Construction and Verification of Software

2022 - 2023

MIEI - Integrated Master in Computer Science and Informatics
Consolidation block

Lecture 8 - Lists and Trees in Separation Logic

Bernardo Toninho (btoninho@fct.unl.pt)
based on previous editions by **João Seco** and **Luís Caires**

NOVA SCHOOL OF
SCIENCE & TECHNOLOGY

Outline

- Recap on Separation Logic
- Lists and Trees in Separation Logic
- Arrays in Separation Logic

Part I
Separation Logic (recap)

Separation Logic (recap)

Separation logic assertions used in our CVS course are described by the following grammar:

$$
\begin{array}{rclll}
A & ::= & & & \text{Separation Logic Assertions} \\
 & & & L \mapsto V & \text{Memory Access} \\
 & & | & A * A & \text{Separating Conjunction} \\
 & & | & \texttt{emp} & \text{Empty heap} \\
 & & | & B & \text{Boolean condition (pure, not spatial)} \\
 & & | & B ? A : A & \text{Conditional} \\
 & & & & \\
B & ::= & \multicolumn{3}{l}{B \wedge B \mid B \vee B \mid V = V \mid V \neq V \mid \ldots} \\
V & ::= & \ldots & & \text{Pure Expressions} \\
L & ::= & x.\ell & & \text{Object field}
\end{array}
$$

Separation Logic (recap)

- The assignment rule in separation logic is

$$\{x \mapsto V\}\ x := E\ \{x \mapsto E\}$$

- Follows the small footprint principle, the precondition refers exactly to the part of the memory used by the fragment!

Separation Logic (recap)

- The assignment rule in separation logic is

$$\{x.\ell \mapsto V\}\; x.\ell := E\; \{x.\ell \mapsto E\}$$

- Follows the small footprint principle, the precondition refers exactly to the part of the memory used by the fragment!
- The memory locations here are fields of objects. We also have to model dynamically allocated arrays.

Example of Separation Logic in Verifast

```
/*@
  predicate Node(Node t; Node n, int v) = t.nxt |-> n &*& t.val |-> v;

  predicate List(Node n;) =  n == null ? emp : Node(n, ?nn, _) &*& List(nn);

  predicate StackInv(Stack t;) = t.head |-> ?h &*& List(h);
@*/

class Stack {
  private Node head;

  public Stack()
  //@ requires true;
  //@ ensures StackInv(this);
  { head = null; }

  public int pop()
  //@ requires NonEmptyStackInv(this);
  //@ ensures StackInv(this);
  {   int val = head.getValue(); head = head.getNext(); return val; }

  public boolean isEmpty()
  //@ requires StackInv(this);
  //@ ensures result?StackInv(this):NonEmptyStackInv(this);
  { return head == null; }
…
```

```
public static void main(String args[])
//@ requires true;
//@ ensures true;
{
  Stack s = new Stack();
  s.push(0);
  s.pop();
  if( ! s.isEmpty() ) {
    //@ open NonEmptyStackInv(_);
    s.push(1);
    s.pop();
  }
}
```

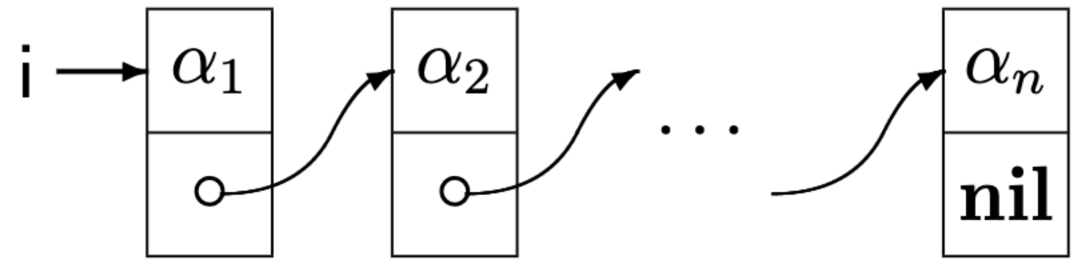
Part II
Lists in Separation Logic

Lists in Separation Logic

- Separation logic is designed to allow us to reason about pointer manipulating programs!
- Lets do that then…

Lists in Separation Logic

- We can define the predicate $\mathsf{list}\,\alpha\,i$, denoting that i is list reference representing the logical sequence α:



- By structural induction on α:

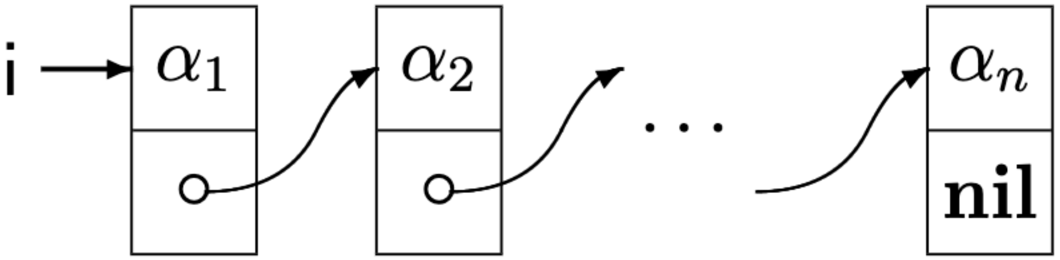
$$\mathsf{list}\,\epsilon\,i \triangleq \mathbf{emp} \wedge i = \mathbf{null}$$
$$\mathsf{list}\,(a \cdot \alpha)\,i \triangleq \exists j\,.\,i \mapsto a, j * \mathsf{list}\,\alpha\,j$$

- We write ϵ for the empty sequence and $i \mapsto a, j$ to denote that the list reference has a field containing a and another containing j.

