Introduction to GNNs and Self-supervised Learning on Graphs

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BigData Academy MADE from Mail.ru Group

Graph Neural Networks and Applications



Class Details

- Instructor: Ilya Makarov
- Tutor: Vitaly Pozdnyakov, Dmitrii Kiselev
- Invited lecturers: to be announced
- Course length: 11-12 lectures/classes
- Lecturer's Telegram: @iamakarov (urgent)
- Tutor's Telegram: @pozdnyakov_vitaliy (urgent), @dkiselev (urgent)
- Discord (questions, deadlines, grading)
- Programming: Python, iPython notebooks, Anaconda distribution
- Python libraries: NetworkX, pyG, DGL

Prerequisites

- Network Science
- Deep Learning
- Machine Learning
- Differential Equations
- Programming in Python

- "CS224W: Machine Learning with Graphs". Jure Leskovec. Stanford, 2020-2021.
- "SNAP Project". http://snap.stanford.edu/
- "PyG & PyG Temporal".
 https://pytorch-geometric.readthedocs.io/en/latest/,
 https://pytorch-geometric-temporal.readthedocs.io/en/latest/
- "Network Science", Albert-Laszlo Barabasi, Cambridge University Press, 2016. http://networksciencebook.com
- "Network Science" course by Leonid E. Zhukov.

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Topics (max list)

- Self-supervised learning on graphs
- Subgraph Embedding & Deep Sets
- Scaling GNNs: Efficient models, Sampling models
- Ompressing GNNs: Quantization, Distillation
- Deep Generative Graph Models & adversarial robustness
- Temporal graph embeddings
- GNN Explanations
- GNN & RecSys & KG
- Query Embeddings for KGs
- Transport GNNs + Combinatorics
- Guest lectures: Applications to Biology, Medicine, Bioinformatics, Chemistry, Physics
- OGB benchmark (optional challenge)

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Network Science

Recap

→ Go to Lecture 1: SSL on Graphs

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Network Science

- Real-world networks
- Random graphs
- Ocentrality & Prestige node measures
- Structural similarity in networks
- Community detection
- Modeling dynamics on graphs
 - continuous: diffusion, epidemics
 - discrete: information propagation, segregation
- Graph Machine Learning:
 - label propagation and node similarities
 - node and edge embeddings
 - graph neural networks
- Knowledge Graphs

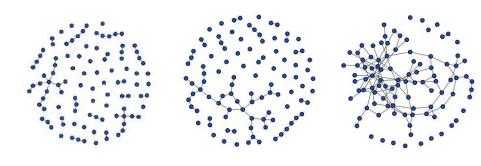
Complex networks (nor regular, nor random)

- 1 Power law node degree distribution: "scale-free" networks
- Small diameter and average path length: "small world" networks
- 4 Hight clustering coefficient: transitivity

Random graph model

Consider $G_{n,p}$ as a function of p

- p = 0, empty graph $\langle k \rangle = 0$
- p=1, complete (full) graph $\langle k \rangle = n-1$
- n_G -largest connected component, $s = \frac{n_G}{n}$

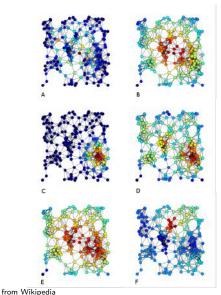


Random graph models comparison

	Random	BA model	WS model	Empirical networks
P(k)	$\frac{\lambda^k e^{-\lambda}}{k!}$	k^{-3}	poisson like	power law
C	$\langle k \rangle / N$	$N^{-0.75}$	const	large
$\langle L \rangle$	$\frac{\log(N)}{\log(\langle k \rangle)}$	$\frac{\log(N)}{\log\log(N)}$	log(N)	small

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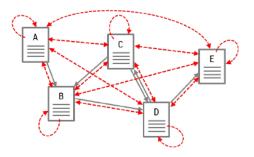
Centrality examples



- A) Betweenness centrality
- B) Closeness centrality
- C) Eigenvector centrality
- D) Degree centrality
- F) Harmonic centrality
- E) Katz centrality

PageRank

"PageRank can be thought of as a model of user behavior. We assume there is a "random surfer" who is given a web page at random and keeps clicking on links, never hitting "back" but eventually gets bored and starts on another random page. The **probability** that the random surfer visits a page is its **PageRank**."



