# Assignment 1 FMAN95

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## 1 Exercises

Exercise 1. $X_1 = \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix} \xrightarrow{/2} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \rightarrow x_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

Homogeneous 2D cartesian coordinate

$$X_2 = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \xrightarrow{/-1} \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} \rightarrow x_2 = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

$$X_3 = \begin{pmatrix} 4\lambda \\ -2\lambda \\ 2\lambda \end{pmatrix} \xrightarrow{/2\lambda} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \rightarrow x_3 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$X_4$ means that no transformation / translation can be applied to $X_4$ as the last row has 0 which will lead to the same matrix after multiplication.

Exercise 2.

with $\begin{cases} l_1^T x = 0 \\ l_2^T x = 0 \end{cases} \rightarrow \begin{cases} x + y + z = 0 \\ 3x + 2y + z = 0 \end{cases} \rightarrow \begin{matrix} 2x + y = 0 \\ 0 = 0 \end{matrix} \rightarrow \begin{cases} x = t \\ y = -2t \\ z = t \end{cases}$ point $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

$\begin{cases} l_3^T x = 0 \\ l_4^T x = 0 \end{cases} \rightarrow \begin{cases} x + 2y + 3z = 0 \\ x + 2y + z = 0 \end{cases} \rightarrow 2z = 0 \rightarrow \begin{cases} x = -2t \\ y = t \\ z = 0 \end{cases}$ point $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

Because $z = 0$ it's a point at infinity as division by 0 is not possible. In euclidean geometry two parallell lines will never meet (per definition). In projective geometry however, they will intersect in point at infinity.

$X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ & $X_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \rightarrow$ $R_x$ using previous answer $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Exercise 3.

Intersection point $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ because its an intersection from the rows in $M$ then $M \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = 0$ should give a non-zero solution. It satisfies $Mx = 0$

Infinitely many vectors in the nullspace $\rightarrow$ Infinite points

Exercise 4.

$$H = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \quad X_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad X_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$y_1$ & $y_2 \sim Hx_1$ & $Hx_2$

$$y_1 \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \left.\begin{matrix} x & = 0 \\ x + y + z & = 0 \end{matrix}\right\} \rightarrow \begin{cases} x = 0 \\ y = t \\ z = -t \end{cases}$$

$$y_2 \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \longrightarrow l_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\left.\begin{matrix} x + z = 0 \\ y + z = 0 \end{matrix}\right\} \longrightarrow \left.\begin{matrix} x - y = 0 \\ 0 = 0 \end{matrix}\right\} \longrightarrow \begin{cases} x = t \\ y = t \\ z = -t \end{cases} \quad l_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$(H^{-1})^T l_1 = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad \text{Same as } l_2$$

Proof: $0 = l_1^T x = l_1^T H^{-1} H x = ((H^{-1})^T l_1) Hx \sim l_2^T y$

Exercise 5.

Projection is given by $P \cdot X$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = \text{projection of } X_1$$

Projection of $X_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ & $X_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

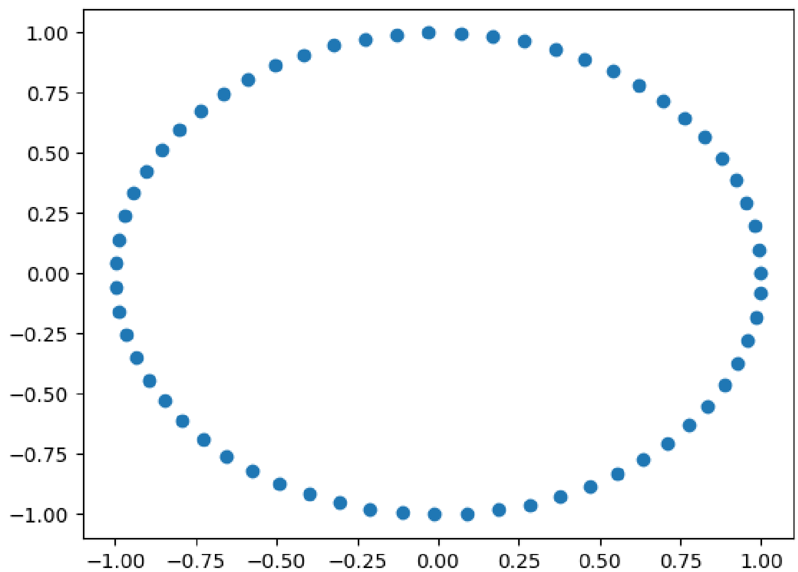
Geometric interpretation the projection is a vanishing point. Where two parallell lines converge.

