Exercise Class - Statistics Review

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Overview on integration

This is meant to be a VERY simplified overview on integration to give you the instruments to solve basic exercises as the ones below (for more on this, you can refer to William Neilson's Overview on Math for Economists). So, two most important things on integration:

- integration is the opposite of differentiation,
- integration finds the area under a curve.

To see this denote with $F(x)$ a function whose derivative is $f(x)$. The following two statements provide the fundamental relationship between derivatives and integrals:

$$\int_a^b f(x)dx = F(b) - F(a) \tag{1}$$

$$\int f(x)dx = F(x) + c \tag{2}$$

where c is a constant. The integral in (1) is a definite integral, and its distinguishing feature is that the integral is taken over a finite interval. The integral in (2) is an indefinite integral, and it has no endpoints. The reason for the names is that the solution in (1) is unique, or definite, while the solution in (2) is not unique. This occurs because when we integrate the function $f(x)$, all we know is the slope of the function $F(x)$, and we do not know anything about its height. If we choose one function that has slope $f(x)$, call it $F(x)$, and we shift it upward by one unit, its slope is still $f(x)$. The role of the constant c in (2), then, is to account for the indeterminacy of the height of the curve when we take an integral. The two equations (1) and (2) are consistent with each other. To see why, notice that

$$\int f(x)dx = \int_{-\infty}^{\infty} f(x)dx \tag{3}$$

so an indefinite integral is really just an integral over the entire real line $(-\infty, \infty)$. Some important properties to remember:

- Additive properties: $\int_a^a f(x)dx = 0$; $\int_a^b f(x)dx = -\int_b^a f(x)dx$; $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
- Scaling by a constant and integral of a sum: $\int_a^b cf(x)dx = c\int_a^b f(x)dx$; $\int_a^b \big(f(x) + g(x)\big)dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

The former point says that a constant inside of the integral can be moved outside of the integral and that the integral of the sum of two functions is the sum of the two integrals. Together they say that integration is a linear operation.
The most straightforward integrals, those that you can find in future exercises, are the follows (note that you can easily check by differentiating the right hand side)

- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$
- $\int \frac{1}{x} dx = ln(x) + c$
- $\int e^{rx} dx = \frac{e^{rx}}{r} + c$