Hall Effect Thrusters 2!!!

Discharge Power Model

The discharge power can be broken down to analyse how it is used in the thruster:

$$P_d = P_b + P_w + P_a + P_R + P_i$$

P_b is the beam power

P_{w} is the power to the walls due to ion + electron losses (heating power)

P_{a} is the anode power due to electron collection

P_{R} is the radiative power loss from the plasma

P_{i} is the power required to produce ions, that hit the wall and form part of the beam

Beam Power and Anode Power

$$P_b = I_b V_b$$

$$P_{\mathrm{a}} = 2\mathrm{T}_{\mathrm{eV}}\mathrm{I}_{\mathrm{a}} \approx 2T_{eV}I_d$$

I_{a} is the electron current to the anode

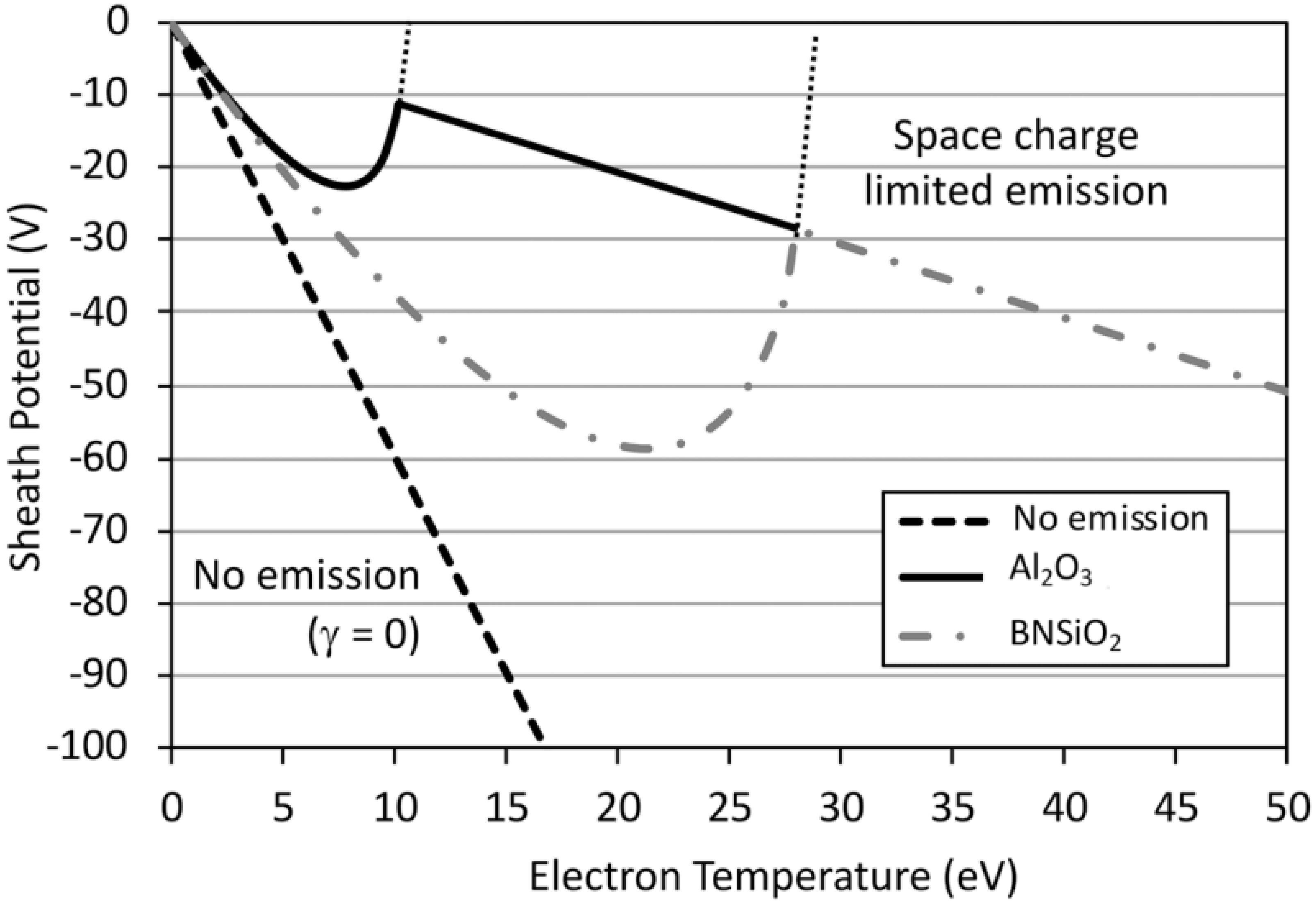
Wall Power

$$P_w = n_e e A \left[\left(\frac{kT_e}{e} \right)^{3/2} \left(\frac{e}{2\pi m_e} \right)^{1/2} e^{\frac{e\phi_s}{kT_e}} + \frac{1}{2} \sqrt{\frac{kT_e}{m_i}} (\varepsilon - \phi_s) \right]$$

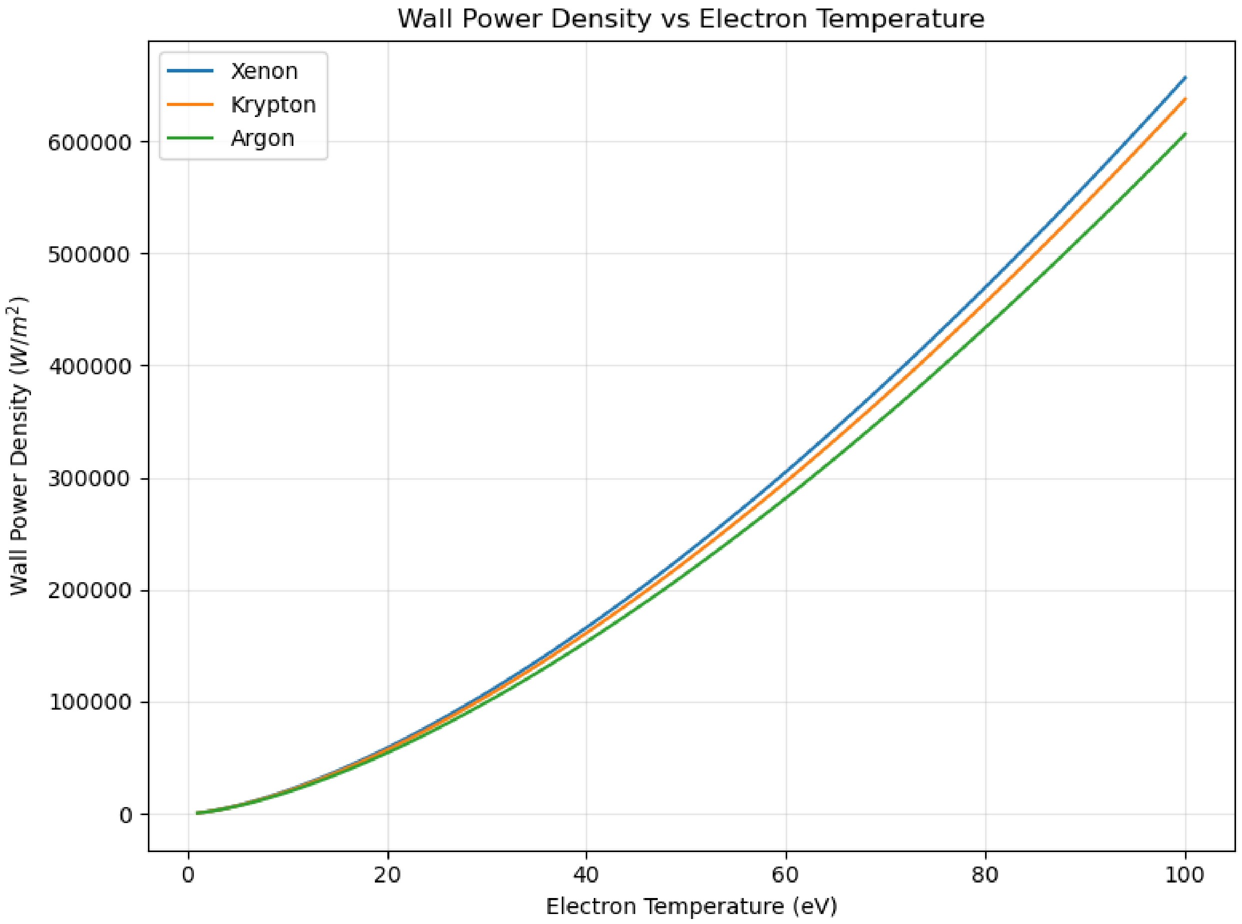
Number density can be “approximated” using $I_b = n_e A_c v_b e$