Lists in Separation Logic

- Consider the following program:

```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```

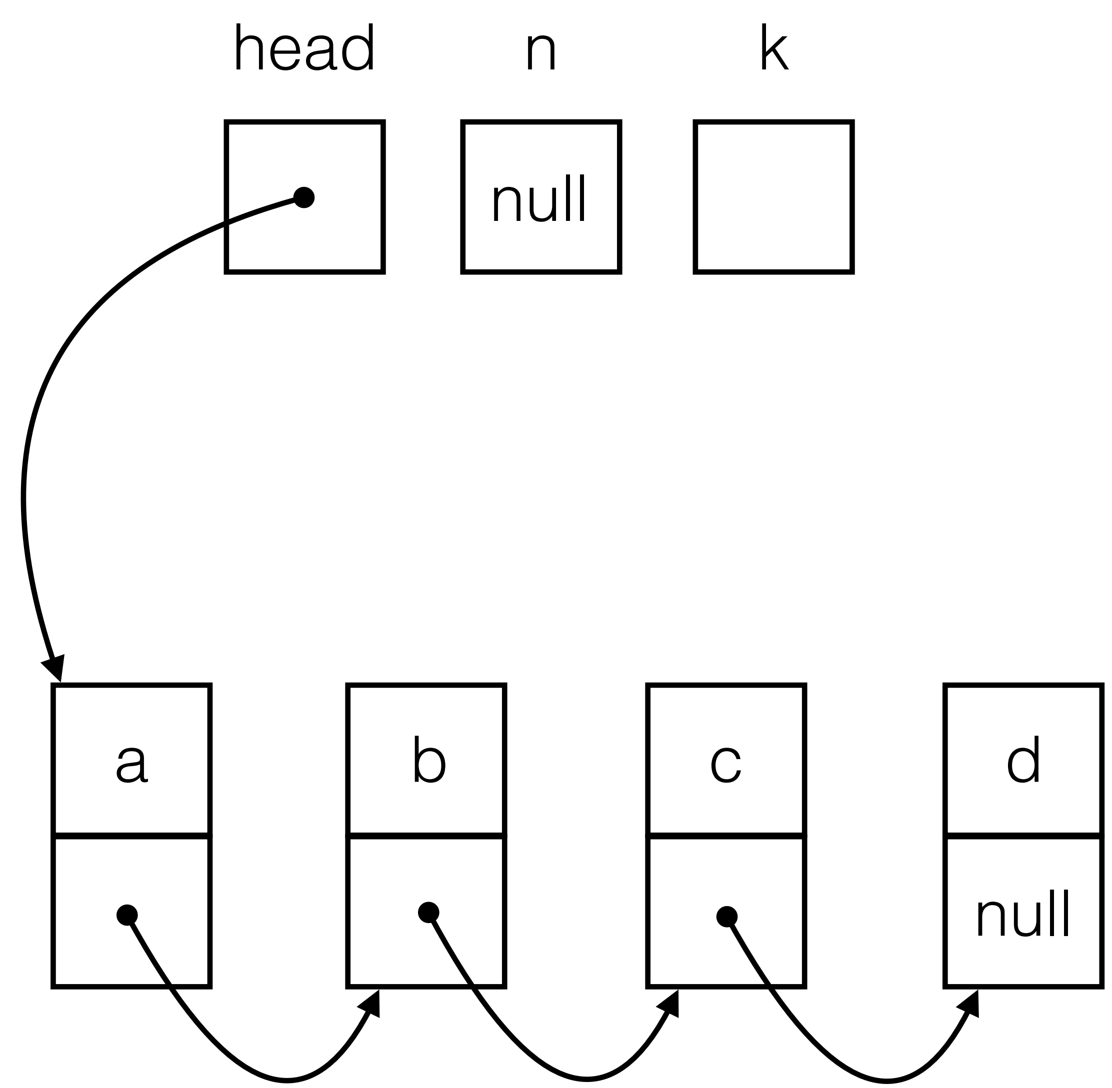


What does this method do?

Lists in Separation Logic

- Consider the following program:

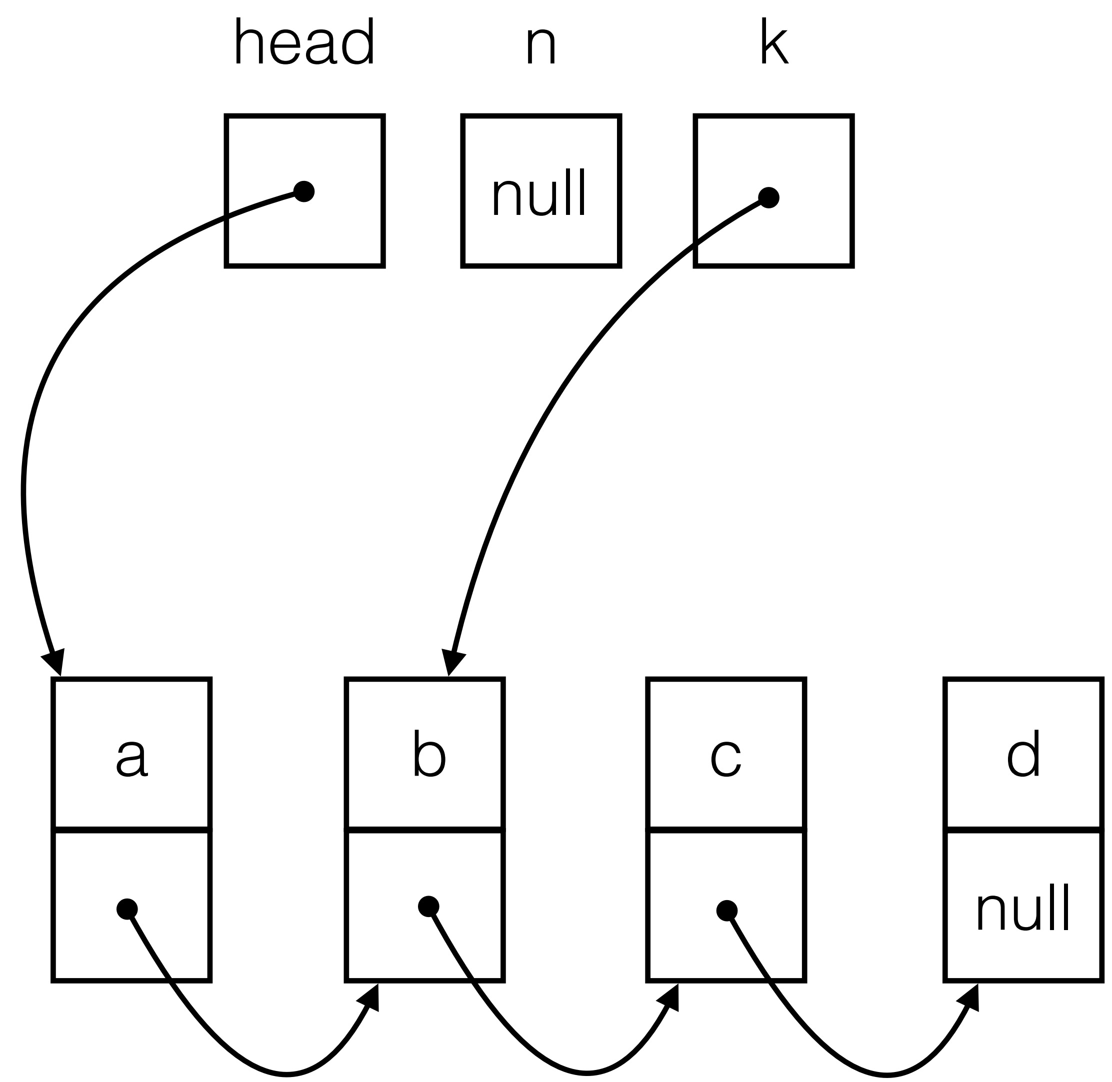
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

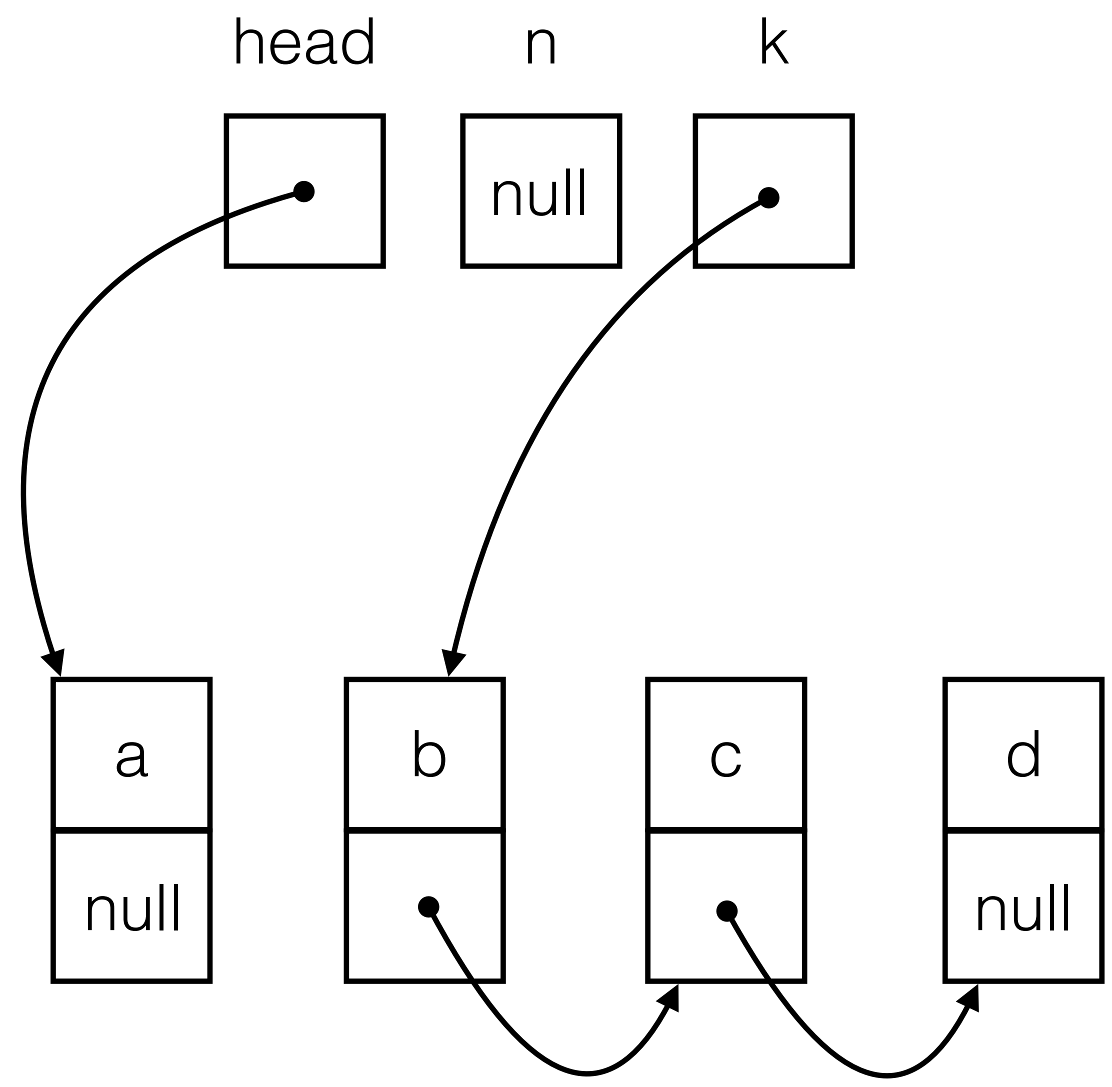
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

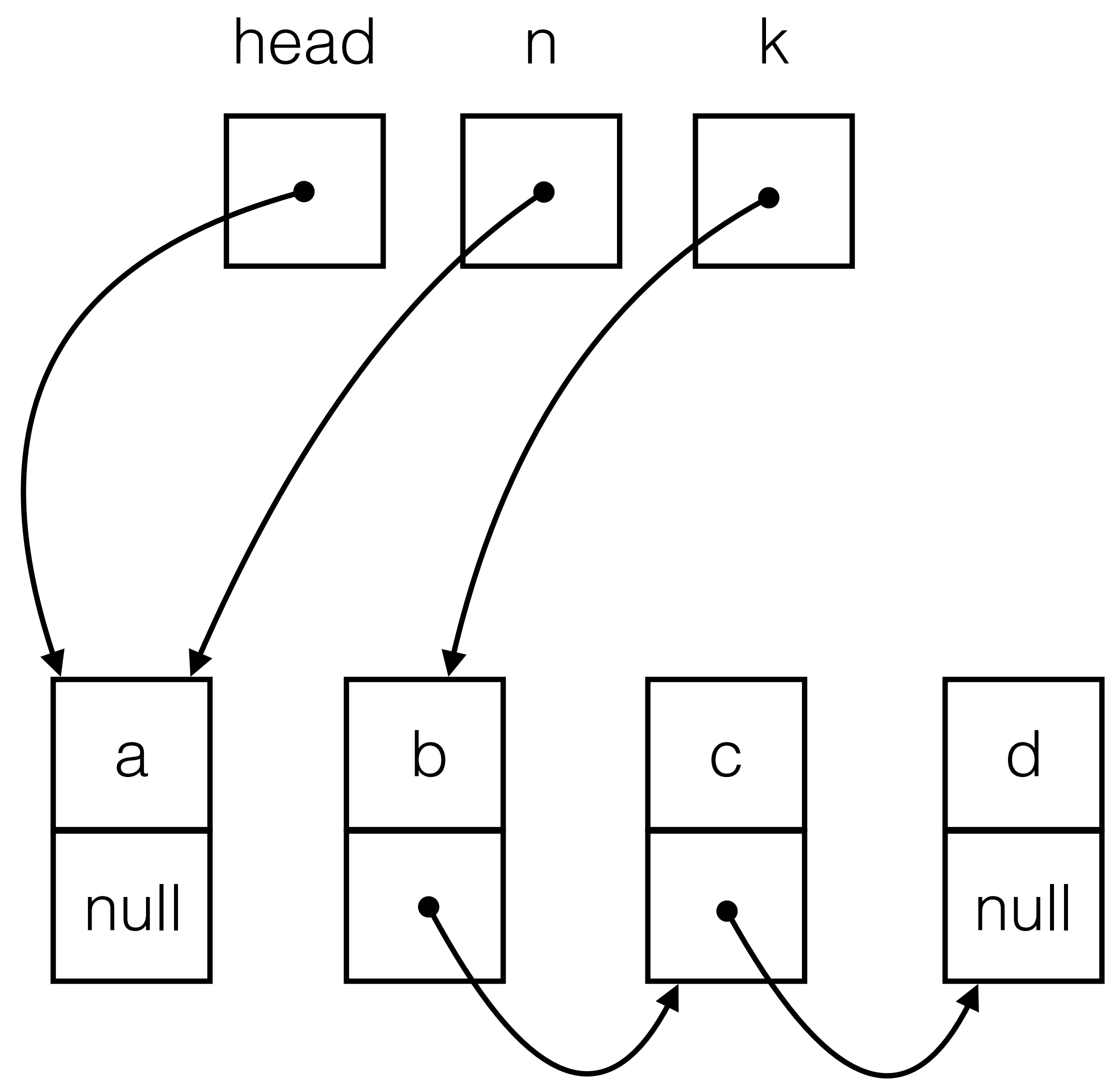
