Ay 190 Worksheet 7

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pp-Chain Nucleosynthesis

Figures 1, 2, and 3 show the evoluton of ^{1}H and ^{4}He for central temperatures of 1×10^7, 2×10^7, and 3×10^7 K, respectively. If the main sequence lifetime of the Sun is 10^{10} years and there should be a remaining mass hydrogen mass fraction of 0.02, then based on these plots the temperature should be between 2×10^7 and 3×10^7 K. The actual central temperature is 1.5×10^7.

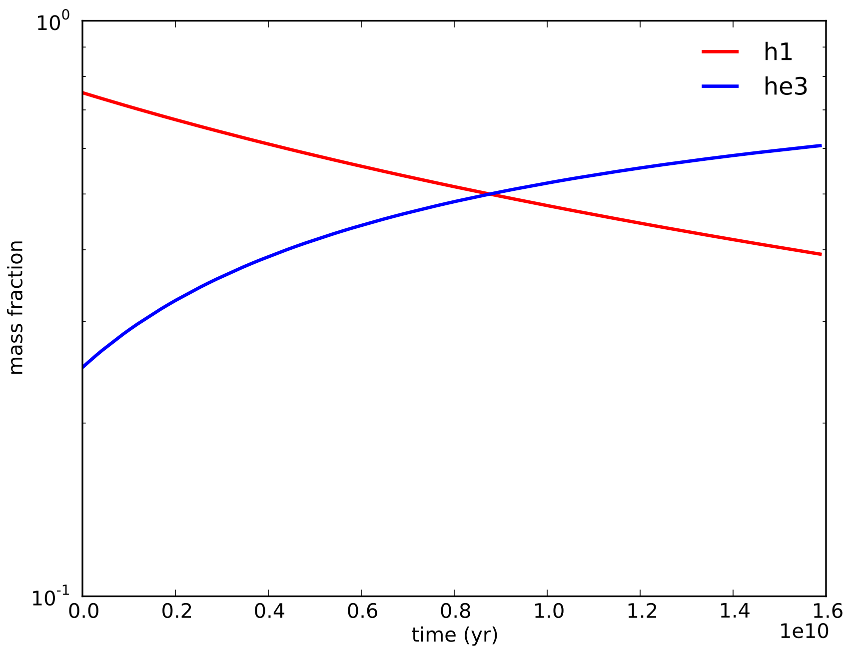

Fig. 1.—

The pp chain starts as follows:

$$ {}^1_1\mathrm{H} + {}^1_1\mathrm{H} \longrightarrow {}^2_2\mathrm{He} \tag{1} $$

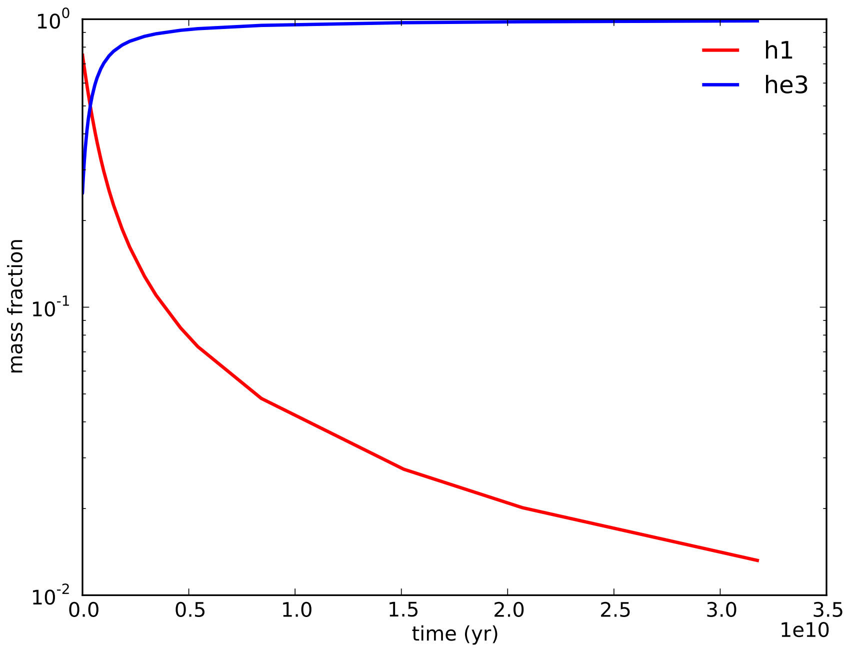

Fig. 2.—

$$ {}^2_2\mathrm{He} \longrightarrow {}^2_1\mathrm{D} + \mathrm{e}^+ + \nu_\mathrm{e} \tag{2} $$