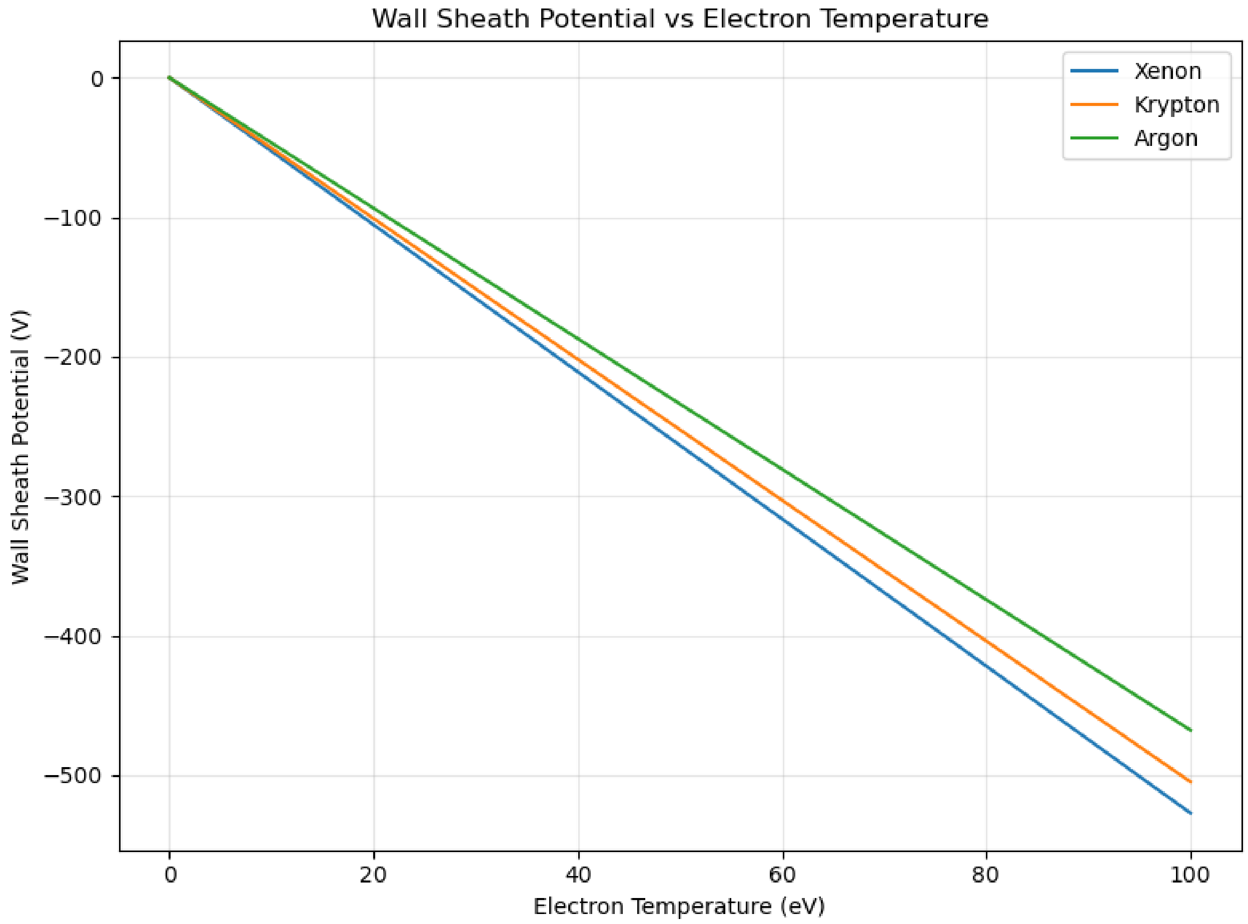
The other variables are their standard representation were T_e is in Kelvin ε is the ion energy and ϕ_s is the sheath potential between the plasma and the walls. Both quantities are calculatable for a range of electron temperatures.

Wall Sheath Potential



Wall Power Density and Sheath Potential Assuming secondary electron emission is zero





Radiative Power

$$P_R = n_o n_e \langle \sigma_* v_e \rangle V$$

$\langle \sigma_* v_e \rangle$ the excitation reaction rate

n_o is the neutral number density

n_e is the electron number density

V is the high temperature plasma volume

Power to Produce Ions

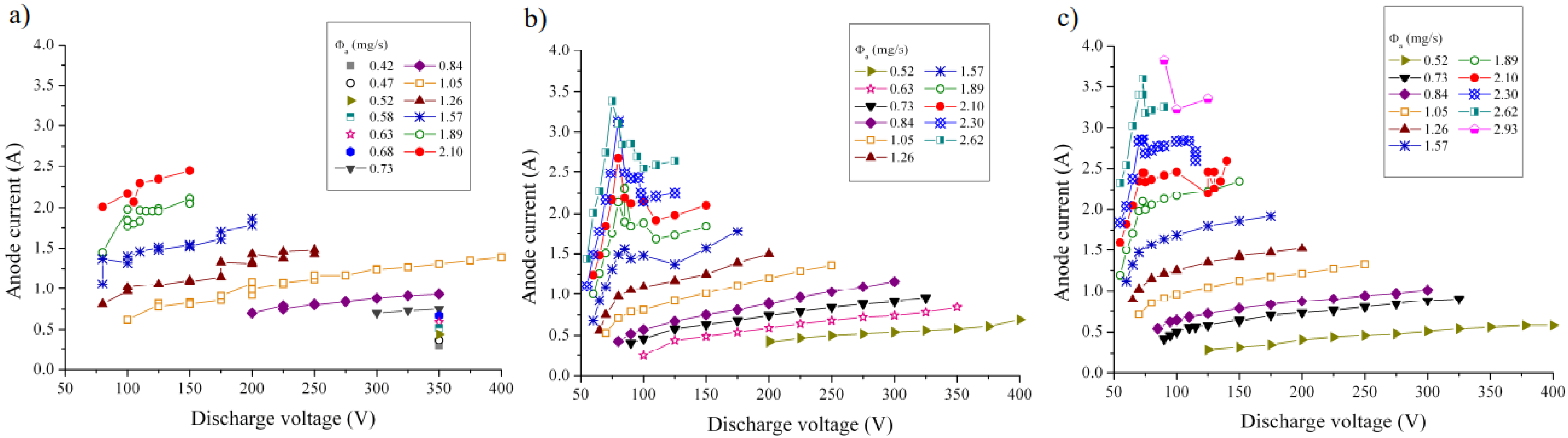
$$P_i = (I_b + I_{iw})U^+ = [\eta_b I_d + I_{ew}(1 - \gamma)]U^+$$

I_{iw} is the ion current to the walls

I_{ew} is the electron current to the walls

U^+ is the ionisation potential

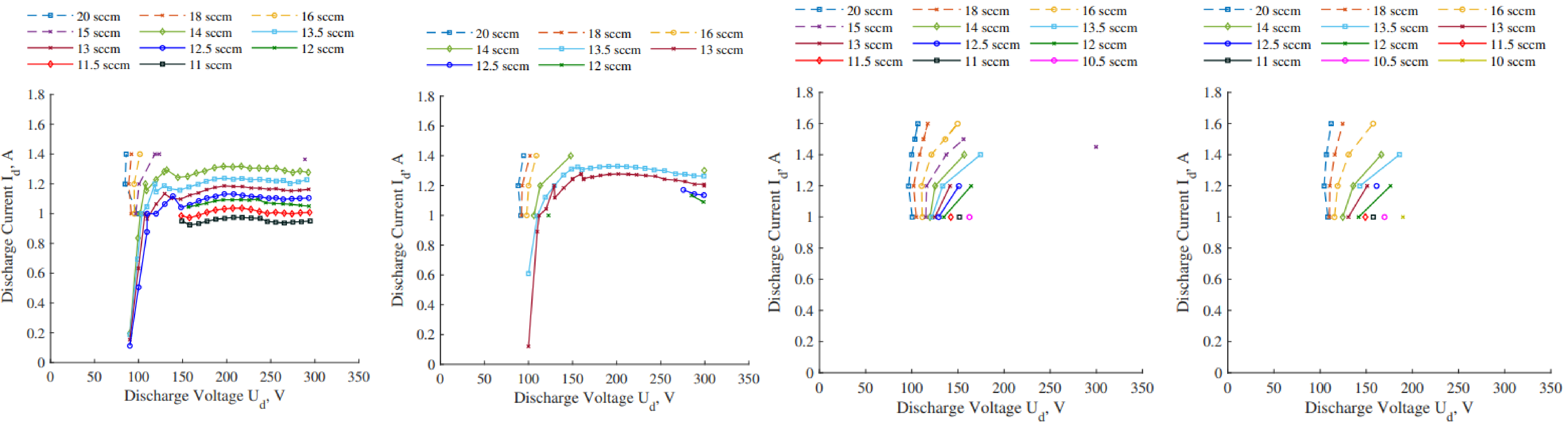
Current-Voltage Experimental Behaviour



Effect of channel width: (a) has the smallest, (c) has the largest, (b) is in-between.

