



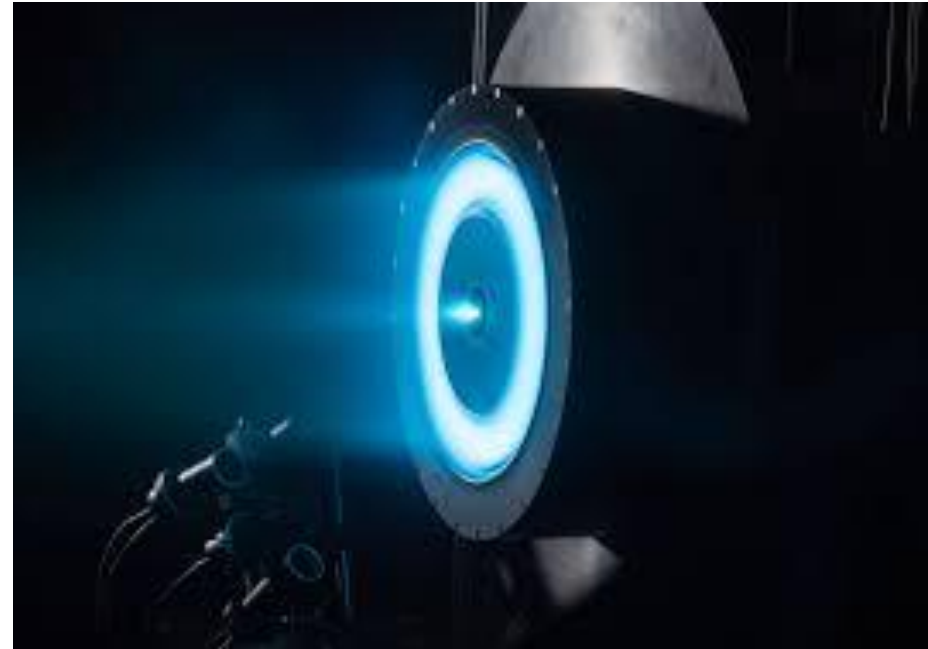
Hall Effect Thrusters...

pt1

Recap

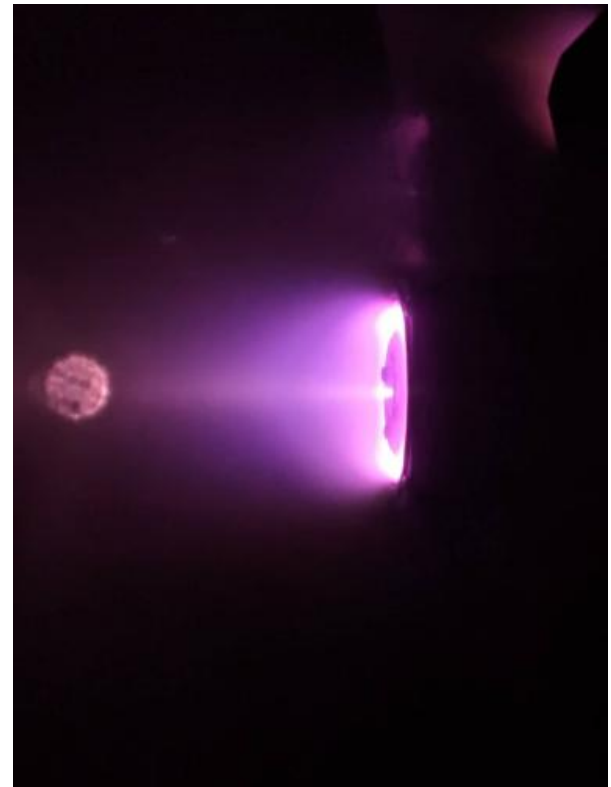
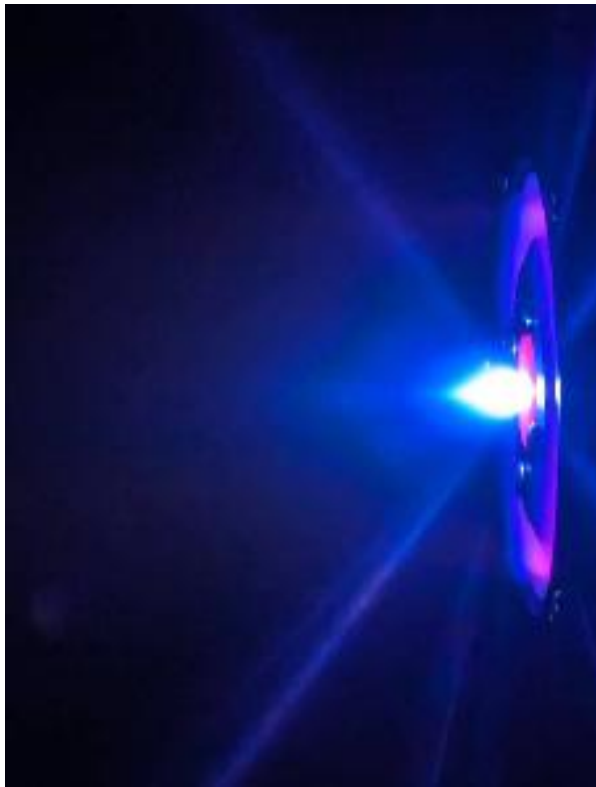


Thrust: (1 – 10000000) N's
Isp: (150 – 550)s
Efficiency: > 90%
Reusability: 5/10
Coolness: 7/10