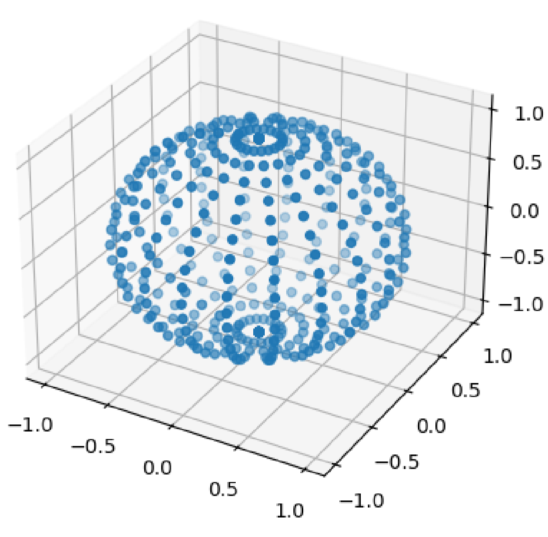
Camera center by finding null space. $\rightarrow$ $\begin{matrix} x & = 0 \\ y & = 0 \\ z + w & = 0 \end{matrix} \rightarrow \begin{cases} x = 0 \\ y = 0 \\ z = -1 \\ w = 1 \end{cases}$

Camera center: $(0 \quad 0 \quad -1 \quad 1)$

The principal axis $= (0 \quad 0 \quad 1)$ & viewing direction.

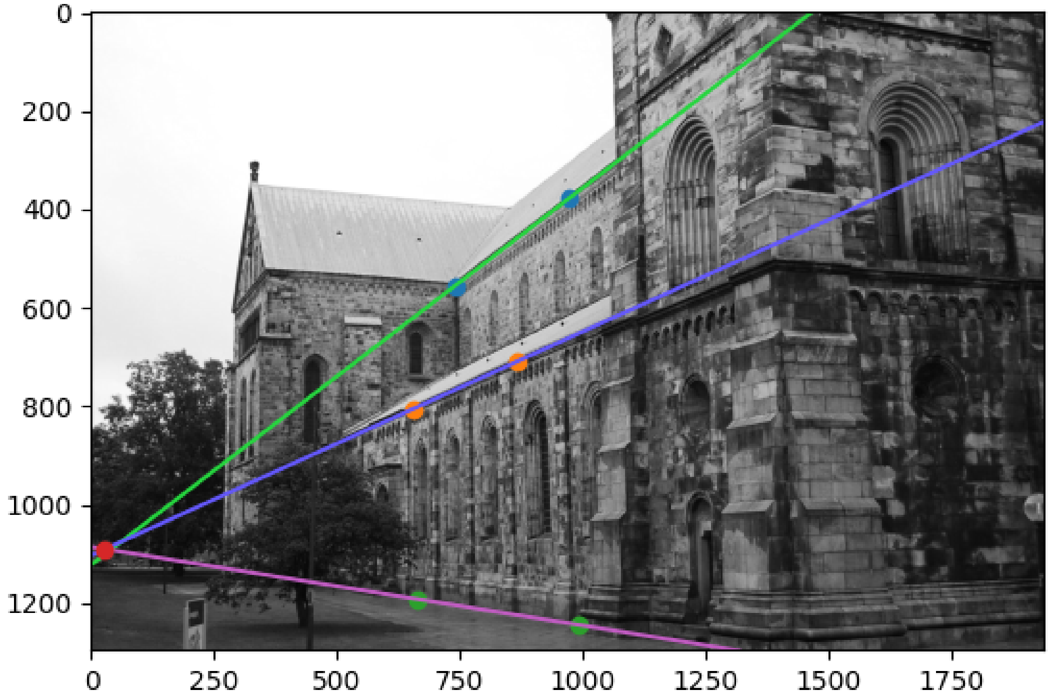
## 2 Points in Homogeneous Coordinates



## 3 Lines

The lines seem to be parallel in 3D as they are the upper edges of walls on the same side. The distance between the intersection point and the first line is close but not the same. This means that the first points are not parallel to the other two points, but very close. If the points were parallel then they would converge into the same vanishing point. The distance between the first line and the intersection point for line 2 and 3 is approximately 8.



## 4 Projective Tranformations

**Which of the transformations preserve lengths between points?**
**Which preserve angles between lines?**
**Which map parallel lines to parallel lines?**

H1: It looks like there has been a rotation only of the image. The angles and length of the lines has been preserved. The parallel lines are preserved. This is a Euclidean transformation as the distance between the points are still the same.