Sergey Brin and Larry Page, 1998

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PageRank formulation

Power iterations:

$$\mathbf{p} \leftarrow \alpha \mathbf{P}^T \mathbf{p} + (1 - \alpha) \frac{\mathbf{e}}{\mathbf{n}}, \quad \alpha \text{ - teleportation coefficient}$$

Sparse linear system:

$$(I - \alpha P^T)p = (1 - \alpha)\frac{e}{n}$$

• Eigenvalue problem ($\lambda = 1$):

$$(\alpha P^T + (1 - \alpha)E) p = \lambda p$$

$$P = D^{-1}A$$

PageRank variations

Power iterations

$$\begin{aligned} \mathbf{p} \leftarrow \alpha \mathbf{P}^T \mathbf{p} + (1 - \alpha) \mathbf{v}, & \mathbf{v} & \text{- teleportation vector} \\ \mathbf{P}' &= \alpha \mathbf{P} + (1 - \alpha) \mathbf{e} \mathbf{v}^T \\ & \mathbf{p} \leftarrow {\mathbf{P}'}^T \mathbf{p}, \ ||\mathbf{p}|| = 1 \end{aligned}$$

• Topic specific PageRank

v - set of pages on specific topics

TrustRank

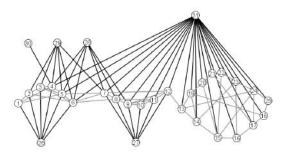
Personalized PageRank

v - set of personal preference pages

Structural similarity

Definition

Two nodes are similar to each other if they share many neighbors.



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Similarity measures

Jaccard similarity

$$J(v_i, v_j) = \frac{|\mathcal{N}(v_i) \cap \mathcal{N}(v_j)|}{|\mathcal{N}(v_i) \cup \mathcal{N}(v_j)|}$$

Cosine similarity (vectors in n-dim space)

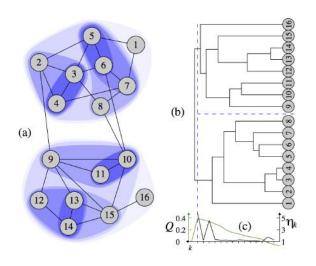
$$\sigma(v_i, v_j) = cos(\theta_{ij}) = \frac{v_i^T v_j}{|v_i| |v_j|} = \frac{\sum_k A_{ik} A_{kj}}{\sqrt{\sum_i A_{ik}^2} \sqrt{\sum_i A_{jk}^2}}$$

Pearson correlation coefficient:

$$r_{ij} = \frac{\sum_{k} (A_{ik} - \langle A_i \rangle) (A_{jk} - \langle A_j \rangle)}{\sqrt{\sum_{k} (A_{ik} - \langle A_i \rangle)^2} \sqrt{\sum_{k} (A_{jk} - \langle A_j \rangle)^2}}$$

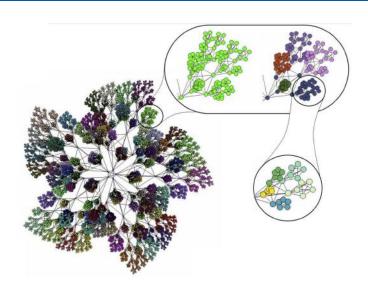
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Walktrap



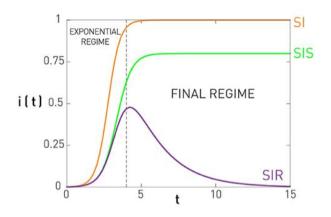
P. Pons and M. Latapy, 2006

Fast community unfolding



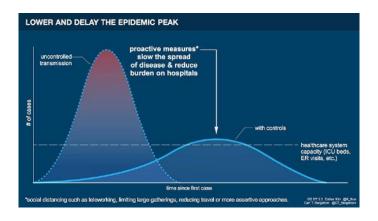
V. Blondel et.al., 2008

Compartmental models summary



from Barabasi, 2016

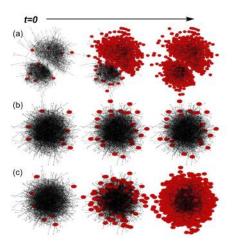
Flatten the curve!



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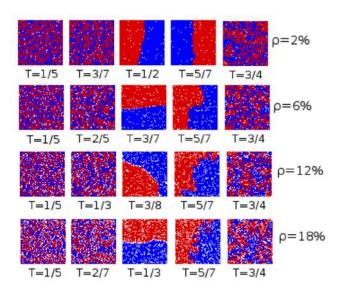
Cascades in random networks

multiple seed nodes



(a) Empirical network; (b), (c) - randomized network P. Singh, 2013

Spatial segregation



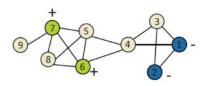
L. Gauvin et.al. 2009

Graph machine learning

- Node classification (attribute inference)
- Link prediction (missing/hidden links inference)
- Community detection (clustering nodes in graph)
- Graph visualization (cluster projections)