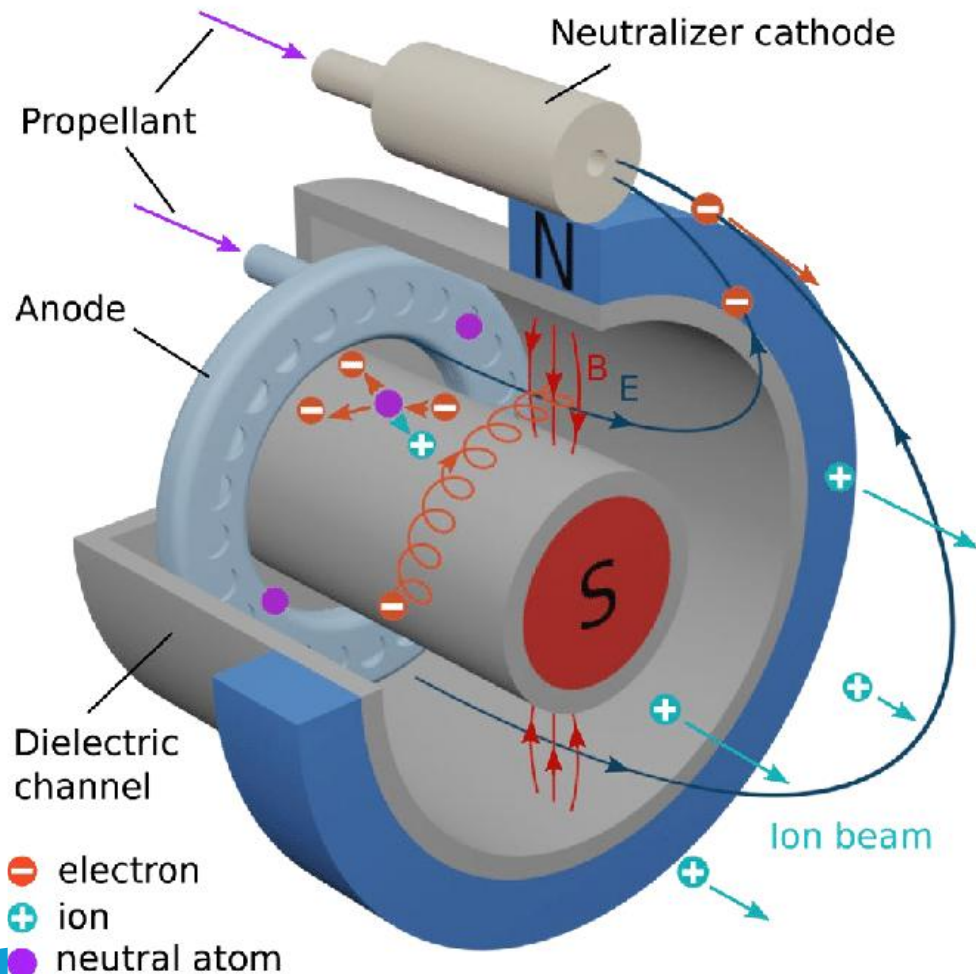


Thrust: (0.001 - 10) N's
Isp: (500 – 4000)s
Efficiency: (1 – 70)%
Reusability: 10/10
Coolness: 10/10

So cool...



Operating Principle



- Neutral gas is injected into a channel
- The neutral gas gets ionised then accelerated downstream
- Electrons are produced by a cathode
- Thruster uses the $E \times B$ drift to “trap” electrons
- Electrons are what is used for ionisation
- Ions get neutralised by electrons in the $E \times B$ region as well as further downstream electrons

Operating Principle

We can immediately calculate certain performance variables for the HET.

$$eV = \frac{1}{2} m_i v_i^2 \Rightarrow v_i = \sqrt{\frac{2eV_b}{m_i}}$$
$$F = \dot{m}_i v_i = \dot{m}_i \sqrt{\frac{2eV}{m_i}} = I_b \sqrt{\frac{2m_i V_b}{e}}$$

For a xenon thruster with a beam current and voltage of 4.5A and 300V it means the maximum exhaust velocity and thrust are (assuming 100% efficiency):

- 20989.7m/s = 2140.4s
- 0.1286N = 128.6mN

Relevant Plasma Physics

For a plasma to successfully be sustained several conditions must be obeyed:

$$\begin{aligned}\lambda_D &\ll L \\ \lambda_i &\ll L \\ r_i &\gg L \\ r_e &\ll L \\ \beta = \Omega = \frac{\omega_e}{v_e} &\gg 1\end{aligned}$$



Relevant Plasma Physics

Debye Sheath:

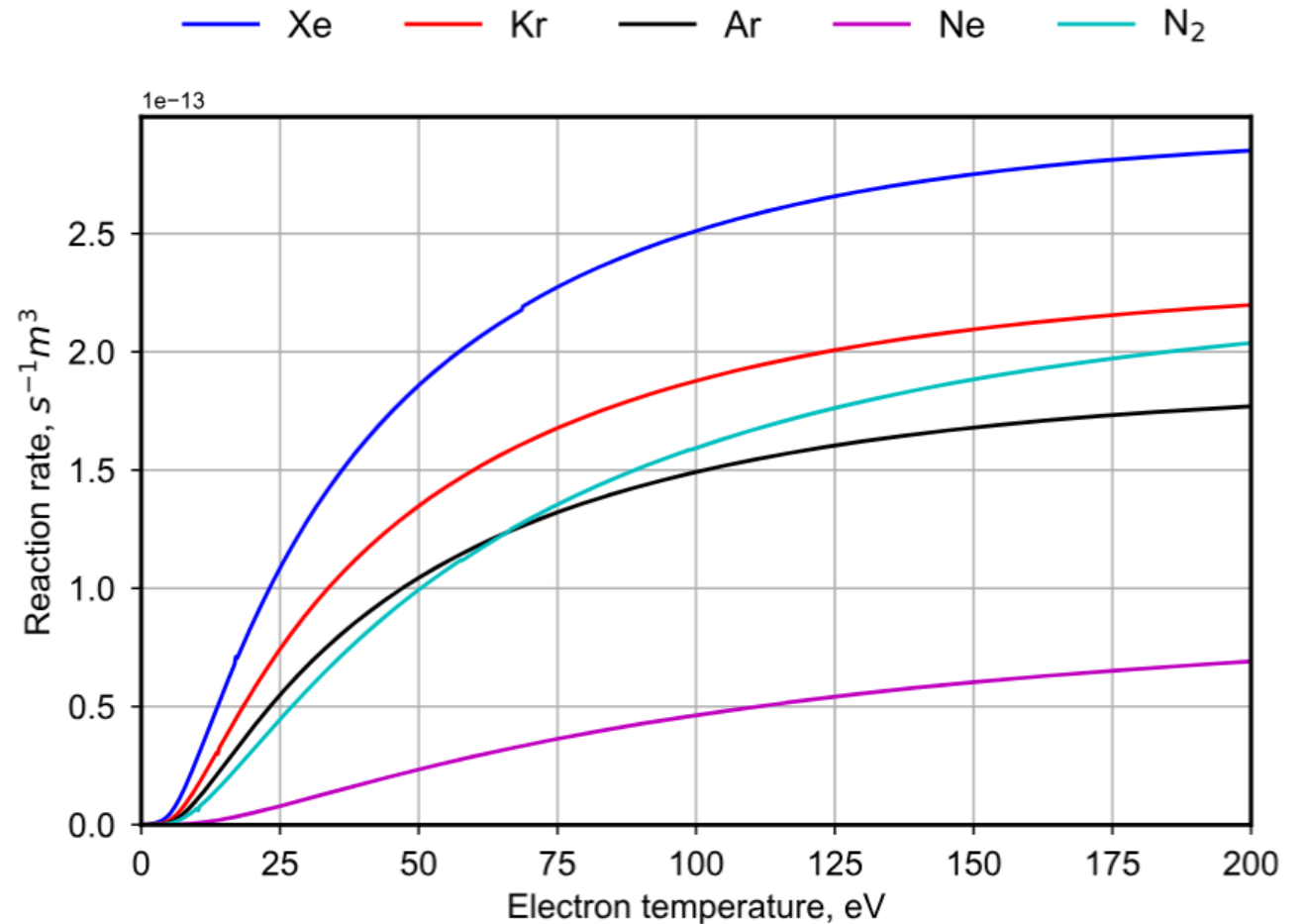
$$\lambda_D = \sqrt{\frac{\epsilon_0 k T_e}{e^2 n_e}}$$

