Assignment 3 FMAN95

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1 Exercises

Exercise 1. $P_1 = [I \; 0]$ and $P_2 = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$[t]_x = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}$ $\qquad t$

$F = [t]_x A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & -2 & 0 \end{pmatrix}$ ← Fundamental M

$X = \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Epipolar line of x in P_1: $\lambda A_1 x + t$

$\underbrace{\lambda \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{Ax} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$ = Epipolar line

$l = e_2 \times Ax = t \times Ax = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$

$l^T \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = 0 \qquad l^T \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 0 \qquad l^T \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} = 4$

Points $(2,0)$ & $(2,1)$

Exercise 2.

$P_1 \rightarrow C_1 = (0,0,0,1)$

P_2: Nullspace: $\left.\begin{matrix} x + y + z + 2w = 0 \\ 2y + 2w = 0 \\ z = 0 \end{matrix}\right\} \begin{matrix} x = -w \\ y = -w \\ z = 0 \end{matrix} \Bigg\} \begin{matrix} x = -t \\ y = -t \\ z = 0 \\ w = t \end{matrix}$

$C_2 = (-1, -1, 0, 1)$

$P_1 C_2 = [I \; 0] \begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = e_1$ ← epipole

$P_2 C_1 = \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = e_2$

$F = [t]_x A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix}$

Scanned with CamScanner

Exercise 2.

$\det(F) = 0$

$$Fe_1 = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$e_2^T F = \begin{pmatrix} 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

Exercise 3: $\tilde{x}_1 \sim N_1 x_1$ & $\tilde{x}_2 \sim N_2 x_2$

$N_2^T x_2^T \tilde{F} N_1 x_1 \longrightarrow F = N_2^T \tilde{F} N_1$

Exercise 4: $F = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

Get e_2 by computing nullspace of $F^T = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$

$$\left.\begin{aligned} y &= 0 \\ x + z &= 0 \\ x + z &= 0 \end{aligned}\right\} \begin{aligned} x &= t \\ y &= 0 \\ z &= -t \end{aligned} \rightarrow e_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \rightarrow [e_2]_\times = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$[e_2]_\times F = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & -2 \\ 1 & 0 & 0 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

Add 1 to row 4 to make it homogenous

$$x_{11} = P_1 X_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad x_{12} = P_2 X_1 = \begin{pmatrix} 2 \\ -10 \\ 0 \end{pmatrix}$$

$$x_{12}^T F x_{11} = (2 \ -10 \ 0) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (-10 \ 2 \ 2) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 0$$

$$x_{21} = P_1 X_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \qquad x_{22} = P_2 X_2 = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$$

$$x_{22}^T F x_{21} = (4 \ -6 \ 2) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = (-6 \ 6 \ 6) \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = 0$$

$$x_{31} = P_1 X_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \qquad x_{32} = P_2 X_3 = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

$$x_{32}^T F x_{31} = (2 \ -2 \ 0) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = (-2 \ 2 \ 2) \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 0$$

Confirmed the epipolar constraint for all of the 3D scene points

Camera center of P_2 is a point at infinity.

From: $0 = \left[[e_2]_\times F \quad e_2 \right] \begin{bmatrix} e_1 \\ 0 \end{bmatrix}$

Exercise 6.

$$UV^T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1 & 0 \end{pmatrix}\begin{matrix} 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 \\ 0 & -1 \end{matrix} \rightarrow \det(UV^T) = 1$$

$$E = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 0 & 0 \end{pmatrix}$$

Check plausible by checking the epipolar constraint $x_2^T E x_1 = 0$

$$(1 \;\; 1 \;\; 1)\begin{pmatrix} \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = -\sqrt{2}/2 + \sqrt{2}/2 + 0 = 0$$

$$P_1 X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \\ s \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \longrightarrow \text{Homogenous coordinate of } x_1 = (0, 0).$$

$$[UWV^T \;\; u_3] = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} = L_1$$

$$[UWV^T \;\; -u_3] = \begin{pmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} = L_2 \qquad [UW^TV^T \;\; u_3] = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} = L_3$$

$$[UW^TV^T \;\; -u_3] = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} = L_4$$

$$\underbrace{L_1 X(s) = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ s \end{pmatrix}}_{s = 1/\sqrt{2}} \qquad L_2 X(s) = \underbrace{\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ -s \end{pmatrix}}_{s = -1/\sqrt{2}} \qquad L_3 X(s) = \underbrace{\begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ s \end{pmatrix}}_{s = -1/\sqrt{2}} \qquad L_4 X(s) = \underbrace{\begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ -s \end{pmatrix}}_{s = 1/\sqrt{2}}$$

We want the homogenous x_2 to be $x_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\rightarrow$ The twisted pair $L_1 = [UWV^T \;\; u_3]$ and $L_3 = [UW^TV^T \;\; u_3]$ has the point $X(s)$ in front of both cameras