Node classification

- Node classification labeling of all nodes in a graph structure
- Subset of nodes is labeled: categorical/numeric/binary values
- Extend labeling to all nodes on the graph (class/class probability/regression)
- Classification in networked data, network classification, structured inference, relational learning



Label propagation

Algorithm: Label propagation, Zhu et. al 2002

Input: Graph
$$G(V, E)$$
, labels Y_I

Output: labels
$$\hat{Y}$$

Compute
$$D_{ii} = \sum_{j} A_{ij}$$

Compute
$$P = D^{-1}A$$

Initialize
$$Y^{(0)} = (Y_I, 0)$$
, t=0

repeat

$$Y^{(t+1)} \leftarrow P \cdot Y^{(t)}$$

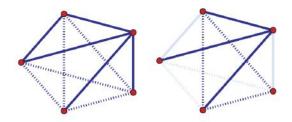
 $Y^{(t+1)}_{l} \leftarrow Y^{(t)}_{l}$

until $Y^{(t)}$ converges;

$$\hat{Y} \leftarrow Y^{(t)}$$

Solution:
$$\hat{Y} = \lim_{t \to \infty} Y^{(t)} = (I - P_{uu})^{-1} P_{ul} Y_l$$

Link prediction



- Graph G(V,E)
- Number of "missing edges": |V|(|V|-1)/2 |E|
- ullet In sparse graphs $|E| \ll |V|^2$, Prob. of correct random guess $O(rac{1}{|V|^2})$

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Local similarity indices

Local neighborhood of v_i and v_j

Number of common neighbors:

$$s_{ij} = |\mathcal{N}(v_i) \cap \mathcal{N}(v_j)|$$

Jaccard's coefficient:

$$s_{ij} = \frac{|\mathcal{N}(v_i) \cap \mathcal{N}(v_j)|}{|\mathcal{N}(v_i) \cup \mathcal{N}(v_j)|}$$

Resource allocation:

$$s_{ij} = \sum_{w \in \mathcal{N}(v_i) \cap \mathcal{N}(v_i)} \frac{1}{|\mathcal{N}(w)|}$$

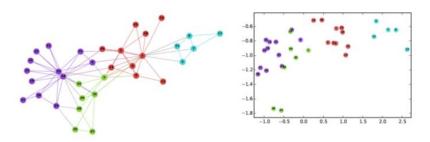
Adamic/Adar:

$$s_{ij} = \sum_{w \in \mathcal{N}(v_i) \cap \mathcal{N}(v_i)} \frac{1}{\log |\mathcal{N}(w)|}$$

Liben-Nowell and Kleinberg, 2003

Graph Embeddings

- Necessity to automatically select features
- Reduce domain- and task- specific bias
- Unified framework to vectorize network
- Preserve graph properties in vector space
- ullet Similar nodes o close embeddings

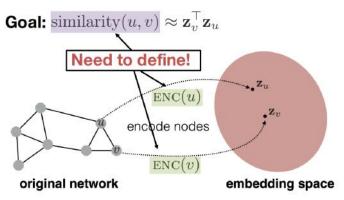


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http://snap.stanford.edu/proj/embeddings-www/

Graph Embeddings

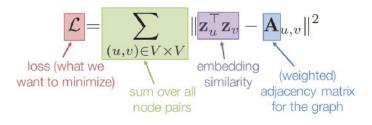
- Define Encoder
- Define Similarity/graph feature to preserve graph properties
- Define similarity/distance in the embedding space
- Optimize loss to fit embedding with similarity computed on graph



from Leskovec et al., 2018

First-order Proximity

- Similarity between u and v is A_{uv}
- MSE Loss
- Variant of Matrix Decomposition

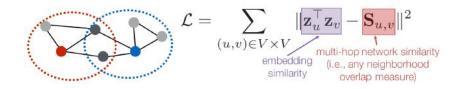


from Leskovec et al., 2018

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Multi-order Proximity

• Similarity score S_{uv} as Jaccard/Common Neighbours, etc. (HOPE)