Mean free path of ions:

$$\lambda_i = \frac{v_n}{n_e \langle \sigma_i v_e \rangle}$$

Peak electron temperature:

$$T_e = 0.12 V_d$$



Relevant Plasma Physics

Ion and electron Larmor radius differ significantly due to the difference in ion and electron thermal velocity:

$$v_i = \sqrt{\frac{2eV_b}{m_i}}, \quad v_{\text{th}} = \sqrt{\frac{8k_B T_e}{\pi m_e}}$$

$$r_L = \frac{mv_{\perp}}{qB}$$



Relevant Plasma Physics

The hall parameter is a dimensionless variable that can tell you about the electron behaviour of the thruster.

$$\Omega = \frac{\omega_e}{\nu_e} = \frac{qB}{m_e \nu_e}$$

A high hall parameter implies a high magnetic field strength and or low collision frequency, we want the hall parameter to be large to give the electron a lower diffusion and hence extend the period that it can make interactions.

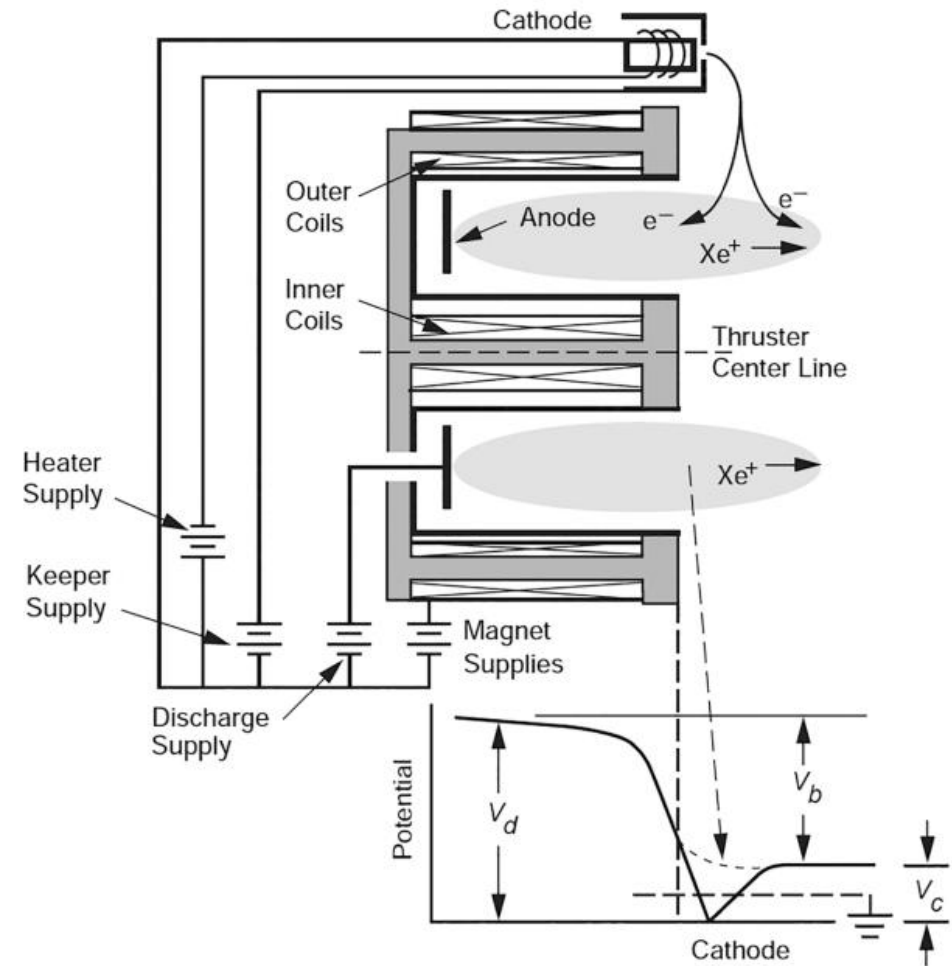
Discharge Voltage Characteristics

Ions are accelerated by the beam voltage V_b .

The total input voltage to the thruster is the discharge voltage V_d .

$$V_d = V_b + V_c$$

Where V_c is the cathode voltage



Discharge Current Characteristics

The thruster and plasma form one massive circuit, therefore the current can be analysed like with traditional circuit laws.

