LaboratoryWork2

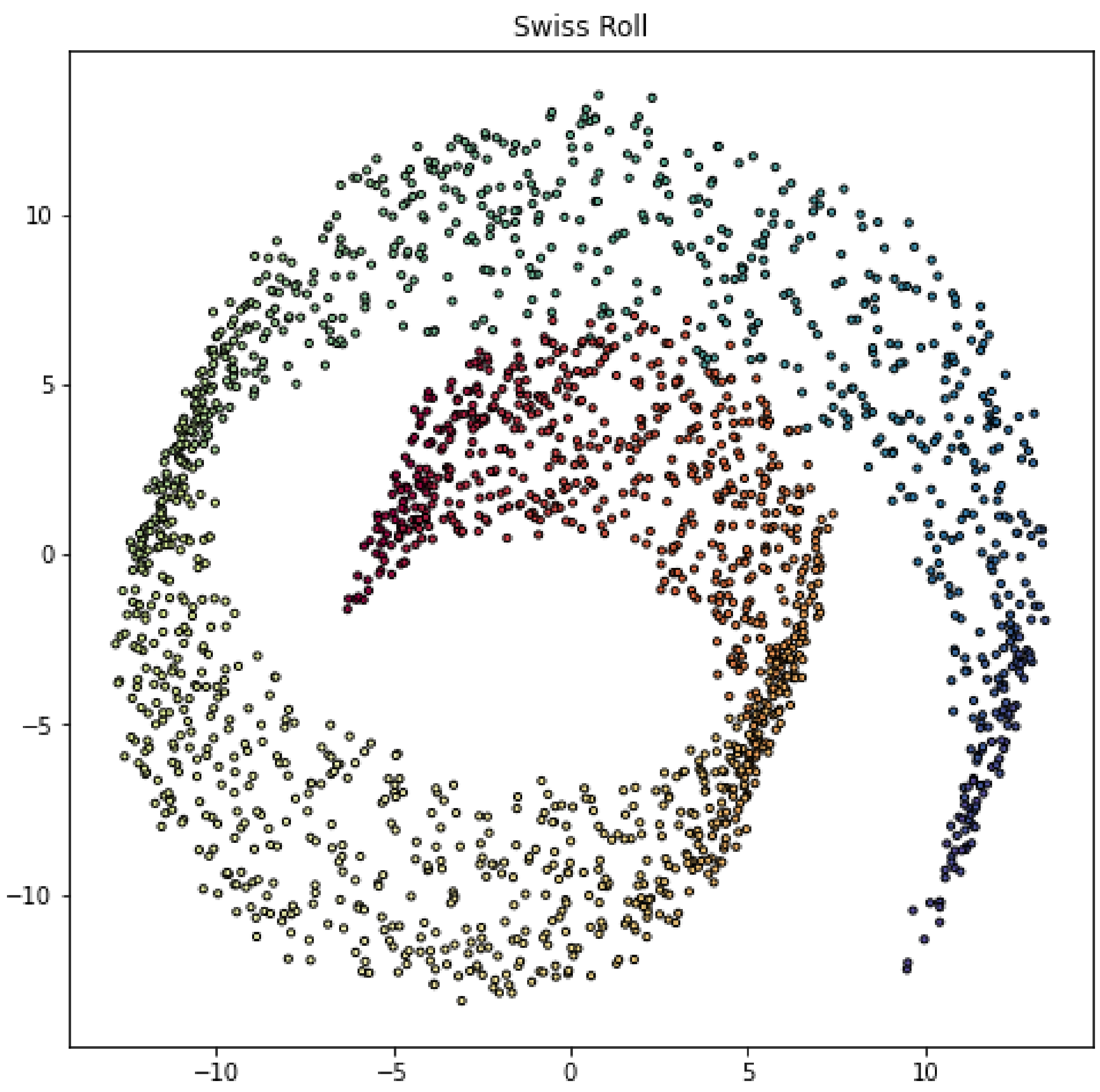
January 4, 2022

0.0.1 Part I. Swiss-roll datase

```
[ ]: import numpy as np
     import matplotlib.pyplot as plt
     from sklearn.datasets import make_swiss_roll
     from sklearn.decomposition import PCA
     from sklearn.manifold import LocallyLinearEmbedding
     from sklearn.manifold import MDS
     from sklearn.manifold import TSNE
     from sklearn.datasets import load_digits
     from sklearn.tree import DecisionTreeClassifier
     from sklearn.neighbors import KNeighborsClassifier
     from sklearn.metrics import accuracy_score
```