$\dot{m} \propto I_d$

Current-Voltage Experimental Behaviour



(a) CVC at 17 mT magnetic flux density

(b) CVC at 22 mT magnetic flux density

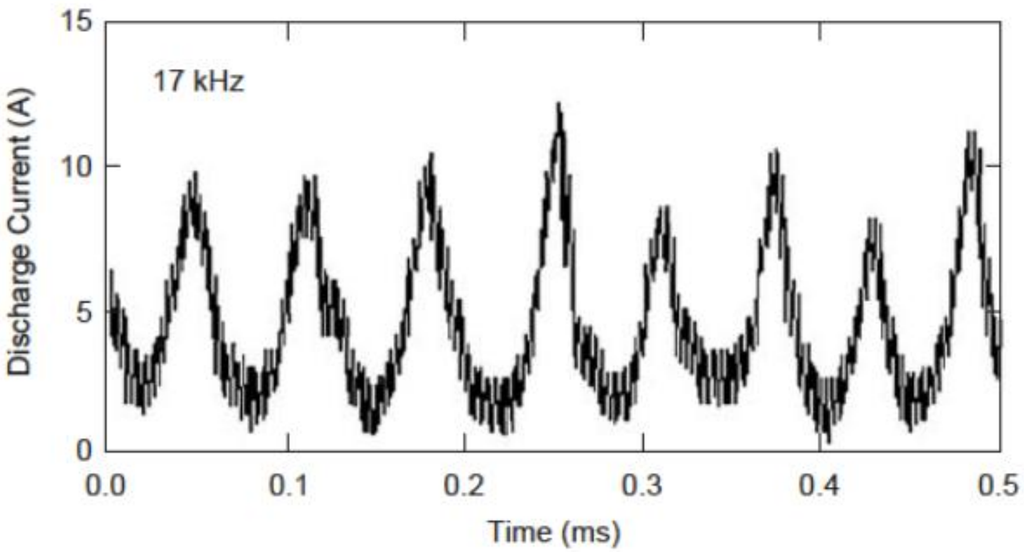
(c) CVC at 25 mT magnetic flux density

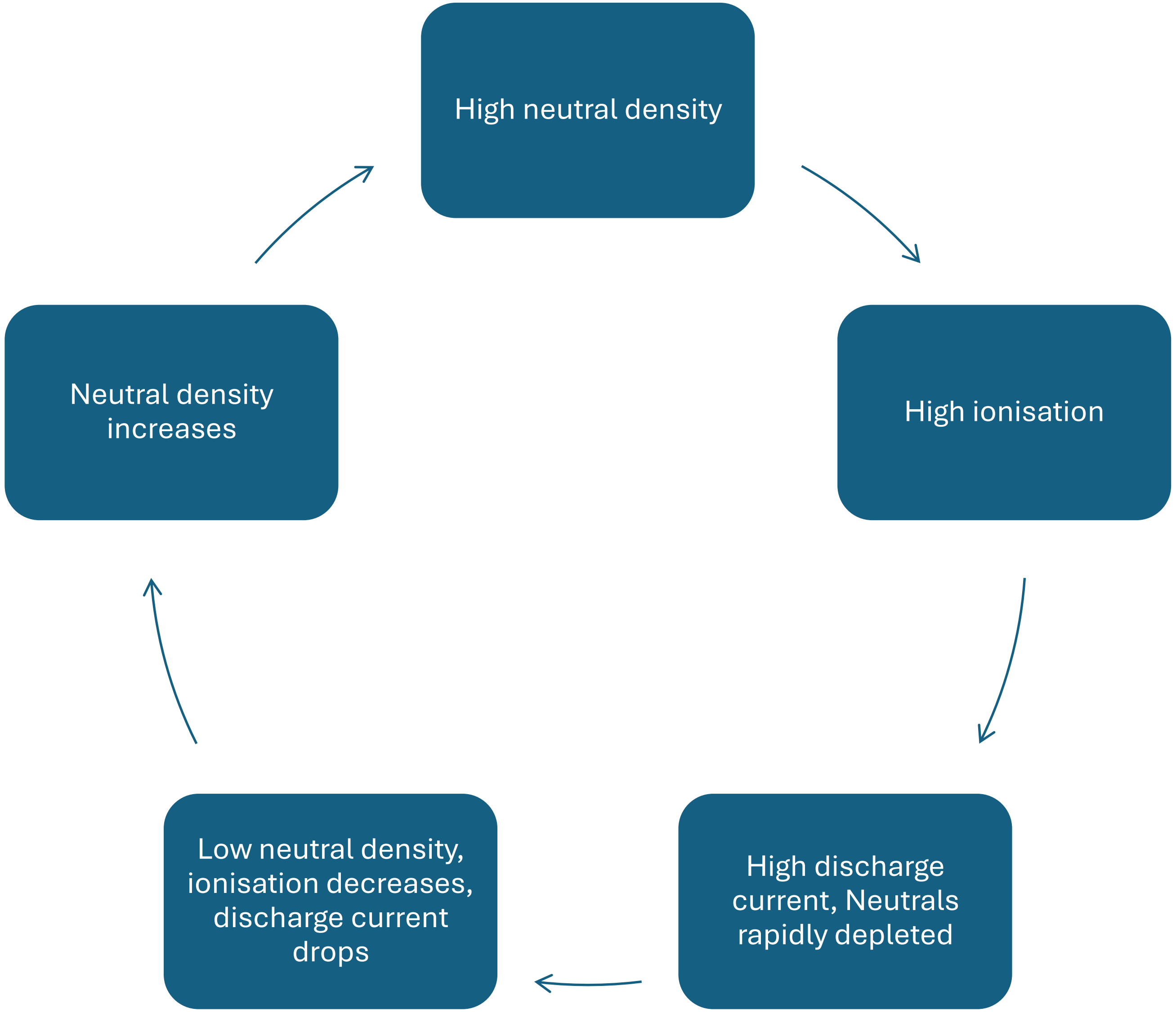
(d) CVC at 28 mT magnetic flux density

Magnetic field has a significant effect on thruster performance; it appears that as the field becomes stronger the voltage operational range becomes limited; thus, the field strength should be carefully selected

Breathing Mode Oscillations

The breathing mode isn't entirely fully understood but the current belief is that occurs due to the following behaviour:





Breathing Mode Oscillations

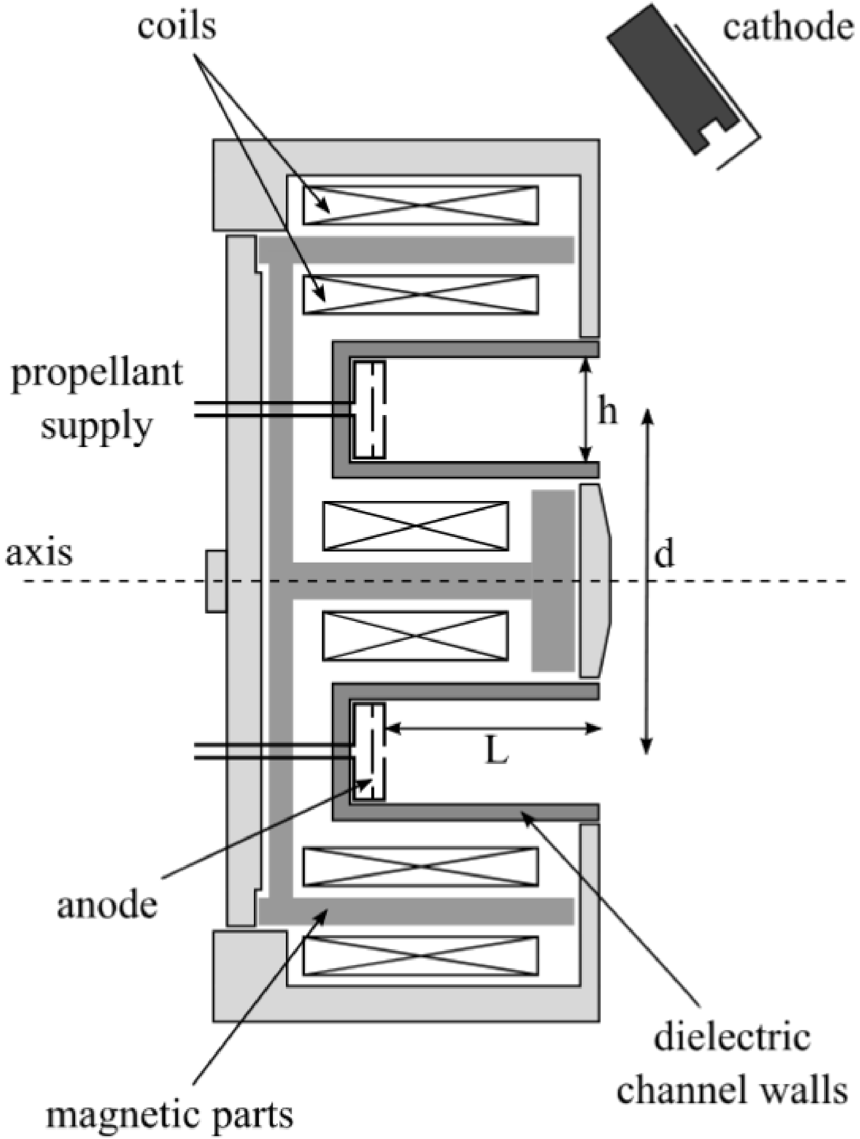
Mathematically speaking, neutral particle conservation:

$$\frac{\partial n_n}{\partial \mathrm{t}} = -n_i n_n \langle \sigma_i v_e \rangle + \frac{n_n v_{\mathrm{o}}}{L}$$

