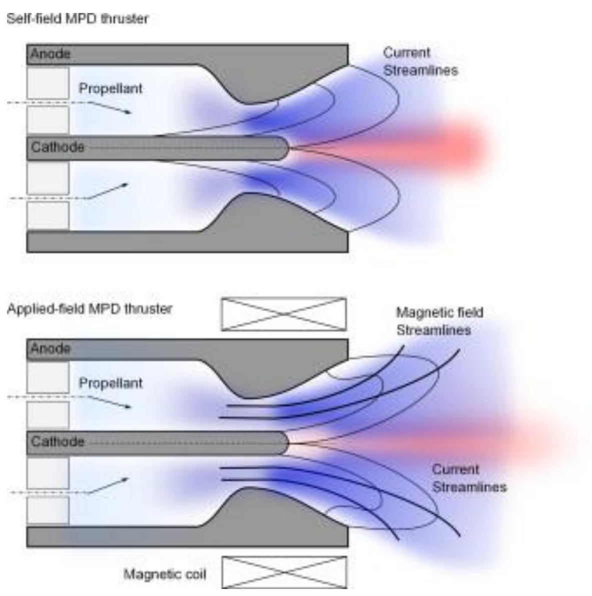
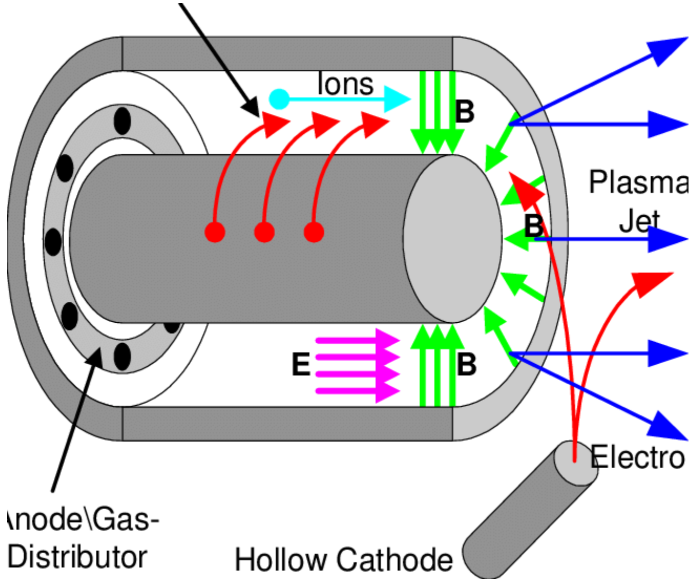
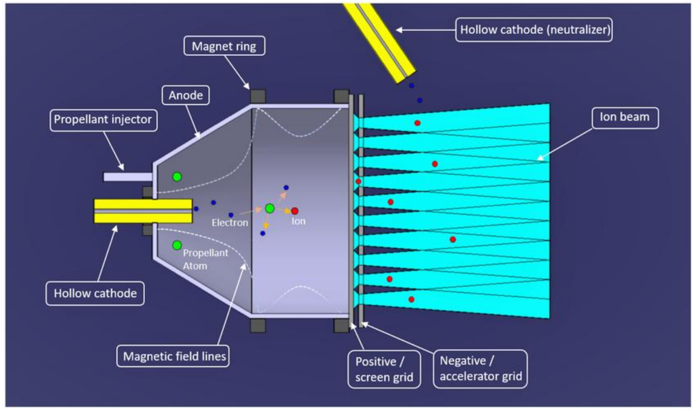
Electro-Magnetism