The combination of these two reactions becomes:

$$ {}^1_1\mathrm{H} + {}^1_1\mathrm{H} \longrightarrow {}^2_1\mathrm{D} + \mathrm{e}^+ + \nu_\mathrm{e} + 0.42\ \mathrm{MeV} \tag{3} $$

We also have:

$$ \mathrm{e}^- + \mathrm{e}^+ \longrightarrow 2\gamma + 1.02\ \mathrm{MeV} \tag{4} $$

and

$$ {}^2_1\mathrm{D} + {}^1_1\mathrm{H} \longrightarrow {}^3_2\mathrm{He} + \gamma + 5.49\ \mathrm{MeV}. \tag{5} $$

Then, for the pp I branch,

$$ {}^3_2\mathrm{He} + {}^3_2\mathrm{He} \longrightarrow {}^4_2\mathrm{He} + 2\,{}^1_1\mathrm{H} + 12.86\ \mathrm{MeV}. \tag{6} $$

1. Other pp-Chain Branches

The other two branches of the pp chain start with Equations 1 and 2 (or Equation 3). They then proceed in different ways:

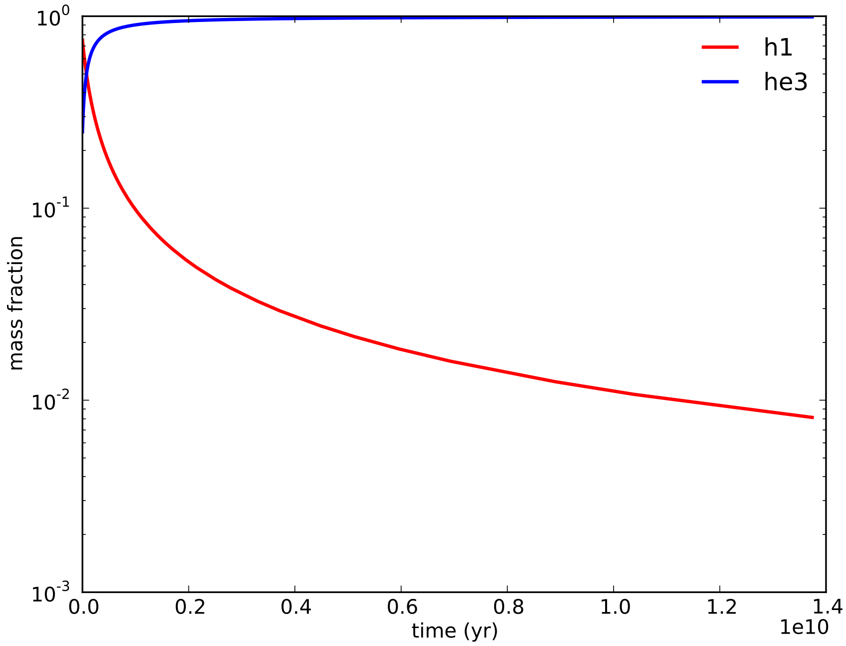

Fig. 3.—

1.1. pp II

$$^3_2\text{He} +^4_2 \text{He} \longrightarrow^7_4 \text{Be} + \gamma \tag{7}$$

$$^7_4\text{Be} + \text{e}^- \longrightarrow^7_3 \text{Li} + \nu_\text{e} + (0.861 \text{ MeV}, 0.383 \text{ MeV}) \tag{8}$$

$$^7_3\text{Li} +^1_1 \text{H} \longrightarrow 2^4_2\text{He} \tag{9}$$

In Equation 8, the two different energies correspond to the production of lithium in either the ground state (90% of the time) or in an excited state (10 % of the time).