Weighted k-hop paths with different k (GraRep)

$$\tilde{\mathbf{A}}_{i,j}^k = \max \left(\log \left(\frac{(\mathbf{A}_{i,j}/d_i)}{\sum_{l \in V} (\mathbf{A}_{l/j}/d_l)^k} \right)^k - \alpha, 0 \right)$$
 node degree constant shift

from Leskovec et al., 2018

Even worse complexity

- Similarity between u and v is probability to co-occur on a random walk
- Sample each vertex u neighborhood $N_R(u)$ (multiset) by short random walks via strategy R
- Optimize similarity considering independent neighbor samples via MLE (remind Word2Vec)

$$\mathcal{L} = \sum_{u \in V} \sum_{v \in N_R(u)} -\log(P(v|\mathbf{z}_u))$$

from Leskovec et al., 2018

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Random Walks

• $P(v|z_u)$ is approximated via softmax over similarity $z_u^T \cdot z_v$

$$\mathcal{L} = \sum_{u \in V} \sum_{v \in N_R(u)} -\log \left(\frac{\exp(\mathbf{z}_u^{\top} \mathbf{z}_v)}{\sum_{n \in V} \exp(\mathbf{z}_u^{\top} \mathbf{z}_n)} \right)$$

- ullet Problem in second Σ over all nodes
- Hard to find optimal solution

• Use Negative Sampling to approximate denominator

$$\begin{split} \log \left(\frac{\exp(\mathbf{z}_u^\top \mathbf{z}_v)}{\sum_{n \in V} \exp(\mathbf{z}_u^\top \mathbf{z}_n)} \right) & \text{random distribution} \\ & \approx \log(\sigma(\mathbf{z}_u^\top \mathbf{z}_v)) - \sum_{i=1}^k \log(\sigma(\mathbf{z}_u^\top \mathbf{z}_{n_i})), n_i \sim P_V \end{split}$$

from Leskovec et al., 2018

34 / 84

- Sample in proportion to node degree
- Experiment with k to impact negative prior and robustness
- No need to sample non-connected edges same as random

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Feature representation

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(MADE)

- How to construct pair of nodes representation having node embeddings?
- Will it be more efficient than $\sigma(z_i^t \cdot z_j)$

Symmetry operator	Definition		
Average	$\frac{f_i(u) + f_i(v)}{2}$		
Hadamard	$f_i(u) \cdot f_i(v)$		
Weighted-L ₁	f(u) - f(v)		
Weighted-L ₂	$(f(u) - f(v))^2$		
Neighbor Weighted-L ₁	$\left \frac{\sum_{u\in N(u)\cup\{u\}}f_i(w)}{ N(u) +1}-\frac{\sum_{t\in N(v)\cup\{v\}}f_i(t)}{ N(v) +1}\right $		
Neighbor Weighted-L ₂	$\left(\frac{\sum_{u \in N(u) \cup \{u\}} f_i(u)}{ N(u) + 1} - \frac{\sum_{t \in N(v) \cup \{v\}} f_i(t)}{ N(v) + 1}\right)^2$		

DOI: 10.7717/peerj-cs.172/table-2

from Makarov et al., 2019

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35 / 84

Lecture 1

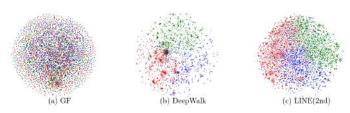
Random Walk Strategies

- Finite unbiased random walks (DeepWalk)
- 1-hops & 2-hops for half of the embedding, arbitrary random walks for the second half of the embedding (LINE)
- Diffusion for sampling (Diff2Vec)
- Biased random walks combining BFS and DFS (Node2Vec)

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DeepWalk & LINE

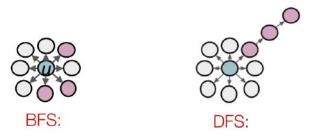
- DeepWalk: Unbiased random walks with fixed length and number
- LINE: Combination of first-order and second-order proximity aggregation via concatenation



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Node2Vec

BFS samples local neighborhood, DFS goes for global features

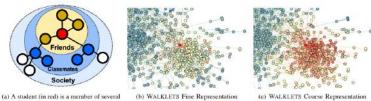


- \bullet Two parameters p and q to control sampling
- Second order Markov process

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Grarep & Walklets

- Grarep: Approximate normalized A^k efficiently
- Walklets: approximate attention over k-distance neighbors

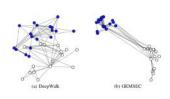


increasing larger social communities.