2 The Fundamental Matrix

Computer exercise 1

The determinant of F is very close to zero, 9.35*e(-19) (after using formula 7.33 from the lecture notes), same with the epipolar constraint with them hovering around zero. Also checked the euclidean norm of Mv to be 1.88*e-17 which is close to zero. The mean distance is 18.3 when normalized and 59 without normalization. Unfortunately I'm having issues with translating rital.m to python code. Currently trying to make axline work which I used in assignment 1.

$$\text{The fundamental matrix: } \begin{bmatrix} 0.2 & -16.1 & 41 \\ 13.3 & -0.8 & -186.5 \\ -35 & 184.6 & 1 \end{bmatrix}$$

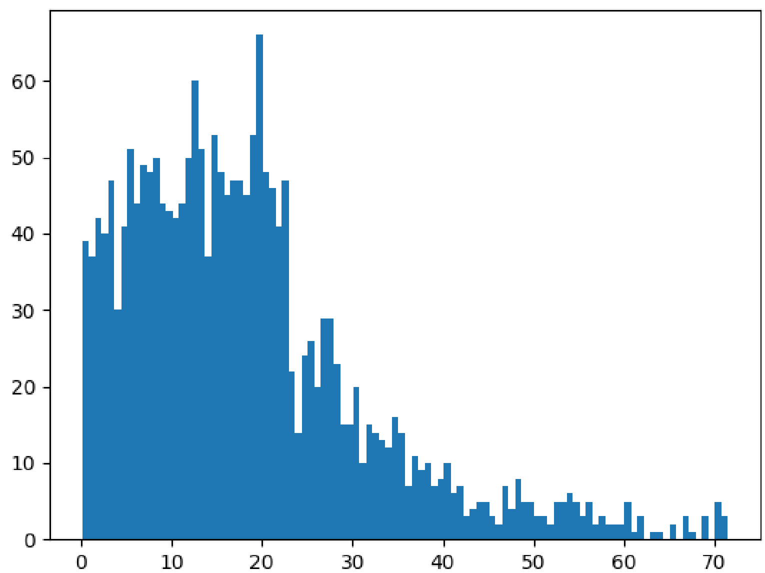

Figure 1: The distances between a point and it's epipolar line using normalized F, presented in a histogram.

Computer exercise 2

Figure 3-4 looks to be correct. Figure 5 shows the 3D points which is not at all what I expected. If the two parts of the image points are divided into the upper and lower cluster, they together form a parenthesis. I interpret it as the points being behind and in front of the camera (as the smaller cluster looks to be upside down like in figure 6.

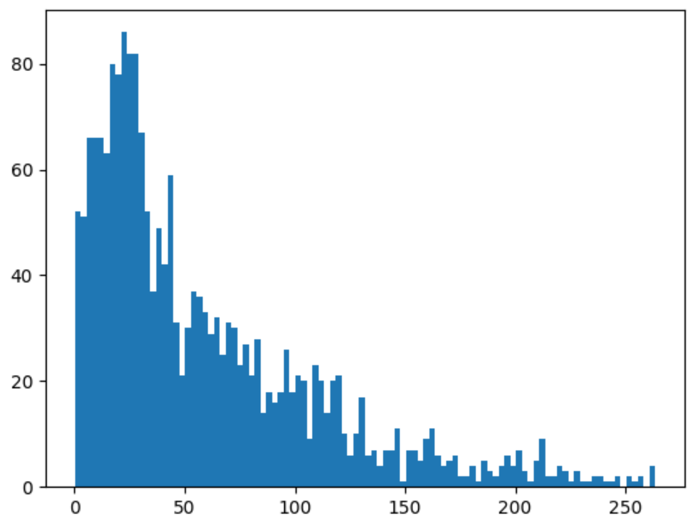

Figure 2: The distances between a point and it's epipolar line without normalizing F, presented in a histogram.

2.1 The Essential Matrix

Computer exercise 3
The determinant of E is 8.74*e-18 which is close to zero. The epipolar constraint is also hovering around zero ± 0.03. We also check that Mv is close to zero being 0.007.

The fundamental matrix: $\begin{bmatrix} -8.9 & 1252.5 & -472.8 \\ -1005.8 & 78.4 & 2550.2 \\ 377.1 & -2448.2 & 1 \end{bmatrix}$

The mean average distance from point to epipolar line is 33.8 which is between the normalized and un-normalized values in computer exercise 1. The distances from the points x2 to epipolar lines are definitely more evenly distributed compared to the first computer exercise. The problem with rital and getting the epipolar lines to properly show still exists here.