$$I_d = I_e = I_{ec} + I_{eb}$$

$$I_{ei} = I_{ib}$$

$$I_{ib} = I_{eb} = I_b$$

$$I_d = I_{ei} + I_{ec}$$

I_d is the discharge current

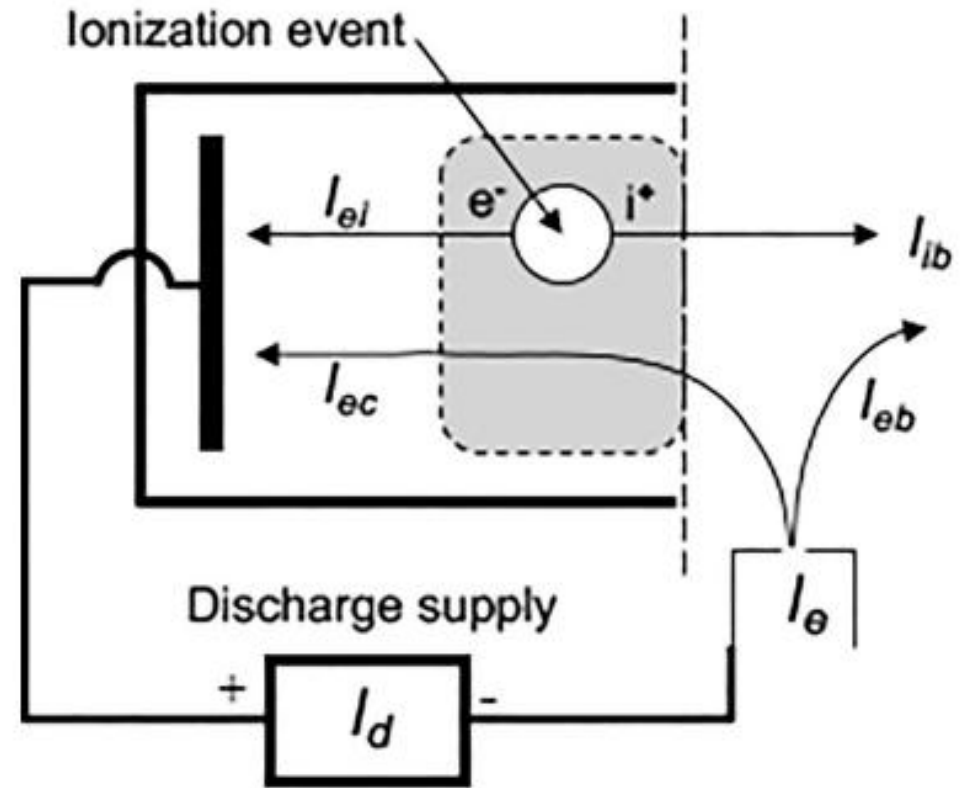
I_e is the electron current

I_{ib} is the ion beam current

I_{eb} is the electron beam current

I_{ec} is the electron cathode current

I_{ei} is the electron ionisation current



Efficiencies

Hall thruster efficiency is broken down and characterised by several forms of efficiency that allows for deeper analysis into how the thruster performs.

$$\eta = \frac{F^2}{2\dot{m}_a P_d} = \eta_b \eta_q \eta_m \eta_v \eta_d$$

η_b is the beam current utilisation

η_q is the charge utilisation

η_m is the mass utilisation

η_v is the voltage utilisation

η_d is the plume divergence

Beam Current Utilisation Efficiency

$$\eta_b = \frac{I_b}{I_d} = \frac{I_d - I_e}{I_d}$$

The beam current utilisation is maximised when the electron current is minimised this reflects the need for the rate of flow of electrons to be minimised

Charge Utilisation Efficiency

$$\eta_q = \left(\sum_n \frac{\Omega_n}{\sqrt{Z_n}} \right)^2 / \sum_n \frac{\Omega_n}{Z_n}$$

Z_n is the charge state of the n th ion species

$\Omega_n = \frac{I_n}{I_d}$ is the current fraction of the n th ion species

This efficiency improves more when all the particles have the same energy

Mass Utilisation Efficiency

$$\eta_m = \frac{\dot{m}_b}{\dot{m}_a}$$

This efficiency measures the fact not all species will be ionised. Because it is often normalised to a charge state of +1 this efficiency can exceed 100% due to multiply charged species.

Voltage Utilisation Efficiency

$$\eta_v = \frac{\overline{V_b}}{V_d}$$

$\overline{V_b}$ is the average accelerating voltage

This efficiency is due to the fact not all discharge voltage accelerates the ions as some ions are formed further downstream than others.

Plume Divergence Efficiency

$$\eta_d = (\cos\theta_d)^2$$

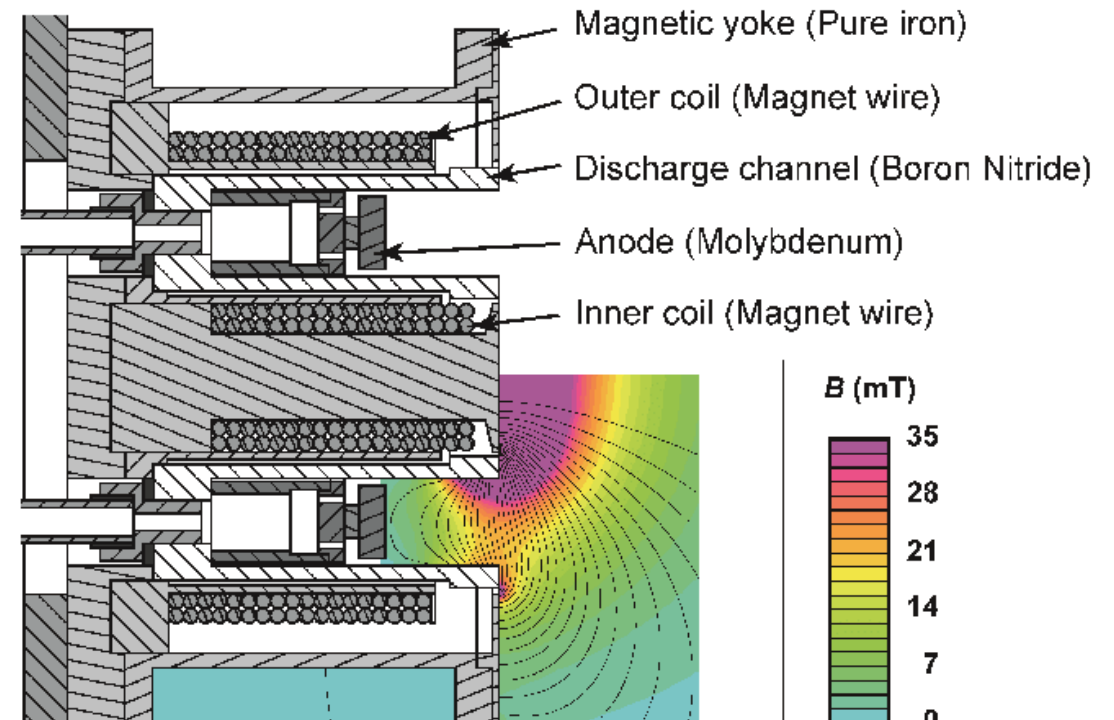
θ_d is the plume divergence, typically sits around 30° .