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

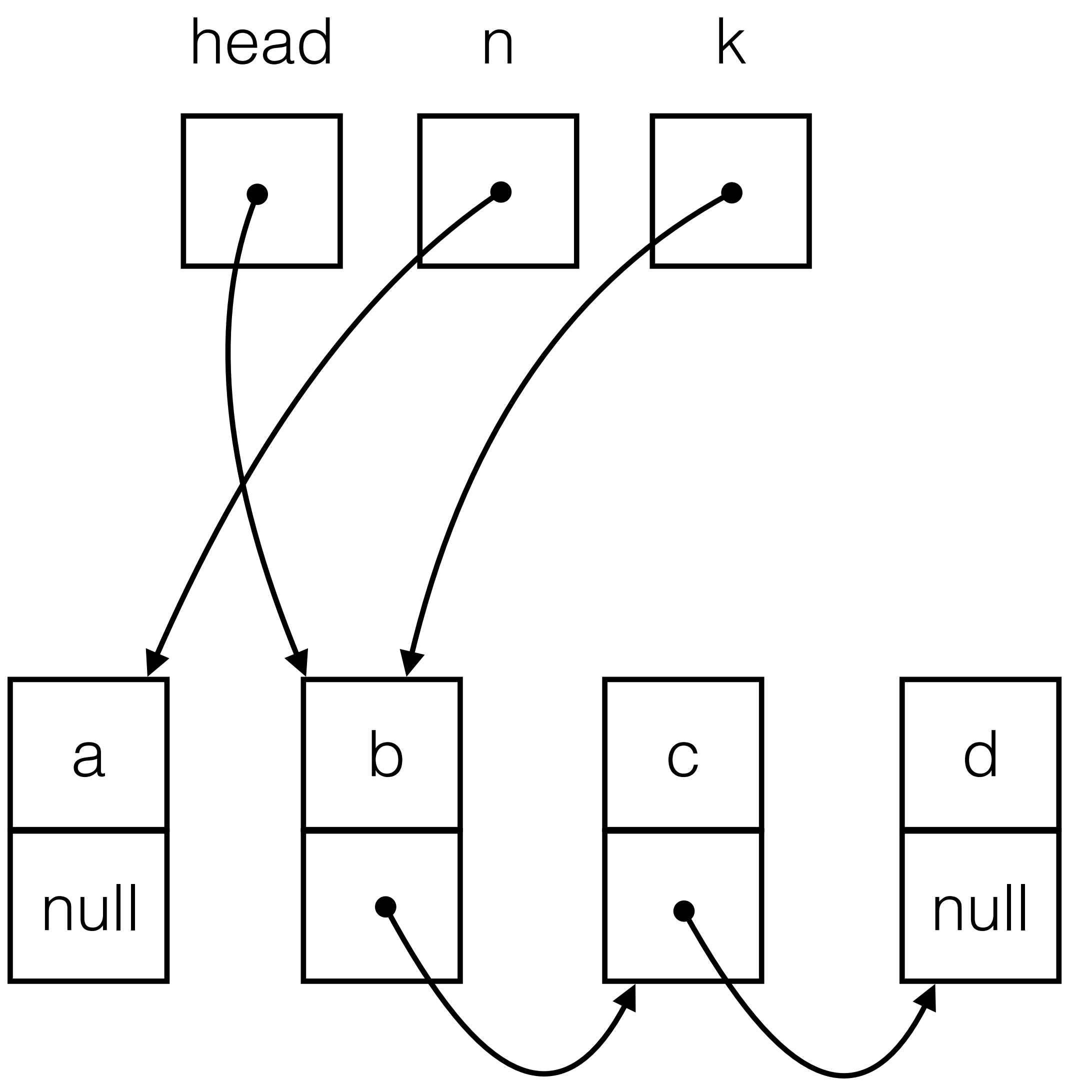
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

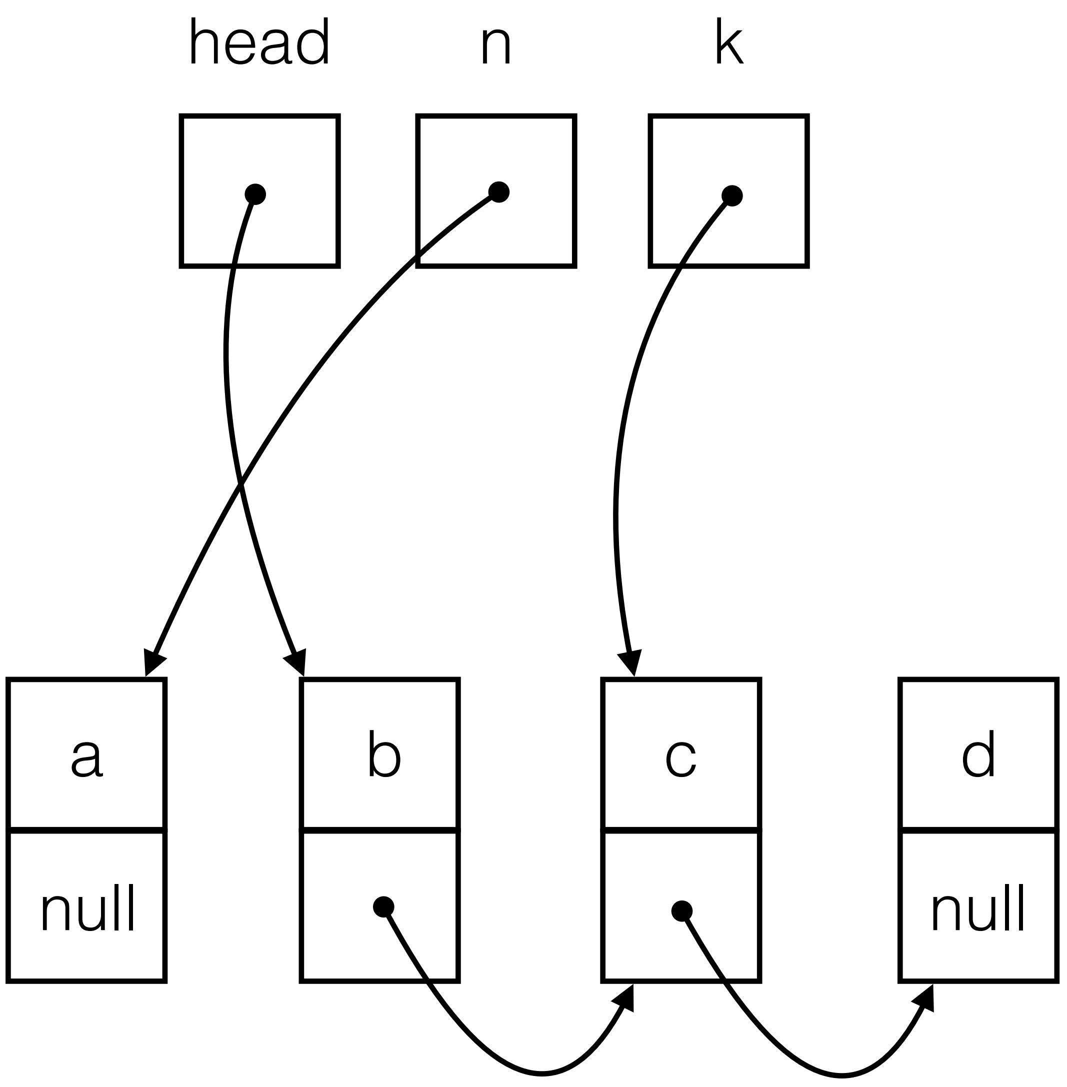
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

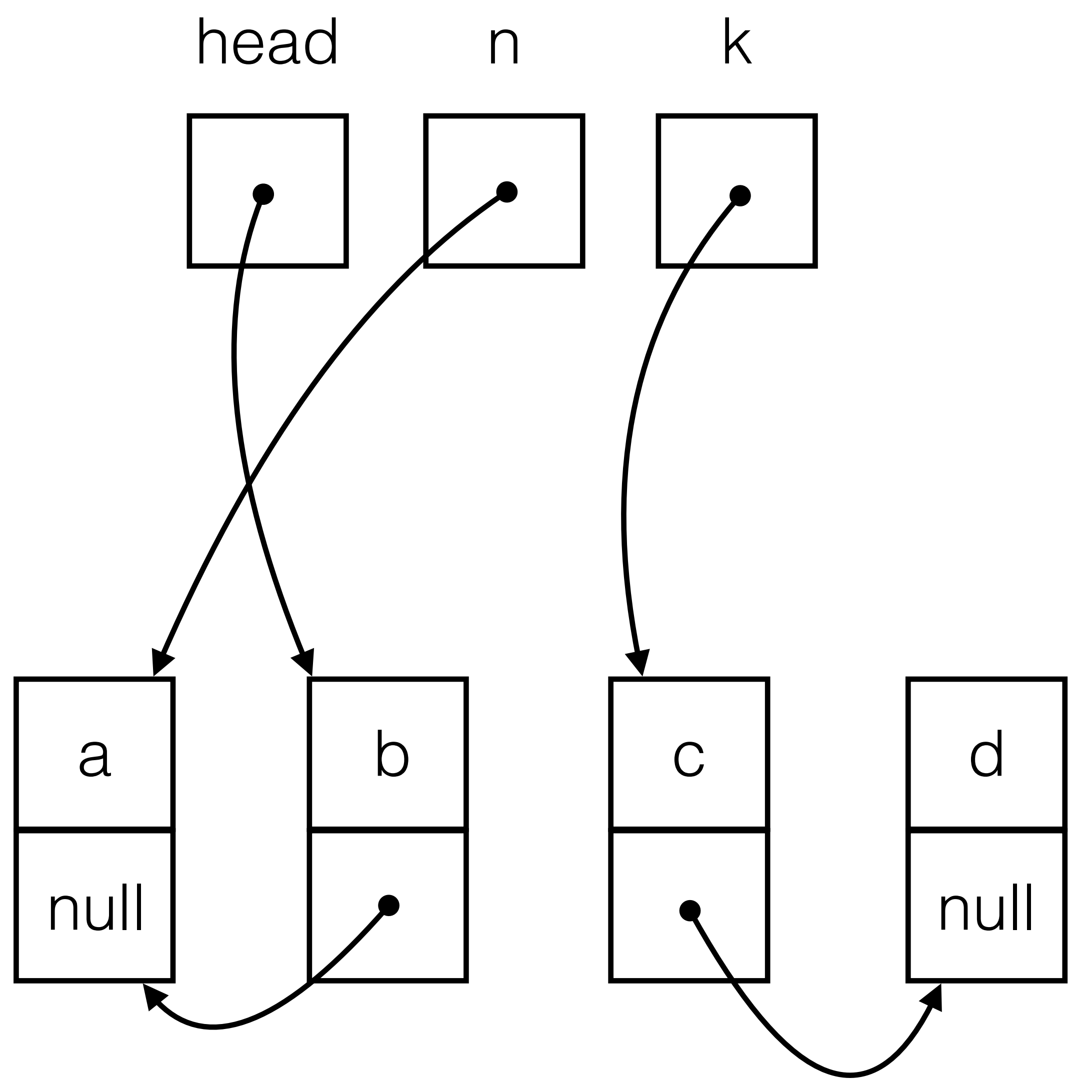
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

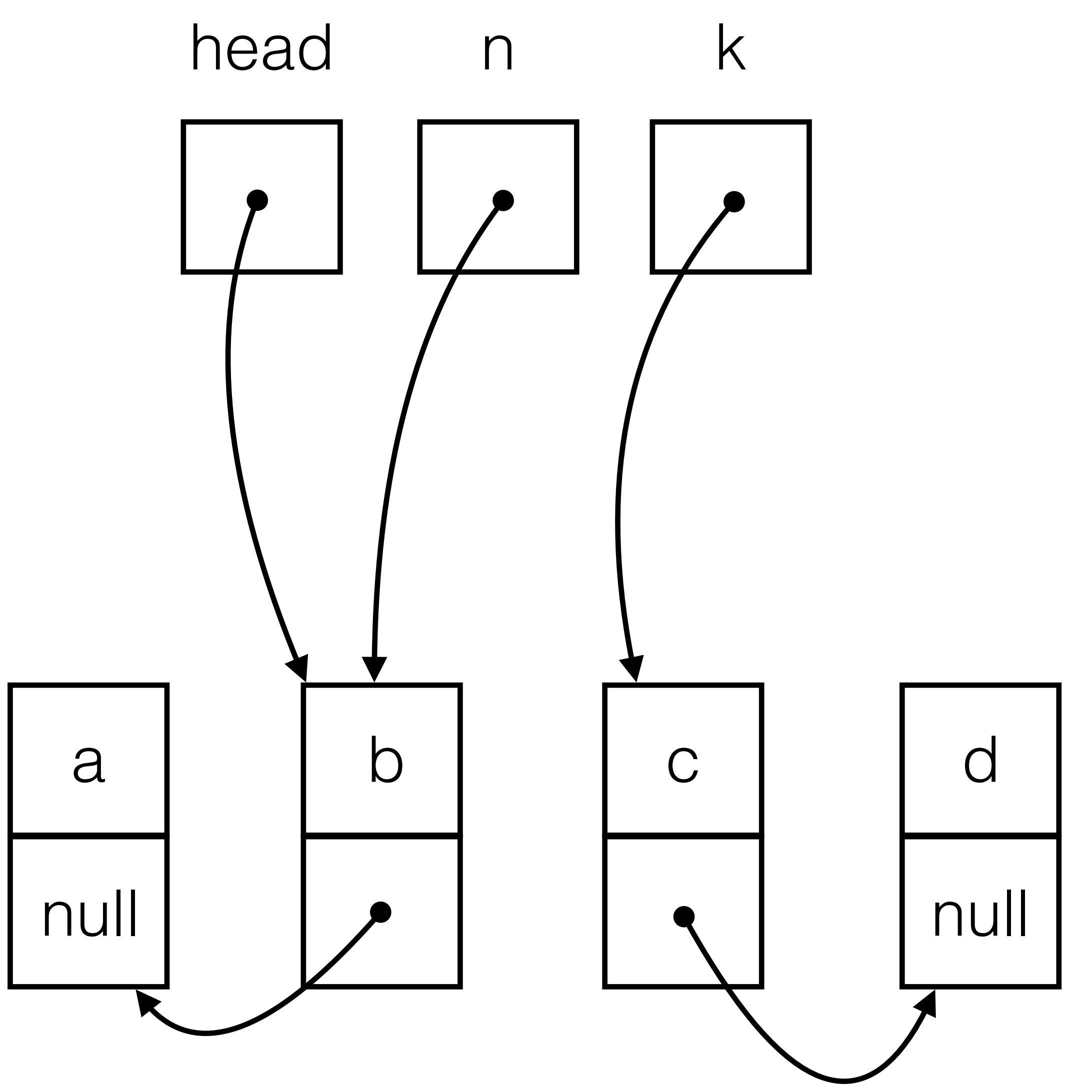
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