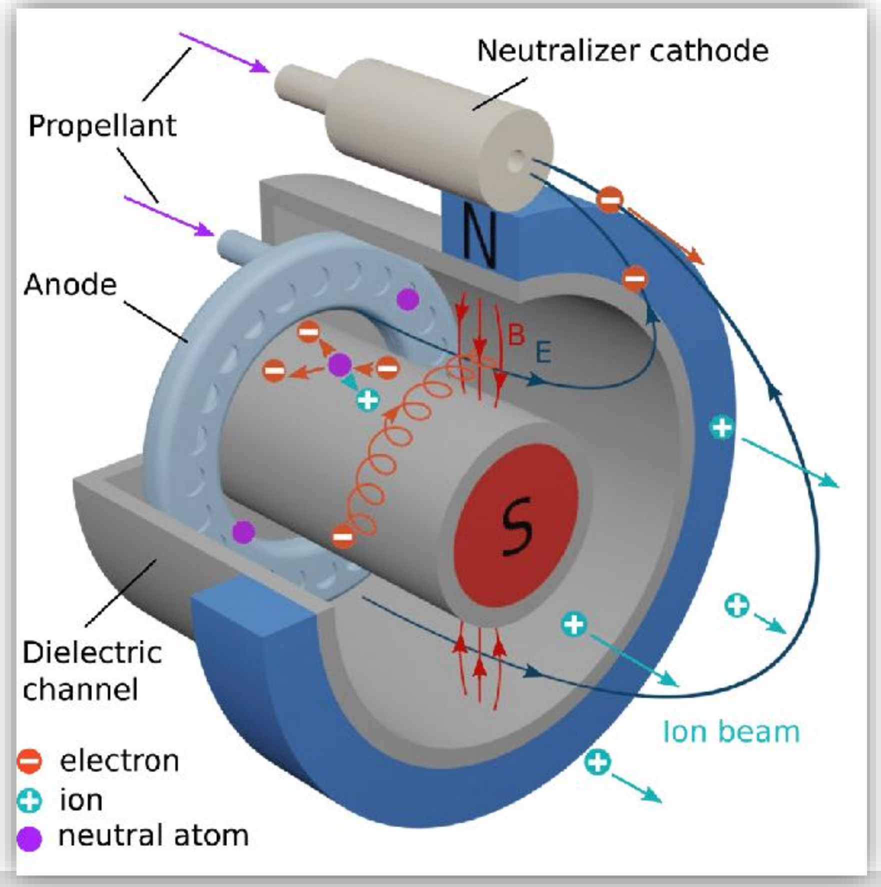
Why it matters in Electric Propulsion

Duh… Electric… Propulsion

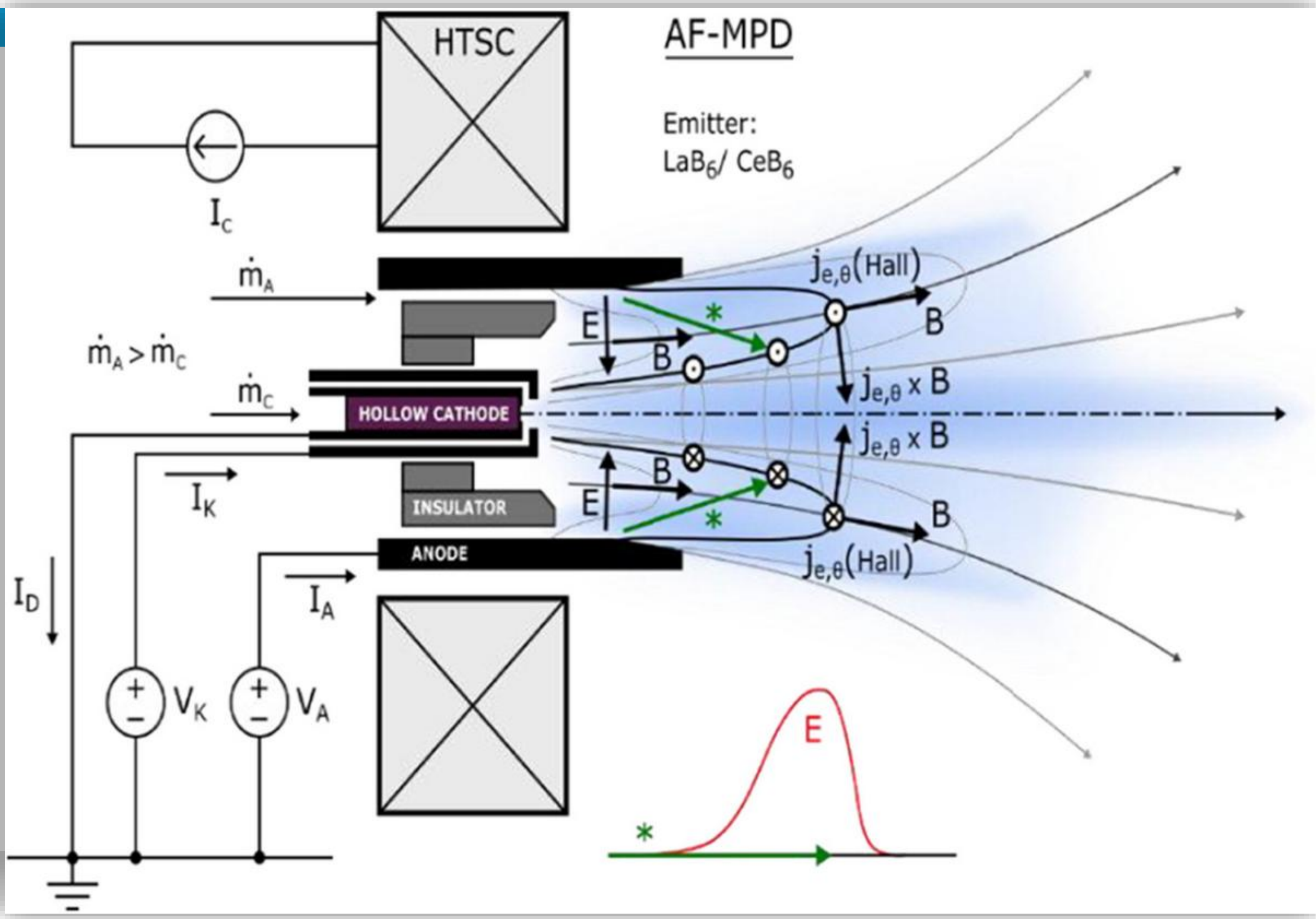
How are Electric and Magnetic Fields Used?

