Exercise 1. $P_1 = [I \;\; 0]$ and $P_2 = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ $\begin{array}{c} \\ \\ \\ \tilde{t} \end{array}$

$[t]_x = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}$

$F = [t]_x A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & -2 & 0 \end{pmatrix}$ ↖ Fundamental M

$X = \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Epipolar line of x in P_1: $\lambda A_1 x + t$

$\underbrace{\lambda \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{Ax} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$ = Epipolar line

$l = e_2 \times Ax = t \times Ax = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$

$l^T \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = 0 \qquad l^T \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 0 \qquad l^T \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} = 4$

Points $(2,0)$ & $(2,1)$

Exercise 2.

$P_1 \longrightarrow C_1 = (0,0,0,1)$

P_2: Nullspace: $\left.\begin{array}{l} x + y + z + 2w = 0 \\ 2y + 2w = 0 \\ z = 0 \end{array}\right\} \begin{array}{l} x = -w \\ y = -w \\ z = 0 \end{array} \left.\right\} \begin{array}{l} x = -t \\ y = -t \\ z = 0 \\ w = t \end{array}$

$C_2 = (-1, -1, 0, 1)$

$P_1 C_2 = [I \;\; 0] \begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = e_1$ ← epipole

$P_2 C_1 = \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = e_2$

$F = [t]_x A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix}$

Exercise 2.

$\det(F) = 0$

$$F e_1 = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$e_2^T F = \begin{pmatrix} 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

Exercise 3: $\tilde{x}_1 \sim N_1 x_1$ & $\tilde{x}_2 \sim N_2 x_2$

$$N_2^T x_2^T \tilde{F} N_1 x_1 \longrightarrow F = N_2^T \tilde{F} N_1$$

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Exercise 4: $F = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

Get e_2 by computing nullspace of $F^T = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$

$$\left.\begin{aligned} y &= 0 \\ x + z &= 0 \\ x + z &= 0 \end{aligned}\right\} \begin{aligned} x &= t \\ y &= 0 \\ z &= -t \end{aligned} \rightarrow e_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \rightarrow [e_2]_x = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$[e_2]_x F = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & -2 \\ 1 & 0 & 0 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

Add 1 to row 4 to make it homogeneous

$$x_{11} = P_1 X_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad x_{12} = P_2 X_1 = \begin{pmatrix} 2 \\ -10 \\ 0 \end{pmatrix}$$

$$x_{12}^T F x_{11} = (2\ -10\ 0) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (-10\ 2\ 2) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 0$$

$$x_{21} = P_1 X_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \qquad x_{22} = P_2 X_2 = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$$

$$x_{22}^T F x_{21} = (4\ -6\ 2) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = (-6\ 6\ 6) \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = 0$$

$$x_{31} = P_1 X_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \qquad x_{32} = P_2 X_3 = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

$$x_{32}^T F x_{31} = (2\ -2\ 0) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = (-2\ 2\ 2) \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 0$$

Confirmed the epipolar constraint for all of the 3D scene points

Camera center of P_2 is a point at infinity.

From: $0 = \left[[e_2]_x F \quad e_2 \right] \begin{bmatrix} e_1 \\ 0 \end{bmatrix}$

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Exercise 6.

$$UV^T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1 & 0 \end{pmatrix}\begin{matrix} 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 \\ 0 & -1 \end{matrix} \rightarrow \det(UV^T) = 1$$

$$E = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 0 & 0 \end{pmatrix}$$

Check plausible by checking the epipolar constraint $x_2^T E x_1 = 0$

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = -\sqrt{2}/2 + \sqrt{2}/2 + 0 = 0$$

$$P_1 X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \\ s \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \longrightarrow \text{Homogenous coordinate of } x_1 = (0, 0).$$

$$[UWV^T \ u_3] = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} = L_1$$

$$[UWV^T \ -u_3] = \begin{pmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} = L_2 \qquad [UW^TV^T \ u_3] = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} = L_3$$

$$[UW^TV^T \ -u_3] = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} = L_4$$

$$L_1 X(s) = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ s \end{pmatrix} \quad L_2 X(s) = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ -s \end{pmatrix} \quad L_3 X(s) = \begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ s \end{pmatrix} \quad L_4 X(s) = \begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ -s \end{pmatrix}$$

$s = 1/\sqrt{2}$ $\qquad$ $s = -1/\sqrt{2}$ $\qquad$ $s = -1/\sqrt{2}$ $\qquad$ $s = 1/\sqrt{2}$

We want the homogenous x_2 to be $x_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

The twisted pair $L_1 = [UWV^T \ u_3]$ and $L_3 = [UW^TV^T \ u_3]$ has the point $X(s)$ in front of both cameras