This efficiency accounts for the fact not all ions travel perfectly axially meaning some momentum is lost.

Magnetic Field Design

Magnetic field design is one of the most critically important parts to HET design.

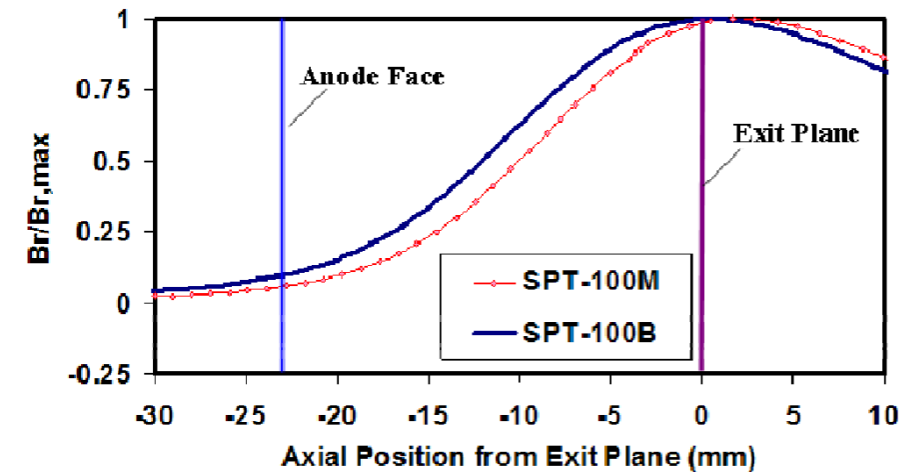
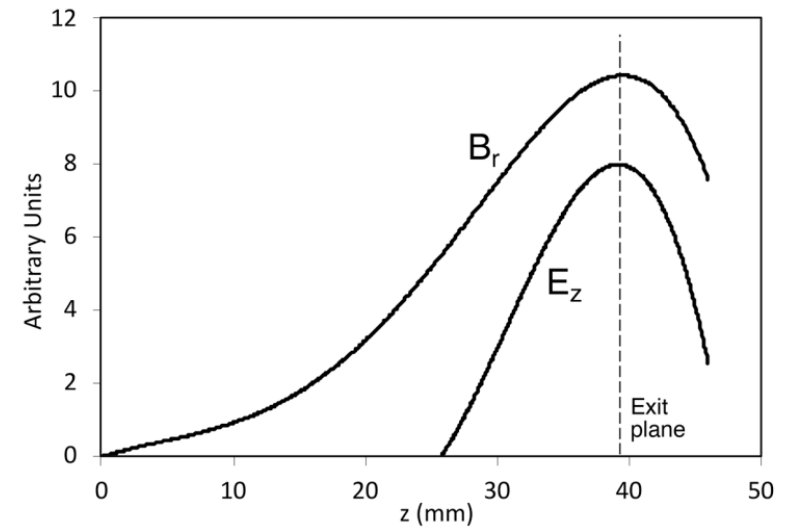
Field topology can dramatically effect performance, that being said there isn't necessarily one correct way to do it.



Magnetic Field Design

The magnetic field down the centreline of the channel should be as radial as possible, alternatively written as:

$$B_z = 0 \quad B_r = B(r, z)$$

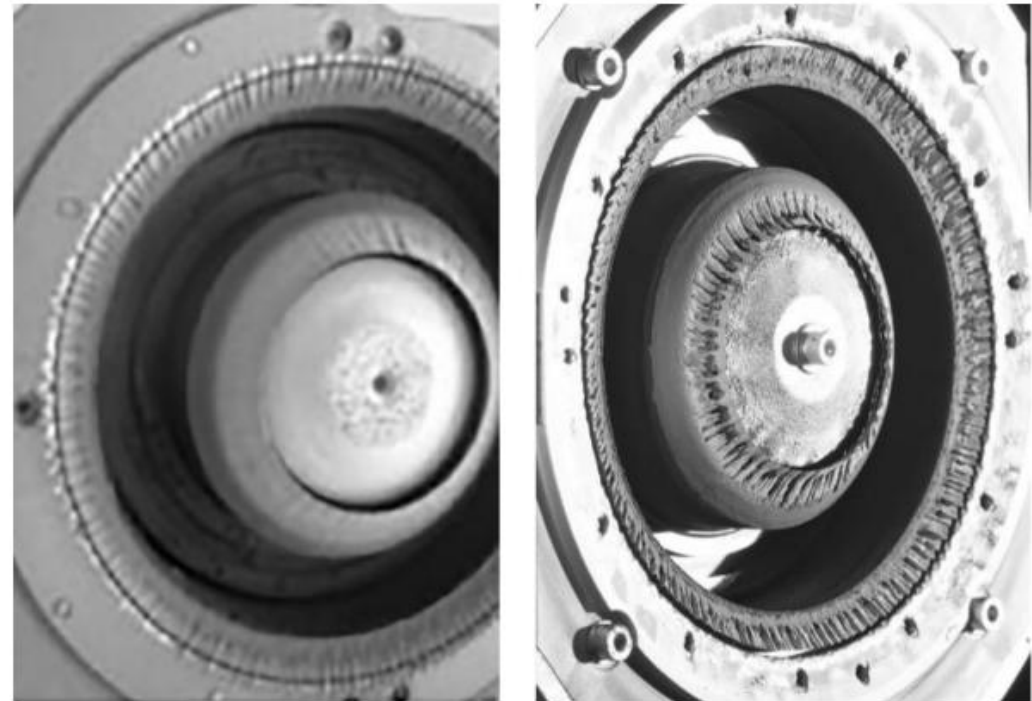
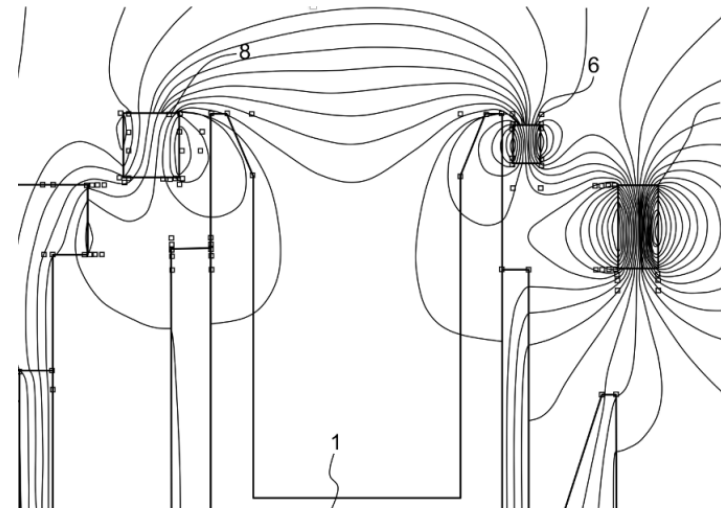


