrix}}_{s = -1/\sqrt{2}} \quad \underbrace{L_2 X(s) = \begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ -s \end{pmatrix}}_{s = \frac{1}{\sqrt{2}}} \quad \underbrace{L_3 X(s) = \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ s \end{pmatrix}}_{s = 1/\sqrt{2}} \quad \underbrace{L_4 X(s) = \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ -s \end{pmatrix}}_{s = -1/\sqrt{2}}$$

We want the homogenous x_2 to be $x_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Only L_3 is in front of both cameras as it's non negative.

2 The Fundamental Matrix

Computer exercise 1

The determinant of F is very close to zero, 9.35*e(-19) (after using formula 7.33 from the lecture notes), same with the epipolar constraint with them hovering around zero. Also checked the euclidean norm of Mv to be 1.88*e-17 which is close to zero. The mean distance is 0.36 when normalized and 6.1 without normalization.

The fundamental matrix: $\begin{bmatrix} 0 & 0 & 0.0058 \\ 0 & 0 & -0.027 \\ -0.07 & 0.026 & 1 \end{bmatrix}$

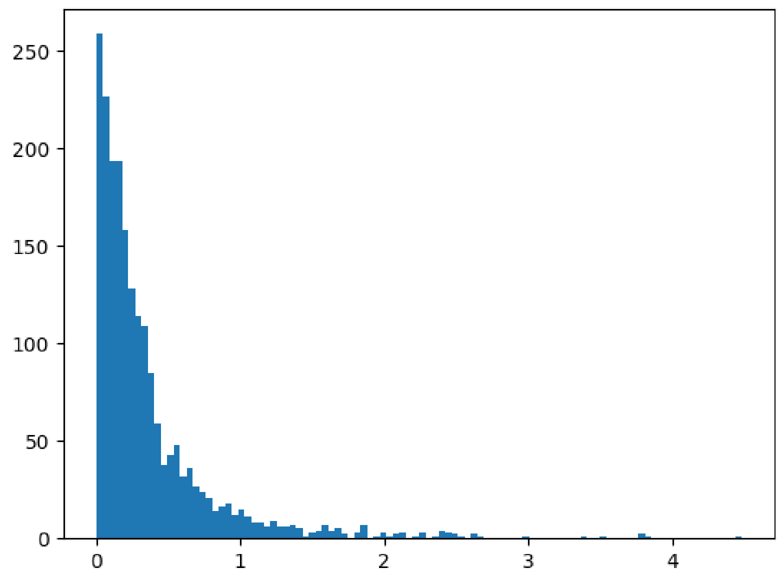

Figure 1: The distances between a point and it's epipolar line using normalized F, presented in a histogram.

Computer exercise 2

Figure 3-4 looks to be correct. Figure 5 shows the 3D points which is not at all what I expected. If the two parts of the image points are divided into the upper and lower cluster, they together form a parenthesis. I interpret it as the points being behind and in front of the camera (as the smaller cluster looks to be upside down like in figure 6.

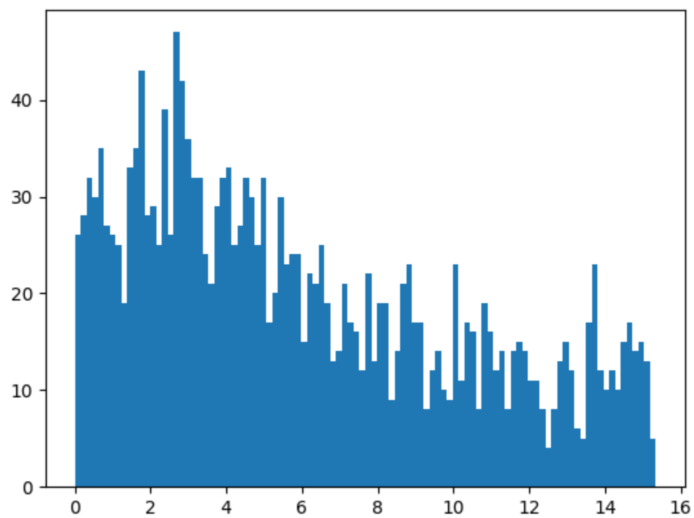

Figure 2: The distances between a point and it's epipolar line using N2 = N1 = 1

3 The Essential Matrix

Computer exercise 3
The determinant of E is zero. The epipolar constraint is also hovering around zero ± 0.002. We also check that Mv is close to zero being 0.0066.