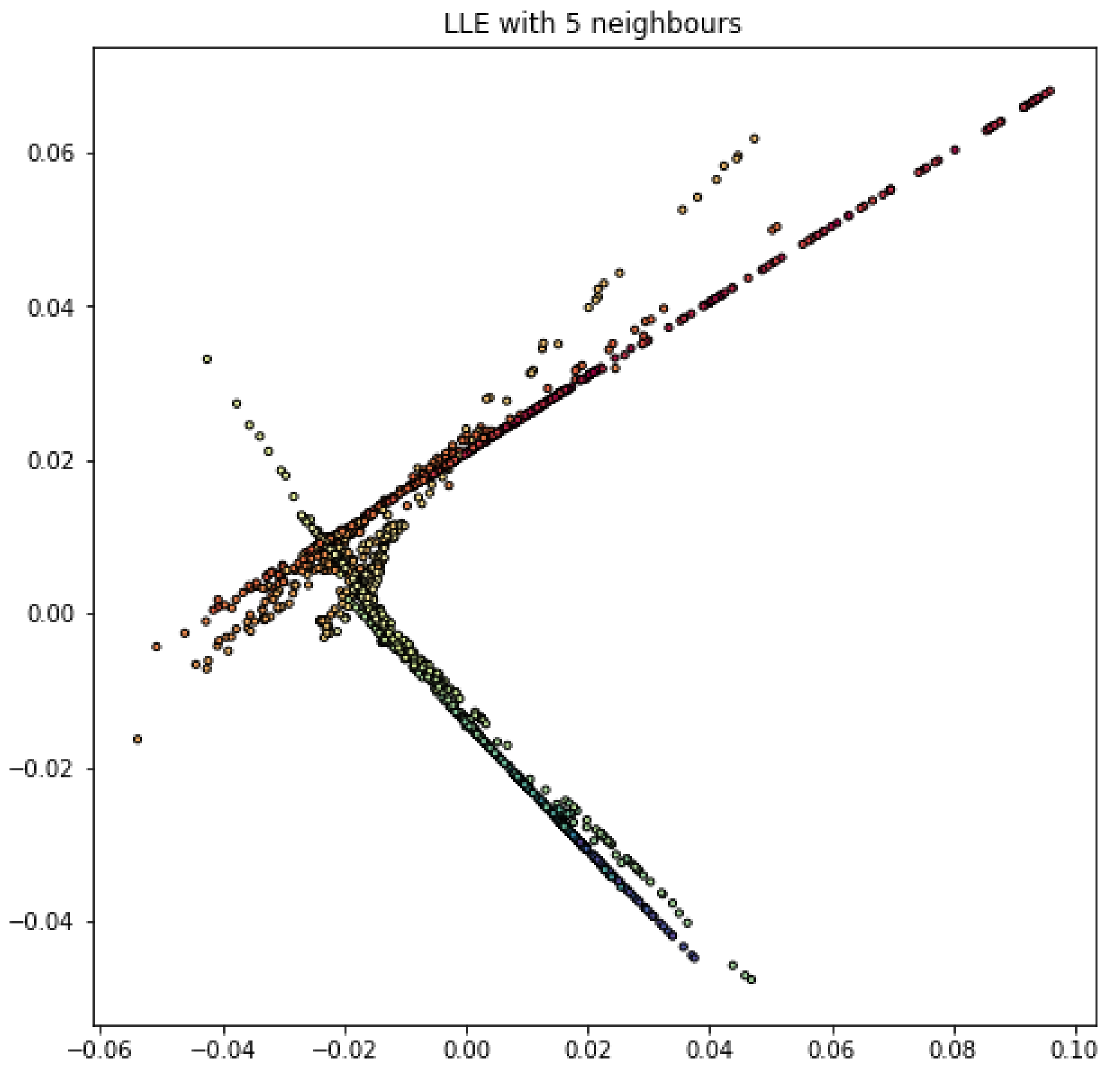
```
[ ]: def plot_swiss_roll(X, y, title):
         """
         Plot the first 2000 points of the swis-roll dataset.
         """
         pca = PCA(n_components=2)
         X_pca = pca.fit_transform(X)
         plt.figure(figsize=(8, 8))
         plt.scatter(X_pca[:, 0], X_pca[:, 1], c=y, s=10, edgecolor='k', cmap=plt.cm.
      ↪get_cmap('Spectral'))
         plt.title(title)
         plt.show()
```

```
[ ]: # Generate the swis-roll dataset with 2000 points using the functio datasets.
      ↪make_swiss_roll
     # and plot the first 100 points.

     X, y = make_swiss_roll(n_samples=2000, noise=0.2)
     plot_swiss_roll(X, y, "Swiss Roll")
```



```
[ ]: def plot(X, y, title):
         """
         Plot the first 2000 points of the swis-roll dataset.
         """
         pca = PCA(n_components=2)
         X_pca = pca.fit_transform(X)
         plt.figure(figsize=(8, 8))
         plt.scatter(X_pca[:, 0], X_pca[:, 1], c=y, s=10, edgecolor='k', cmap=plt.cm.
      ↪get_cmap('Spectral'))
         plt.title(title)
         plt.show()
```

```
[ ]: # Apply LLE (Local Linear Embedding) with 5 neighbours by printing the error

     lle = LocallyLinearEmbedding(n_neighbors=5, n_components=2)
     X_lle = lle.fit_transform(X)
     plot(X_lle, y, "LLE with 5 neighbours")
     print(lle.reconstruction_error_)
```



