#### Scaling GNNs: Efficient models, Sampling models

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#### BigData Academy MADE from Mail.ru Group

**Graph Neural Networks and Applications** 



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#### **Topics**

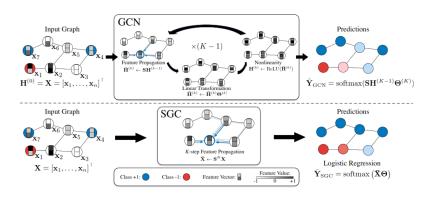
- Efficient training
- 2 Finding proper architectures
- Graph coarsening and large-scale training
- Sampling GNN models

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### Efficient training

#### Simplifying Graph Convolutional Networks

- How to solve oversmoothing problem with sparse labels?
- 2 Let's approximate K-layer GCN with  $A^K$  feature propagation



from Weinberger et al., 2019

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## Adaptive Label Smoothing To Regularize Large-Scale Graph Training

- Adapt label smoothing via two-stage PageRank approximation
- Consistent labels between prediction and ground truth, propagation and ground truth, and regularization on uniform distribution for propagation

$$\mathcal{L}^{LS}(\theta) = \frac{1}{|\mathcal{B}|} \sum_{i \in \mathcal{B}} H(\boldsymbol{y}_{i}^{LS}, \hat{\boldsymbol{y}}_{i}) = \frac{1}{|\mathcal{B}|} \sum_{i \in \mathcal{B}} (1 - \alpha) H(\boldsymbol{y}_{i}, \hat{\boldsymbol{y}}_{i}) + \alpha H(\mathbf{1}/C, \hat{\boldsymbol{y}}_{i})$$

$$\boldsymbol{Y}^{(k+1)} = (1 - \beta) \boldsymbol{D}^{-1} \boldsymbol{A} \boldsymbol{Y}^{(k)} + \beta \boldsymbol{Y}^{(0)} \qquad \boldsymbol{y}_{i}^{\text{soft}} = \operatorname{Softmax}(\boldsymbol{W} \boldsymbol{y}_{i}^{(K)})$$

$$\boldsymbol{y}_{i}^{ALS} = (1 - \alpha) \boldsymbol{y}_{i} + \alpha \boldsymbol{y}_{i}^{\text{soft}}$$

$$\mathcal{L}^{ALS}(\theta, \boldsymbol{W}) = \frac{1}{|\mathcal{B}|} \sum_{i \in \mathcal{B}} H(\boldsymbol{y}_{i}^{ALS}, \hat{\boldsymbol{y}}_{i}) + \gamma \operatorname{KL}(\boldsymbol{y}_{i}^{\text{soft}}, \mathbf{1}/C)$$

$$= \frac{1}{|\mathcal{B}|} \sum_{i \in \mathcal{B}} (1 - \alpha) H(\boldsymbol{y}_{i}, \hat{\boldsymbol{y}}_{i}) + \alpha H(\boldsymbol{y}_{i}^{\text{soft}}, \hat{\boldsymbol{y}}_{i}) + \gamma \operatorname{KL}(\boldsymbol{y}_{i}^{\text{soft}}, \mathbf{1}/C)$$

from Hu et al., 2021

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### Adaptive Filters and Aggregator Fusion for Efficient Graph Convolutions