The essential matrix: $\begin{bmatrix} -8.9 & -1005.8 & 377.1 \\ 1252.5 & 78.4 & -2448.2 \\ -472.8 & 2550.2 & 1 \end{bmatrix}$

The mean average distance from point to epipolar line is 2.1 which is between the normalized and un-normalized values in computer exercise 1. The distances from the points x2 to epipolar lines are definitely more evenly distributed compared to the first computer exercise. The problem with rital and getting the epipolar lines to properly show still exists here.

Computer exercise 4
The camera matrix P2a

$$[UWV^T u3]$$

had 2008 points in front of the camera.
The camera matrix P2b

$$[UWV^T - u3]$$

had 0 points in front of the camera.

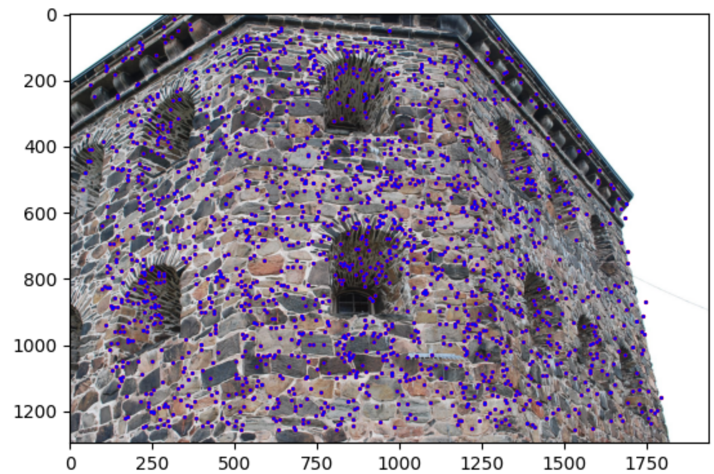

Figure 3: The image points and the projected 3D points for image 1. Blue are the projected.

The camera matrix P2c

$$[UW^TV^Tu3]$$

had 1839 points in front of the camera.
The camera matrix P2d

$$[UW^TV^T - u3]$$

had 169 points in front of the camera.

Looking at figure 7 and 8 we can see that the projected 3D points translated still looks very good compared to the original ones.

The best projection looks like figure 9 and looks like the three walls from the images. (NOTE: Python SVD returns the transpose of V)

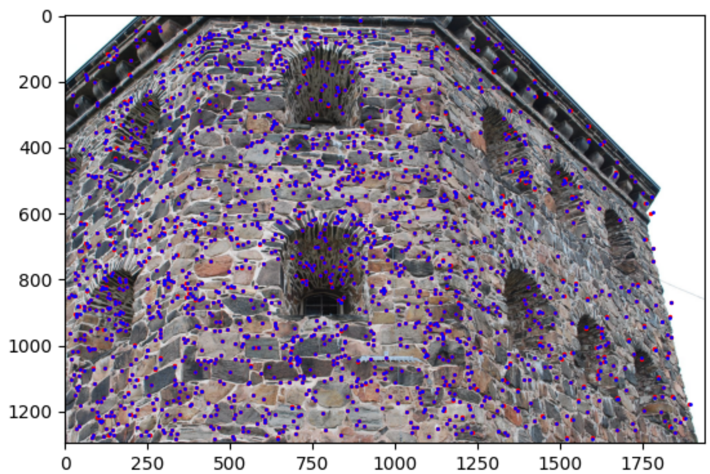

Figure 4: The image points and the projected 3D points for image 2. Blue are the projected.