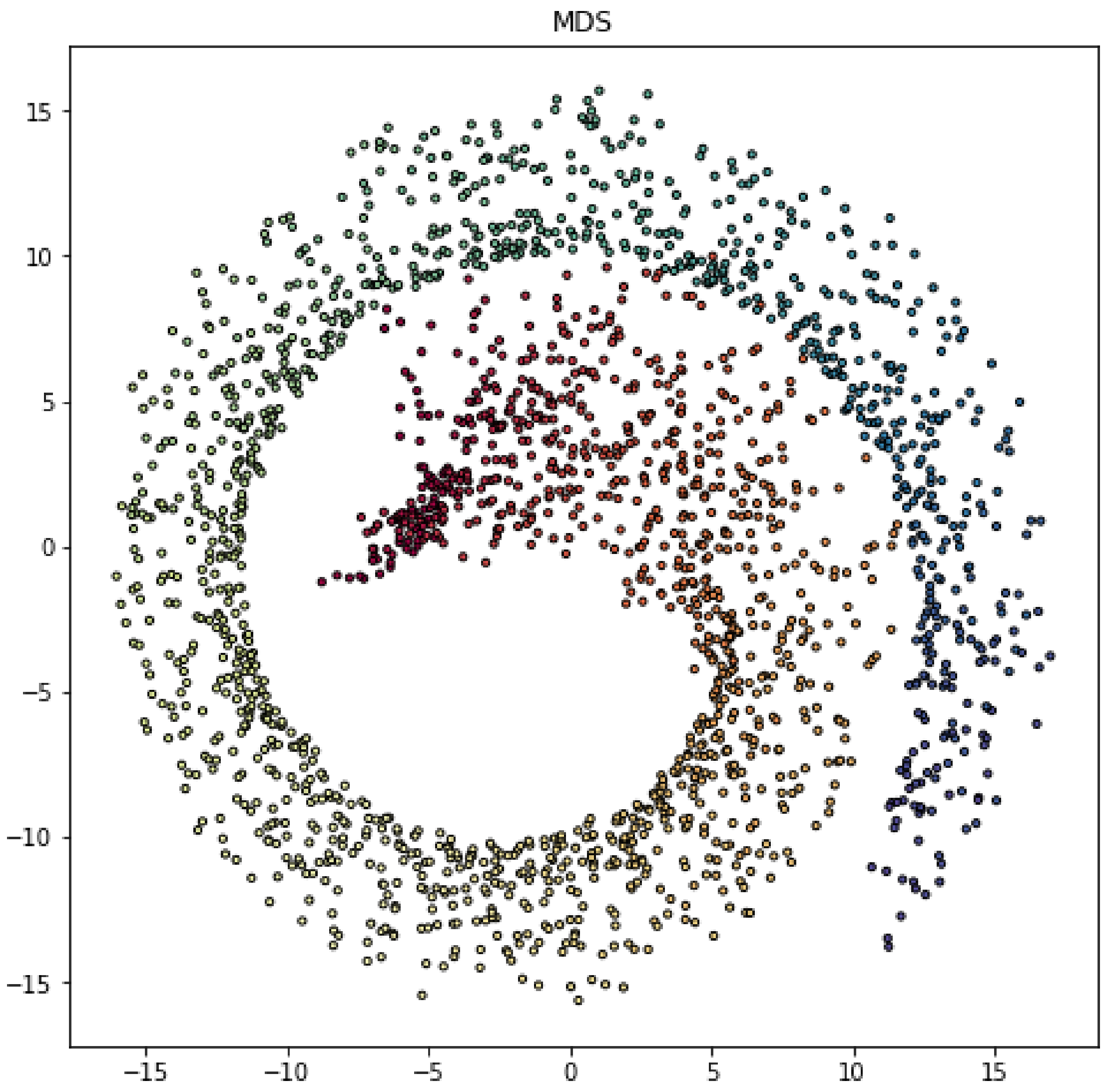
```
2.4360173452880945e-12
```

```
[ ]: # Change the number of neighbours from 2 to 15 and plot the error line. Which␣
      ↪is the best number of neighbours ?

     error = np.Infinity
     best_neighbours = None
```

```
for n_neighbors in range(2, 15):
    lle = LocallyLinearEmbedding(n_neighbors=n_neighbors, eigen_solver='dense')
    X_lle = lle.fit_transform(X)
    if lle.reconstruction_error_ < error:
        error = lle.reconstruction_error_
        best_neighbours = n_neighbors
    print("n_neighbours = {}: Reconstruction error = {}".format(n_neighbors,␣
 ↪lle.reconstruction_error_))

print("The best number of neighbors is {}.".format(best_neighbours))
```