Magnetic Shielding

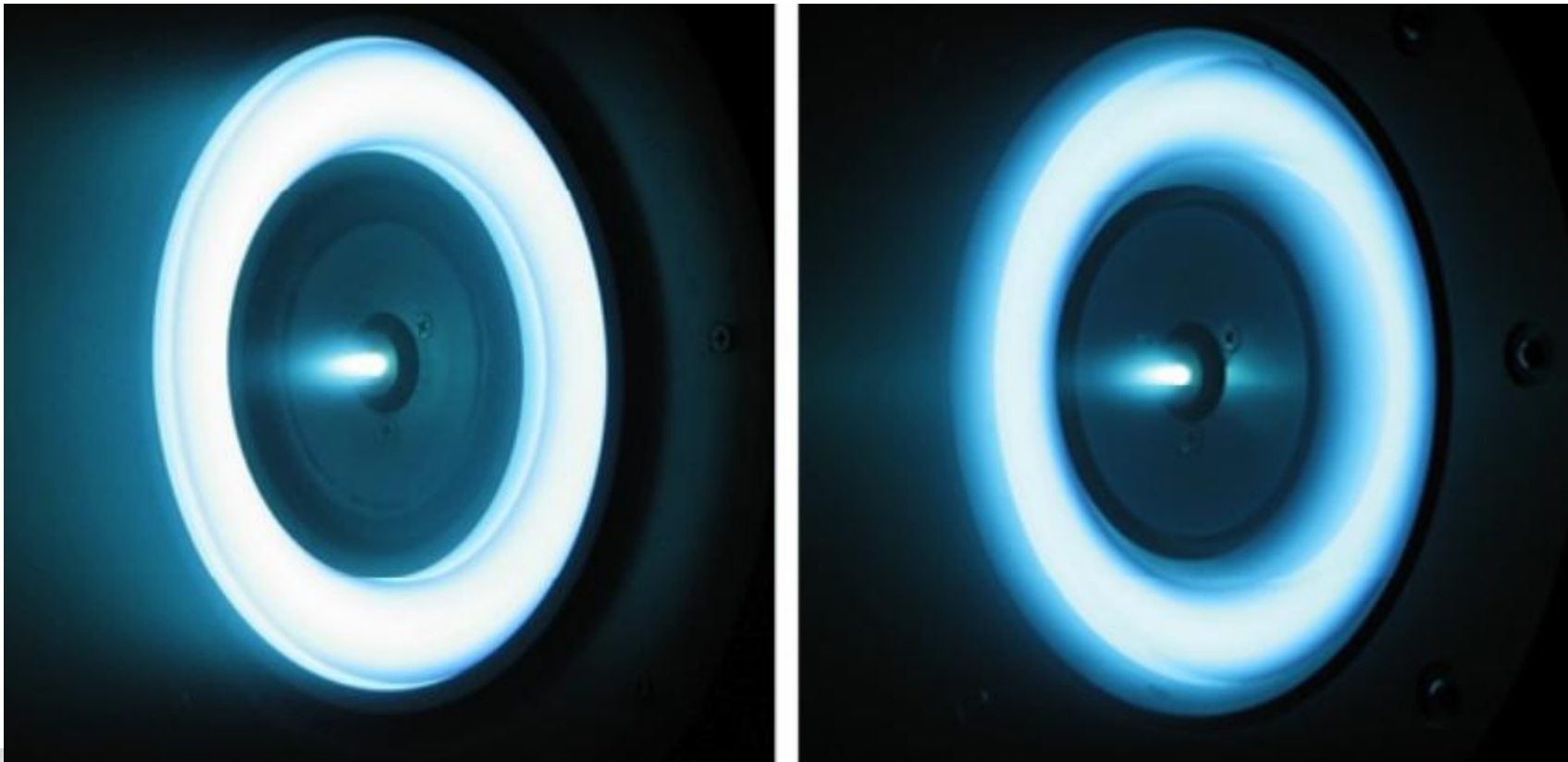
A method used to extend the operational lifetime of thrusters by reducing erosion.

Also reduces power flux to the walls, reducing thermal loads.