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GEMSEC

- GEMSEC model cluster information adding regularization term over community labels given number of classes
- \bullet parameter γ balance cluster error and cluster structure given model hyper-parameters



$$\mathcal{L} = \underbrace{\sum_{v \in V} \left[\ln \left(\sum_{u \in V} \exp(f(v) \cdot f(u)) \right) - \sum_{n_i \in N_S(v)} f(n_i) \cdot f(v) \right]}_{\text{Embedding cost}} + \underbrace{\gamma \cdot \sum_{v \in V} \min_{c \in C} \left\| f(v) - \mu_c \right\|_2}_{\text{Clustering cost}}.$$
 (5)

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VERSE for Structural Similarity

- VERSE uses conditional probability not on the embedding, but on the similarity rank.
- O(|E|) performance

$$\begin{split} sim_{\mathbf{E}}(v,\cdot) &= \frac{\exp(W_v W^\top)}{\sum_{i=1}^n \exp\left(W_v \cdot W_i\right)} \\ \mathcal{L} &= -\sum_{v \in V} sim_{\mathbf{G}}(v,\cdot) \log\left(sim_{\mathbf{E}}\left(v,\cdot\right)\right) \\ \mathcal{L}_{NCE} &= \sum_{u \sim \mathcal{P}} \left\lfloor \log \Pr_{W}(D=1|sim_{\mathbf{E}}(u,v)) + \sum_{v \sim sim_{\mathbf{G}}(u,\cdot)} \log \Pr_{W}(D=0|sim_{\mathbf{E}}(u,\widetilde{v})) \right\rfloor \end{split}$$

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Graph Neural Networks

GNN

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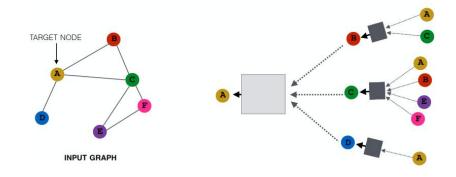
Graph Neural Network: Setting

- We have a graph G(V,E) defined by adjacency matrix A and feature matrix $X \in \mathbb{R}^{f,|V|}$
- Confirmed relation between closeness of feature space and graph structure
- Non-graph features are vectorized separately (images, texts, one-hot encoding for labels, numeric features)

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Graph Neural Network: Idea

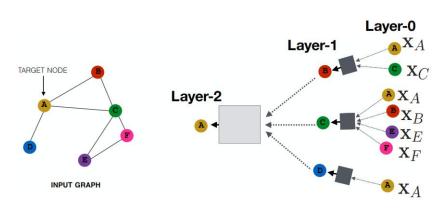
- Assign weights only to information obtained from neighbors
- Include node itself via loop with trainable weight
- Each node generate its own computational graph



from Leskovec et al., 2018

Graph Neural Network: Layer structure

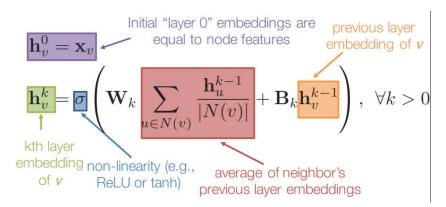
- Each aggregation defines new layer
- Zero-level embedding is non-graph feature
- Arbitrary depth but remember on "law of six handshakes"



from Leskovec et al., 2018

Graph Neural Network: Basic Approach

- Aggregation over weighted sum of neighbor input and node itself under non-linearity
- Use simple neural network construction



from Leskovec et al., 2018

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Graph Convolutions

GCN

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Graph Convolutional Network

- Aggregation over shared weights between node and its neighbors
- Normalization to stabilize training for high-degree nodes

Basic Neighborhood Aggregation

$$\mathbf{h}_{v}^{k} = \sigma \left(\mathbf{W}_{k} \sum_{u \in N(v)} \frac{\mathbf{h}_{u}^{k-1}}{|N(v)|} + \mathbf{B}_{k} \mathbf{h}_{v}^{k-1} \right)$$

VS.

GCN Neighborhood Aggregation

$$\mathbf{h}_{v}^{k} = \sigma \left(\mathbf{W}_{k} \sum_{u \in N(v) \cup v} \frac{\mathbf{h}_{u}^{k-1}}{\sqrt{|N(u)||N(v)|}} \right)$$

same matrix for self and neighbor embeddings

per-neighbor normalization

Graph Convolutional Network

- Efficient batch computation in matrix form
- Obtained O(|E|) complexity (see pyG, DGL libraries)

$$\mathbf{H}^{(k+1)} = \sigma \left(\mathbf{D}^{-\frac{1}{2}} \tilde{\mathbf{A}} \mathbf{D}^{-\frac{1}{2}} \mathbf{H}^{(k)} \mathbf{W}_k \right)$$
$$\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{I}$$
$$\mathbf{D}_{ii} = \sum_{j} \mathbf{A}_{i,j}$$

from Leskovec et al., 2018

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Graph ATtention

GAT

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Graph ATtention Network

Not all the neighbors are equal

$$\begin{split} e_{ij} &= a(\mathbf{W}\vec{h}_i, \mathbf{W}\vec{h}_j) \\ \alpha_{ij} &= \mathrm{softmax}_j(e_{ij}) = \frac{\exp(e_{ij})}{\sum_{k \in \mathcal{N}_i} \exp(e_{ik})} \\ \alpha_{ij} &= \frac{\exp\left(\mathrm{LeakyReLU}\left(\vec{\mathbf{a}}^T[\mathbf{W}\vec{h}_i \| \mathbf{W}\vec{h}_j]\right)\right)}{\sum_{k \in \mathcal{N}_i} \exp\left(\mathrm{LeakyReLU}\left(\vec{\mathbf{a}}^T[\mathbf{W}\vec{h}_i \| \mathbf{W}\vec{h}_i]\right)\right)} \end{split}$$

is the concatenation operation.