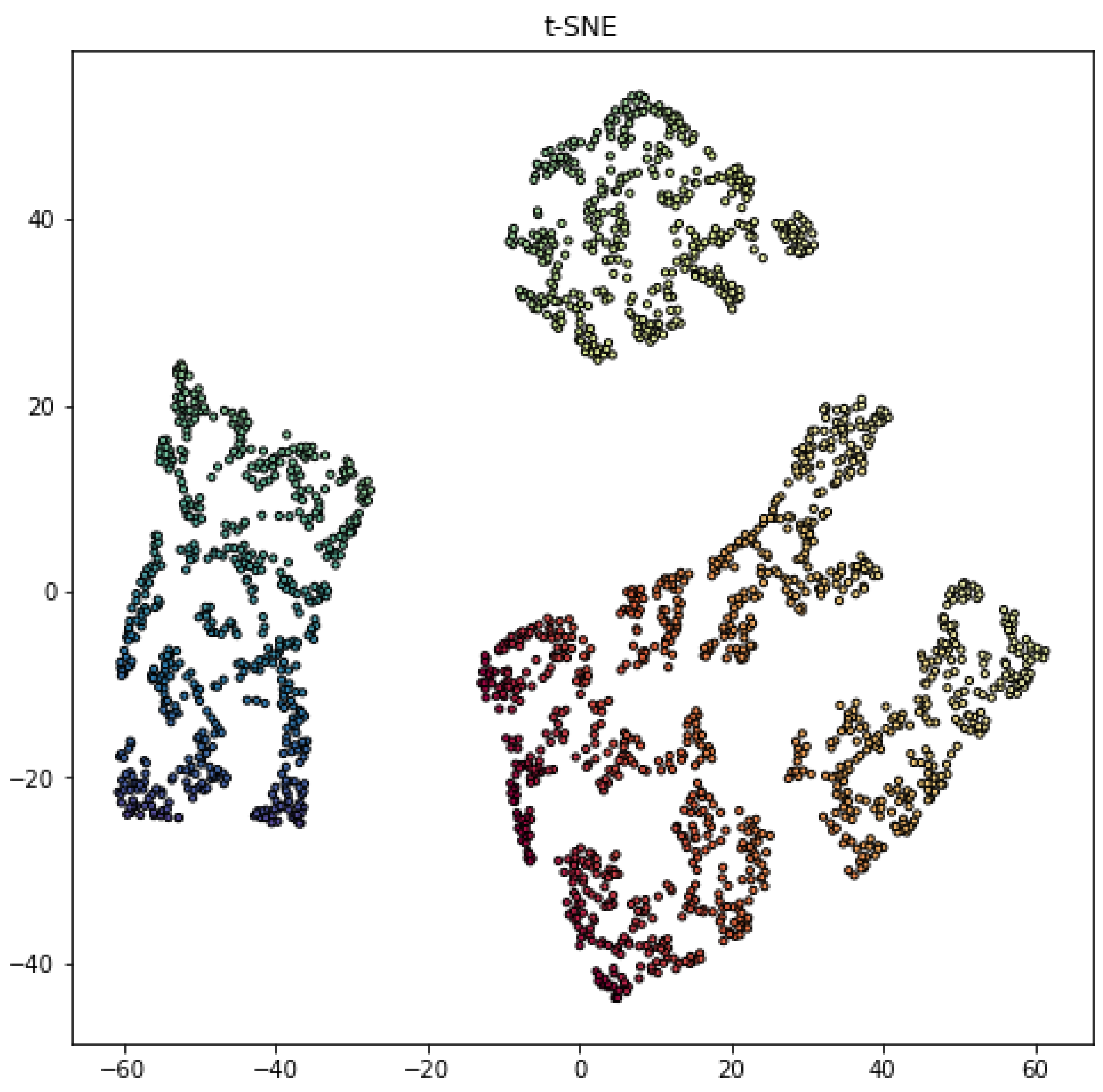
```
n_neighbours = 2: Reconstruction error = -6.289026807873649e-15
n_neighbours = 3: Reconstruction error = -1.8366789880515 94e-16
n_neighbours = 4: Reconstruction error = -2.611543152542782e-16
n_neighbours = 5: Reconstruction error = 2.4351417956970172e-12
n_neighbours = 6: Reconstruction error = 1.4595895512299734e-10
n_neighbours = 7: Reconstruction error = 2.5605038867055727e-10
n_neighbours = 8: Reconstruction error = 2.962623980351711e-09
n_neighbours = 9: Reconstruction error = 8.233248359376546e-10
n_neighbours = 10: Reconstruction error = 7.446463699008286e-09
n_neighbours = 11: Reconstruction error = 2.8208150443870565e-08
n_neighbours = 12: Reconstruction error = 3.525462456301093e-08
n_neighbours = 13: Reconstruction error = 3.150649230884524e-08
n_neighbours = 14: Reconstruction error = 3.242402318890731e-08
The best number of neighbors is 2.
```

