Exercise 2.

with $\begin{cases} l_1^T x = 0 \\ l_2^T x = 0 \end{cases} \rightarrow \begin{cases} x+y+z=0 \\ 3x+2y+z=0 \end{cases} \rightarrow \begin{matrix} 2x+y=0 \\ 0=0 \end{matrix} \rightarrow \begin{cases} x=t \\ y=-2t \\ z=t \end{cases} \rightarrow$ point $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

$\begin{cases} l_3^T x = 0 \\ l_4^T x = 0 \end{cases} \rightarrow \begin{cases} x+2y+3z=0 \\ x+2y+z=0 \end{cases} \rightarrow 2z=0 \rightarrow \begin{cases} x=-2t \\ y=t \\ z=0 \end{cases}$ point $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

Because $z=0$ it's a point at infinity as division by 0 is not possible

In euclidean geometry two parallell lines will never meet (per definition).

In projective geometry however, they will intersect in point at infinity.

$x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ & $x_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \rightarrow$ Re using previous answer $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$z = -t$

$$(H^{-1})^T l_1 = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad \text{Same as } l_2$$

Proof: $0 = l_1^T x = l_1^T H^{-1} H x = ((H^{-1})^T l_1) H x \sim l_2^T y$

Exercise 5.

Projection is given by $P \cdot X$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = \text{projection of } X_1$$