1.2. pp III

$$^3_2\text{He} +^4_2 \text{He} \longrightarrow^7_4 \text{Be} + \gamma \tag{10}$$

$$^7_4\text{Be} +^1_1 \text{H} \longrightarrow^8_5 \text{Be} + \gamma \tag{11}$$

$$^8_5\text{Be} \longrightarrow^8_4 \text{Be} + \text{e}^+ + \nu_\text{e} \tag{12}$$

$$ {}^{8}_{4}\mathrm{Be} \longrightarrow 2\,{}^{4}_{2}\mathrm{He} \tag{13} $$

2. Formulation of the pp I Reaction Network

Let's use Equations 1, 2, 4, 5, and 6. The rate of change in number density for ${}^{1}_{1}$H, ${}^{2}_{2}$He, ${}^{2}_{1}$D, ${}^{3}_{2}$He, and ${}^{4}_{2}$He are:

$$ \frac{dn_{\mathrm{H}}}{dt} = -n_{\mathrm{H}}^2 < \sigma v >_{\mathrm{HH}} - n_{\mathrm{H}} n_{\mathrm{D}} < \sigma v >_{\mathrm{HD}} + n_{\mathrm{He3}}^2 < \sigma v >_{\mathrm{He3He3}} \tag{14} $$

$$ \frac{dn_{\mathrm{He2}}}{dt} = -\lambda_{\mathrm{He2}} n_{\mathrm{He2}} \tag{15} $$

$$ \frac{dn_{\mathrm{D}}}{dt} = -n_{\mathrm{H}} n_{\mathrm{D}} < \sigma v >_{\mathrm{HD}} + \lambda_{\mathrm{He2}} n_{\mathrm{He2}} \tag{16} $$

$$ \frac{dn_{\mathrm{He3}}}{dt} = n_{\mathrm{H}} n_{\mathrm{D}} < \sigma v >_{\mathrm{HD}} - n_{\mathrm{He3}}^2 < \sigma v >_{\mathrm{He3He3}} \tag{17} $$

$$ \frac{dn_{\mathrm{He4}}}{dt} = n_{\mathrm{He3}}^2 < \sigma v >_{\mathrm{He3He3}} \tag{18} $$

We can use (from the notes)

$$ \frac{dn_i}{dt} = \rho N_A \frac{dY_i}{dt} + N_A Y_i \frac{d\rho}{dt} = \rho N_A \frac{dY_i}{dt} \tag{19} $$

if $d\rho/dt = 0$. Then we can rewrite these equations as

$$ f_{\mathrm{H}} = \frac{dY_{\mathrm{H}}}{dt} = -N_A \rho \lambda_{\mathrm{HH}} Y_{\mathrm{H}}^2 - N_A \rho \lambda_{\mathrm{HD}} Y_{\mathrm{H}} Y_{\mathrm{D}} + N_A \rho \lambda_{\mathrm{He3He3}} Y_{\mathrm{He3}}^2 \tag{20} $$

$$ f_{\mathrm{He2}} = \frac{dY_{\mathrm{He2}}}{dt} = -\lambda_{\mathrm{He2}} n_{\mathrm{He2}} \tag{21} $$

$$ f_{\mathrm{D}} = \frac{dY_{\mathrm{D}}}{dt} = N_A \rho \lambda_{\mathrm{HD}} Y_{\mathrm{H}} Y_{\mathrm{D}} + \lambda_{\mathrm{He2}} n_{\mathrm{He2}} \tag{22} $$

$$ f_{\mathrm{He2}} = \frac{dY_{\mathrm{He2}}}{dt} = N_A \rho \lambda_{\mathrm{HD}} Y_{\mathrm{H}} Y_{\mathrm{D}} - N_A \rho \lambda_{\mathrm{He3He3}} Y_{\mathrm{He3}}^2 \tag{23} $$

$$ f_{\mathrm{He4}} = \frac{dY_{\mathrm{He4}}}{dt} = N_A \rho \lambda_{\mathrm{He3He3}} Y_{\mathrm{He3}}^2 \tag{24} $$

Following the derivation in the notes, we can set up a system of equations

$$ \Delta_i - \sum_j \frac{df_i}{dY_j} \Delta_j \Delta t = f_i(t) \Delta t \tag{25} $$

where $\Delta_i = Y_i(t + \Delta t) - Y_i(t)$.