Exercise 1. $x_1 = \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix} \xrightarrow{/2} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \rightarrow x_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

Homogeneous — 2D cartesian coordinate

$x_2 = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \xrightarrow{/-1} \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} \rightarrow x_2 = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$x_3 = \begin{pmatrix} 4\lambda \\ -2\lambda \\ 2\lambda \end{pmatrix} \xrightarrow{/2\lambda} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \rightarrow x_3 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

x_4 means that no transformation / translation can be applied to x_4 as the last row has 0 which will lead to the same matrix after multiplication.

Exercise 2.

with $\begin{cases} l_1^T x = 0 \\ l_2^T x = 0 \end{cases} \rightarrow \begin{cases} x + y + z = 0 \\ 3x + 2y + z = 0 \end{cases} \rightarrow \begin{matrix} 2x + y = 0 \\ 0 = 0 \end{matrix} \rightarrow \begin{cases} x = t \\ y = -2t \\ z = t \end{cases} \rightarrow$ point $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

$\begin{cases} l_3^T x = 0 \\ l_4^T x = 0 \end{cases} \rightarrow \begin{cases} x + 2y + 3z = 0 \\ x + 2y + z = 0 \end{cases} \rightarrow 2z = 0 \rightarrow \begin{cases} x = -2t \\ y = t \\ z = 0 \end{cases}$ point $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

Because $z=0$ it's a point at infinity as division by 0 is not possible. In euclidean geometry two parallell lines will never meet (per definition). In projective geometry however, they will intersect in point at infinity.

$x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ & $x_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \rightarrow$ l_i using previous answer $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Exercise 3.

Intersection point $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ because its an intersection from the rows in M then $M\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = 0$ should give a non-zero solution. It satisfies $Mx=0$

Infinitely many vectors in the nullspace $\rightarrow$ Infinite points

Exercise 4.

$$H = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \quad X_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad X_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$y_1 \,\&\, y_2 \sim Hx_1 \,\&\, Hx_2$

$$y_1 \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \longrightarrow \left.\begin{matrix} x \qquad\quad = 0 \\ x + y + z = 0 \end{matrix}\right\} \to \begin{cases} x = 0 \\ y = t \\ z = -t \end{cases}$$

$$y_2 \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad l_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\left.\begin{matrix} x + \quad z = 0 \\ y + z = 0 \end{matrix}\right\} \longrightarrow \left.\begin{matrix} x - y = 0 \\ 0 = 0 \end{matrix}\right\} \longrightarrow \begin{cases} x = t \\ y = t \\ z = -t \end{cases} \quad l_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$(H^{-1})^T l_1 = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad \text{same as } l_2$$

Proof: $0 = l_1^T x = l_1^T H^{-1} H x = ((H^{-1})^T l_1) Hx \sim l_2^T y$

Exercise 5.

Projection is given by $P \cdot X$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = \text{projection of } X_1$$

Projection of $X_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ & $X_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

Geometric interpretation the projection is a vanishing point. Where two parallell lines converge.

Camera center by finding null space. $\longrightarrow$ $\begin{matrix} x \quad = 0 \\ y \quad = 0 \\ z + w = 0 \end{matrix} \to \begin{cases} x = 0 \\ y = 0 \\ z = -1 \\ w = 1 \end{cases}$

Camera center: $(0 \;\; 0 \;\; -1 \;\; 1)$

The principal axis $= (0 \;\; 0 \;\; 1)$ $\leftarrow$ Viewing direction.