Computer exercise 4
The camera matrix P2a

$$[UWV^T u3]$$

had 1249 points in front of the camera (see figure 8).
The camera matrix P2b

$$[UWV^T - u3]$$

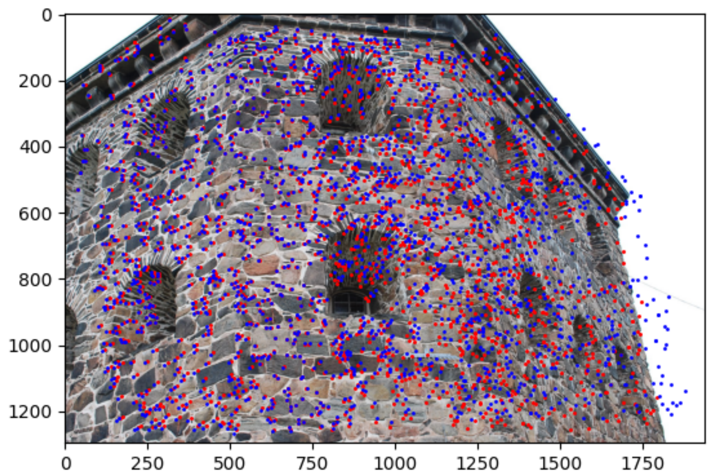

Figure 3: The image points and the projected 3D points for image 1. Blue are the projected.

had 759 points in front of the camera (see figure 9).
The camera matrix P2c

$$[UW^TV^Tu3]$$

had 2008 points in front of the camera (see figure 10).
The camera matrix P2d

$$[UW^TV^T - u3]$$

had 0 points in front of the camera (see figure 11).

The plots of the first two cameras are hard to understand but in P2c and P2d you can see the contour of the building. The camera matrix had all of the points in front of the camera which is the best of all of them.

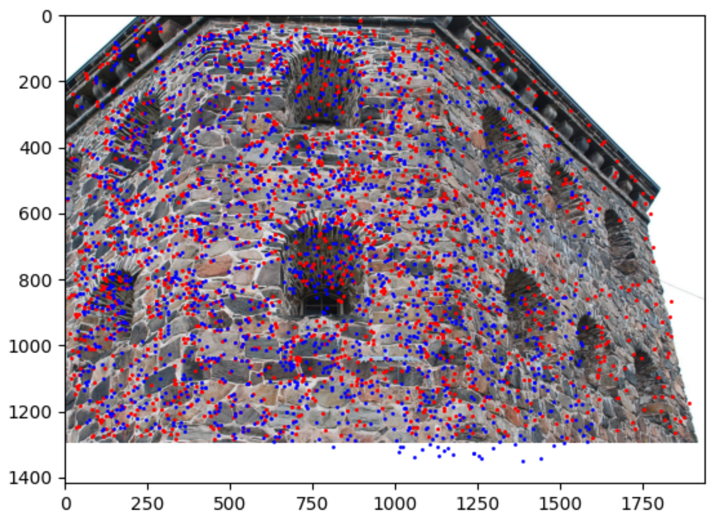

Figure 4: The image points and the projected 3D points for image 2. Blue are the projected.