With some maths and re-arrangement from Fundamentals of EP (my beloved):

$$f_i = \frac{1}{2\pi}\sqrt{n_{i,o} n_{n,o} \langle \sigma_i v_e \rangle^2} \approx \frac{\sqrt{v_i v_o}}{2\pi L}$$

Channel Geometry

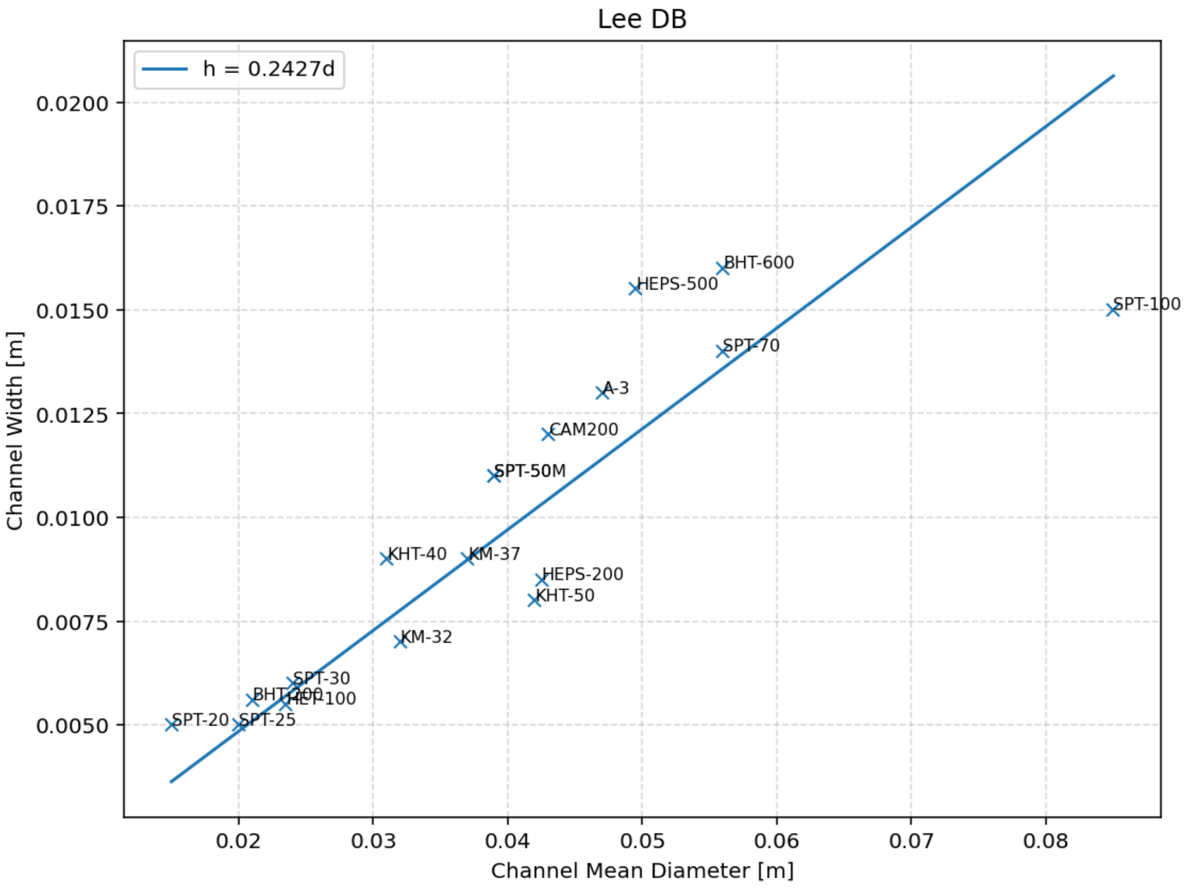


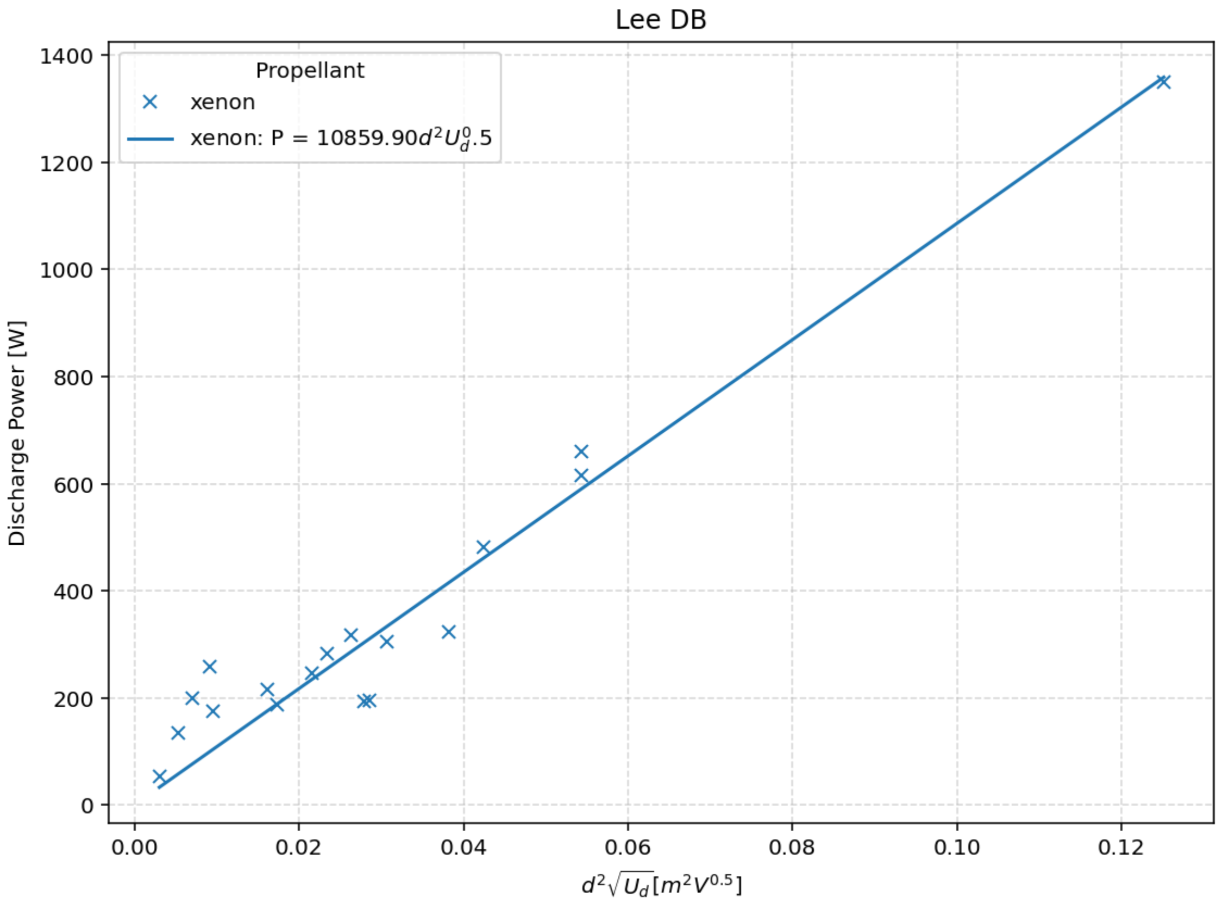
The following set of equations are typically used to scale a hall thruster:

$$T = C_{T1}\dot{m}_n\sqrt{V_d}$$
$$T = C_{T2}d^2\sqrt{V_d}$$
$$P = C_{\mathrm{P}}d^2\sqrt{V_d}$$
$$h = C_{hd}d$$
$$L = C_L\lambda_i$$