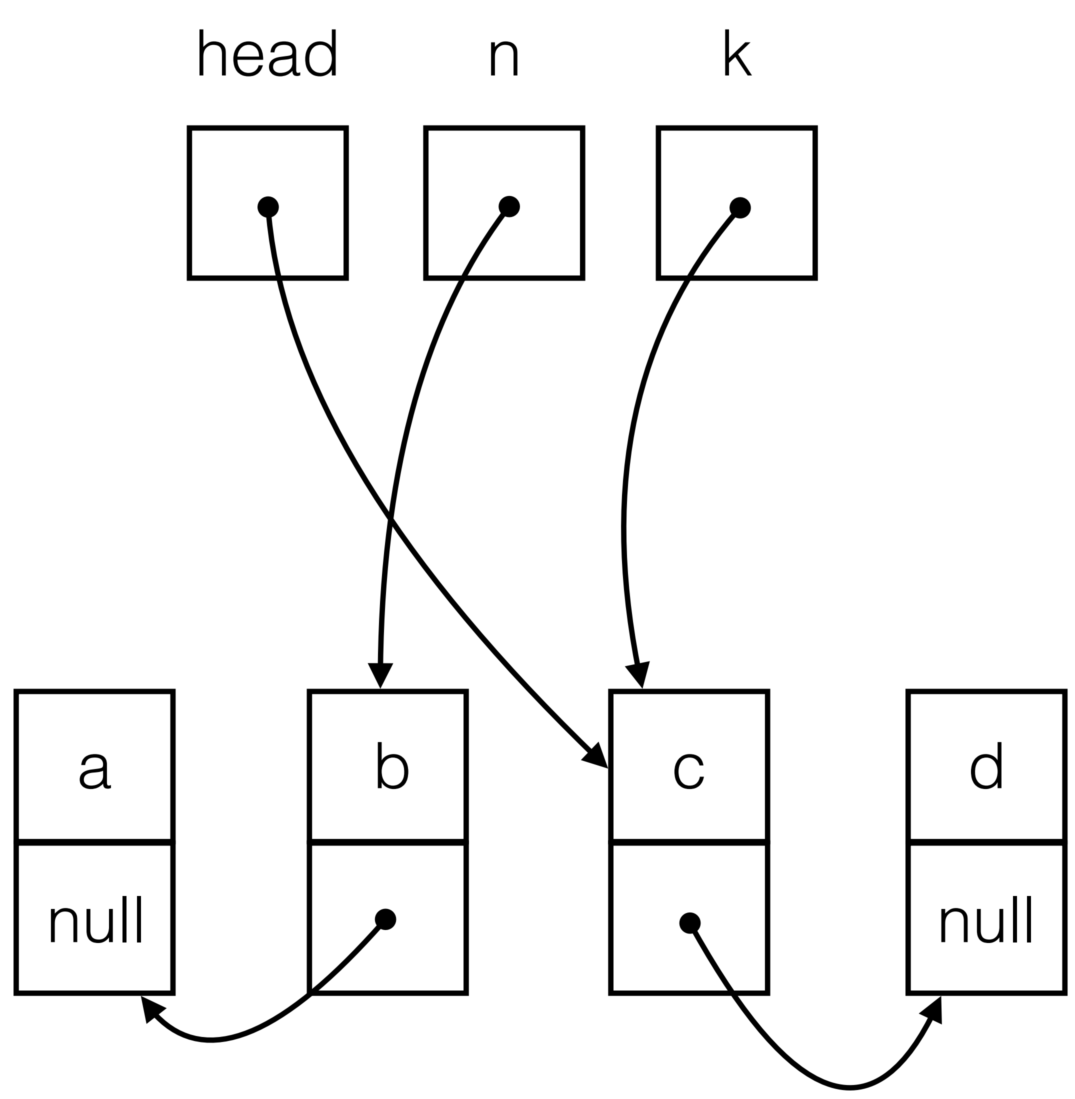
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

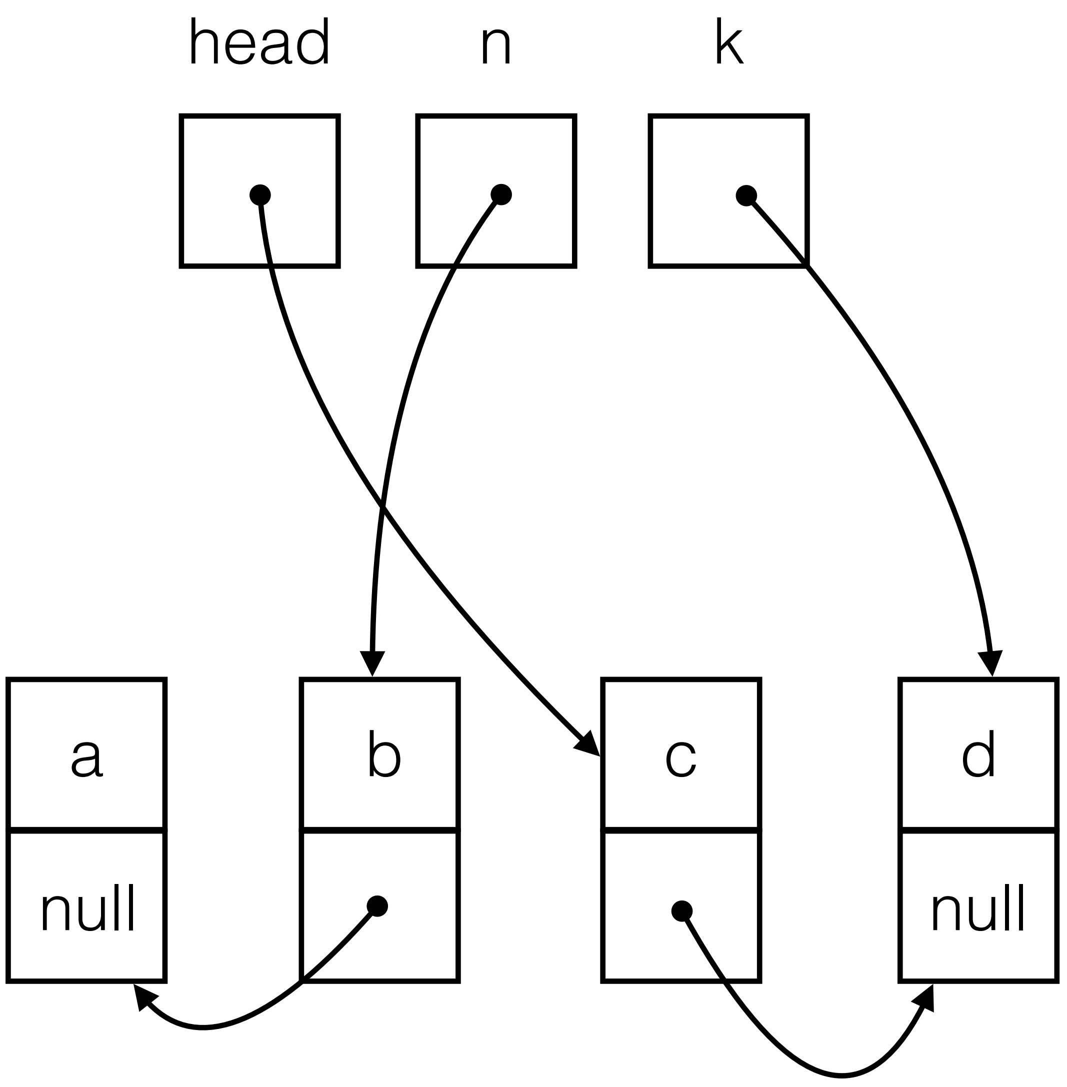
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

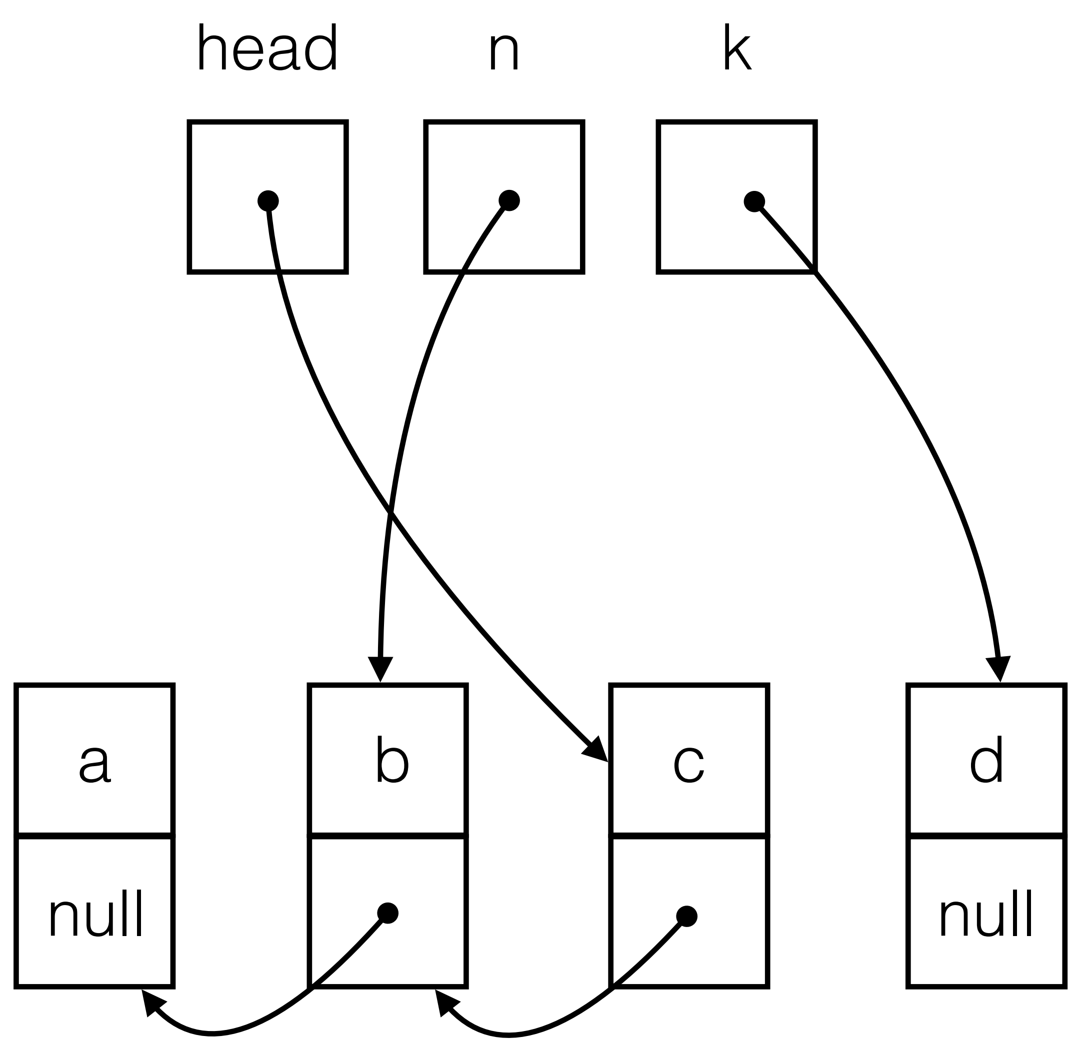
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

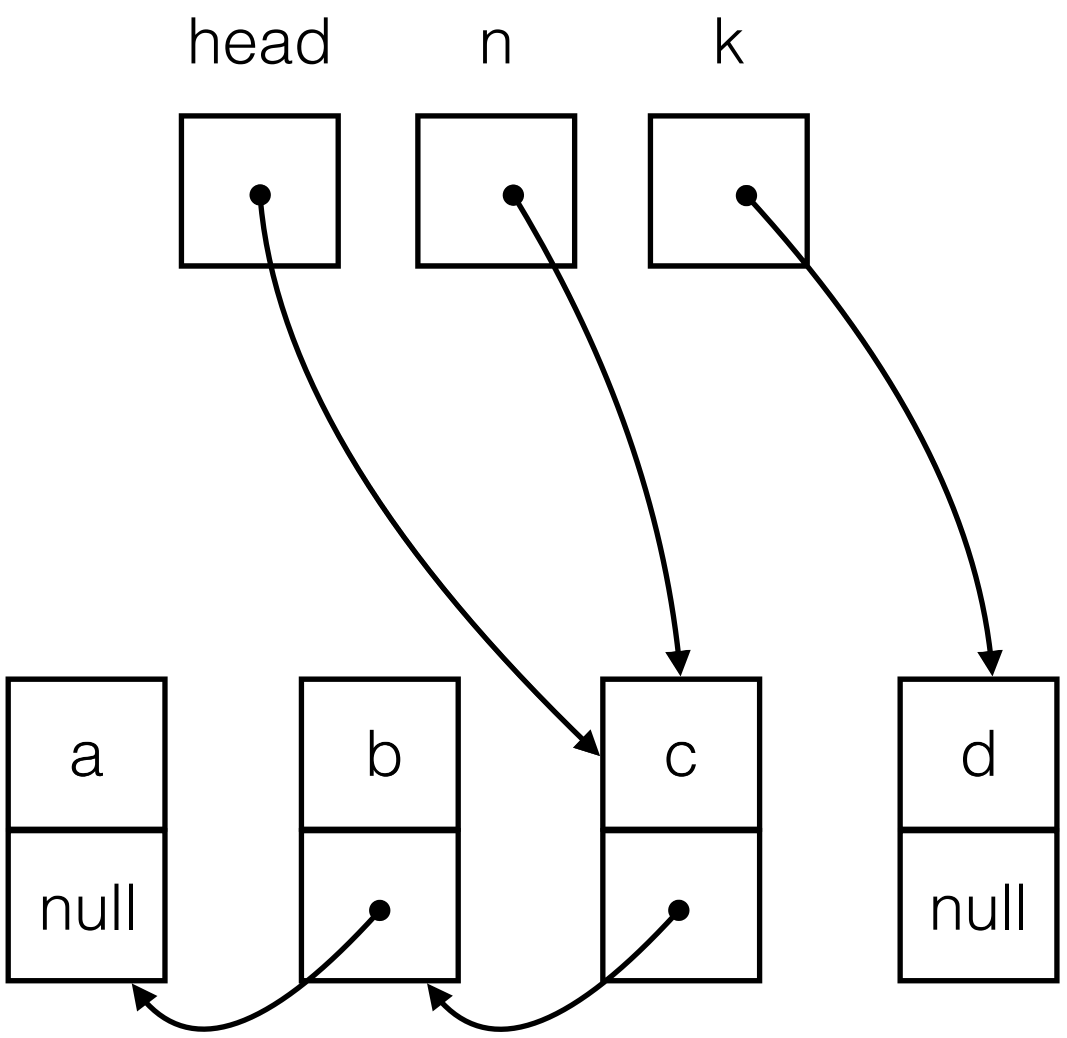
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

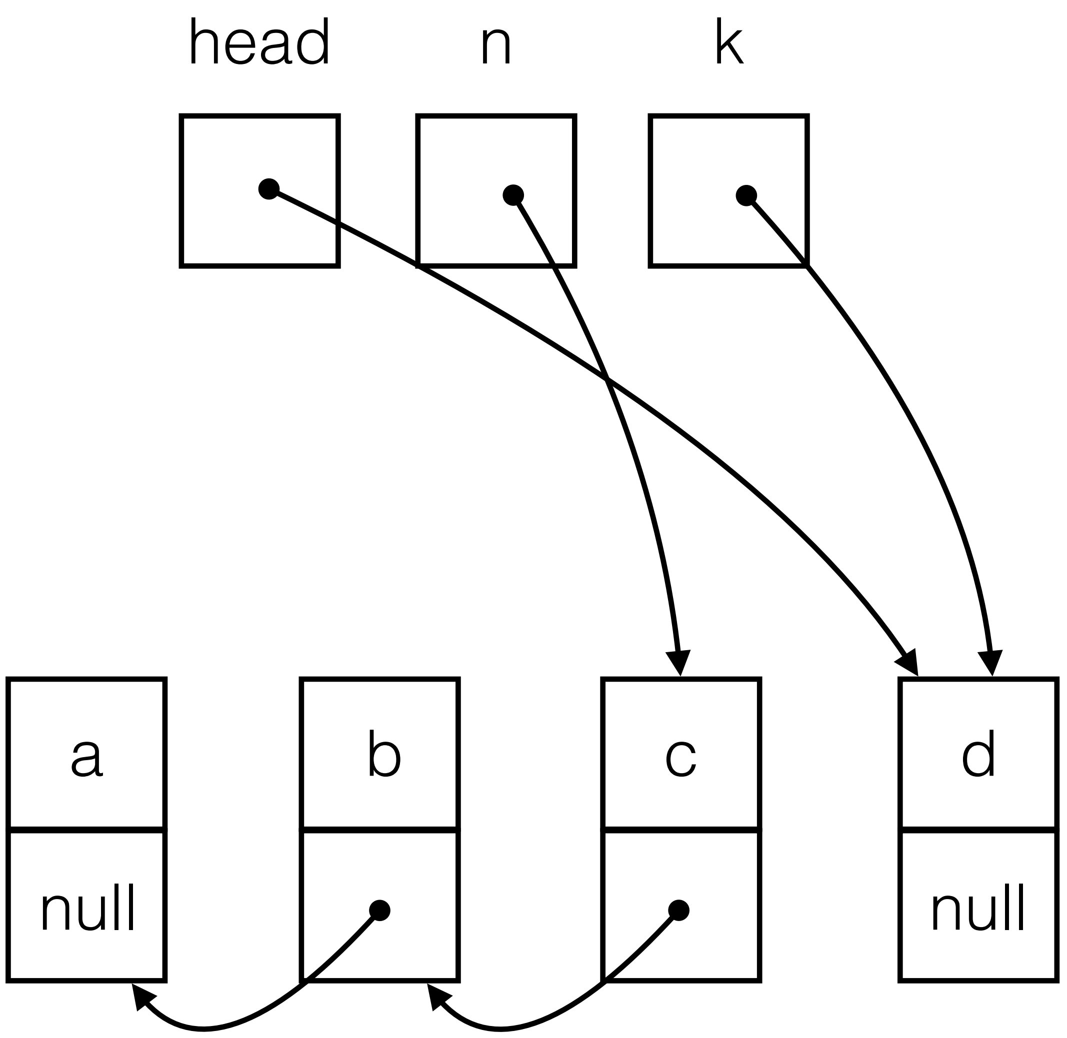
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