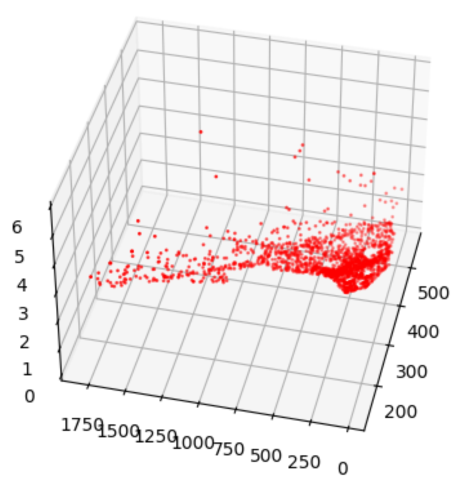

Figure 5: The 3D points

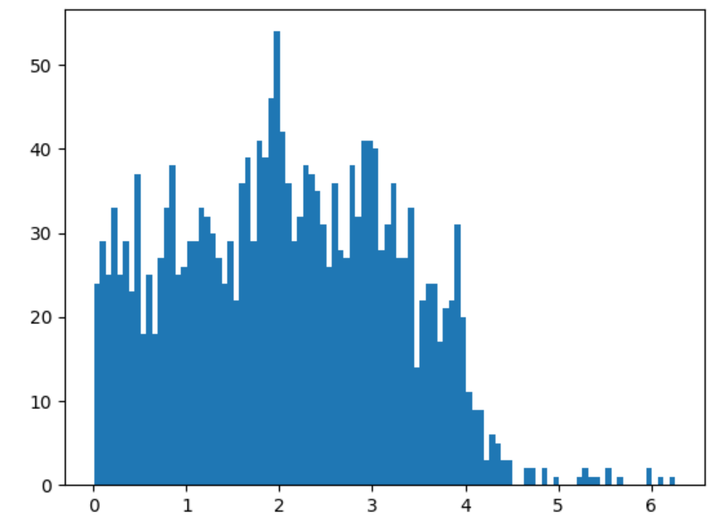

Figure 6: Distances from points to corresponding epipolar line

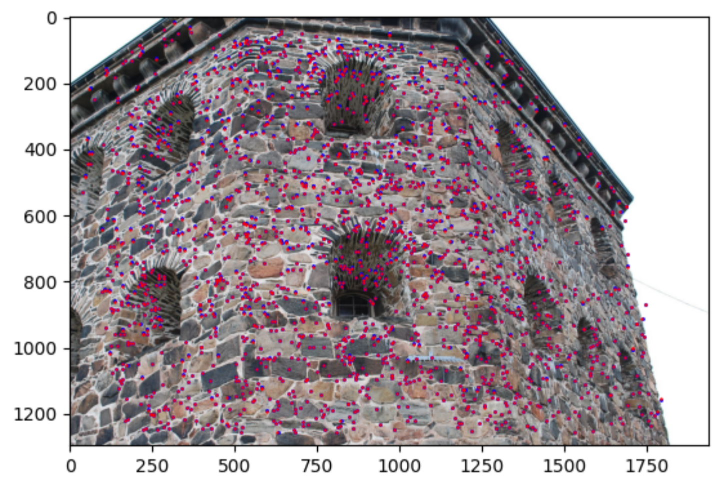

Figure 7: The projected points and the original points

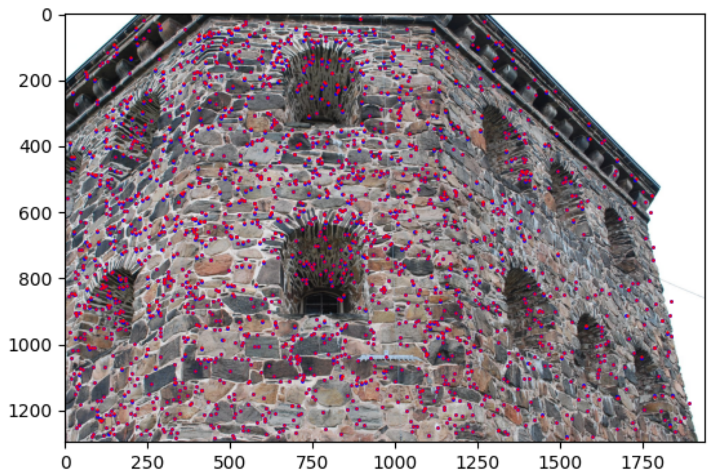

Figure 8: Projected points and the original points

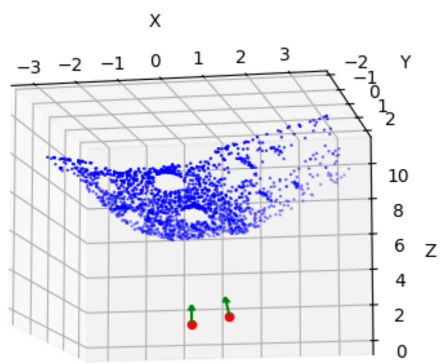

Figure 9: The projection of the image.