#### Different aggregation and fusion schema

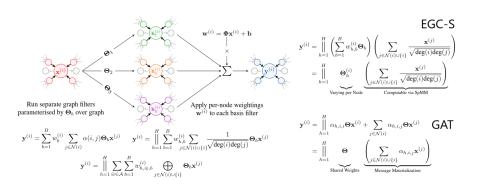
Method	Propagation Rule	Memory	Notes	
GCN (Kipf & Welling, 2017)	$\mathbf{y}^{(i)} = \mathbf{\Theta} \sum_{j \in \mathcal{N}(i) \cup \{i\}} \frac{1}{\sqrt{\deg(i)\deg(j)}} \mathbf{x}^{(j)}$	$\mathcal{O}( V )$	Formally defined for undirected graphs with self-loops; mot vated by graph signal processing.	
GAT (Veličković et al., 2018)	$\mathbf{y}^{(i)} = \alpha_{i,i} \mathbf{\Theta} \mathbf{x}^{(i)} + \sum_{j \in \mathcal{N}(i)} \alpha_{i,j} \mathbf{\Theta} \mathbf{x}^{(j)}$	$\mathcal{O}( E )$	Attention coefficients calculated using dot-product attention: $\alpha_{i,j} = \frac{\exp(\operatorname{leakyReLU}(\alpha^T   \Theta \mathbf{x}^{(i)}   \  \mathbf{e} \mathbf{x}^{(j)}]))}{\sum_{k \in \mathcal{N}(i) \cup \{i\}} \exp(\operatorname{leakyReLU}(\alpha^T   \mathbf{e} \mathbf{x}^{(i)}   \  \mathbf{e} \mathbf{x}^{(k)}]))}.$ Common to define multiple attention heads and concatenate.	
GIN (Xu et al., 2019)	$\mathbf{y}^{(i)} = f_{\Theta}[(1+\epsilon)\mathbf{x}^{(i)} + \sum_{j\in\mathcal{N}(i)}\mathbf{x}^{(j)}]$	$\mathcal{O}( V )$	$f$ is a learnable function, typically parameterized as an MLP or linear layer; $\epsilon$ may be fixed or learned.	
MPNN (Gilmer et al., 2017)	$\mathbf{y}^{(i)} = U(\mathbf{x}^{(i)}, \bigoplus_{j \in \mathcal{N}(i)} M(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}, \mathbf{e}_{ij}))$	$\mathcal{O}( E )$	$U,M$ typically defined as linear layers acting on concatenated features; $\bigoplus$ may be any valid aggregator, typically sum or max.	
PNA (Corso et al., 2020)	$\mathbf{y}^{(i)} = U(\mathbf{x}^{(i)}, \bigoplus_{j \in \mathcal{N}(i)} M(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}, \mathbf{e}_{ij}))$	$\mathcal{O}( E )$	Similar to MPNN, but with $\bigoplus$ defined to use 4 aggregators (mean, standard deviation, max, and min) scaled by 3 different functions of node degree, resulting in 12 different aggregations.	

from Lane et al., 2021

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### Adaptive Filters and Aggregator Fusion for Efficient Graph Convolutions

#### General model for GCN, GAT and GraphSAGE/GIN

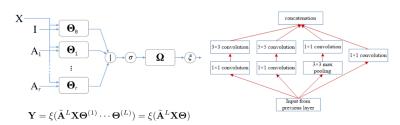


from Lane et al., 2021

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#### SIGN: Scalable Inception Graph Neural Networks

 General simplified GCN and feature propagation under unified framework similar to Inception block in CNNs



from Monti et al., 2020

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#### Training Graph Neural Networks with 1000 Layers

 Use Residual blocks with post-normalization to remove vanishing/exploding gradient problem

$$\begin{split} \langle X_1, X_2, ..., X_C \rangle \\ X'_0 &= \sum_{i=2}^C X_i \\ X'_i &= f_{w_i}(X'_{i-1}, A, U) + X_i, \ i \in \{1, \cdots, C\} \\ \widehat{X}_i &= \operatorname{Dropout}(\operatorname{ReLU}(\operatorname{Norm}(X'_{i-1}))) \\ \widehat{X}_i &= \operatorname{GraphConv}(\widehat{X}_i, A, U). \end{split} \qquad \begin{aligned} Z' &= \operatorname{GraphConv}(Z_{\operatorname{in}}, A, U) \\ Z'' &= \operatorname{Norm}(Z' + X) \\ Z''' &= \operatorname{GraphConv}(\operatorname{Dropout}(\operatorname{ReLU}(Z'')), A, U) \\ Z_o &= \operatorname{Norm}(\operatorname{ReLU}(Z''' + Z')), \end{aligned}$$