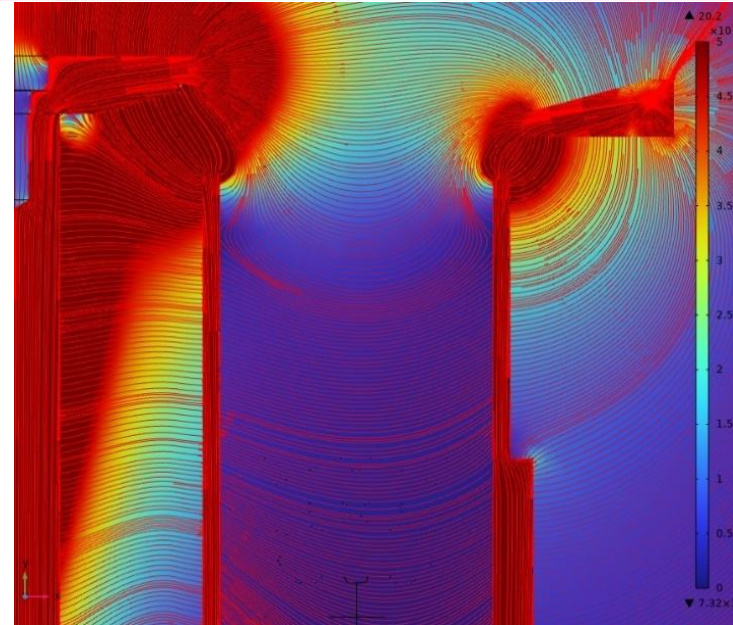
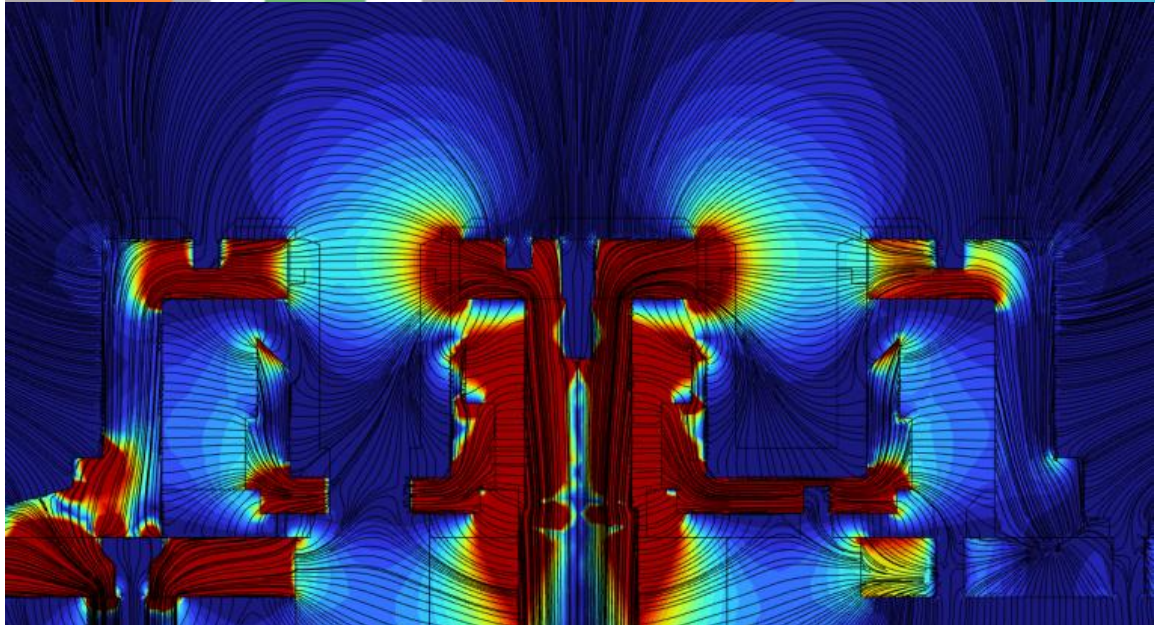
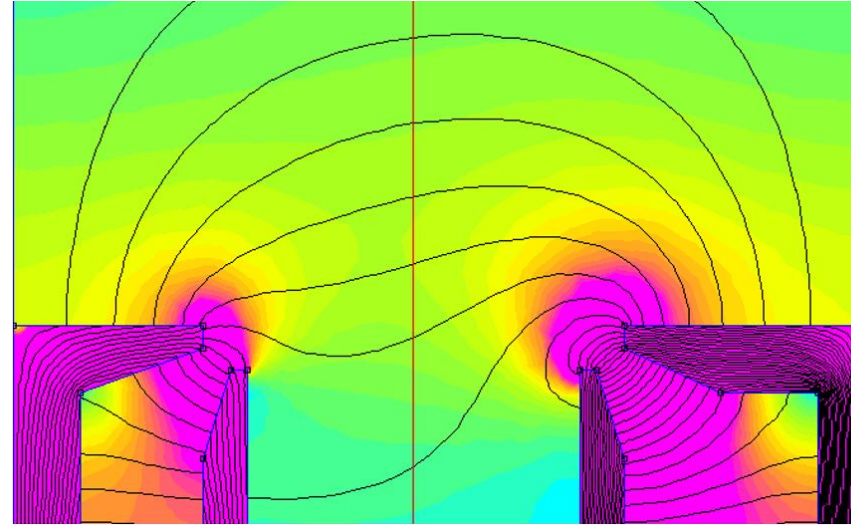
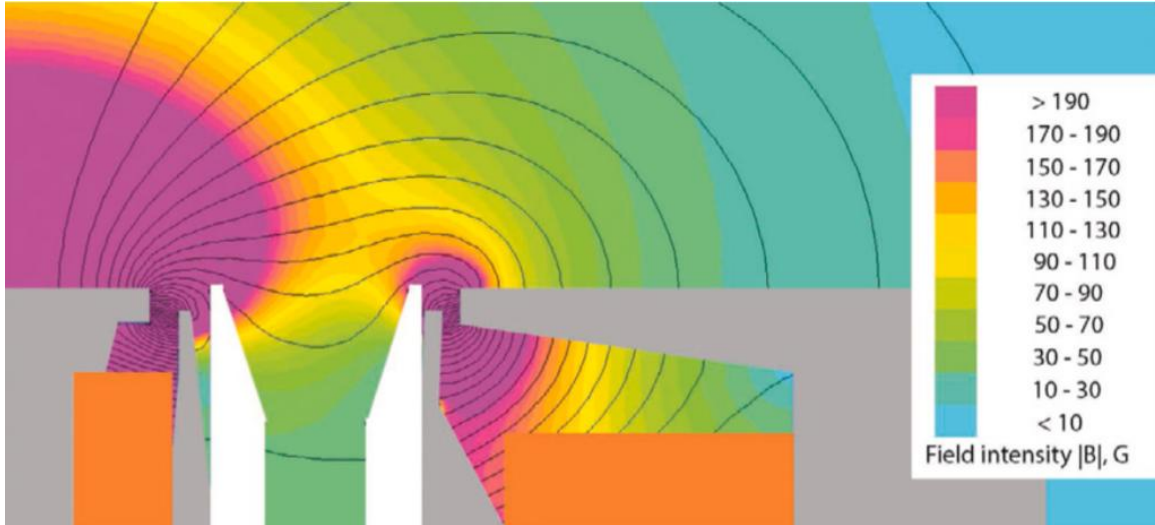
Two thrusters without magnetic shielding can be seen on the right.