Electric and magnetic fields have two primary uses in most EP

- Acceleration
- Confinement



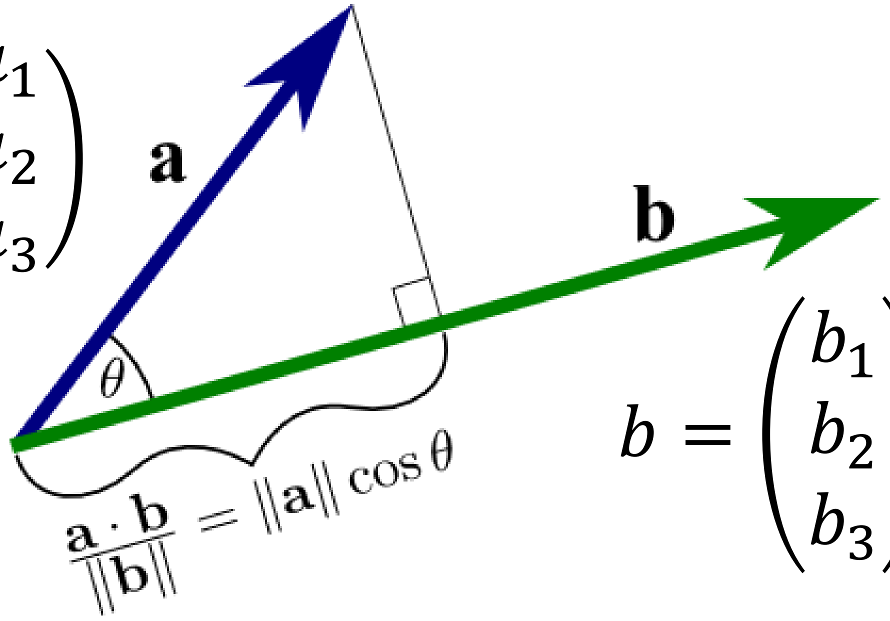
MPD Example



HTSC
AF-MPD
Emitter:
LaB6/ CeB6
I_C
$\dot{m}_A$
$\dot{m}_A > \dot{m}_C$
$\dot{m}_C$
HOLLOW CATHODE
INSULATOR
ANODE
I_K
I_D
I_A
V_K
V_A
E
B
$j_{e,\theta}$(Hall)
$j_{e,\theta} \times B$
E

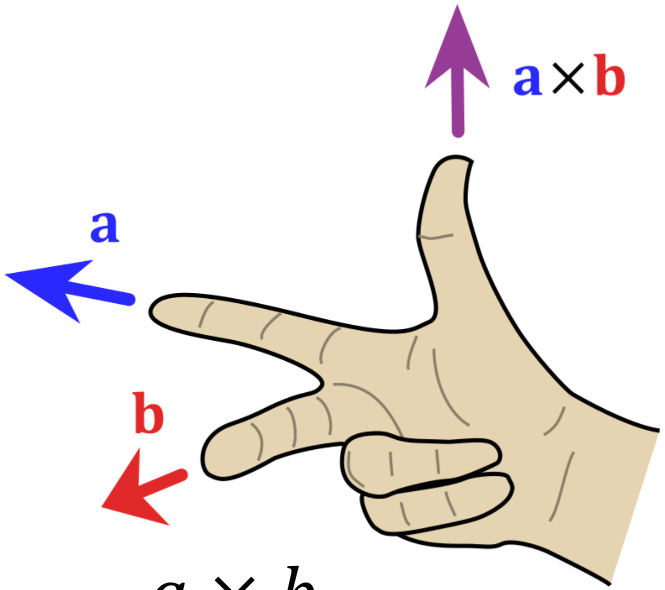
Basic Vector Operations

$$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$



$$b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$$



$$a \times b$$
$$= (a_2 b_3 - a_3 b_2)\hat{\imath}$$
$$- (a_1 b_3 - a_3 b_1)\hat{\jmath}$$
$$+ (a_1 b_2 - a_2 b_1)\hat{k}$$