$$\vec{h}_i' = \sigma \left(\sum_{j \in \mathcal{N}_i} \alpha_{ij} \mathbf{W} \vec{h}_j \right)$$

Graph ATtention Network

- Multi-head attention works better like in different convolution filters
- Final layer require pooling isntead of concatenation

$$\vec{h}'_i = \sigma \left(\sum_{j \in \mathcal{N}_i} \alpha_{ij} \mathbf{W} \vec{h}_j \right)$$

$$\vec{h}'_i = \prod_{k=1}^K \sigma \left(\sum_{j \in \mathcal{N}_i} \alpha_{ij}^k \mathbf{W}^k \vec{h}_j \right)$$

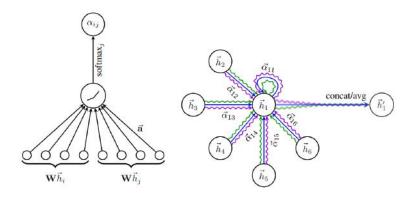
$$\vec{h}'_i = \sigma \left(\frac{1}{K} \sum_{k=1}^K \sum_{j \in \mathcal{N}_i} \alpha_{ij}^k \mathbf{W}^k \vec{h}_j \right)$$

from Bengo et al., 2018

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Graph ATtention Network

- Feature aggregation via attention over learned weights
- Different patterns for the same structure



from Bengo et al., 2018

GraphSAGE

GraphSAGE: Feature Pyramid

- Vary feature space across layers
- Aggregate from neighbors and concatenate with self-representation

Simple neighborhood aggregation:

$$\mathbf{h}_{v}^{k} = \sigma \left(\mathbf{W}_{k} \sum_{u \in N(v)} \frac{\mathbf{h}_{u}^{k-1}}{|N(v)|} + \mathbf{B}_{k} \mathbf{h}_{v}^{k-1} \right)$$

GraphSAGE:

concatenate self embedding and neighbor embedding

$$\mathbf{h}_{v}^{k} = \sigma\left(\left[\mathbf{W}_{k} \cdot \overline{\mathbf{AGG}\left(\left\{\mathbf{h}_{u}^{k-1}, \forall u \in N(v)\right\}\right)}, \mathbf{B}_{k} \mathbf{h}_{v}^{k-1}\right]\right)$$

generalized aggregation

from Leskovec et al., 2018

Mean:

$$AGG = \sum_{u \in N(v)} \frac{\mathbf{h}_u^{k-1}}{|N(v)|}$$

Pool

$$\mathrm{AGG} = \sqrt{\left(\{\mathbf{Q}\mathbf{h}_u^{k-1}, \forall u \in N(v)\}\right)}$$

LSTM:

Apply LSTM to random permutation of neighbors.

$$AGG = LSTM ([\mathbf{h}_u^{k-1}, \forall u \in \pi(N(v))])$$

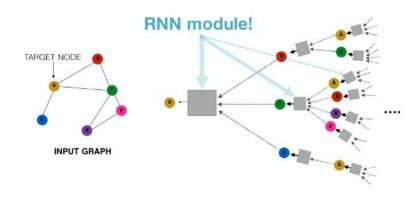
from Leskovec et al., 2018

How to fight dimension curse

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Model Depth

- Usually 2-3 layers for GCN / GraphSAGE
- More layers make method global
- Computation graph exceed memory limits
- Overfitting, vanishing gradient



from Leskovec et al., 2018

Gated GNN

 Use recurrent model with shared weights across all the layers, support any depth

1. Get "message" from neighbors at step k:

$$\mathbf{m}_v^k = \mathbf{W} \sum_{u \in N(v)} \mathbf{h}_u^{k-1}$$
 aggregation function does not depend on \mathbf{k}

2. Update node "state" using <u>Gated Recurrent</u> <u>Unit (GRU)</u>. New node state depends on the old state and the message from neighbors:

$$\mathbf{h}_v^k = \mathrm{GRU}(\mathbf{h}_v^{k-1}, \mathbf{m}_v^k)$$

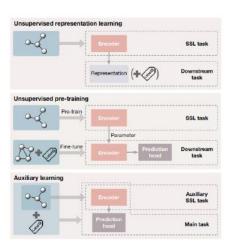
from Leskovec et al., 2018

Already too complex? What comes next?