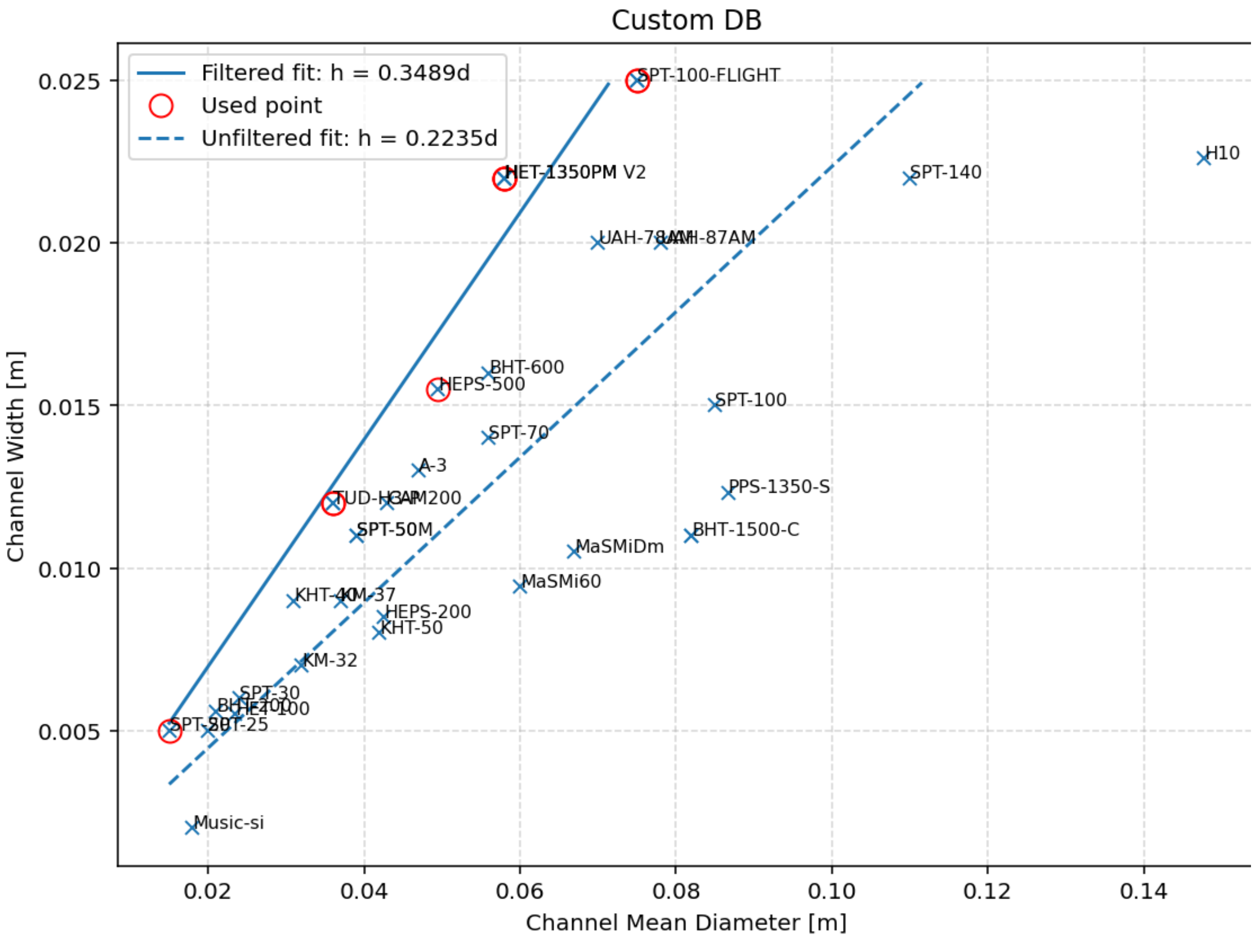
$$d = \frac{d_o + d_i}{2} \qquad \lambda_i = \frac{v_n}{n_e\langle\sigma_i v_e\rangle}$$