Slightly Less Basic Vector Operations

$$\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$$

$$\nabla A = \frac{\partial A}{\partial x}\hat{\imath} + \frac{\partial A}{\partial y}\hat{\jmath} + \frac{\partial A}{\partial z}\hat{k}$$

$$\nabla \cdot B = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$$

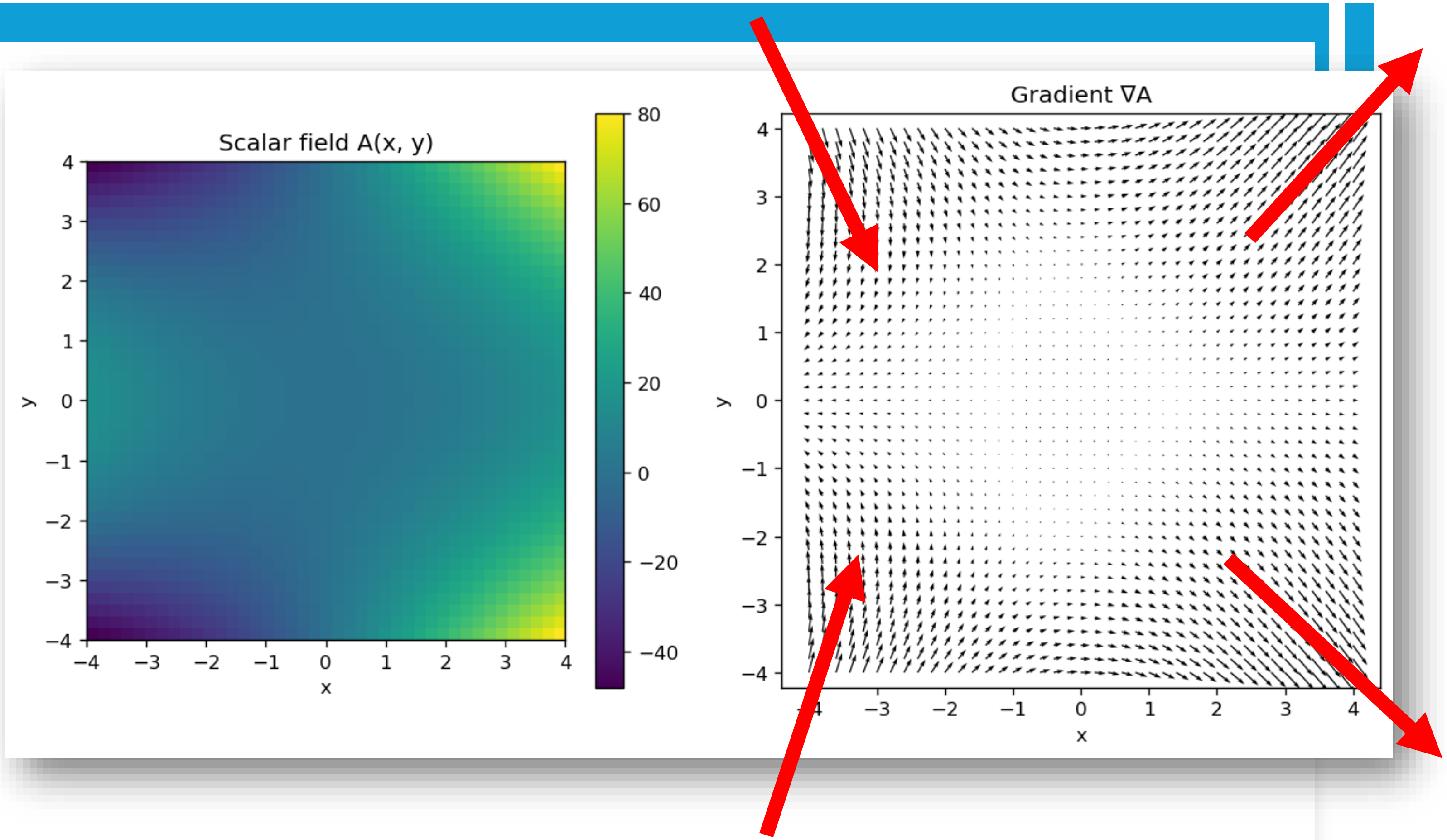
$$\nabla \times B = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)\hat{\imath} - \left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}\right)\hat{k} + \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)\hat{k}$$

$$\nabla \cdot (\nabla \mathrm{A}) = \nabla^2 A = \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2}$$

Gradient Operation

$$\nabla A = \frac{\partial A}{\partial x}\hat{\imath} + \frac{\partial A}{\partial y}\hat{\jmath} + \frac{\partial A}{\partial z}\hat{k}$$