I. Makarov et al. (MADE) Lecture 1 07.10.2021 60 / 84

Self-supervised Learning

• Train data representation before using it in downstream tasks



from Ji et al., 2021

Self-supervised Learning

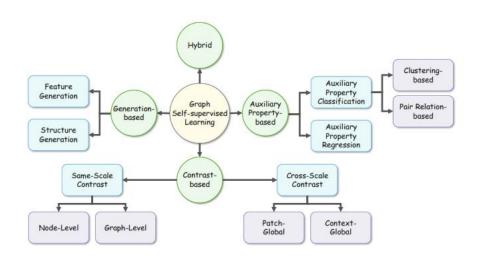
• Various data type lead to different data augmentation strategies



from Zhou et al., 2021

I. Makarov et al. (MADE) Lecture 1 07.10.2021 62 / 84

SSL for Graphs



from Zhou et al., 2021

SSL for Graphs

(a) Contrastive Method

$$F(t) \rightarrow F(t) \rightarrow$$

(b) Generative Method

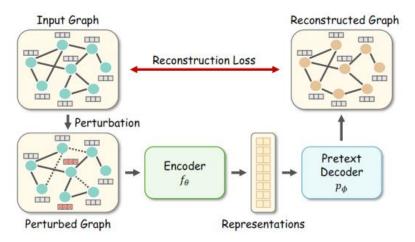
$$\begin{array}{c|c} \text{Label} & \text{Self-generated} \\ \text{Generation} & \text{Labels} \end{array}$$

(c) Predictive Method

from Li S. et al., 2021

1. Generative SSL

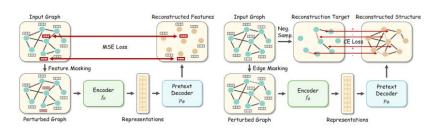
- Use generative models trained via reconstruction loss
- Model input is generated by (optional) graph perturbation. Decoder tries to recover the original graph



I. Makarov et al. (MADE) Lecture 1 07.10.2021 65 / 84

1. Generative SSL methods

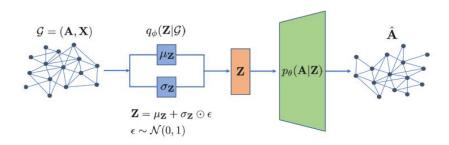
Attribute or structure reconstruction/prediction



from Zhou et al., 2021

1. Generative SSL: Variational Graph Auto-Encoder

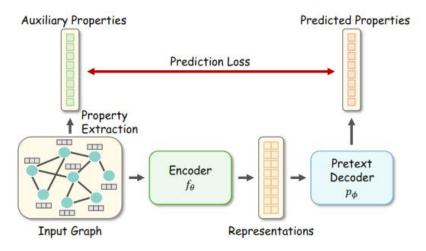
- VGAE is a variational counter-part of GAE
- Instead of direct reconstruction in GAE via LPP, VGAE learns a conditional distribution over adjacency matrices generated from embeddings



from Hamilton W.L., 2020

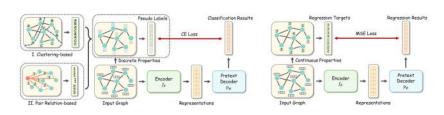
2. Predictive SSL

- Auxiliary properties are extracted from graphs freely
- \bullet Classification or regression is trained via CE/MSE loss



2. Predictive SSL methods

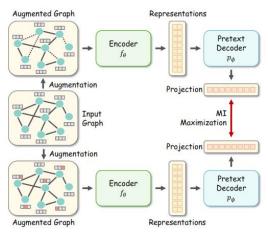
• Clustering/similarity/centrality/prestige based labels for CE/MSE loss



from Zhou et al., 2021

3. Contrastive SSL

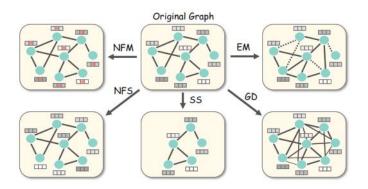
- Two different augmented views are constructed from the original graph.
- Model is trained by maximizing the Mutual Information (MI) between two views.