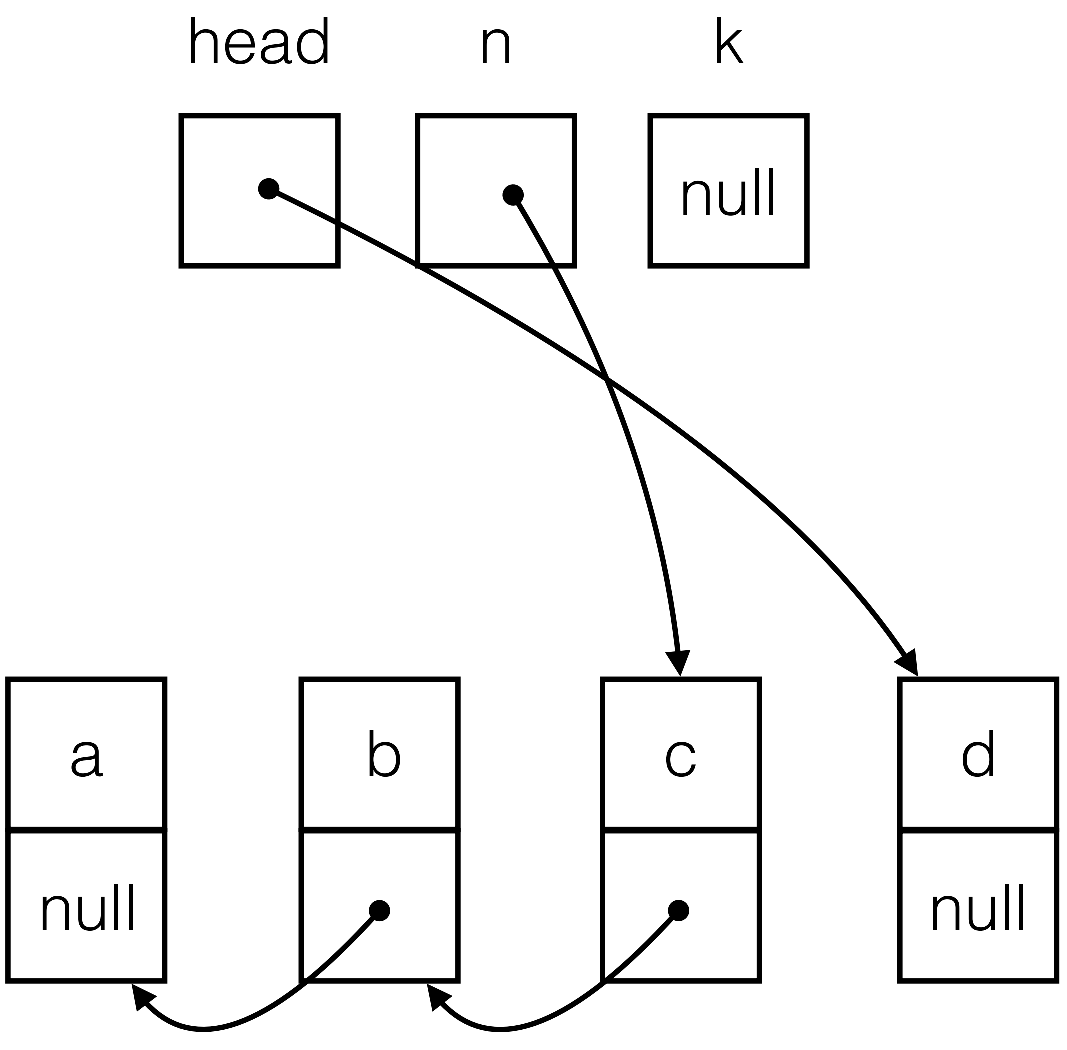
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

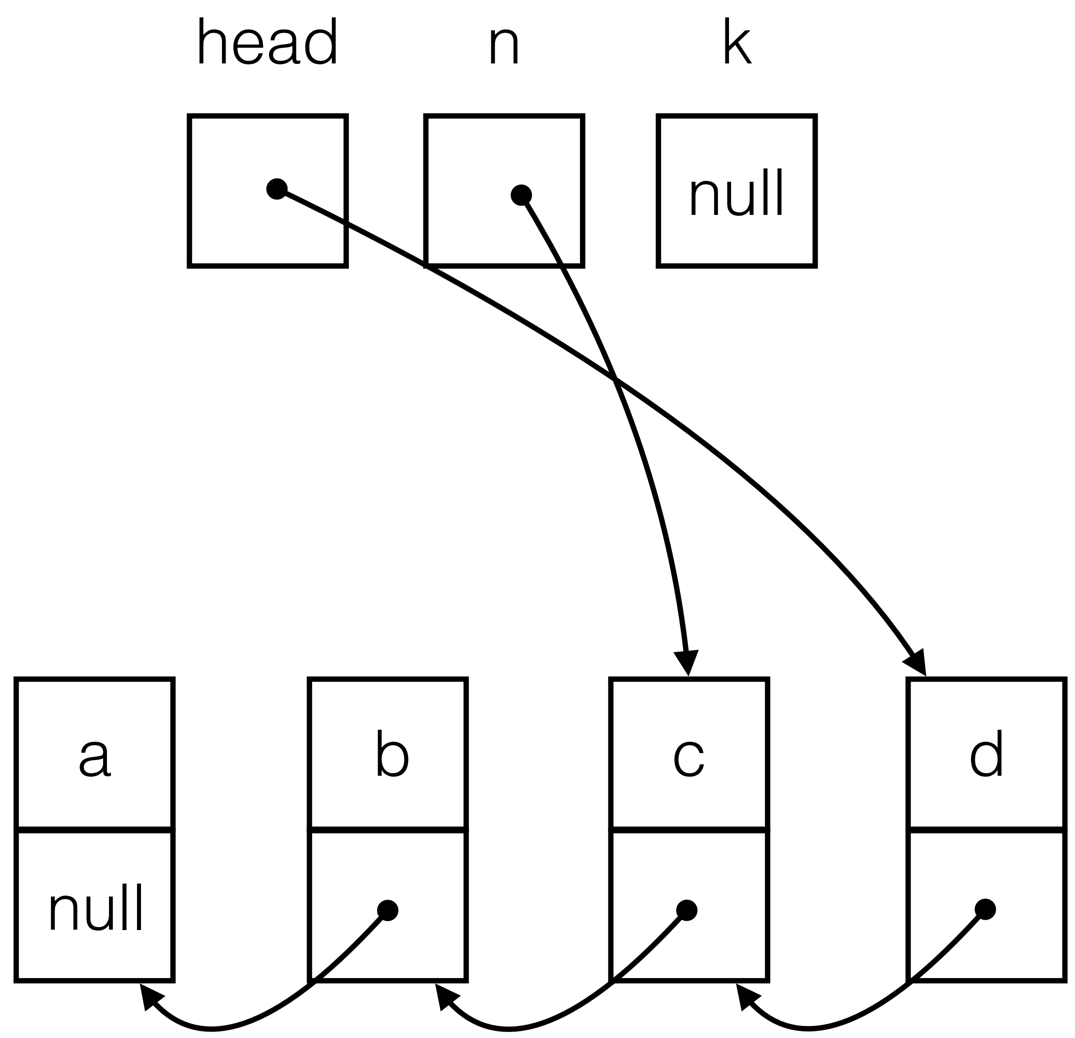
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

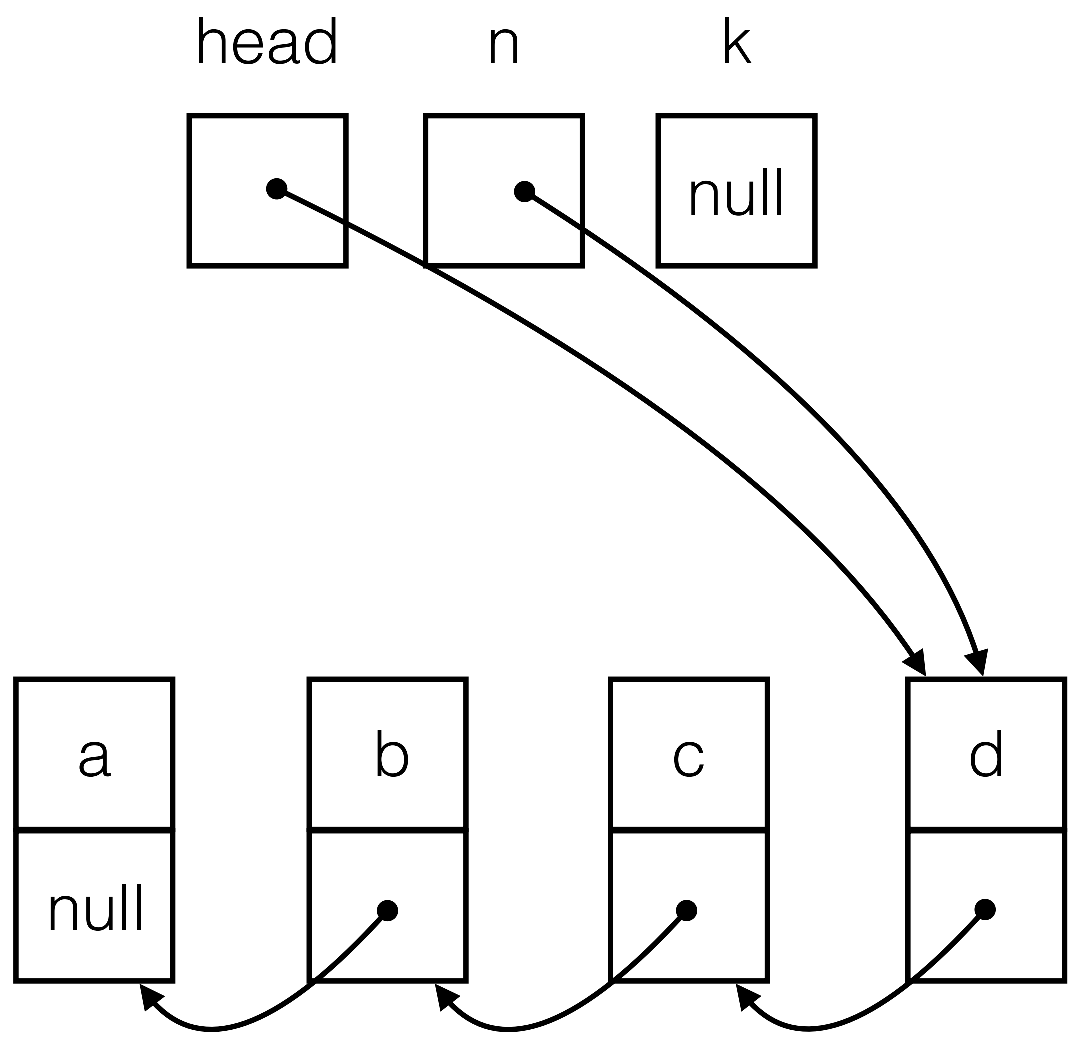
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

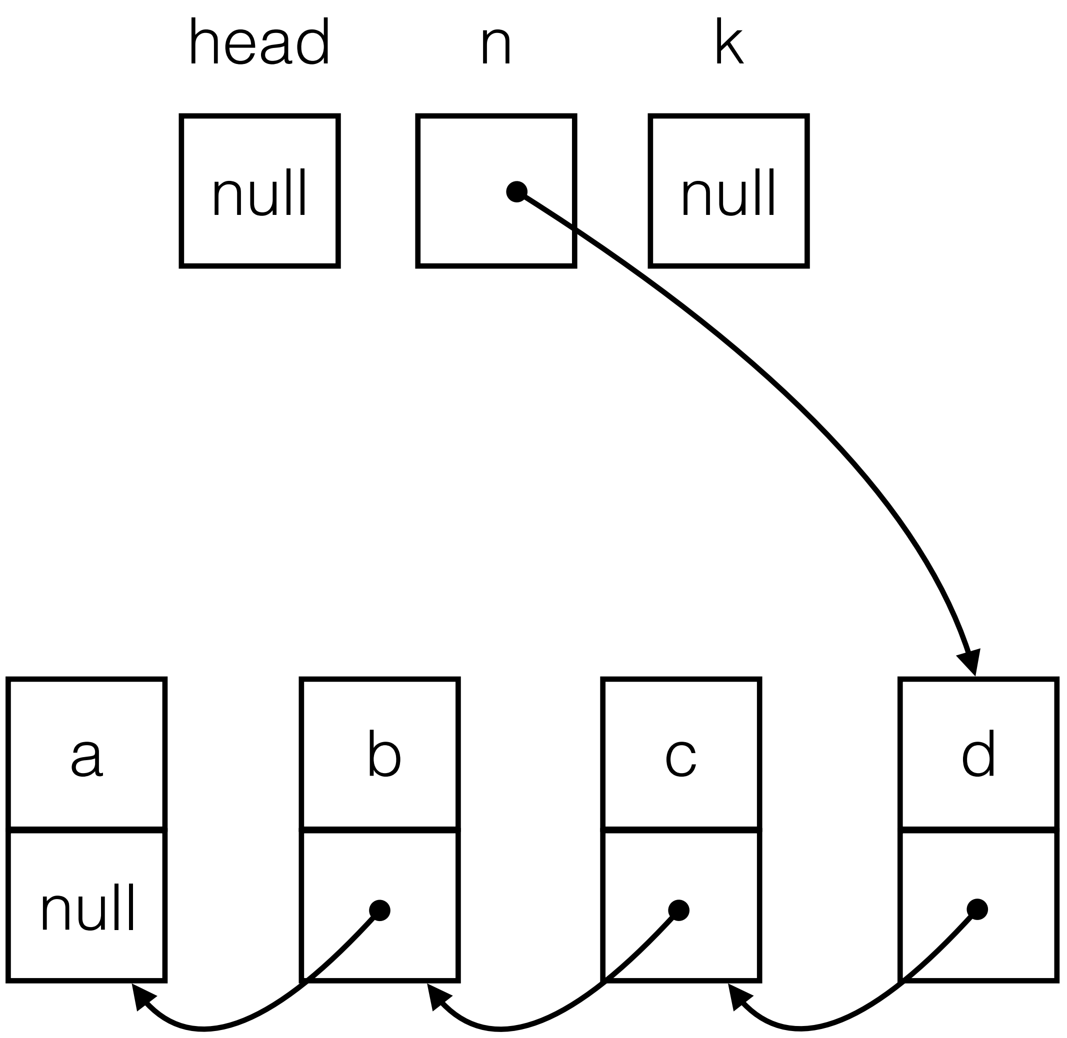
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

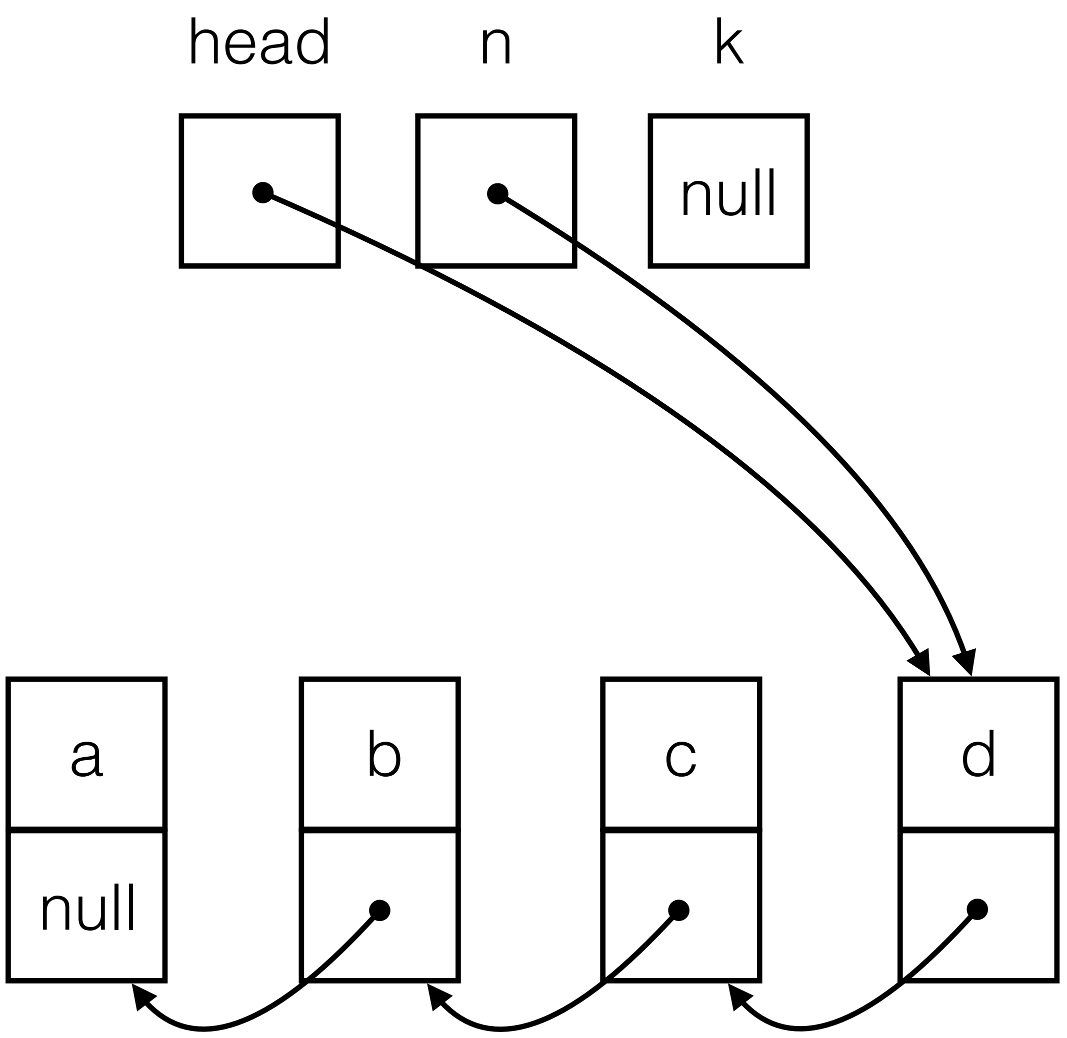
```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