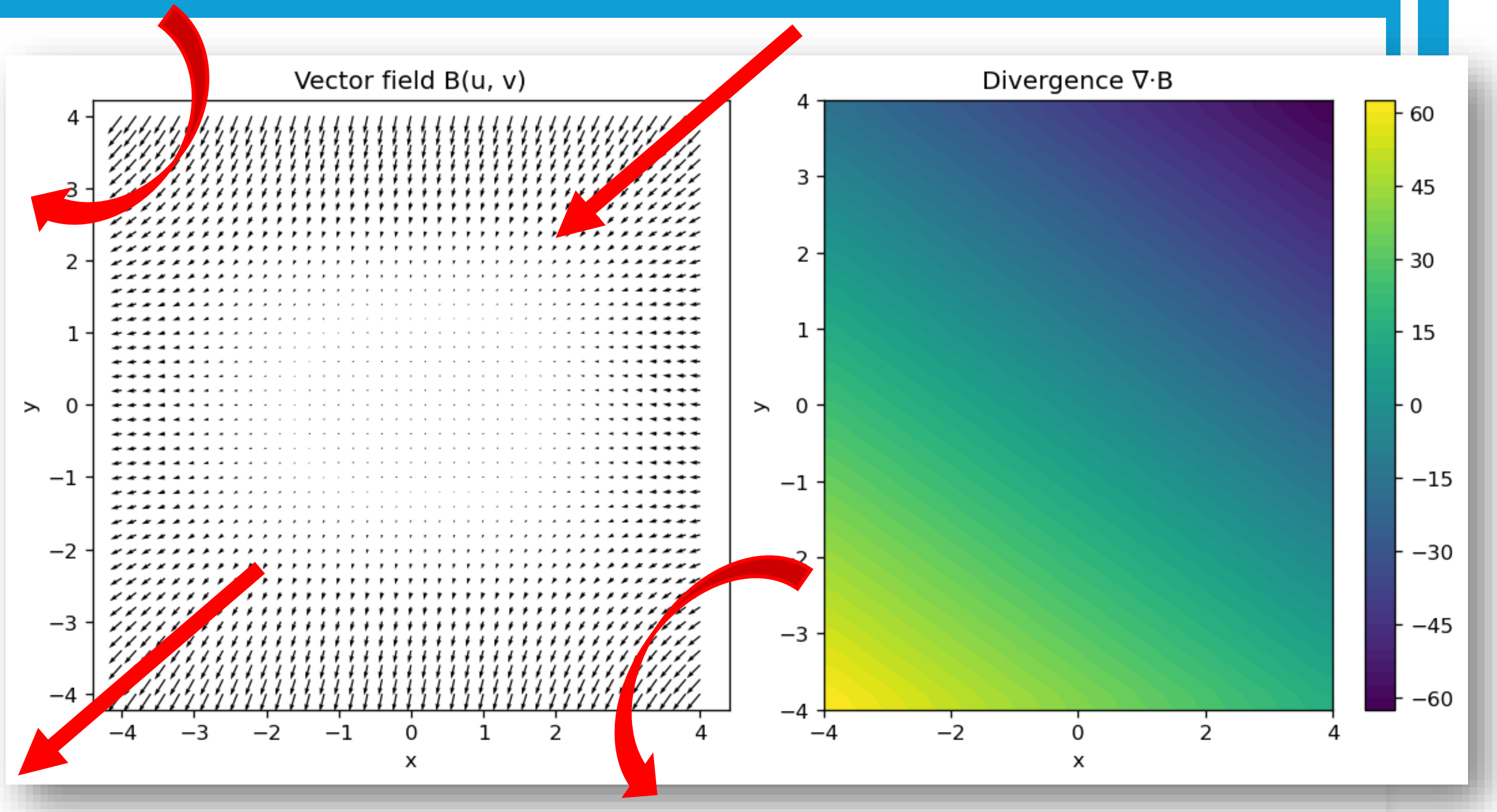
$$let\ A = x^2 + xy^2$$



Divergence Operation

