Exercise 4.
$$H = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \quad x_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad x_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

y_1 & $y_2 \sim Hx_1$ & Hx_2

$$y_1 \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \longrightarrow \left.\begin{matrix} x = 0 \\ x + y + z = 0 \end{matrix}\right\} \to \begin{cases} x = 0 \\ y = t \\ z = -t \end{cases}$$

$$l_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$y_2 \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\left.\begin{matrix} x + z = 0 \\ y + z = 0 \end{matrix}\right\} \longrightarrow \left.\begin{matrix} x - y = 0 \\ 0 = 0 \end{matrix}\right\} \longrightarrow \begin{cases} x = t \\ y = t \\ z = -t \end{cases} \quad l_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$(H^{-1})^T l_1 = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad \text{Same as } l_2$$

Proof: $0 = l_1^T x = l_1^T H^{-1} H x = ((H^{-1})^T l_1) Hx \sim l_2^T y$