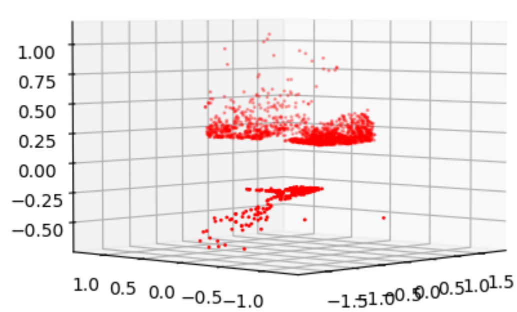

Figure 5: The 3D points

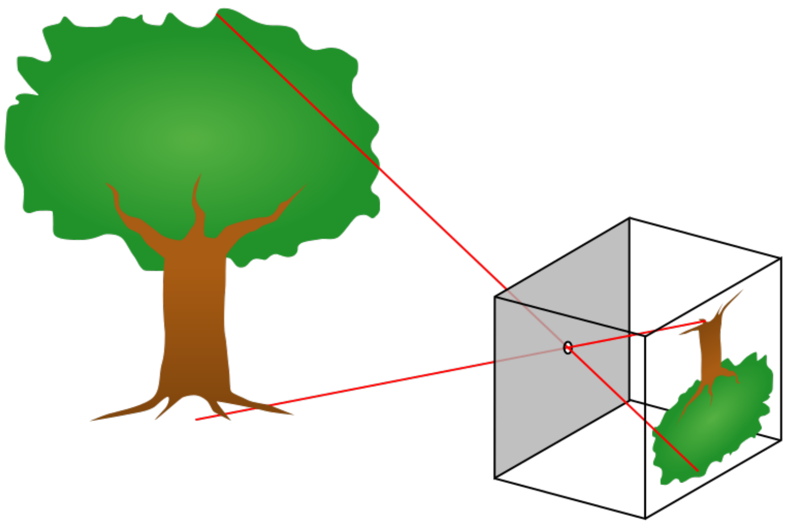

Figure 6: Pinhole camera example

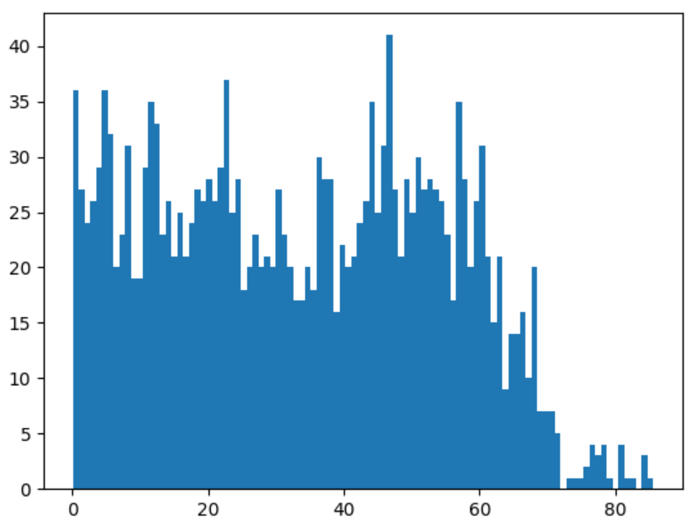

Figure 7: Distances from points to corresponding epipolar line

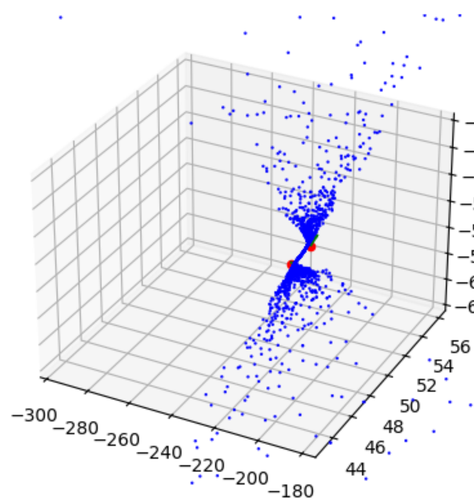

Figure 8: 3D points gotten from triangulation using P2a

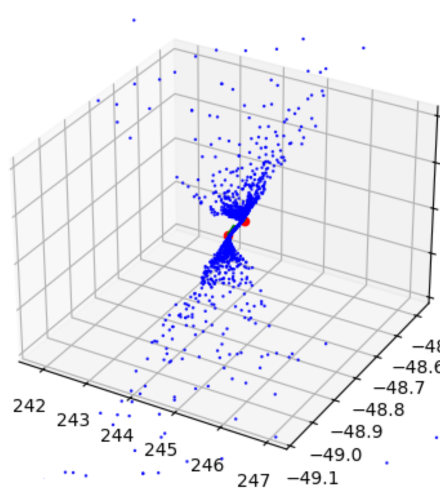

Figure 9: 3D points gotten from triangulation using P2b

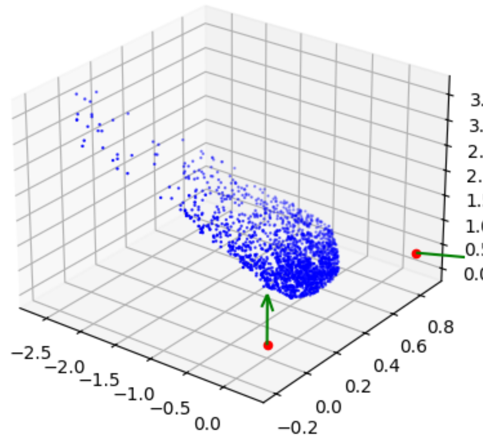

Figure 10: 3D points gotten from triangulation using P2c

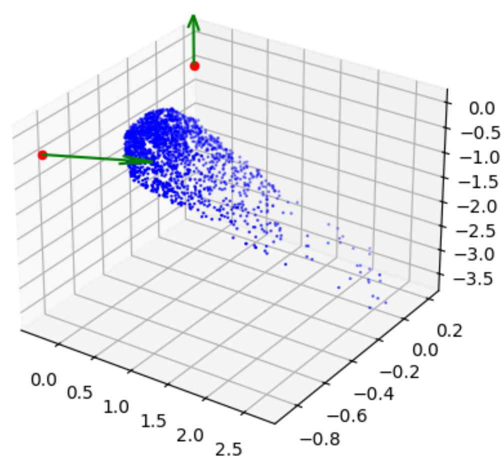

Figure 11: 3D points gotten from triangulation using P2d