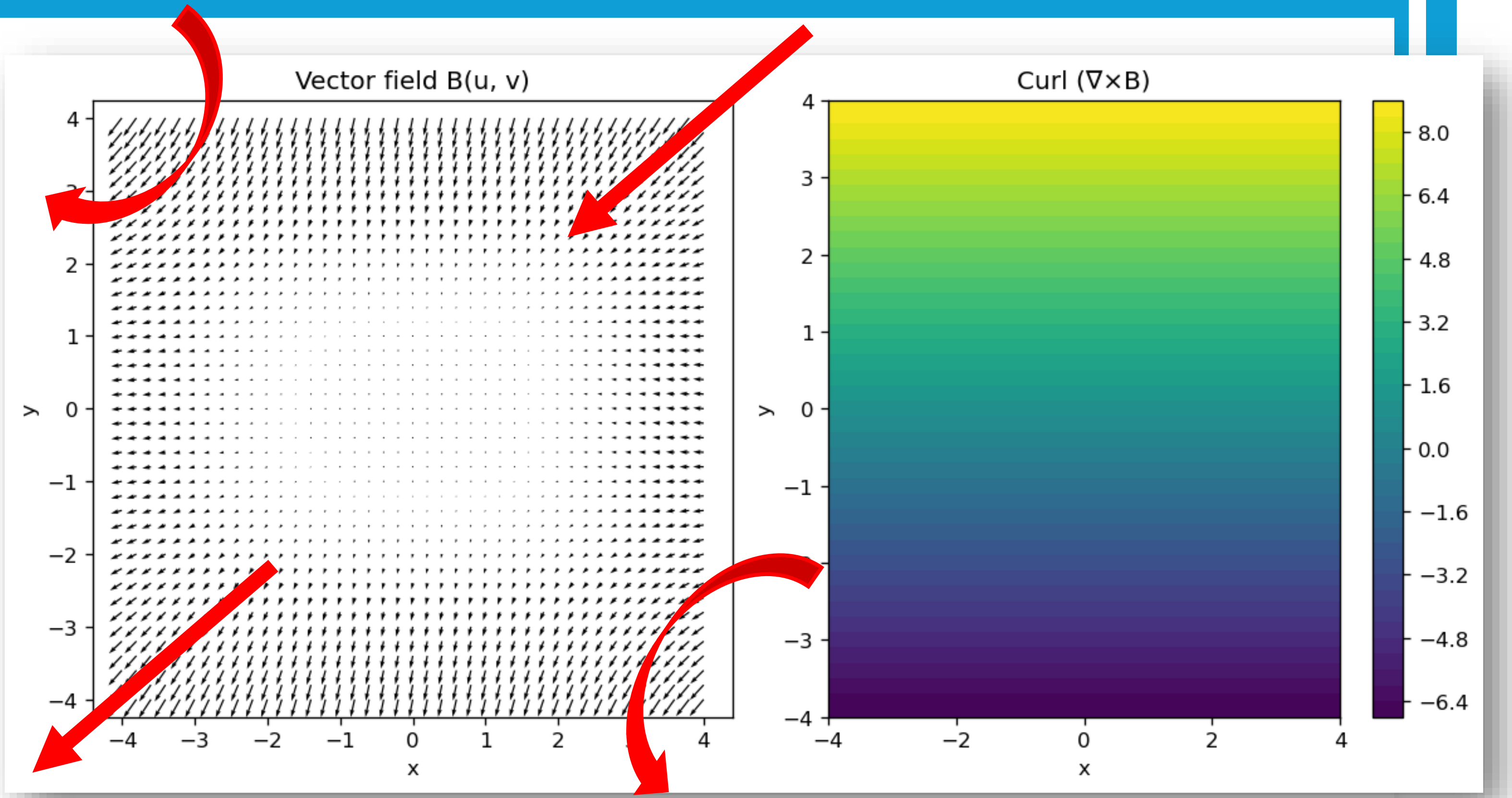
$$\nabla \cdot B = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$$