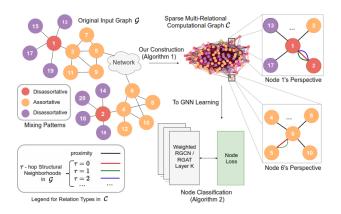
from Koltun et al., 2021

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## Breaking the Limit of Graph Neural Networks by Improving the Assortativity of Graphs with Local Mixing Patterns

Model uses both proximity and structural information to build a computation graph on which a GNN is run

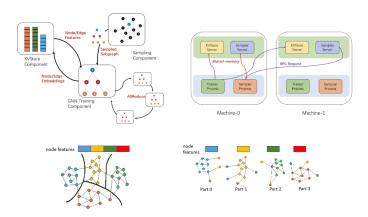


from Ma et al., 2021

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## DistDGL: Distributed Graph Neural Network Training for Billion-Scale Graphs

Distributed DGL (or e.g., DistGNN), smart partitions with HALO vertices



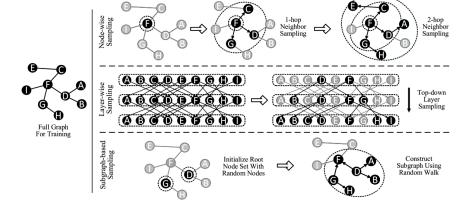
from Karypis et al., 2020

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### **Graph Sampling**

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- Sample neighborhood
- Aggregate message
- Ombine with self representation



from Fan et al., 2021

#### Various sampling strategies

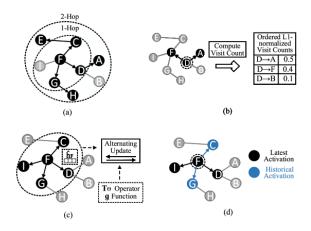
TABLE I: Categories and correlative works of the sampling methods

Categories	Works		
Node-wise sampling method	GraphSAGE [35], PinSage [44], SSE [47], VR-GCN [48]		
Layer-wise sampling method	FastGCN [49], AS-GCN [50], LADIES [51]		
Subgraph-based sampling method	Cluster-GCN [52], GraphSAINT [53],		
Subgraphi-based sampling method	RWT [54], Parallelized Graph Sampling [55]		
Heterogeneous sampling method	Time-related sampling [56], HetGNN [57],		
neterogeneous sampning method	HGSampling [58], Text Graph Sampling [59]		

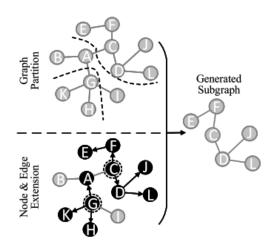
from Fan et al., 2021

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- (a) 2-hop neighbor sampling in GraphSAGE.
- (b) Importance-based neighbor sampling in PinSage.
- (c) Alternating sampling for embedding and update in SSE.
- (d) Neighbor sampling leveraging historical activation in VR-GCN.



Subgraph-based sampling



from Fan et al., 2021

#### Heterogeneous sampling

TABLE VIII: Summary of the comparisons between heterogeneous sampling methods

Method	Sampling Target	Sampling Condition	Extra Trick
Time-related Sampling [56]	6] Comments (Edges)	Sample the Closest Comments	Use of Padded Placeholders
Time-related Sampling [50]		in terms of Publish Time	Use of Fadded Flaceholders
HetGNN [57]	Nodes in (random) All Types	Sample the Most Frequently	Random Walk with Restart
Helonin [57]		Visited Nodes in RWR(v)	Grouping by Types
HGSampling [58]	Nodes in (ordered) All Types	Sampling Probability Various	Budget Stores
ricisampning [36]	Nodes in (ordered) All Types	in Different Node Types	Nodes by Types
Text Graph Sampling [59]	Document Nodes and Word Nodes	Sample the Most Intimate	Strategy to Calculate
Text Graph Sampling [59]	Document Nodes and Word Nodes	Nodes by Types	the Intimacy Matrix

from Fan et al., 2021

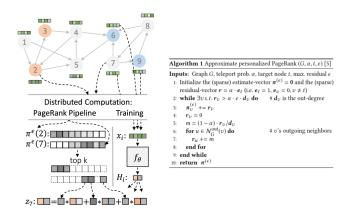
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### Sampling Models