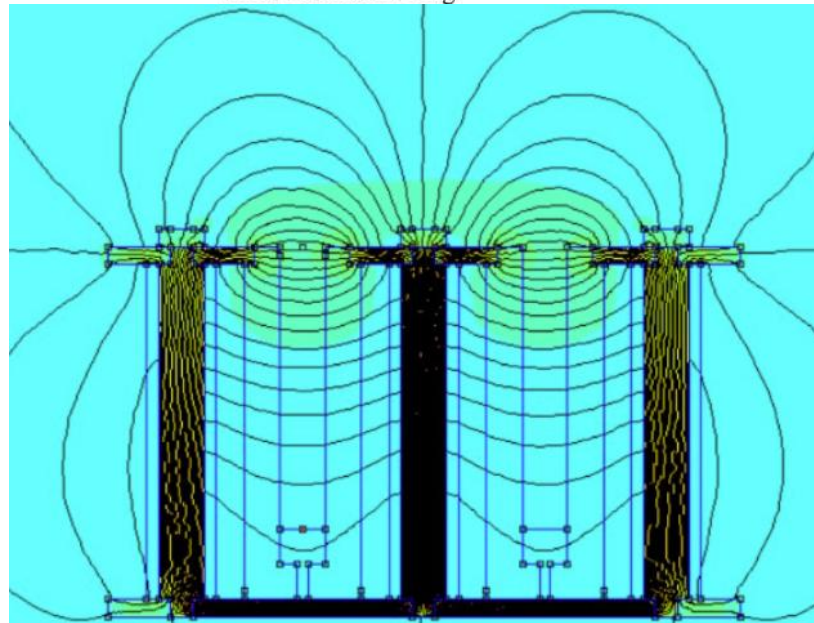
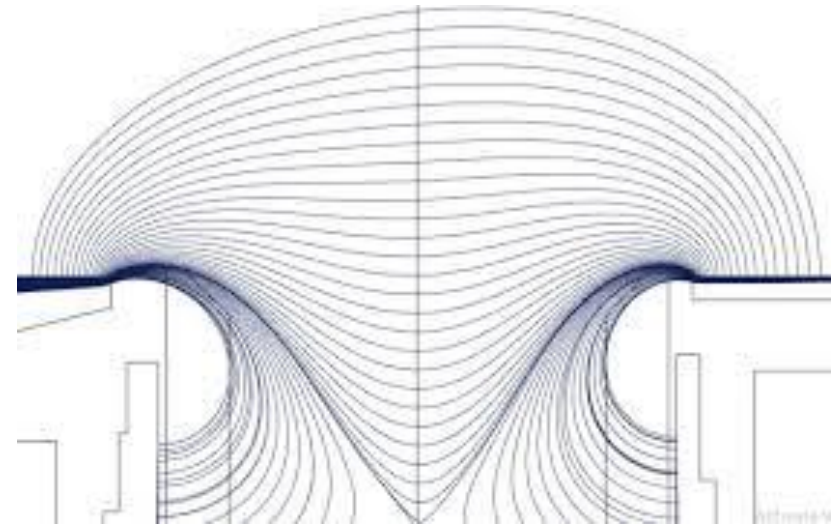
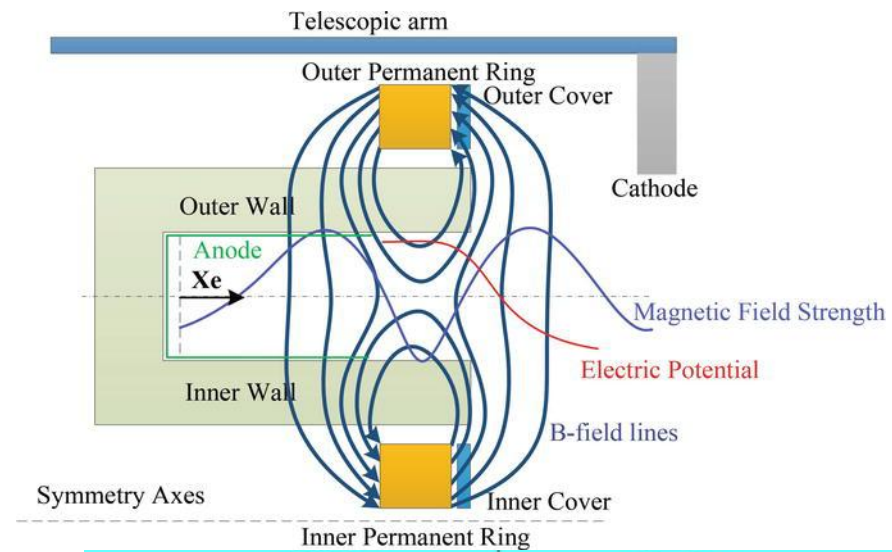
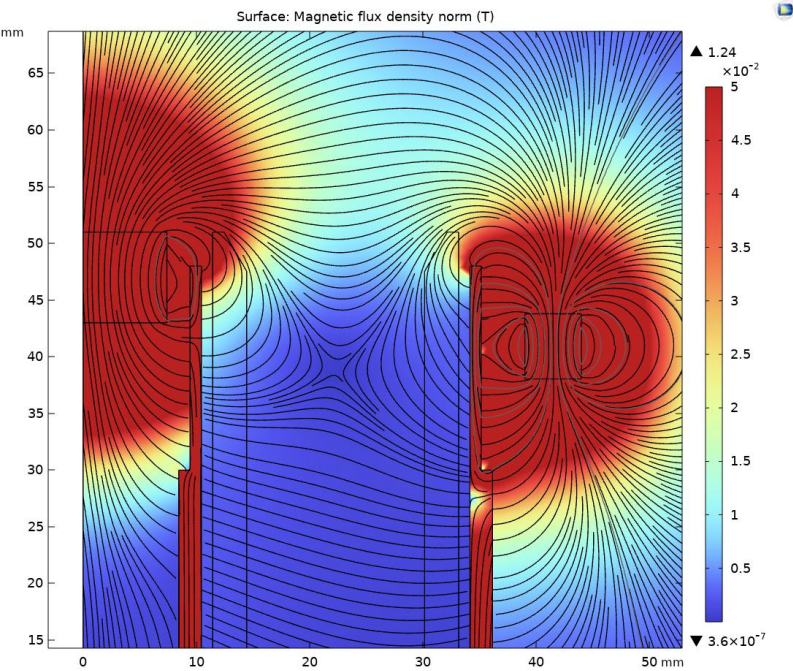
Magnetic Shielding



Examples of Magnetic Field Design



More examples...



Reference:

[1] T. F. Munro-O'Brien and C. N. Ryan, "Performance of a low power Hall effect thruster with several gaseous propellants," *Acta Astronautica*, vol. 206, pp. 257-273, Jan. 2023. [Online]. Available: https://eprints.soton.ac.uk/475781/1/1_s2.0_S0094576523000449_main.pdf

Reading:

D. M. Goebel and I. Katz, *Fundamentals of Electric Propulsion: Ion and Hall Thrusters*, 2nd ed. Hoboken, NJ, USA: Wiley, 2024.