$$let\ B(u, v)$$
$$u = -3x^2 - y^2$$
$$v = x - 5y^2$$



Curl Operation

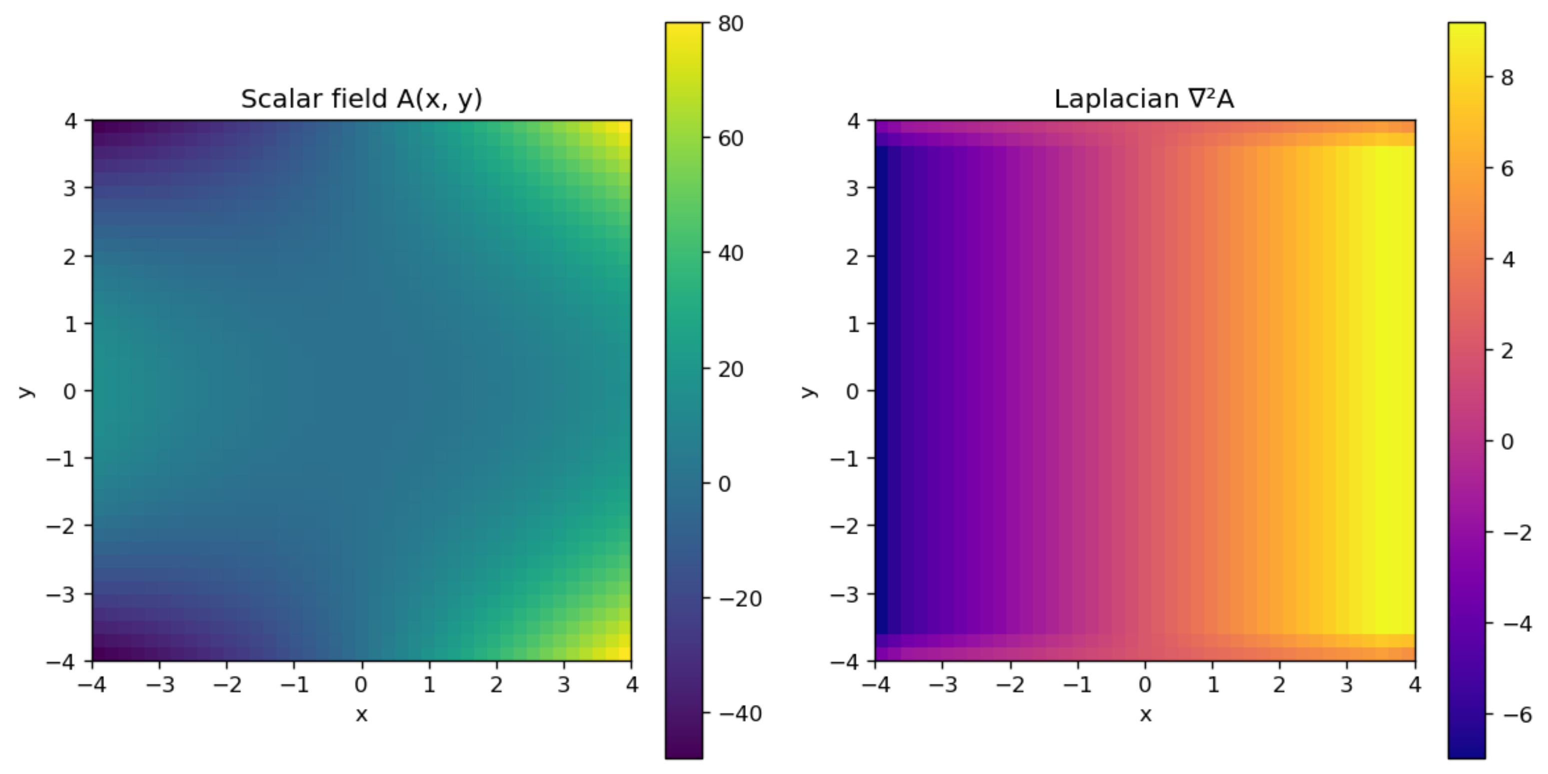
$$\nabla \times B$$
$$= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\imath}$$
$$- \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)\hat{\jmath}$$
$$+ \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{k}$$
$$let\ B(u, v)$$
$$u = -3x^2 - y^2$$
$$v = x - 5y^2$$



Laplacian Operation

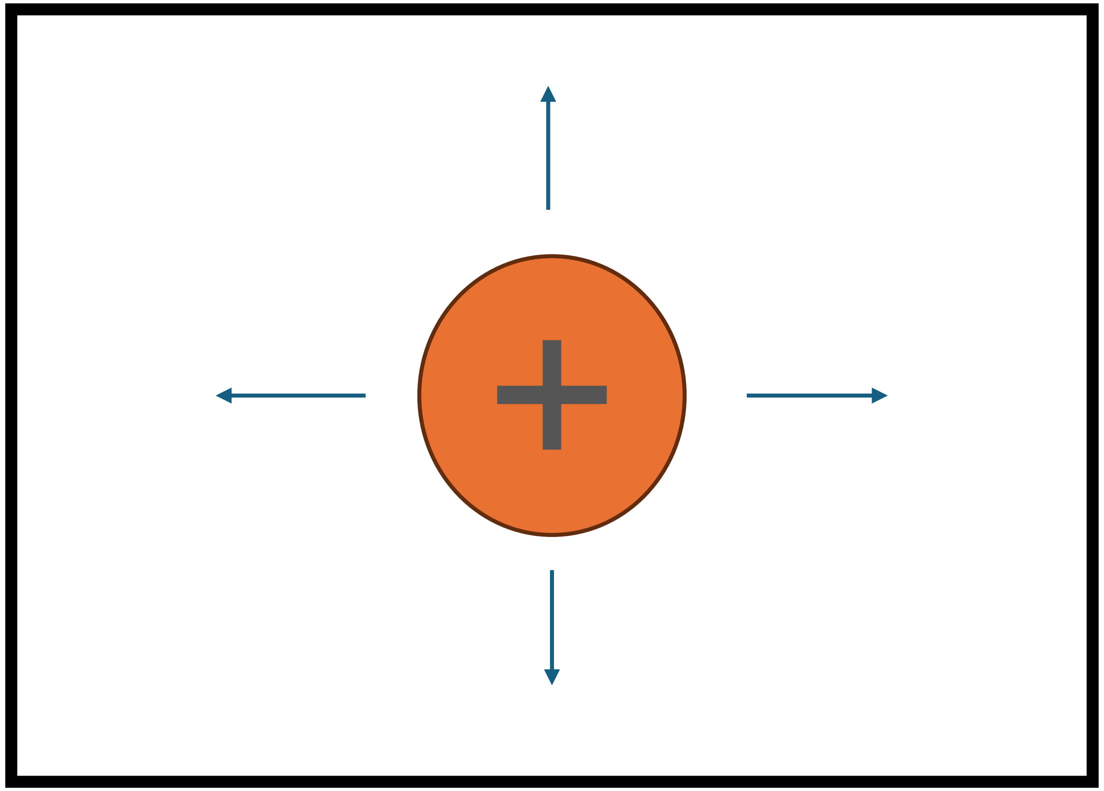
$$\nabla^2 A = \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2}$$