[ ]:
```
# Use Multi Dimensional Scaling with manifold.MDS and visualize the dataset in␣
 ↪2 dimension

mds = MDS(n_components=2, max_iter=100, n_init=1)
X_mds = mds.fit_transform(X)
plot(X_mds, y, "MDS")
```



[ ]:
```
# Apply t-SNE model to the same dataset with manifold.TSNE

tsne = TSNE(n_components=2, init='pca', random_state=0)
X_tsne = tsne.fit_transform(X)
plot(X_tsne, y, "t-SNE")
```

```
/home/ion/.local/lib/python3.9/site-packages/sklearn/manifold/_t_sne.py:790:
FutureWarning: The default learning rate in TSNE will change from 200.0 to
'auto' in 1.2.
  warnings.warn(
/home/ion/.local/lib/python3.9/site-packages/sklearn/manifold/_t_sne.py:982:
FutureWarning: The PCA initialization in TSNE will change to have the standard
deviation of PC1 equal to 1e-4 in 1.2. This will ensure better convergence.
  warnings.warn(
```



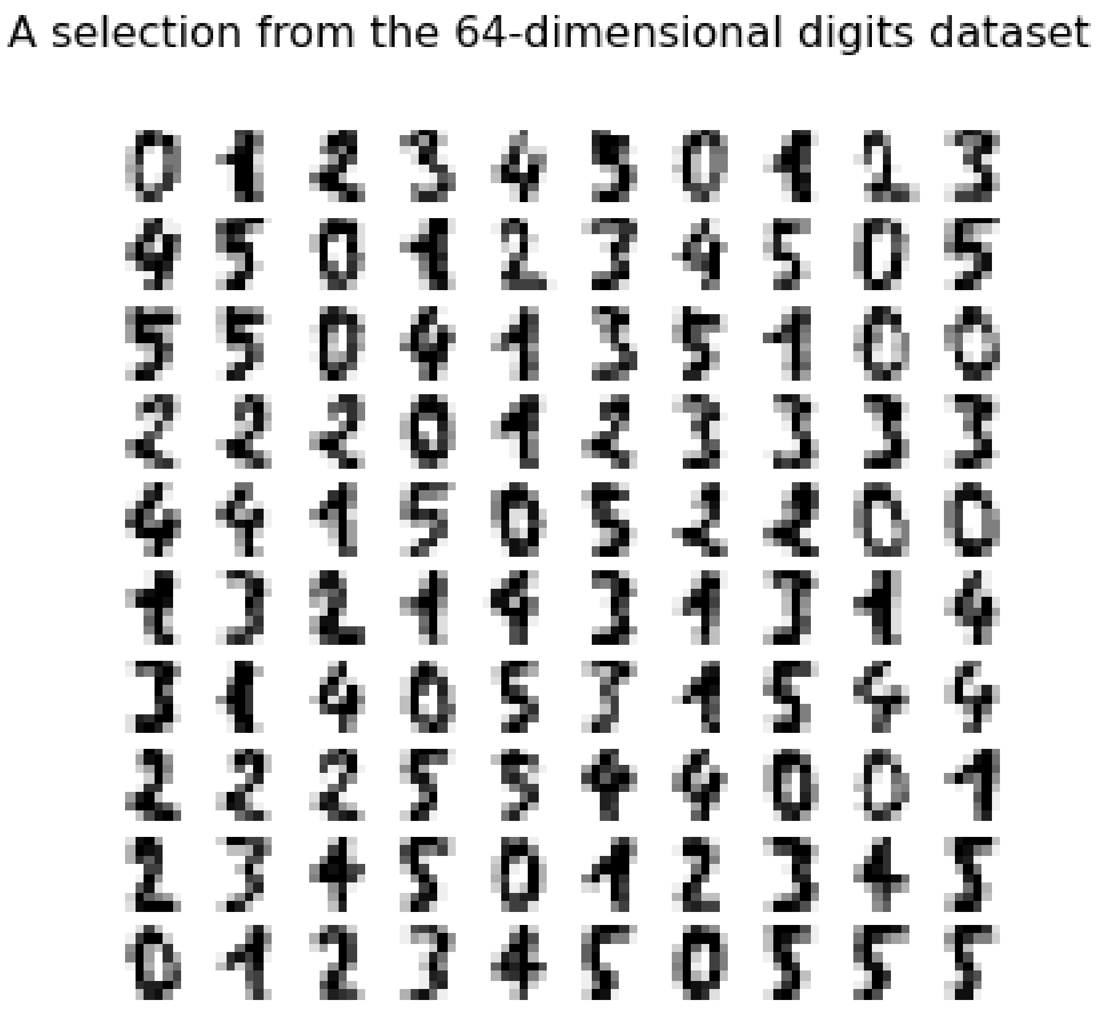
The best model is t-SNE because it could group the data into two dimensions and create clusters that contains pointsof the same color. Even if we reexecute the code, there will be other clusters that may collide, but the model will always be able to create distiguishable clusters. With MDS the are no clusters but the model could create a spiral and separated the red points from the blue points.