H2: There has been a rotation and scaling of the figure. The angles of the lines has been preserved but the lines are scaled up. Parallel lines still stay as parallel lines. Therefore it should be a similarity transformation.

H3: Affine transformation. Parallel lines are mapped to parallel lines.

H4: Projective transformation as it projects on a 3D space. None of the questions apply to this transformation in 2D.

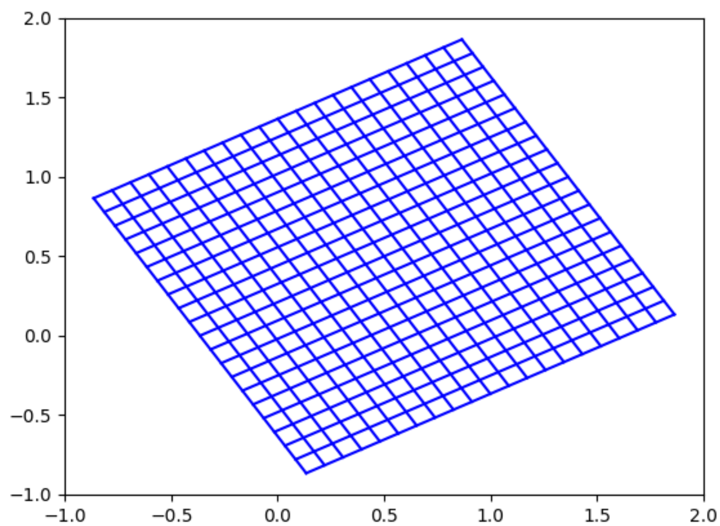



Figure 1: H1 transformation

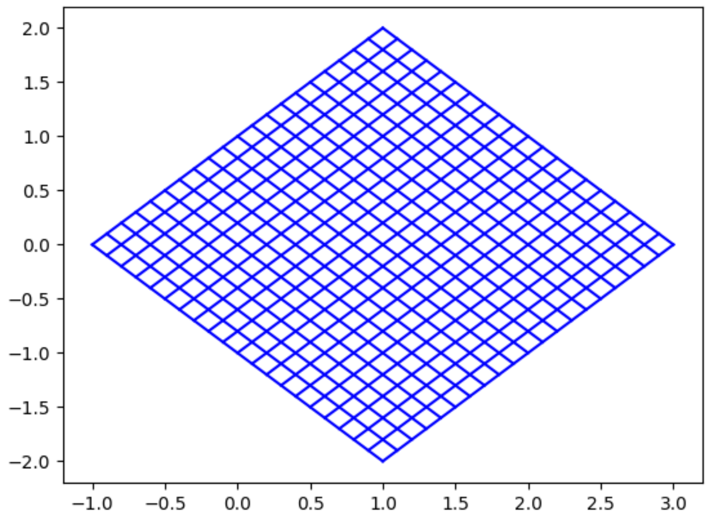



Figure 2: H2 transformation