Lists in Separation Logic

- Consider the following program:

```
public class List {

   Node head;

   void mistery()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```

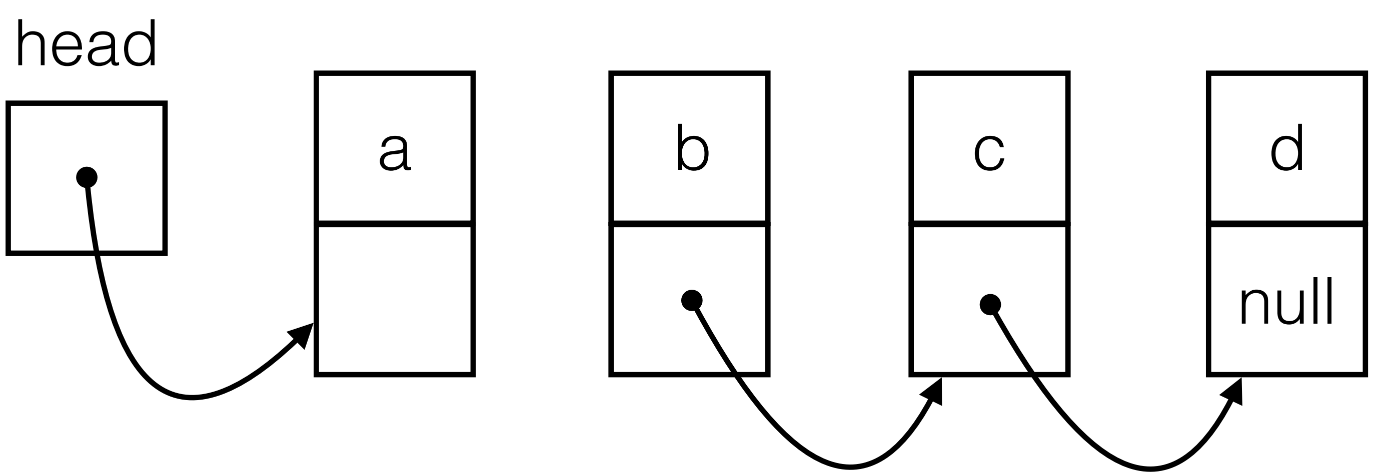


Lists in Separation Logic

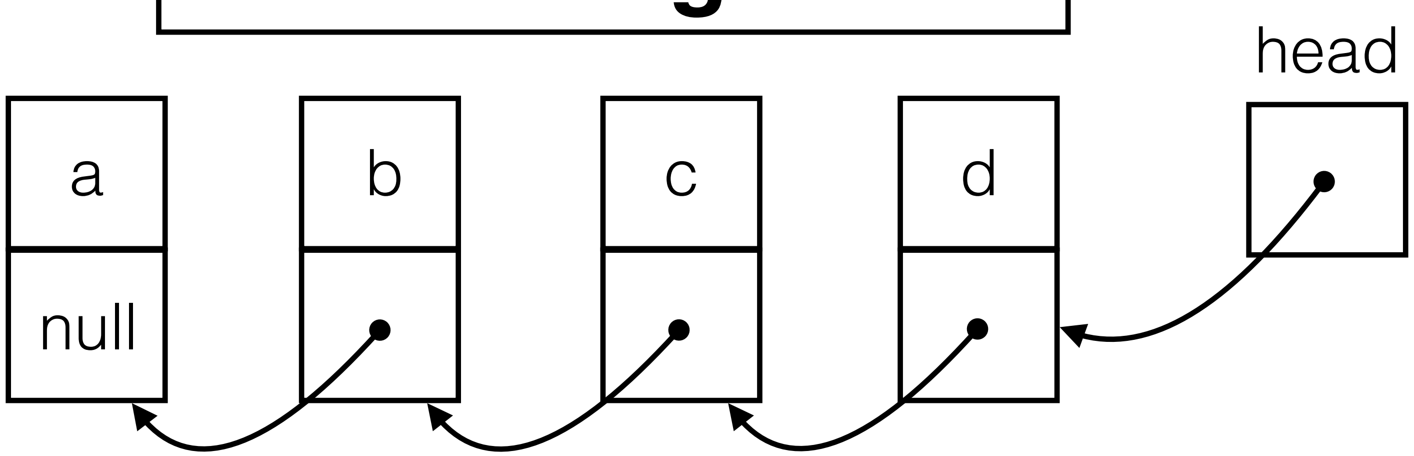
- Consider the following program:

```
public class List {

   Node head;

   void reversal()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```

Initial Configuration



Final Configuration

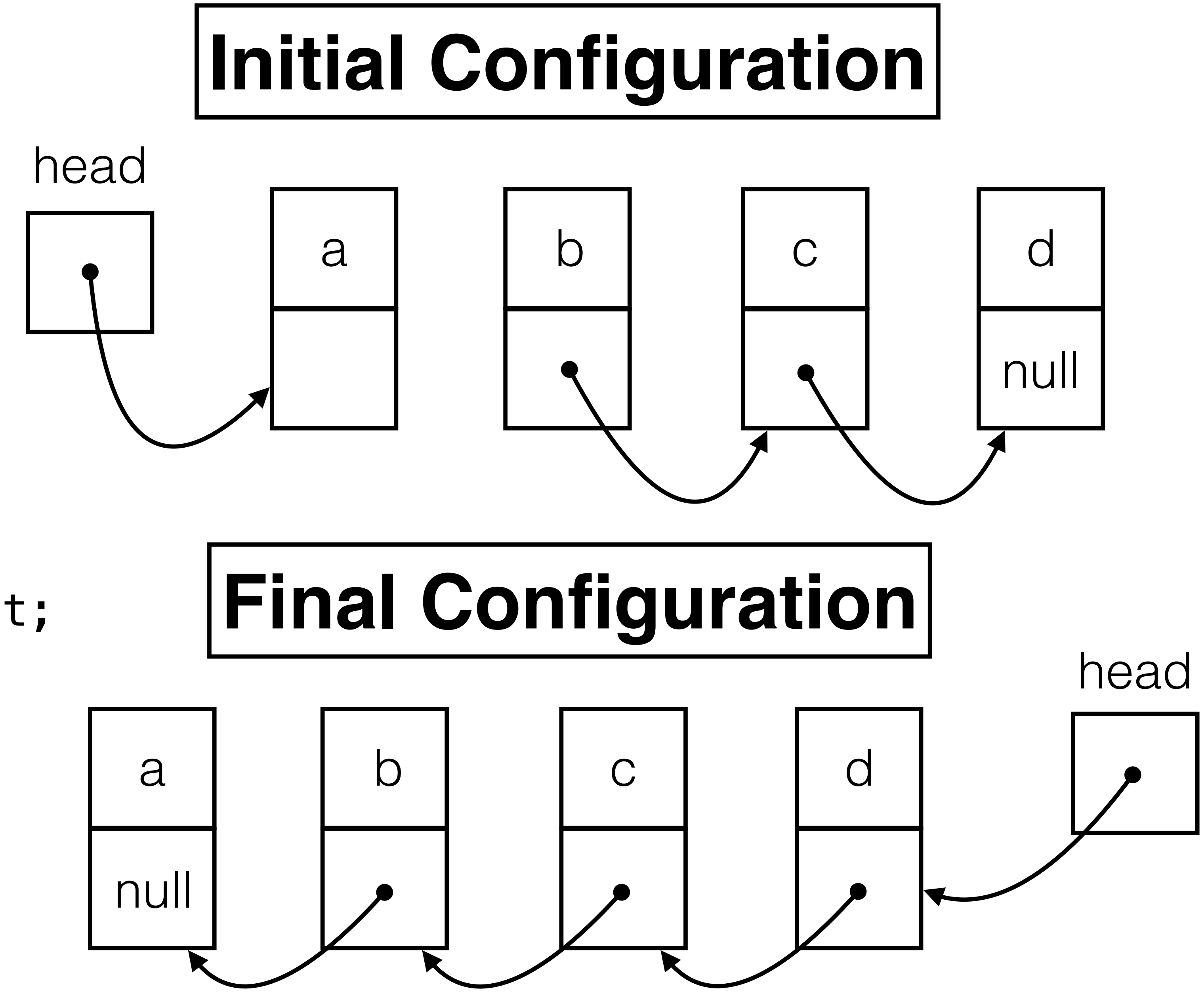


In-place list reversal!

Lists in Separation Logic

- Consider the following program:

```
public class List {

   Node head;

   void reversal()
   {
      Node n = null;

      while (head != null)
      {
         Node k = head.next;
         head.next = n;
         n = head;
         head = k;
      }
      head = n;
   }
   …
}
```



What are the invariants?

Lists in Separation Logic

- Specification:

$$\{\mathsf{List}\,\alpha\,\texttt{head}\}\;\texttt{reverse}()\;\{\mathsf{List}\,\alpha^{\dagger}\,\texttt{head}\}$$

where $\alpha^{\dagger}$ is the sequence obtained by reversing α.

$$\{\mathsf{List}\,\alpha\,\texttt{head}\}$$
$$\{\mathsf{List}\,\alpha\,\texttt{head} \ast (\mathbf{emp} \wedge \texttt{null} = \texttt{null})\}$$

```
Node n = null;
```

$$\{\mathsf{List}\,\alpha\,\texttt{head} \ast (\mathbf{emp} \wedge \texttt{n} = \texttt{null})\}$$
$$\{\mathsf{List}\,\alpha\,\texttt{head} \ast \mathsf{List}\,\epsilon\,\texttt{n}\}$$
$$\{\exists \sigma, \tau\,.\,(\mathsf{List}\,\sigma\,\texttt{head} \ast \mathsf{List}\,\tau\,\texttt{n}) \wedge \alpha = \tau^{\dagger} \cdot \sigma\}$$

Loop Invariant

```
while (head != null)
{
   Node k = head.next;
   head.next = n;
   n = head;
   head = k;
}
head = n;
}
}
```

Lists in Separation Logic

Loop Invariant

```
Node n = null;
while (head != null)
{ {∃σ,τ.(List σ head * List τ n) ∧ α = τ† · σ}
  {∃σ,τ.(List (a · σ) head * List τ n) ∧ α = τ† · (a · σ)}
  {∃σ,τ,k.(head ↦ a,k * List σ k * List τ n) ∧ α = τ† · (a · σ)}
  Node k = head.next;
  {∃σ,τ.(head ↦ a,k * List σ k * List τ n) ∧ α = τ† · (a · σ)}
  head.next = n;
  {∃σ,τ.(head ↦ a,n * List σ k * List τ n) ∧ α = τ† · (a · σ)}
  {∃σ,τ.(List (a · τ) head * List σ k) ∧ α = τ† · (a · σ)}
  {∃σ,τ.(List (a · τ) head * List σ k) ∧ α = (a · τ)† · σ}
  {∃σ,τ.(List σ k * List τ head) ∧ α = τ† · σ}
  n = head;
  head = k;
  {∃σ,τ.(List σ head * List τ n) ∧ α = τ† · σ}
}
head = n;
}
}
```

Loop Invariant

Lists in Separation Logic

$\{\mathsf{List}\,\alpha\,\mathtt{head}\}$

```
Node n = null;
while (head != null)
{
   Node k = head.next;
   head.next = n;
   n = head;
   head = k;
}
```

$\{\,\exists\tau\,.\,(\mathsf{List}\,\epsilon\,\mathtt{head}\ *\ \mathsf{List}\,\tau\,\mathtt{n})\wedge\alpha=\tau^{\dagger}\cdot\epsilon\}$
$\{\,\exists\tau\,.\,(\mathsf{List}\,\epsilon\,\mathtt{head}\ *\ \mathsf{List}\,\tau\,\mathtt{n})\wedge\alpha=\tau^{\dagger}\}$
$\{\,\exists\tau\,.\,(\mathsf{List}\,\tau\,\mathtt{n})\wedge\alpha=\tau^{\dagger}\}$
$\{\,\exists\tau\,.\,(\mathsf{List}\,\tau\,\mathtt{n})\wedge\alpha^{\dagger}=\tau^{\dagger^{\dagger}}\}$
$\{\,\exists\tau\,.\,(\mathsf{List}\,\tau\,\mathtt{n})\wedge\alpha^{\dagger}=\tau\}$
$\{\,\mathsf{List}\,\alpha^{\dagger}\,\mathtt{n}\}$

```
head = n;
```

$\{\,\mathsf{List}\,\alpha^{\dagger}\,\mathtt{head}\}$

$$\{\mathsf{List}\,\alpha\,\mathtt{head}\}\,\mathtt{reverse()}\,\{\mathsf{List}\,\alpha^{\dagger}\,\mathtt{head}\}$$