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## Scaling Graph Neural Networks with Approximate PageRank

- Approximate PageRank
- Aggregate only from top-k PageRank (similar to PinSAGE)



from Günnemann et al., 2020

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#### **GraphSAINT**

- Efficient sampling via edge covering and node-edge estimation to co-occur in sample
- Unbiased estimator for feature propagation
- Ability to implement residual/attention architectures



#### Algorithm 1 GraphSAINT training algorithm

Input: Training graph  $\mathcal{G}(\mathcal{V}, \mathcal{E}, X)$ ; Labels  $\overline{Y}$ ; Sampler SAMPLE; Output: GCN model with trained weights

- 1: Pre-processing: Setup SAMPLE parameters; Compute normalization coefficients  $\alpha, \lambda$ .
- Ior each minibatch do
   G<sub>s</sub> (V<sub>s</sub>, E<sub>s</sub>) ← Sampled sub-graph of G according to SAMPLE
- 4: GCN construction on  $\mathcal{G}_s$ .
- 5:  $\{y_v \mid v \in \mathcal{V}_s\} \leftarrow$  Forward propagation of  $\{x_v \mid v \in \mathcal{V}_s\}$ , normalized by  $\alpha$  6: Backward propagation from  $\lambda$ -normalized loss  $L(y_v, \overline{y}_v)$ . Update weights. 7: end for

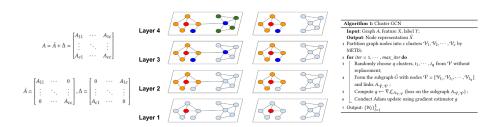
$$\boldsymbol{\zeta}_{v}^{(\ell+1)} = \sum_{u \in \mathcal{V}} \frac{\tilde{\boldsymbol{A}}_{v,u}}{\boldsymbol{\alpha}_{u,v}} \left( \boldsymbol{W}^{(\ell)} \right)^{\mathsf{T}} \boldsymbol{x}_{u}^{(\ell)} \boldsymbol{1}_{u|v} = \sum_{u \in \mathcal{V}} \frac{\tilde{\boldsymbol{A}}_{v,u}}{\boldsymbol{\alpha}_{u,v}} \tilde{\boldsymbol{x}}_{u}^{(\ell)} \boldsymbol{1}_{u|v}. \quad \mathbb{E} \left( L_{\text{batch}} \right) = \frac{1}{|\mathcal{G}|} \sum_{\mathcal{G}_{v} \in \mathcal{G}_{v}} \sum_{k \neq v} \frac{L_{v}}{|\mathcal{V}|} = \frac{1}{|\mathcal{V}|} \sum_{v \in \mathcal{V}} L_{v} = \frac{1}{|\mathcal{V}|} \sum_{v \in \mathcal{V}} L_{v}$$

from Prasanna et al., 2020

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#### Cluster-GCN

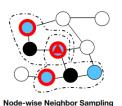
- Use METIS for fast clustering estimation
- Decompose stochastic blocks and connections between
- Optimize block separately, drop intra-cluster connections



from Hsieh et al., 2019

### LADIES: Layer-Dependent Importance Sampling for Training Deep and Large Graph Convolutional Networks

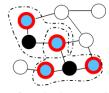
- Node-wise neighbor-sampling method suffers from exponential growing neighbor size
- 2 Layer-wise importance-sampling method discards the neighbor-dependent constraints
- Selects neighborhood nodes, constructs a bipartite subgraph and computes the importance probability followed by sampling.