0.0.2 Part II. Digit dataset

[ ]:
```
def plot_digits(X, y, title):
    """
    Plot the first 100 digits of the dataset.
    """
    plt.figure(figsize=(8, 8))
    plt.imshow(np.reshape(X[0], (8, 8)), cmap=plt.cm.gray)
    plt.title(title)
    plt.show()
```

```
[ ]: # Import the digit dataset containing only 6 classes

     digits = load_digits(n_class=6)
     X, y = digits.data, digits.target
     n_samples, n_features = X.shape
```

```
[ ]: fig, axs = plt.subplots(nrows=10, ncols=10, figsize=(6, 6))
     for idx, ax in enumerate(axs.ravel()):
         ax.imshow(X[idx].reshape((8, 8)), cmap=plt.cm.binary)
         ax.axis("off")
     _ = fig.suptitle("A selection from the 64-dimensional digits dataset",␣
      ↪fontsize=16)
```



```
[ ]: import numpy as np
     from matplotlib import offsetbox
     from sklearn.preprocessing import MinMaxScaler

     def plot_embedding(X, title, ax):
         X = MinMaxScaler().fit_transform(X)
         for digit in digits.target_names:
             ax.scatter(
                 *X[y == digit].T,
                 marker=f"${digit}$",
                 s=60,
                 color=plt.cm.Dark2(digit),
                 alpha=0.425,
                 zorder=2,
             )
         shown_images = np.array([[1.0, 1.0]])  # just something big
         for i in range(X.shape[0]):
             # plot every digit on the embedding
             # show an annotation box for a group of digits
             dist = np.sum((X[i] - shown_images) ** 2, 1)
             if np.min(dist) < 4e-3:
                 # don't show points that are too close
                 continue
             shown_images = np.concatenate([shown_images, [X[i]]], axis=0)
             imagebox = offsetbox.AnnotationBbox(
                 offsetbox.OffsetImage(digits.images[i], cmap=plt.cm.gray_r), X[i]
             )
             imagebox.set(zorder=1)
             ax.add_artist(imagebox)

         ax.set_title(title)
         ax.axis("off")
```

```
[ ]: from sklearn.decomposition import TruncatedSVD
     from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
     from sklearn.ensemble import RandomTreesEmbedding
     from sklearn.manifold import (
         Isomap,
         LocallyLinearEmbedding,
         MDS,
         SpectralEmbedding,
         TSNE,
     )
     from sklearn.neighbors import NeighborhoodComponentsAnalysis
     from sklearn.pipeline import make_pipeline
     from sklearn.random_projection import SparseRandomProjection
```

```
embeddings = {
    "Random projection embedding": SparseRandomProjection(
        n_components=2, random_state=42
    ),
    "Truncated SVD embedding": TruncatedSVD(n_components=2),
    "Linear Discriminant Analysis embedding": LinearDiscriminantAnalysis(
        n_components=2
    ),
    "Isomap embedding": Isomap(n_neighbors=n_neighbors, n_components=2),
    "Standard LLE embedding": LocallyLinearEmbedding(
        n_neighbors=n_neighbors, n_components=2, method="standard"
    ),
    "Modified LLE embedding": LocallyLinearEmbedding(
        n_neighbors=n_neighbors, n_components=2, method="modified"
    ),
    "Hessian LLE embedding": LocallyLinearEmbedding(
        n_neighbors=n_neighbors, n_components=2, method="hessian",␣
↪eigen_solver="dense"
    ),
    "LTSA LLE embedding": LocallyLinearEmbedding(
        n_neighbors=n_neighbors, n_components=2, method="ltsa"
    ),
    "MDS embedding": MDS(n_components=2, n_init=1, max_iter=120, n_jobs=2),
    "Random Trees embedding": make_pipeline(
        RandomTreesEmbedding(n_estimators=200, max_depth=5, random_state=0),
        TruncatedSVD(n_components=2),
    ),
    "Spectral embedding": SpectralEmbedding(
        n_components=2, random_state=0, eigen_solver="arpack"
    ),
    "t-SNE embeedding": TSNE(
        n_components=2,
        init="pca",
        learning_rate="auto",
        n_iter=500,
        n_iter_without_progress=150,
        n_jobs=2,
        random_state=0,
    ),
    "NCA embedding": NeighborhoodComponentsAnalysis(
        n_components=2, init="pca", random_state=0
    ),
}
```