Lists in Separation Logic in Verifast

- Can we “check” this proof using Verifast?

```
predicate Node(Node n; Node nn, int v) = n.next |-> nn &*& n.val |-> v;

predicate List(Node n; list<int> elems) = n == null? (emp &*& elems == nil):
Node(n,?nn,?v) &*& List(nn,?tail) &*& elems == cons(v,tail);

predicate ListInv(List l; list<int> elems) = l.head |-> ?h &*& List(h,elems);
```

```
class Node {
    Node next;
    int val;

    Node(int v, Node next)
    //@requires true;
    //@ensures Node(this,next,v);
     {
       this.next = next;
       val = v;
    }

}
```

```
class List {
  Node head;

  public List()
  //@ requires true;
  //@ ensures ListInv(this,nil);
  {
    head = null;
  }
  void add(int elem)
  //@ requires ListInv(this,?l);
  //@ ensures ListInv(this,cons(elem,l));
  {
    Node next = new Node(elem,head);
    head = next;
  }
```

Lists in Separation Logic in Verifast

- Can we “check” this proof using Verifast?

```
predicate Node(Node n; Node nn, int v) = n.next |-> nn &*& n.val |-> v;
predicate List(Node n; list<int> elems) = n == null? (emp &*& elems == nil):
Node(n,?nn,?v) &*& List(nn,?tail) &*& elems == cons(v,tail);

predicate ListInv(List l; list<int> elems) = l.head |-> ?h &*& List(h,elems);

   void reverseList()
       //@ requires ListInv(this,?l);
       //@ ensures ListInv(this,reverse(l));
       {
           Node n = null;
           //@open ListInv(this,l);
           while (head != null)
           //@ invariant head |-> ?h &*& List(h,?l1) &*& List(n,?l2) &*&  l == append(reverse(l2),l1);
           {
               Node k = head.next;
               head.next = n;
               n = head;
               head = k;
               //@assert l1 == cons(?v,?tail0) &*&  l == append(reverse(l2),cons(v,tail0));
               //@reverse_reverse(cons(v,tail0));
               //@reverse_append( reverse(cons(v,tail0))  , l2 );
               //@append_assoc(reverse(tail0),cons(v,nil),l2);
               //@reverse_append(reverse(tail0),cons(v,l2));
               //@reverse_reverse(tail0);
           }
           //@open List(h,l1);
           head = n;
           //@append_nil(reverse(l2));

       }
```

List Manipulation

Lists in Separation Logic in Verifast

- Can we “check” this proof using Verifast? Yes! :)
- All lemmas used are in the “standard library”:

```
lemma_auto void append_nil<t>(list<t> xs);
    requires true;
    ensures append(xs, nil) == xs;

lemma void append_assoc<t>(list<t> xs, list<t> ys, list<t> zs);
    requires true;
    ensures append(append(xs, ys), zs) == append(xs, append(ys, zs));

lemma void reverse_append<t>(list<t> xs, list<t> ys);
    requires true;
    ensures reverse(append(xs, ys)) == append(reverse(ys), reverse(xs));

lemma_auto void reverse_reverse<t>(list<t> xs);
    requires true;
    ensures reverse(reverse(xs)) == xs;
```

Data Structures in Separation Logic

- Many other pointer-based data structures can be specified and verified using Separation Logic:
  - Doubly-linked Lists
  - Binary trees
  - …
- Sophisticated algorithms:
  - Tree traversals
  - Graph algorithms
  - In-place DFS without using additional memory (Schorr-Waite)

Data Structures in Separation Logic

```
public class Tree{
    public int value;
    public Tree left;
    public Tree right;

    public void add(int x)
    //@ requires tree(this,?b) &*& b!=tnil &*& false==t_contains(b,x) &*& inorder(b)==true;
    //@ ensures tree(this,tree_add(b,x)) &*& inorder(tree_add(b,x))==true;
    {   //@ open tree(this,b);
        int v=this.value;
        Tree l=this.left;
        //@ open tree(l,?bl);
        //@ close tree(l,bl);
        Tree r=this.right;
        //@ open tree(r,?br);
        //@ close tree(r,br);
        if(x < v){
            if(l!=null){
                l.add(x);
                //@ tree_add_inorder(b,x);
                //@ close tree(this,tcons(v,tree_add(bl,x),br));
            }else{
                Tree temp=new Tree(x);
                this.left=temp;
                //@ open tree(l,bl);
                //@ close tree(this,tcons(v,tcons(x,tnil,tnil),br));
                //@ tree_add_inorder(b,x);
            }
        }else{
            if(v < x){
                if(r!=null){
                    r.add(x);
                    //@ tree_add_inorder(b,x);
                    //@ close tree(this,tcons(v,bl,tree_add(br,x)));
                }else{
                    Tree temp=new Tree(x);
                    this.right=temp;
                    //@ open tree(r,br);
                    //@ close tree(this,tcons(v,bl,tcons(x,tnil,tnil)));
                }
            }
        }
    }
```

In-order List Insertion

Data Structures in Separation Logic

- In-place DFS (Schorr-Waite graph marking):
  - Standard garbage collection algorithm
  - Mark reachable nodes of a graph using only “one bit” per node
  - Unmarked nodes can be collected and re-used

- How do find all reachable nodes in a graph? DFS or BFS.
- Depth-first Search:
  - **Recursive** — Requires space proportional to the size of the graph
  - **Iterative** — Requires a stack (keeps nodes to visit)

Data Structures in Separation Logic

- Schorr-Waite graph marking manipulates the pointers in the graph so that the stack of nodes is encoded in the graph itself.

```
/*@
  predicate Tree(Node t, boolean m) =
    t == null ? true : t.marked |-> m &*& t.child |-> ?c &*&
                       t.left |-> ?l &*& t.right |-> ?r &*& Tree(l, m) &*& Tree(r,m);

  predicate Stack(Node t) =
    t == null ? true : t.marked |-> true &*& t.child |-> ?c &*&
                       t.left |-> ?l &*& t.right |-> ?r &*&
                       (c == false ? Stack(l) &*& Tree(r, false) : Stack(r) &*& Tree(l,true));
@*/
```

```
class Node {
    boolean marked;
    boolean child; //false: L , true: R
    Node left;
    Node right;
```

```
void SchorrWaite()
    //@ requires Tree(this, false);
    //@ ensures Tree(_, true);
    {
    Node t = this;
    Node p = null;
    //@close Stack(p);
    //@open Tree(this,false);
    while (p != null || (t!=null && !(t.marked)))
        /*@ invariant (t == null ? true : t.marked |-> ?m &*&
                    t.child |-> ?c &*& t.left |-> ?l &*&
                    t.right |-> ?r &*& Tree(r,m) &*& Tree(l,m)) &*&
                    Stack(p);
          @*/
      {
        if (t == null || t.marked) {
```

Data Structures in Separation Logic

```
void SchorrWaite()
    //@ requires Tree(this, false);
    //@ ensures Tree(_, true);
    {
    Node t = this;
    Node p = null;
    //@close Stack(p);
    //@open Tree(this,false);
    while (p != null || (t!=null && !(t.marked)))
        /*@ invariant (t == null ? true : t.marked |-> ?m &*& t.child |-> ?c &*& t.left |-> ?l &*&
                t.right |-> ?r &*& Tree(r,m) &*& Tree(l,m)) &*&  Stack(p);
          @*/
    { if (t == null || t.marked) {
            //@open Stack(p);
        if (p.child) { //pop
            Node q = t;
            t = p;
            p = p.right;
            t.right = q;
            //@close Tree(q,true);

        } else { //swing
            Node q = t;
            t = p.right;
            p.right = p.left;
            p.left = q;
            p.child = true;
            //@close Tree(q,true);
            //@close Stack(p);
            //@open Tree(t,false);
        }
     }
```

```
else { //push
        Node q = p;
        p = t;
        t = t.left;
        p.left = q;
        p.marked = true;
        p.child = false;
        //@open Tree(t,false);
        //@close Stack(p);
         }
    }
    //@open Stack(p);
    //@close Tree(t,true);
    }
```

Data Structures in Separation Logic

```
void SchorrWaite()
    //@ requires Tree(this, false);
    //@ ensures Tree(_, true);
    {
    Node t = this;
    Node p = null;
    //@close Stack(p);
    //@open Tree(this,false);
    while (p != null || (t!=null && !(t.marked)))
        /*@ invariant (t == null ? true : t.marked |-> ?m &*& t.child |-> ?c &*& t.left |-> ?l &*&
                t.right |-> ?r &*& Tree(r,m) &*& Tree(l,m)) &*&  Stack(p);
          @*/
    { if (t == null || t.marked) {
            //@open Stack(p);
        if (p.child) { //pop
            Node q = t;
            t = p;
            p = p.right;
            t.right = q;
            //@close Tree(q,true);

      } else { //swing
            Node q = t;
            t = p.right;
            p.right = p.left;
            p.left = q;
            p.child = true;
            //@close Tree(q,true);
            //@close Stack(p);
            //@open Tree(t,false);
      }
     }
                        else { //push
                                Node q = p;
                                p = t;
                                t = t.left;
                                p.left = q;
                                p.marked = true;
                                p.child = false;
                                //@open Tree(t,false);
                                //@close Stack(p);
                                 }
                            }
                            //@open Stack(p);
                            //@close Tree(t,true);
                            }
```

Data Structures in Separation Logic

```
void SchorrWaite()
    //@ requires Tree(this, false);
    //@ ensures Tree(_, true);
    {
    Node t = this;
    Node p = null;
    //@close Stack(p);
    //@open Tree(this,false);
    while (p != null || (t!=null && !(t.marked)))
        /*@ invariant (t == null ? true : t.marked |-> ?m &*& t.child |-> ?c &*& t.left |-> ?l &*&
                t.right |-> ?r &*& Tree(r,m) &*& Tree(l,m)) &*&  Stack(p);
          @*/
    { if (t == null || t.marked) {
          //@open Stack(p);
       if (p.child) { //pop
          Node q = t;
          t = p;
          p = p.right;
          t.right = q;
          //@close Tree(q,true);

       } else { //swing
          Node q = t;
          t = p.right;
          p.right = p.left;
          p.left = q;
          p.child = true;
          //@close Tree(q,true);
          //@close Stack(p);
          //@open Tree(t,false);
       }
     }
```

```
else { //push
       Node q = p;
       p = t;
       t = t.left;
       p.left = q;
       p.marked = true;
       p.child = false;
       //@open Tree(t,false);
       //@close Stack(p);
        }
   }
   //@open Stack(p);
   //@close Tree(t,true);
   }
```

Part III
Arrays in Separation Logic

Arrays in Verifast

- The access to an array in Verifast is disciplined by predicates that describe segments of one position:

```
array_element(a, index, v);
```

  to denote that the value (`v`) is stored in position (`index`) of the array value (`a`).

- or more than one position:

```
array_slice(a, 0, n, vs);
```

  to denote the access to the positions of array (`a`) from (`0`) to (`n`) with values given by the specification-level list (`vs`).

Arrays in Verifast

- Properties about the elements of the array:

```
array_slice_deep(a, i, j, P, unit, vs, unit);
```

  to denote that predicate (`P`) is valid for all values (`vs`) stored in the array (`a`) in the positions from (`i`) to (`j`).

- Signature:

```
array_slice_deep<T, A, V>(T[], int, int,
predicate(A, T; V), A; list<T>, list<V>)
```

- Where the predicate has the signature:

```
predicate P<A,T,V>(A a, T v; V n);
```

Arrays in Separation Logic

- Properties about the elements of the array:

  ```
  array_slice_deep(a, i, j, P, unit, vs, unit);
  ```

  to denote that predicate (`P`) is valid for all values (`vs`) stored in the array (`a`) in the positions from (`i`) to (`j`).

- Where the predicate has the signature:

  ```
  predicate P<A,T,V>(A a, T v; V n);
  ```

```
predicate Positive(unit a, int v; unit n) = v >= 0 &*& n == unit;

array_slice_deep(s,0,n,Positive,unit,elems,_)
```

Arrays in Separation Logic

```
fixpoint int sum(list<int> vs) {
  switch(vs) {
    case nil: return 0;
    case cons(h, t): return h + sum(t);
  }
}
```

- With auxiliary functions and definitions that define properties over the values of arrays.

```
public static int sum(int[] a)
//@ requires array_slice(a, 0, a.length, ?vs);
//@ ensures array_slice(a, 0, a.length, vs) &*& result == sum(vs);
{
  int total = 0; int i = 0;
  while(i < a.length)
    //@ invariant 0 <= i &*& i <= a.length &*& array_slice(a, 0, a.length, vs)
        &*& total == sum(take(i, vs));
  {
    int tmp = a[i]; total = total + tmp;
    //@ length_drop(i, vs);
    //@ take_one_more(vs, i);
    i++;
 }
  return total;
}
```

Arrays in Separation Logic

```
lemma void take_one_more<t>(list<t> vs, int i)
  requires 0 <= i && i < length(vs);
  ensures append(take(i, vs), cons(head(drop(i, vs)), nil)) == take(i + 1, vs);
{
  switch(vs) {
    case nil:
    case cons(h, t):
      if(i == 0)
      {
      } else {
        take_one_more(t, i - 1);
      }
  }
}
```

And auxiliary lemmas that can be applied in the course of the proof.

```
lemma_auto(sum(append(xs, ys))) void sum_append(list<int> xs, list<int> ys)
  requires true;
  ensures sum(append(xs, ys)) == sum(xs) + sum(ys);
{
  switch(xs) {
    case nil:
    case cons(h, t): sum_append(t, ys);
  }
}
```

Part III
Managing Arrays in Objects

Verifast Example

- Consider a Bag of integers ADT based on an array with limited capacity.

```
public class Bag {

    int store[];
    int nelems;

    int get(int i) {…}

    int size() {…}

    boolean add(int v) {…}
}
```

-

Verifast Example - Bag

- Fields must be considered in separate heap chunks, pure conditions can be added to assertions and predicates.
- Array access is disciplined by the predicate `array_slice`

```
int get(int i)
  //@ requires this.store |-> ?s &*& array_slice(s,0,?n,_) &*& 0 <= i &*& i < n;
  //@ ensures … ;
{
  return store[i];
}
```