(GraphSAGE)

Layer-wise Importance Sampling

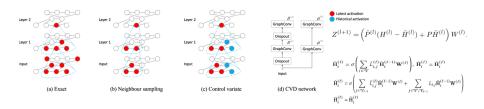
(FastGCN)



Layer-Dependent Importance Sampling (LADIES)

from Gu et al., 2019

- Control layer-wise sampling to efficiently estimate next layer from historical embeddings
- MVS-GNN uses one-shot sampling over nodes and batches with constant number of nodes

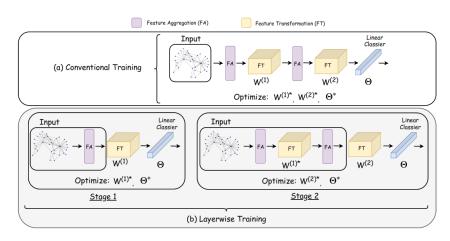


from Song et al., 2018; Mahdavi et al., 2021

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# $L^2$ –GCN: Layer-Wise and Learned Efficient Training of Graph Convolutional Networks

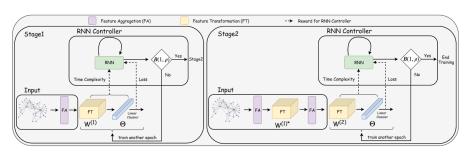
Layer-wise training



from Shen et al., 2020

# $L^2$ –GCN: Layer-Wise and Learned Efficient Training of Graph Convolutional Networks

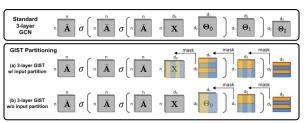
- In each stage the layer-wise trained network generates the training loss,
- 2 The later index as the input for the RNN controller,
- **3** RNN controller outputs the hidden stage for the next epoch, and the stopping probability  $\rho$  for the current epoch



from Shen et al., 2020

### GIST: Distributed Training for Large-Scale Graph Convolutional Networks

- GCN partition into two sub-GCNs. Orange and blue colors depict different feature partitions.
- subGCNs divides the global GCN into several sub-GCNs. Every subGCN is trained by subTrain using mini-batches (smaller sub-graphs) generated by Cluster.
- SubGCN parameters are intermittently aggregated through subAgg.



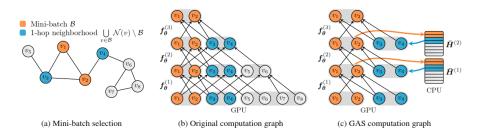


from Kyrillidis et al., 2021

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## GNNAutoScale: Scalable and Expressive Graph Neural Networks via Historical Embeddings

- Orange denotes the nodes in the current mini-batch and blue represents their 1-hop neighbors.
- 4 Historical embeddings avoid exponential growth of computations by pruning entire sub-trees of the computation graph



from Leskovec et al., 2021

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### Comparison of Graphs Sampling models

TABLE XIV: Comprehensive summary of sampling methods in all categories

Category	Method	Random Sampling	Sampling Condition	Target Problem
Node-wise	GraphSAGE [35]	<b>√</b>	Random Neighbor Sampling	Inductive Learning
	PinSage [44]	×	Normalized Visit Counts	Scaling up GCN
Sampling	SSE [47]	<b>√</b>	Random Neighbor Sampling	Steady-state Condition Learning
	VR-GCN [48]	<b>√</b>	Random Neighbor Sampling	Receptive Field Reduction
	FastGCN [49]	×	Layer-independent Sampling	Neighbor Explosion Alleviation
Layer-wise Sampling	AS-GCN [50]	×	Layer-dependent Sampling	Neighbor Explosion Alleviation
	LADIES [51]	×	Layer-dependent Sampling	Sparse Connection Alleviation
	Cluster-GCN [52]	<b>√</b>	Random Cluster Sampling	Constructing Graph Partition
Subgraph-based	Parallelized Graph Sampling [55]	<b>√</b>	Parallel Frontier Sampling	Model parallelizing and Scaling up
Sampling	GraphSAINT [53]	×	Probabilistic Edge Sampling	Constructing Unbiased Subgraph Sampling
	RWT [54]	<b>√</b>	Random Neighbor Expansion	Handling Node Relation and Over-smoothing
Heterogeneous Sampling	Time-related Sampling [56]	×	Publish Time	Adversarial Action Alleviation
	HetGNN [57]	×	Visit Frequency	Heterogeneous Graph Learning
	HGSampling [58]	×	Probabilistic Sampling Nodes by Type	Heterogeneous Graph Learning
	Text Graph Sampling [59]	×	Intimacy Matrix	Heterogeneous Text Graph Learning

from Fan et al., 2021

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