[ ]:
```
from time import time

projections, timing = {}, {}
```

```
for name, transformer in embeddings.items():
    if name.startswith("Linear Discriminant Analysis"):
        data = X.copy()
        data.flat[:: X.shape[1] + 1] += 0.01  # Make X invertible
    else:
        data = X

    print(f"Computing {name}...")
    start_time = time()
    projections[name] = transformer.fit_transform(data, y)
    timing[name] = time() - start_time
```

```
Computing Random projection embedding…
Computing Truncated SVD embedding…
Computing Linear Discriminant Analysis embedding…
Computing Isomap embedding…
Computing Standard LLE embedding…
Computing Modified LLE embedding…
Computing Hessian LLE embedding…
Computing LTSA LLE embedding…
Computing MDS embedding…
```

```
/home/ion/.local/lib/python3.9/site-
packages/scipy/sparse/linalg/eigen/arpack/arpack.py:936: LinAlgWarning: Diagonal
number 350 is exactly zero. Singular matrix.
  self.M_lu = lu_factor(M)
```

```
Computing Random Trees embedding…
Computing Spectral embedding…
Computing t-SNE embeedding…
```

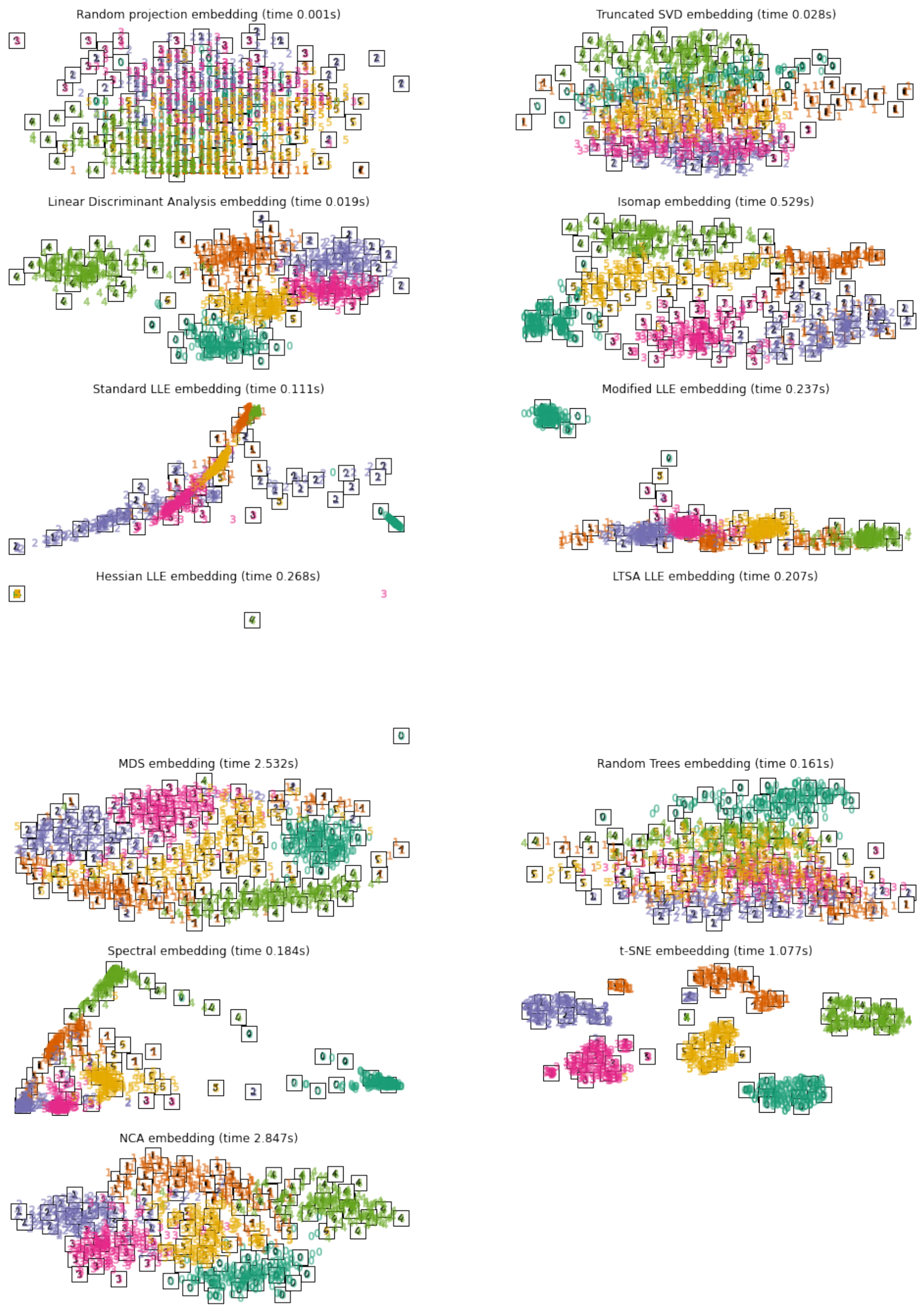
```
/home/ion/.local/lib/python3.9/site-packages/sklearn/manifold/_t_sne.py:982:
FutureWarning: The PCA initialization in TSNE will change to have the standard
deviation of PC1 equal to 1e-4 in 1.2. This will ensure better convergence.
  warnings.warn(
```

```
Computing NCA embedding…
```