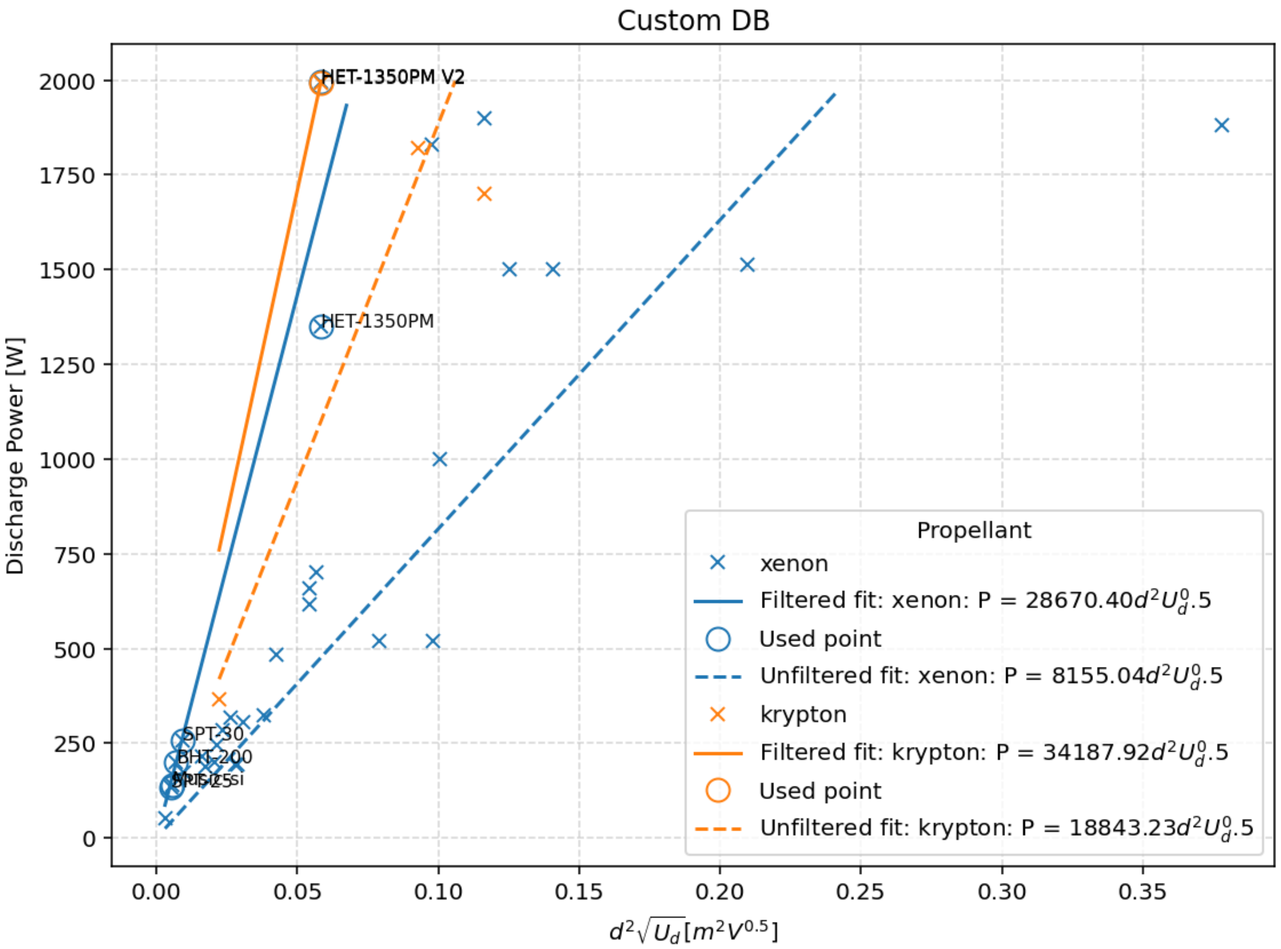
Publicly Available Scaling Data



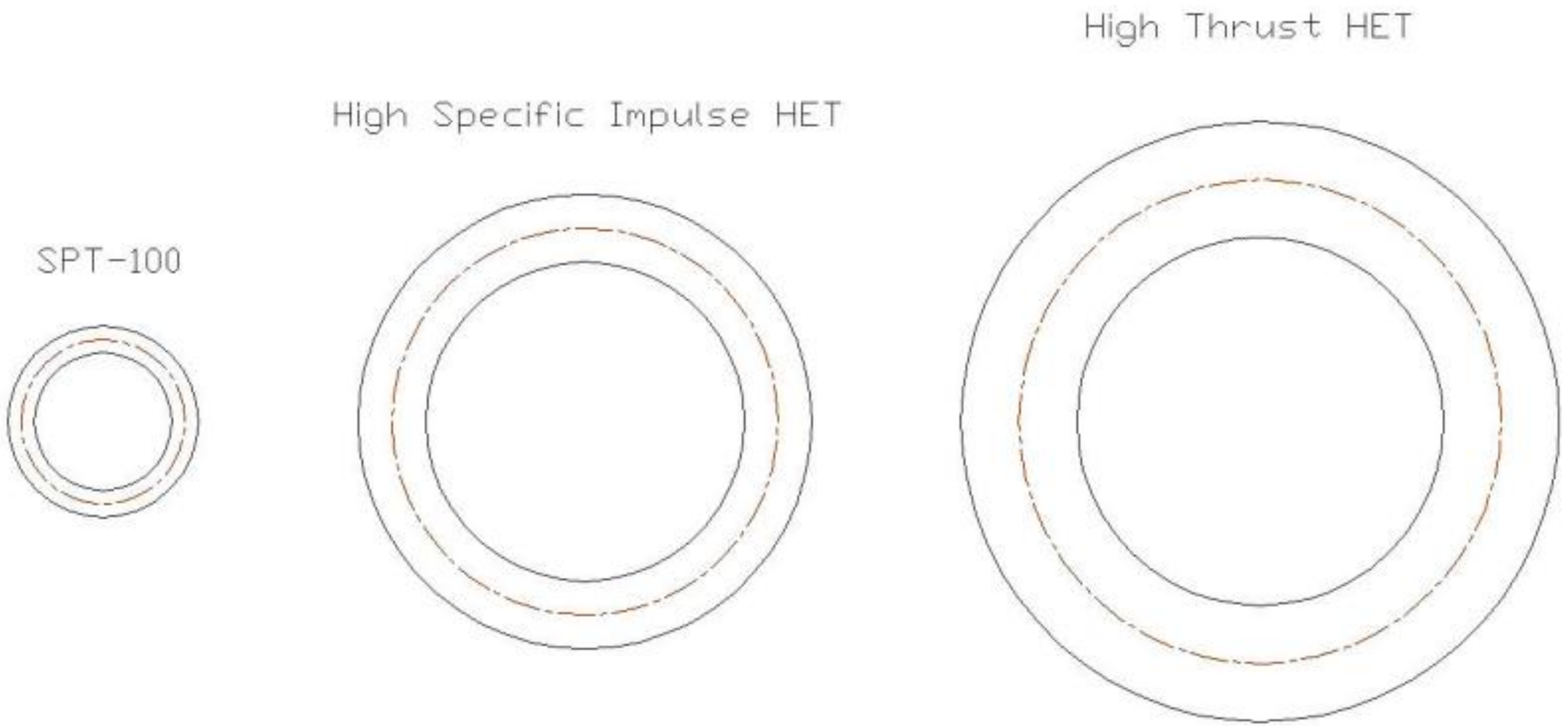


More Personal Scaling Relations

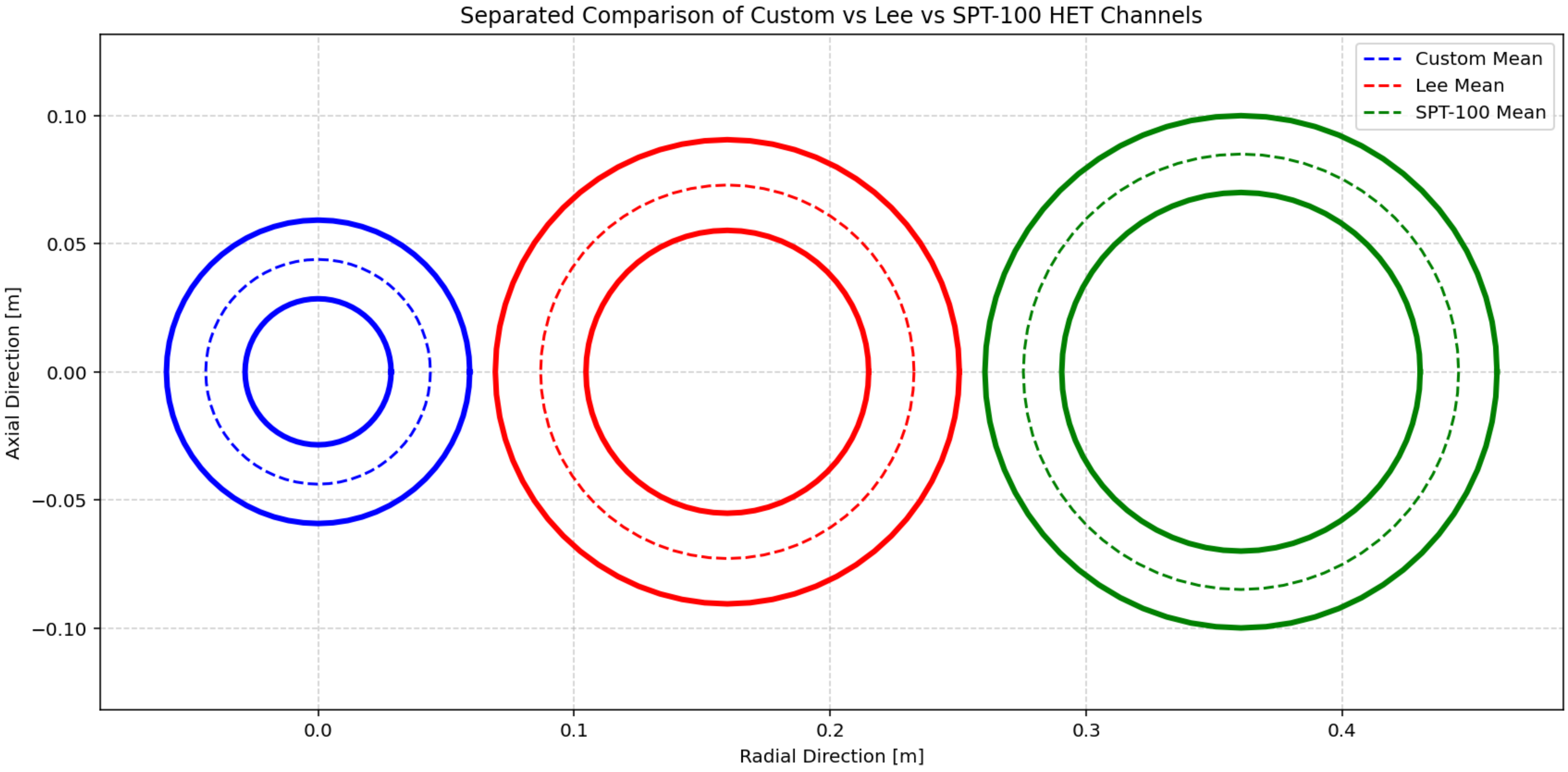




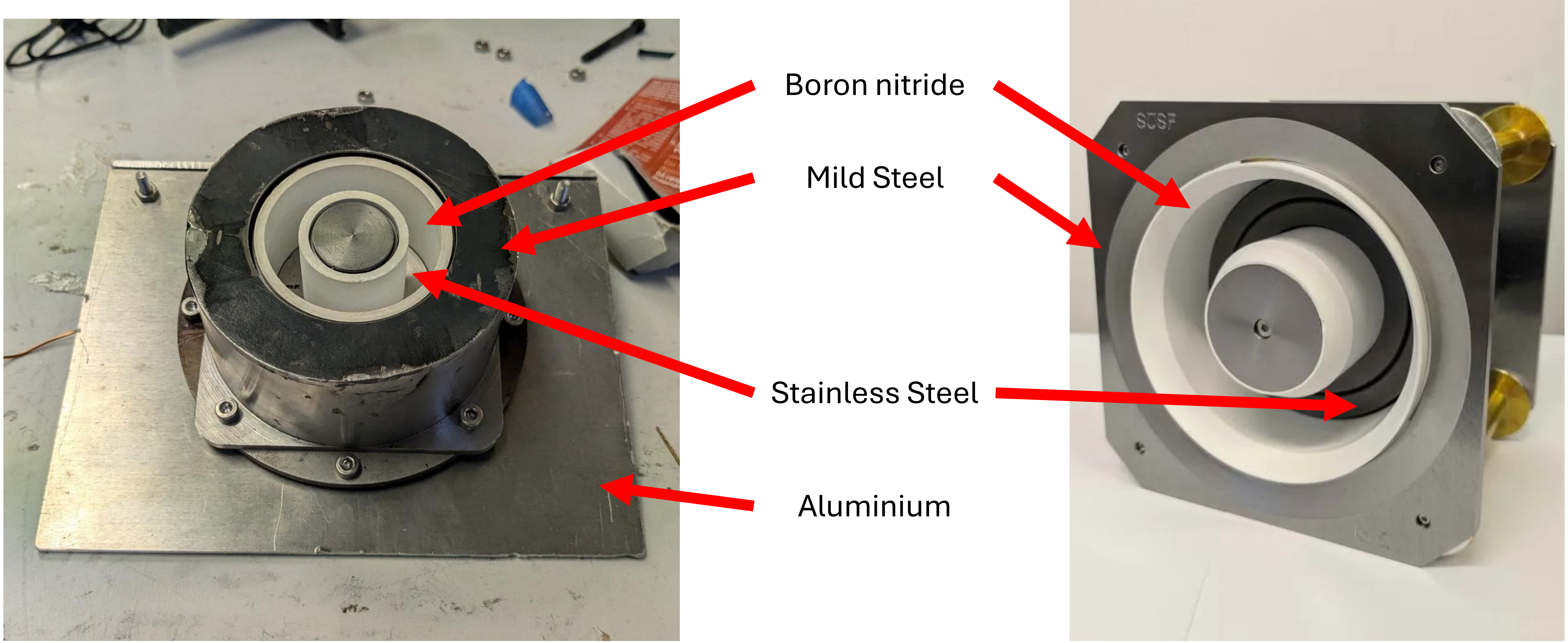
Comparison of Different Scaling Coefficients