I. Makarov et al. (MADE) Lecture 1 07.10.2021 70 / 84

3. Contrastive SSL: Graph augmentations

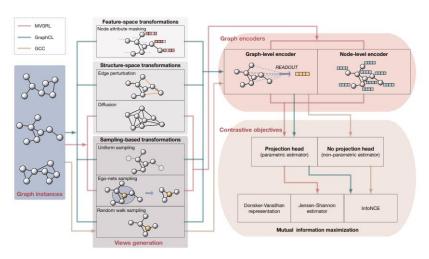
 Typical graph augmentations: Node Feature Masking (NFM), Node Feature Shuffle (NFS), Edge Modification (EM), Graph Diffusion (GD), and Subgraph Sampling (SS).



from Zhou et al., 2021

3. Contrastive SSL: Graph augmentations

• Different combinations of augmentations



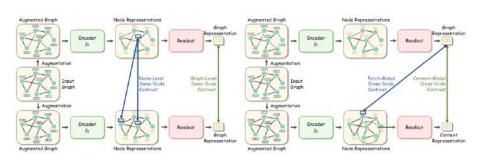
Ji et al., 2021

72 / 84

I. Makarov et al. (MADE) Lecture 1 07.10.2021

3. Contrastive SSL methods

Hierarchical augmentations on different graph levels

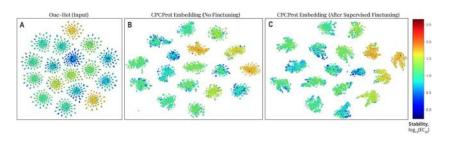


from Zhou et al., 2021

I. Makarov et al. (MADE) Lecture 1 07.10.2021 73 / 84

3. Contrastive SSL methods: SSL for Protein sequences

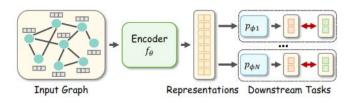
 Encoding protein sequences one can use contrastive learning to group common sequences by either, sequential patterns of predicted labels from downstream tasks



from Moses et al., 2020

4. Hybrid SSL methods

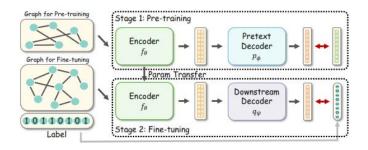
 Multiple pretext tasks are designed to train the model together in a multi-task learning manner.



from Zhou et al., 2021

Training SSL models: Pretrain and Finetune

- Encoder is pretrained with pretext tasks in an unsupervised manner.
- Pretrained parameters are leveraged as the initial parameters in the fine-tuning phase for downstream tasks.

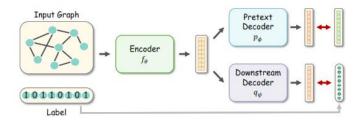


from Zhou et al., 2021

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Training SSL models: Joint Learning

 The model is trained with pretext and downstream tasks simultaneously in a multi-task learning setting.

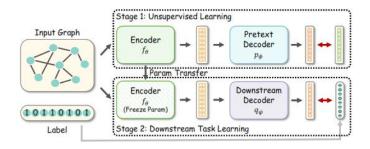


from Zhou et al., 2021

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Training SSL models: Unsupervised Learning

- Training encoder with pretext tasks.
- Use obtained representations to learn the downstream decoder in the second phase.



from Zhou et al., 2021

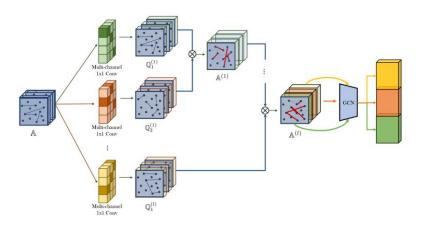
78 / 84

What about transformers?

I. Makarov et al. (MADE) Lecture 1 07.10.2021 79 / 84

Graph Transformers

- Heterogeneous graph aggregation
- SSL Pretraining

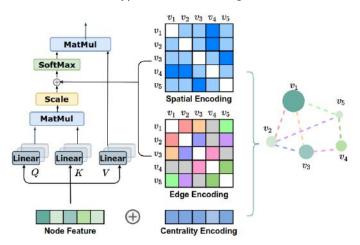


from Kim et al., 2019

80 / 84

Graph Transformers

- Node and edge encodings
- Positional and similarity/distance encodings

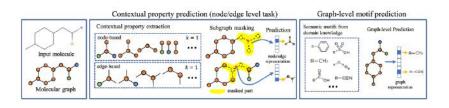


from Liu et al., 2021

81 / 84

SSL for Graph Transformers

- Large molecules dataset
- Physics-/Chemistry- inspired augmentations make more sense than arbitrary augmentations
- Road towards Graph SSL Transformers!



from Huang et al., 2020

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I. Makarov et al. (MADE) Lecture 1 07.10.2021 84 / 84