

**Example 0.1** Consider a product mix problem in which there are 3 basic products ( $P_1, P_2, P_3$ ) and 2 premium ones ( $P_4, P_5$ ). There are 3 machines ( $A, B$ , and  $C$ ) that are used to make the products. Every product must be handled by all the 3 machines. The following table reports the minutes needed for 1000 g of any product on any machine, and the minutes available for any machine.

	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	minutes available
$A$	100	150	120	300	280	3500
$B$	180	160	110	300	290	4200
$C$	130	150	120	290	430	4300

Any produced quantity  $q_i$  must not exceed the upper bound  $u_i$ . At least 10000 g of basic products, and at least 3000 g of premium products must be manufactured. The utility of any product  $i$  is equal to 0 if the produced quantity  $q_i = 0$ , otherwise, it is equal to  $p_i q_i - F_i$  if  $q_i > 0$ .

	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$
$u$	6000	6000	6000	4000	4000
$p$	3	16	10	60	65
$F$	130	500	450	1000	1400

The product mix must satisfy either the following constraints:

- The produced quantity of  $P_1$  must be at least the 25% of the total production.
- The quantity of premium products must be at least 80% of that of basic products.

or the following constraints:

- The produced quantity of  $P_4$  cannot exceed 1000 g.
- The produced quantity of  $P_3$  cannot be less than 5000 g.

Maximize the utility by deciding the production quantity of any product.

**Example 0.2** Assume that the quantities  $q$  can be greater than the upper bounds  $u$ . But if  $q_i > u_i$  then a penalty  $F_i^2$  in the utility occurs.

	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$
$F^2$	2000	2000	1000	3000	2000

Maximize the utility by deciding the production quantity of any product.