Comparison of Different Scaling Coefficients



Materials

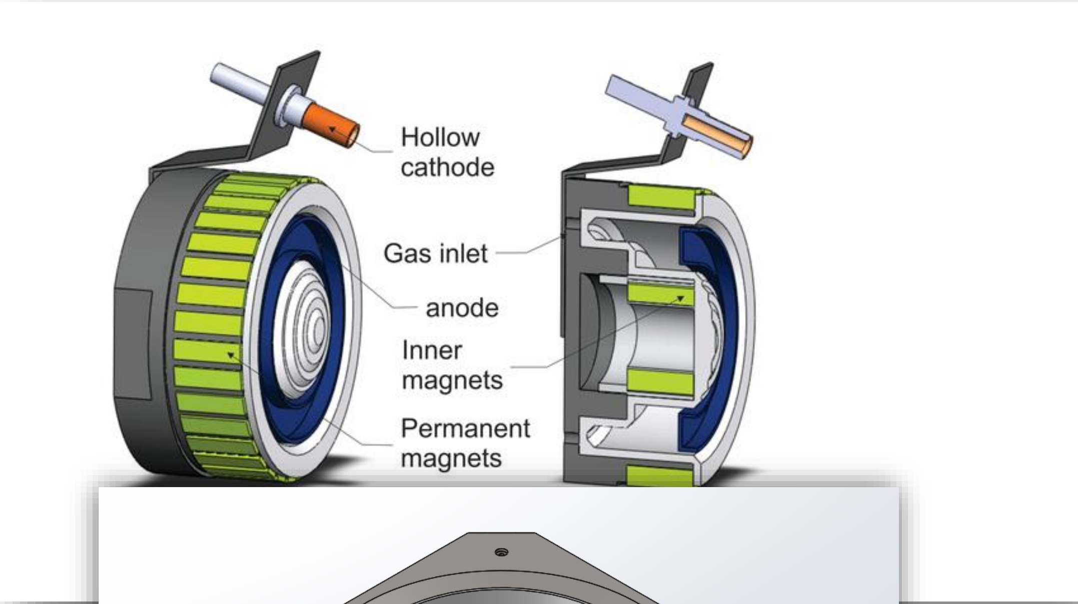


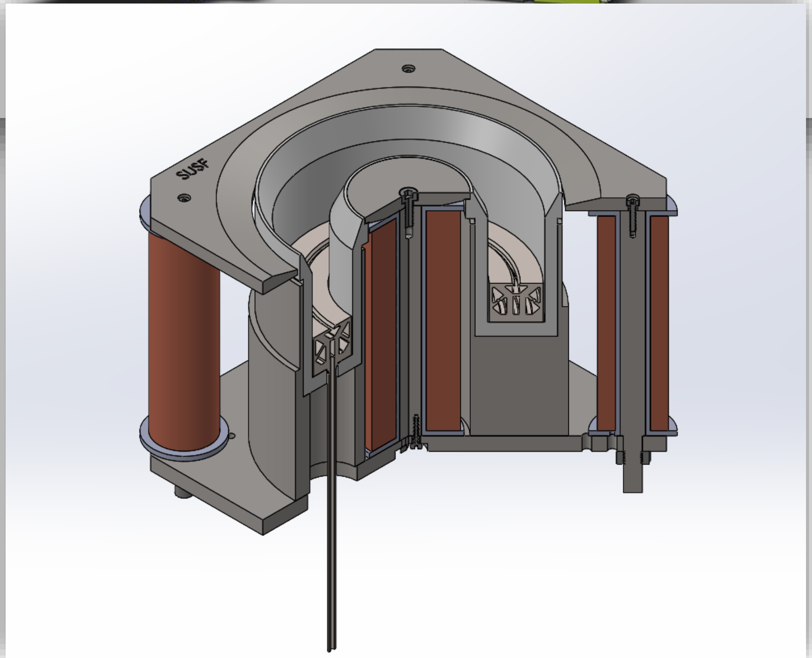
Channel Materials

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt = (z-1)!$$

Material	$\gamma = \Gamma(2+b)a(T_{eV})^b$ [1]			Thermal Conductivity, W/(m k)	Density, g/cm^3	Upper melting point, ℃
Al2O3	$a = 0.145$	$b = 0.650$	$\Gamma(2+b) = 1.49$	20-34	3.90	2000
Boron Nitride	$a = 0.150$	$b = 0.549$	$\Gamma(2+b) = 1.38$	35-85	1.9	2000
BNSiO2	$a = 0.123$	$b = 0.528$	$\Gamma(2+b)$ =1.36	78-130	2.1	~1000
Stainless Steel	$a = 0.040$	$b = 0.610$	$\Gamma(2+b) = 1.44$	15-30	8.00	~1500
Graphite		-		90-140	1.72-1.82	2200-3200

Magnetic Circuits





Field Generation	Advantages	Disadvantages
Permanent Magnets	• Requires less magnetic structure • Easier to adjust for simulation	• Looses magnetism when it gets too hot • Few companies can make custom magnets and they are expensive
Electromagnets	• Have more control over the B field strength	• Enamel can melt causing wires to short circuit

Anode Design

- Required characteristics for a good anode:
  - Few inlets as possible (ideally 1)
  - Spreads the flow out evenly so that it is uniformly injected into the channel
  - Small
  - Light-weight
  - Low cost

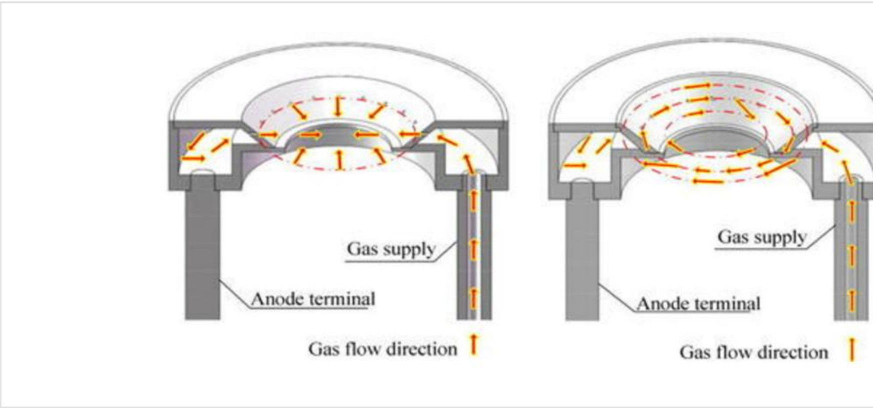
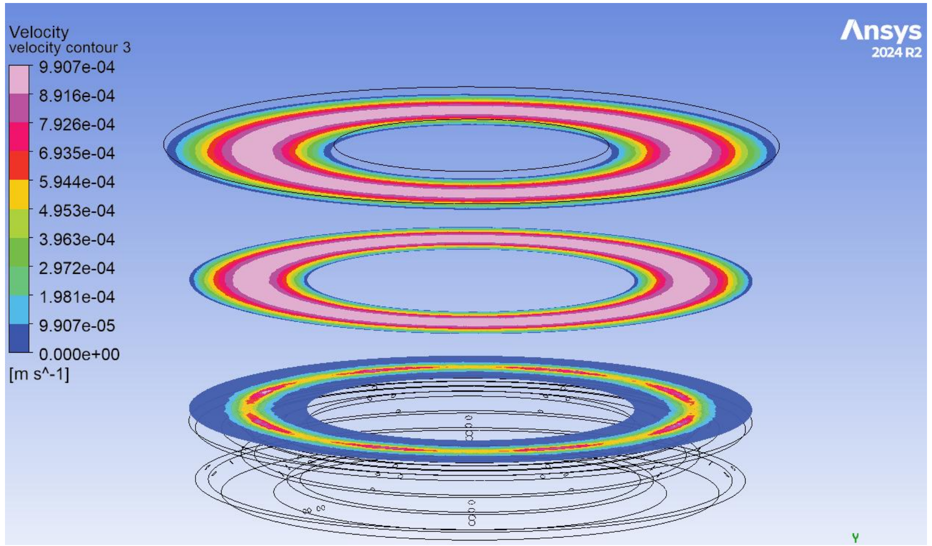
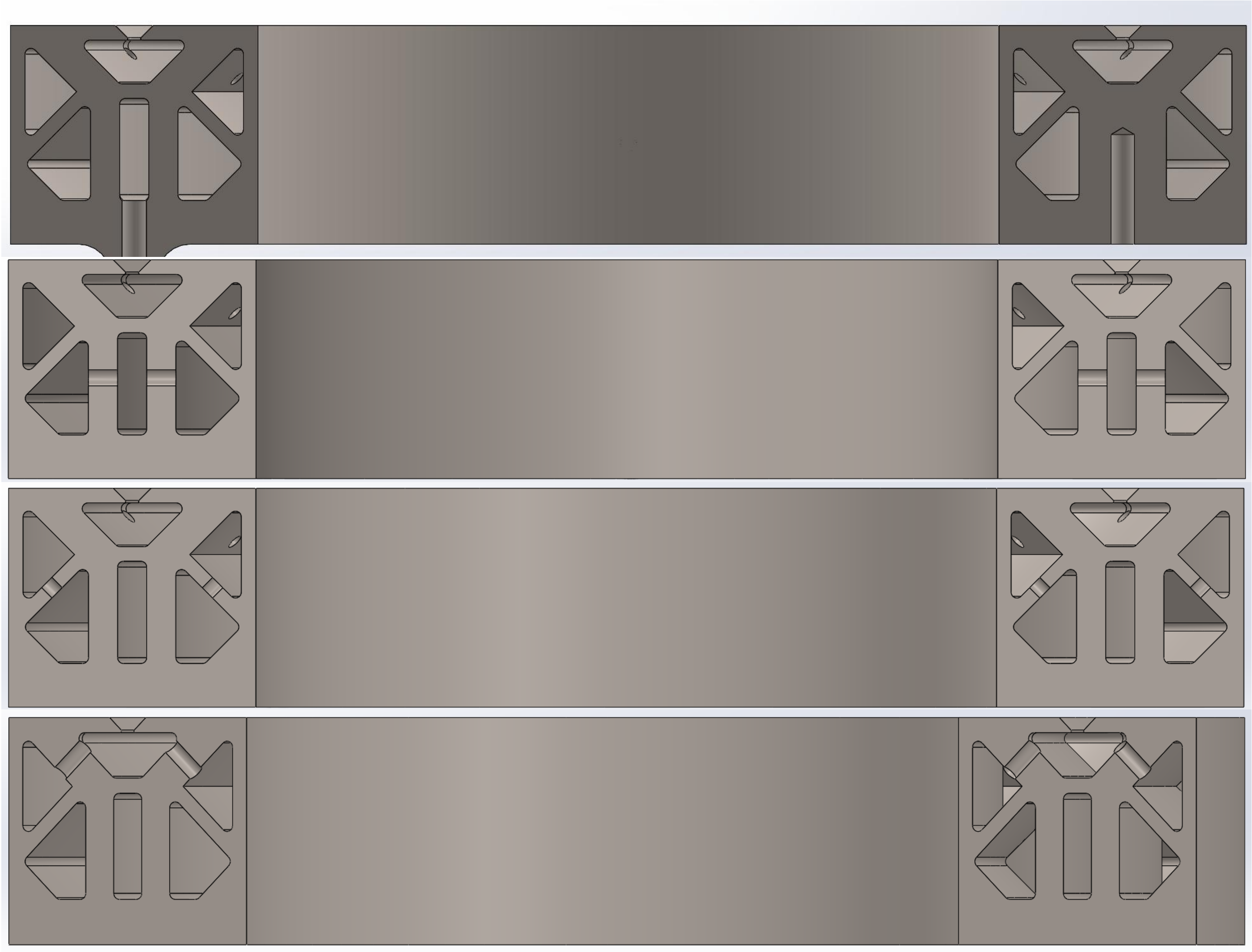