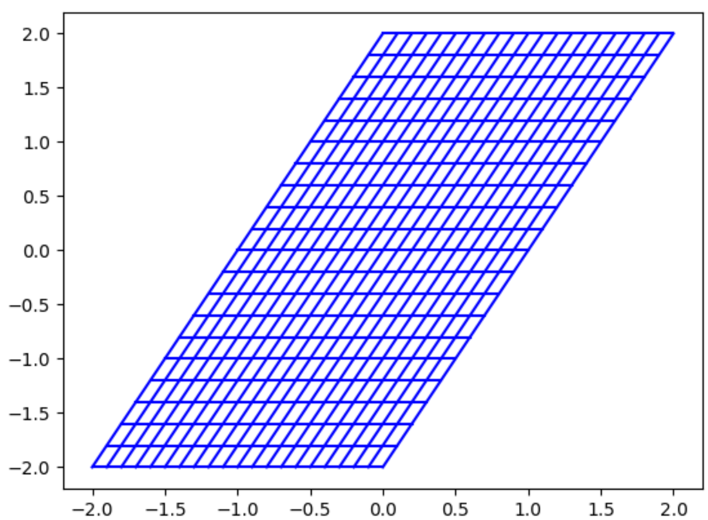



Figure 3: H3 transformation

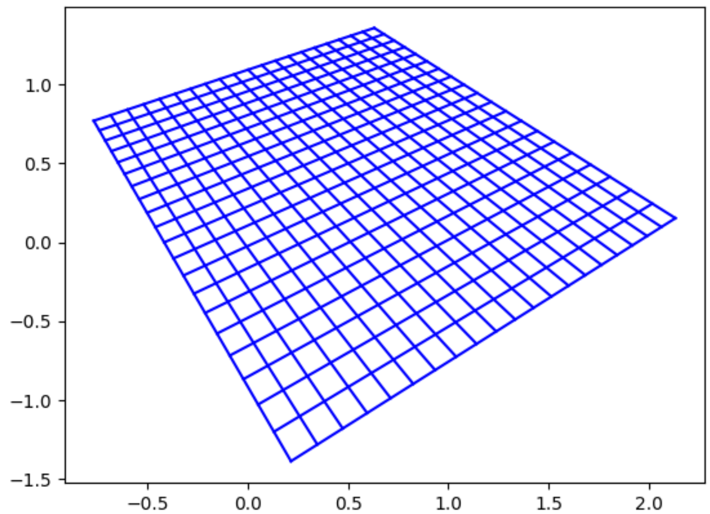



Figure 4: H4 transformation

# 5 The Pinhole Camera

Image 1: Camera center (0,0,0) and principal axes (0.31, 0.95, 0.08)
Image 2: Camera center (6.64, 14.85, -15.07) and principal axes (0.03, 0.34, 0.94)

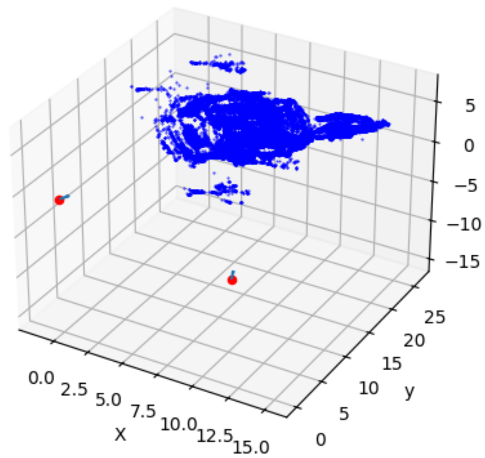



Figure 5: The principal axes and camera centers, together with U

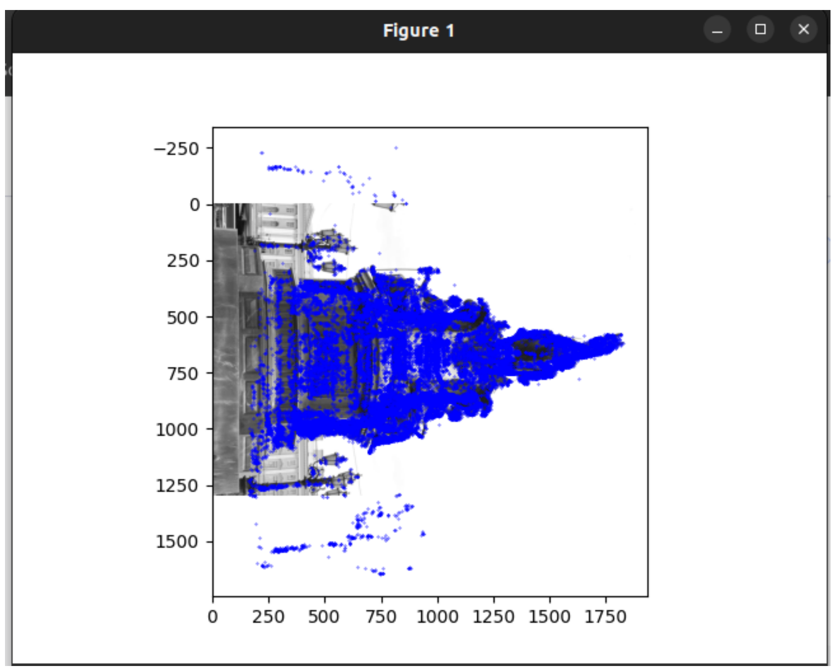



Figure 6: The points U on P1

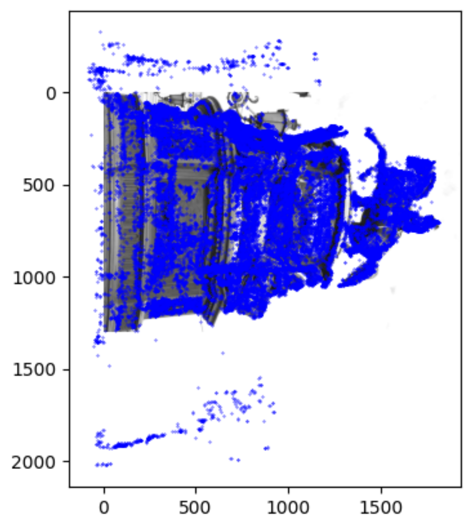



Figure 7: The points U on P2