[ ]:
```
from itertools import zip_longest

fig, axs = plt.subplots(nrows=7, ncols=2, figsize=(17, 24))

for name, ax in zip_longest(timing, axs.ravel()):
    if name is None:
        ax.axis("off")
        continue
    title = f"{name} (time {timing[name]:.3f}s)"
    plot_embedding(projections[name], title, ax)
```

```python
plt.show()
```

```
/home/ion/.local/lib/python3.9/site-packages/sklearn/preprocessing/_data.py:461:
RuntimeWarning: All-NaN slice encountered
  data_min = np.nanmin(X, axis=0)
/home/ion/.local/lib/python3.9/site-packages/sklearn/preprocessing/_data.py:462:
RuntimeWarning: All-NaN slice encountered
  data_max = np.nanmax(X, axis=0)
```



Analyse the results by explaining the models

- The embeding that best grouped the data is t-SNE and it took more time than most of the other embeding like Isomap embedding or Linear discriminant analysis.
- Spectral embedding tends to group the clusters farther from each other and tends to make the clusters dense.
- NCA embedding took the most of the time. It did not create as dense a cluster as Spectral embedding did and the clusters a closer to each other. The results are very similar to MDS embedding which also took a similar amount of time.
- Random tree embedding did not do as a good job as the ones mentioned above and it took much less time. The points are more spread, the clusters collide, and are badly distinguishable.
- Random Projection embedding took the least of time and the points are very spread, with closer clusters that collide. A similar result was given by Truncated SVD embedding, but this one took a bit longer and gave a better result.
- Linear Discriminant Analysis embedding created clusters where the points are closer to each other in comparison to Isomap embedding. LDA is faster and I would say it did a better job than Isomap embedding.
- Hassien LLE creates clusters where the points are in the same position while LTSA LLE didn't create any clusters.
- In the Standart LLE embedding the clusters are more visible and more spread and Modified LLE embedding.

```
[ ]: from sklearn.tree import DecisionTreeClassifier

     # for each projection
     for name, projection in projections.items():
         clf = DecisionTreeClassifier(random_state=0)
         try:
             clf.fit(projection, y)
         except ValueError:
             print(f"{name}: cannot fit a decision tree. Nan Values found in␣
      ↪projection.")
             continue
         score = clf.score(projection, y)
         print(f"{name}: Error = {1 - score:.3f} | Score = {score:.3f}")
```

```
Random projection embedding: Error = 0.282 | Score = 0.718
Truncated SVD embedding: Error = 0.000 | Score = 1.000
Linear Discriminant Analysis embedding: Error = 0.000 | Score = 1.000
Isomap embedding: Error = 0.000 | Score = 1.000
Standard LLE embedding: Error = 0.000 | Score = 1.000
Modified LLE embedding: Error = 0.003 | Score = 0.997
Hessian LLE embedding: Error = 0.829 | Score = 0.171
LTSA LLE embedding: cannot fit a decision tree. Nan Values found in projection.
MDS embedding: Error = 0.000 | Score = 1.000
Random Trees embedding: Error = 0.000 | Score = 1.000
```

```
Spectral embedding: Error = 0.000 | Score = 1.000
t-SNE embeedding: Error = 0.000 | Score = 1.000
NCA embedding: Error = 0.000 | Score = 1.000
```