FIG. 2.

Gas flow direction (Left: radial inlet mode and right: vortex inlet mode).

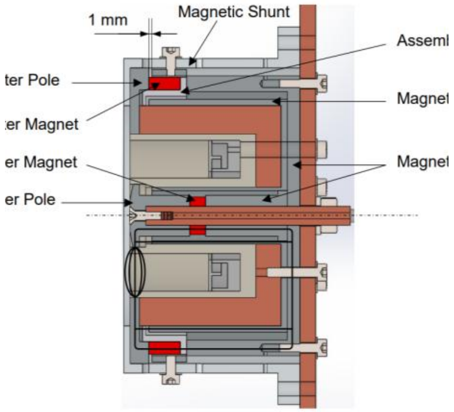


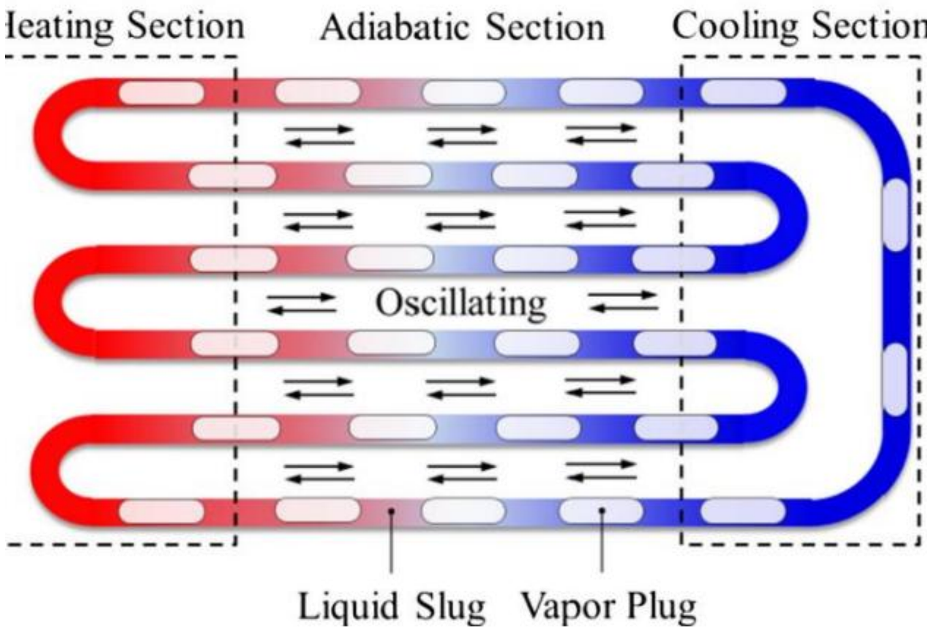
Anode Case Study



Thermal Design

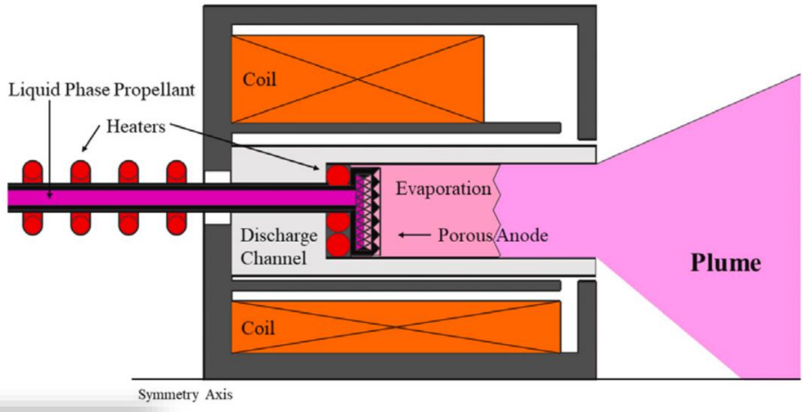
Two categories of cooling: active and passive cooling, the space industry rejects active cooling on almost all electric propulsion due to the extra complexity, cost and power requirement.





Propellant Delivery

Different propellants require different delivery options depending on their storage conditions. All propellants going to the anode must be in gaseous form in order to become ionised.



Porous anode propellant delivery system.

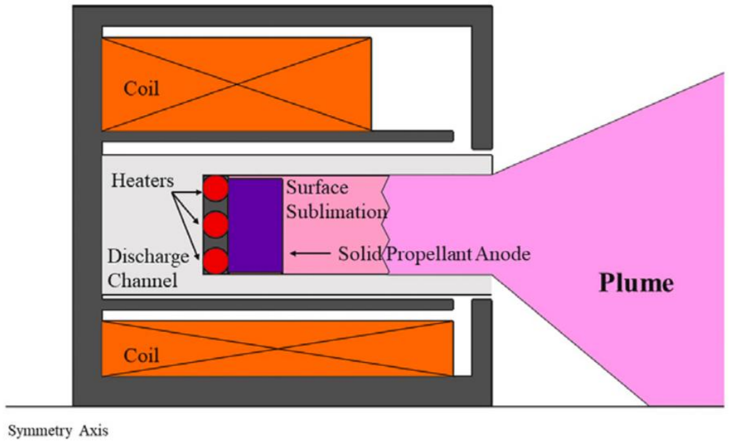


Fig. 9. Solid anode propellant delivery system.

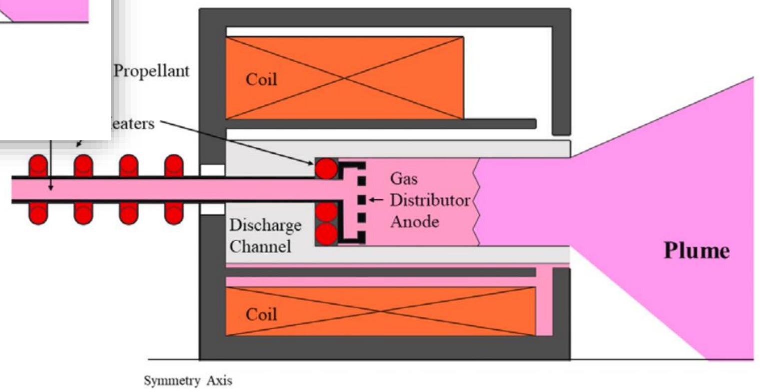


Fig. 11. Gas distributor anode propellant delivery system.

References

[1] D. M. Goebel and I. Katz, Fundamentals of Electric Propulsion: Ion and Hall Thrusters, 2nd ed. Hoboken, NJ, USA: Wiley, 2024.