Verifast Example - Bag

- The representation invariant captures the legal states of the ADT, including the access to the array.

```
public class Bag {

    int store[];
    int nelems;

    /*@
      predicate BagInv(int n) =
          store |-> ?s
      &*& nelems |-> n
      &*& s != null
      &*& 0<=n &*& n <= s.length
      &*& array_slice(s,0,n,?elems)
      &*& array_slice(s,n,s.length,?others)
      ;
    @*/
...
}
```

Verifast Example - Bag

- So…

```
int get(int i)
  //@ requires BagInv(?n) &*& 0 <= i &*& i < n;
  //@ ensures BagInv(n);
{
  return store[i];
}
```

Verifast Example - Bag

- For all methods and fields…

```
int get(int i)
  //@ requires BagInv(?n) &*& 0 <= i &*& i < n;
  //@ ensures BagInv(n);
{
  return store[i];
}

int size()
  //@ requires BagInv(?n) &*& n >= 0;
  //@ ensures BagInv(n) &*& result>=0 ;
{
  return nelems;
}
```

Verifast Example - Bag

- For all methods and fields…

```
public Bag(int size)
  //@ requires size >= 0;
  //@ ensures BagInv(0);
{
  store = new int[size];
  nelems = 0;
}

boolean add(int v)
  //@ requires BagInv(_);
  //@ ensures BagInv(_);
{
  if(nelems<store.length) {
   store[nelems] = v;
   nelems = nelems+1;
   return true;
  } else {
   return false;
  }
}
```

Verifast Example - Bag

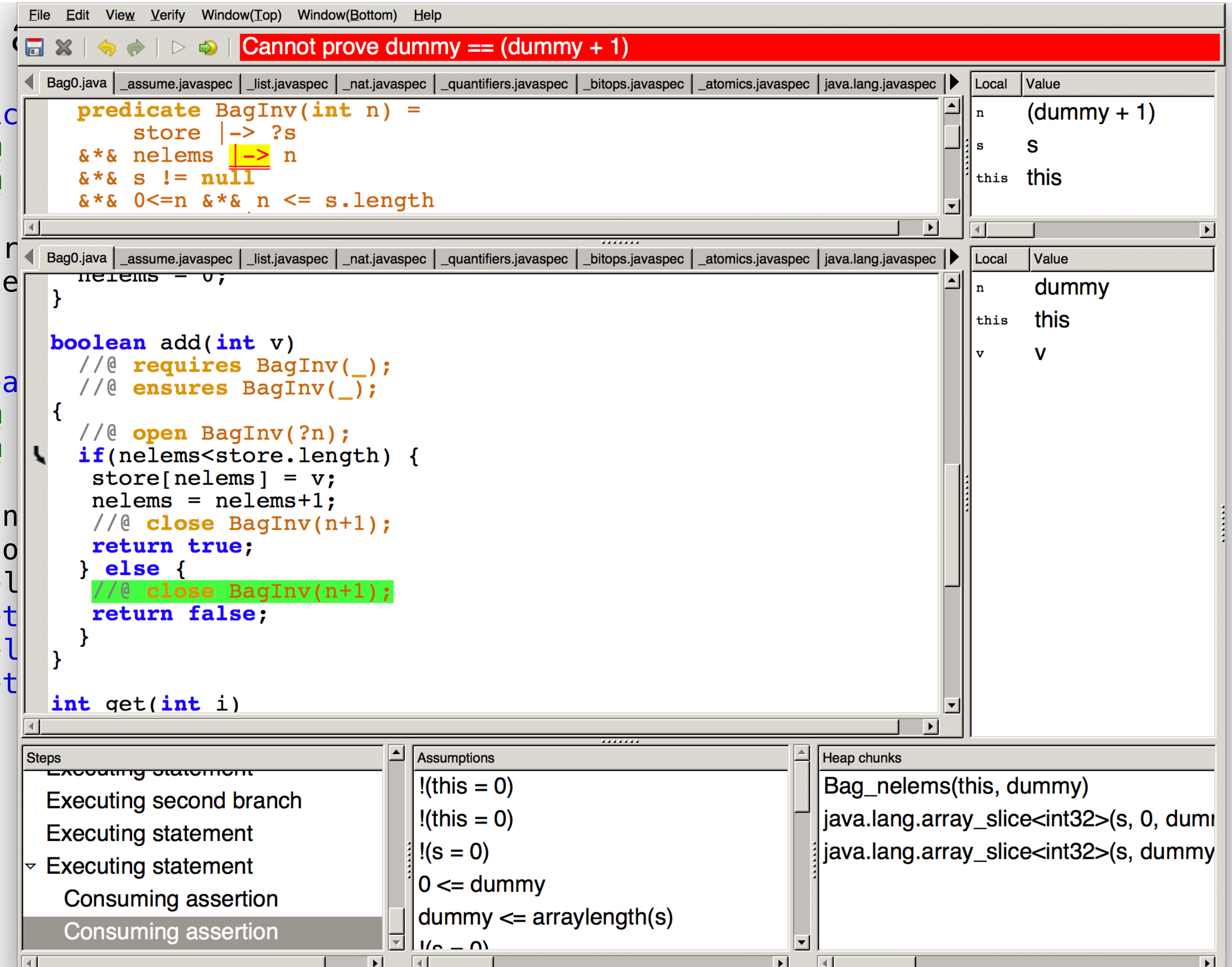
- For all methods and fields…

```
public Bag(int size)
  //@ requires size >= 0;
  //@ ensures BagInv(0);
{
  store = new int[size];
  nelems = 0;
}

boolean add(int v)
  //@ requires BagInv(?n);
  //@ ensures BagInv(n+1); // Does not hold, why?
{
  if(nelems<store.length) {
   store[nelems] = v;
   nelems = nelems+1;
   return true;
  } else {
   return false;
  }
}
```

Verifast Example - Bag

- For a

```
public
   //@
   //@
{
   stor
   nele
}

boolea
   //@
   //@
{
   if(n
    sto
    nel
    ret
   } el
    ret
   }
}
```



Verifast Example - Bag

- The parameters for the representation invariant predicate are specification level and define an abstract state.

```
boolean add(int v)
  //@ requires BagInv(?m);
  //@ ensures result ? BagInv(m+1) : BagInv(m);
{
   //@ open BagInv(?n);
  if(nelems<store.length) {
  store[nelems] = v;
  nelems = nelems+1;
   //@ close BagInv(n+1);
   return true;
  } else {
   //@ close BagInv(n);
   return false;
  }
}
```

Verifast Example - Bag

- The access to an array in Verifast is disciplined by predicates that describe segments of the array:

```
array_element(a, index, v);

array_slice(a, 0, n, vs);

array_slice_deep(a, i, j, P, unit, vs, unit);
```

- The values of the array are accessed through specification level list values and related operations

```
drop( n, vs )

take( n, vs )

append( vs, vs’)
```

Part IV
An example of an ADT
(Bank)

Managing arrays in Separation Logic

- Properties of values stored in arrays can also be captured by the array predicates.
- Examples:
  - Bag of positive integers
  - Array of ADT objects (with representation invariants)

Managing arrays in Separation Logic

```
/*@
predicate AccountInv(Account a;int b) = a.balance |-> b &*& b >= 0;
@*/

public class Account {

    int balance;

    public Account()
    //@ requires true;
    //@ ensures AccountInv(this,0);
    {
      balance = 0;
    }
    ...
}
```

Managing arrays in Separation Logic

- The bank holds an array of accounts…

```
public class Bank {

    Account store[];
    int nelems;
    int capacity;

    Bank(int max)
    {
        nelems = 0;
        capacity = max;
        store = new Account[max];
    }
    ...
}
```

Managing arrays in Separation Logic

- And implements a couple of operations…

```
public class Bank {

    Account store[];
    int nelems;
    int capacity;

    …
    Account retrieveAccount()
    {
        Account c = store[nelems-1];
        store[nelems-1] = null;
        nelems = nelems-1;
        return c;
    }
    ...
}
```

Managing arrays in SL

- And implements a couple of operations…

```
public class Bank {

    Account store[];
    int nelems;
    int capacity;

    ...
    void addnewAccount()
    {
        Account c = new Account();
        store[nelems] = c;
        nelems = nelems + 1;
    }
    ...
}
```

Managing arrays in SL

```
/*@
predicate AccountP(unit a,Account c; unit b) = AccountInv(c,?n) &*& b == unit;
@*/

public class Bank {

/*@
predicate BankInv(int n, int m) =
     this.nelems |-> n
     &*& this.capacity |-> m
     &*& m > 0
     &*& this.store |-> ?accounts
     &*& accounts.length == m
     &*& 0 <= n &*& n<=m
     &*& array_slice_deep(accounts, 0, n, AccountP, unit, _, _)
     &*& array_slice(accounts, n, m,?rest) &*& all_eq(rest, null) == true;
@*/

}
```

array slice assertions

The predicate declared in file `java.lang.javaspec` by

```
predicate array_slice<T>(T[] array, int start, int end; list<T> elements);
```

represents the footprint of the array fragment

```
array[start .. end-1]
```

- `elements` is the list of array "values" v_i such that `a[i] |-> ` v_i
- `elements` is an immutable pure value (like an OCaml list)
- `array_slice(array,start,end,elements)` is equivalent to the assertion
  - `v = [` v_{start}`, ....` v_{end-1} `]`
  - `a[start] |->` v_{start} `&*& a[start+1] |->` $v_{start+1}$ `&*& ... &*& a[end-1] |->` v_{end-1}

array slice assertions

The predicate declared in file `java.lang.javaspec` by

```
predicate array_slice_deep<T, A, V>(
    T[] array,
    int start,
    int end,
    predicate(A, T; V) p,
    A info;
    list<T> elements,
    list<V> values);
```

is as in the (simple) `array_slice` where `elements` is the list of array values v_i such that `a[i] |-> ` v_i, and the predicate $p(a,v_i;o_i)$ holds for each v_i and `values` is the list of all values o_i

Managing arrays in SL

```
public class Bank {

    Account store[];
    int nelems;
    int capacity;

    Bank(int max)
    //@ requires max>0;
    //@ ensures BankInv(0,max);
    {
      nelems = 0;
      capacity = max;
      store = new Account[max];
    }
    ...
}
```

Managing arrays in SL

```
public class Bank {

    Account store[];
    int nelems;
    int capacity;

    Account retrieveLastAccount()
    //@ requires BankInv(?n,?m) &*& n>0;
    //@ ensures  BankInv(n-1,m) &*& AccountInv(result,_);
    {
        Account c = store[nelems-1];
        nelems = nelems-1;
        return c;
        // This does not verify correctly, Why?
    }

}
```

Managing arrays in SL

```
public class Bank {

    Account store[];
    int nelems;
    int capacity;

    Account retrieveLastAccount()
    //@ requires BankInv(?n,?m) &*& n>0;
    //@ ensures  BankInv(n-1,m) &*& AccountInv(result,_);
    {
        Account c = store[nelems-1];
        store[nelems-1] = null;
        nelems = nelems-1;
        return c;
    }

}
```

Managing arrays in SL

```
public class Bank {

    Account store[];
    int nelems;
    int capacity;

    void addnewAccount()
    //@ requires BankInv(?n,?m) &*& n < m;
    //@ ensures  BankInv(n+1,m);
    {
        Account c = new Account();
        store[nelems] = c;
        //@ array_slice_deep_close(store, n, AccountP, unit);
        nelems = nelems + 1;
    }
}
```

array slice "lemmas"

```
lemma void array_slice_deep_close<T, A, V>(
   T[] array, int start, predicate(A, T; V) p, A a);
 requires array_slice<T>(array,start,start+1,?elems) &*& p(a, head(elems), ?v);
 ensures array_slice_deep<T,A,V>(array, start, start+1, p, a, elems, cons(v,nil));
```

- transforms the spec of an array element in a (singleton) `array_slice` spec into a (singleton) `array_slice_deep`
- there are other lemmas, that join together slices
- Verifast is usually able to apply lemmas automatically, but not always, in that case the programmer needs to "help", by calling the needed lemmas.

array slice “lemmas”

```
lemma void array_slice_split<T>(T[] array, int start, int start1);
  requires
    array_slice<T>(array, start, ?end, ?elems) &*&
    start <= start1 &*& start1 <= end;
  ensures
    array_slice<T>(array, start, start1, take(start1 - start, elems)) &*&
    array_slice<T>(array, start1, end, drop(start1 - start, elems)) &*&
    elems == append(take(start1 - start, elems), drop(start1 - start, elems))
```

- this “lemma” splits one array slice assertion into two (sub) array slice assertions.

array slice “lemmas”

```
lemma void array_slice_join<T>(T[] array, int start);
    requires
        array_slice<T>(array, start, ?start1, ?elems1) &*&
        array_slice<T>(array, start1, ?end, ?elems2);
    ensures
        array_slice<T>(array, start, end, append(elems1, elems2));
```

- this “lemma” joins two array slice assertions into a single array slice assertion.

array slice “lemmas”

Secure | https://people.cs.kuleuven.be/~bart.jacobs/verifast/examples/rt/Object.javaspec.html

```
package java.lang;

import java.util.*;

/*@

inductive unit = unit;

inductive pair<a, b> = pair(a, b);

fixpoint a fst<a, b>(pair<a, b> p) {
    switch (p) {
        case pair(x, y): return x;
    }
}

fixpoint b snd<a, b>(pair<a, b> p) {
    switch (p) {
        case pair(x, y): return y;
    }
}

fixpoint t default_value<t>();

inductive boxed_int = boxed_int(int);
fixpoint int unboxed_int(boxed_int i) { switch (i) { case boxed_int(value): return value; } }

inductive boxed_bool = boxed_bool(boolean);
fixpoint boolean unboxed_bool(boxed_bool b) { switch (b) { case boxed_bool(value): return value; } }

predicate array_element<T>(T[] array, int index; T value);
predicate array_slice<T>(T[] array, int start, int end; list<T> elements);
predicate array_slice_deep<T, A, V>(T[] array, int start, int end, predicate(A, T; V) p, A info; list<T> elements

lemma_auto void array_element_inv<T>();
    requires [?f]array_element<T>(?array, ?index, ?value);
    ensures [f]array_element<T>(array, index, value) &*& array != null &*& 0 <= index &*& index < array.length;
```