$$let\ A = x^2 + xy^2$$



Maxwells Equations – Gauss' Law for Electricity

The total Electric flux in any volume is equal to the total charge within that volume



Maxwells Equations – Gauss' Law for Electricity

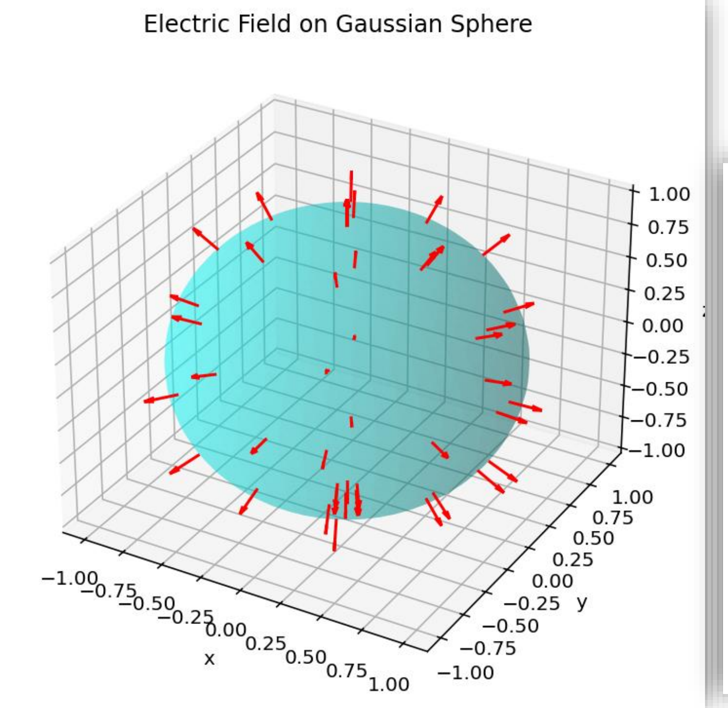
Gauss Law has two main ways of being explained mathematically:

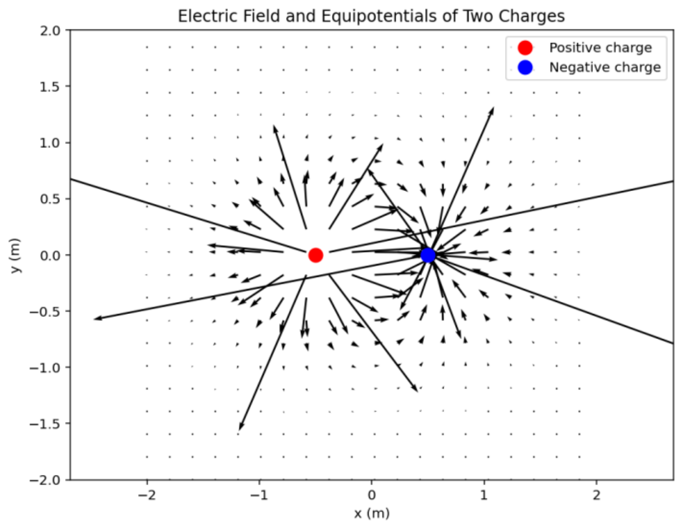
$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

- The first equation (differential form) describes the point charge behaviour
- The integral form describes the electric flux over a defined surface

Programs for the various laws can be found on teams to mess around with.



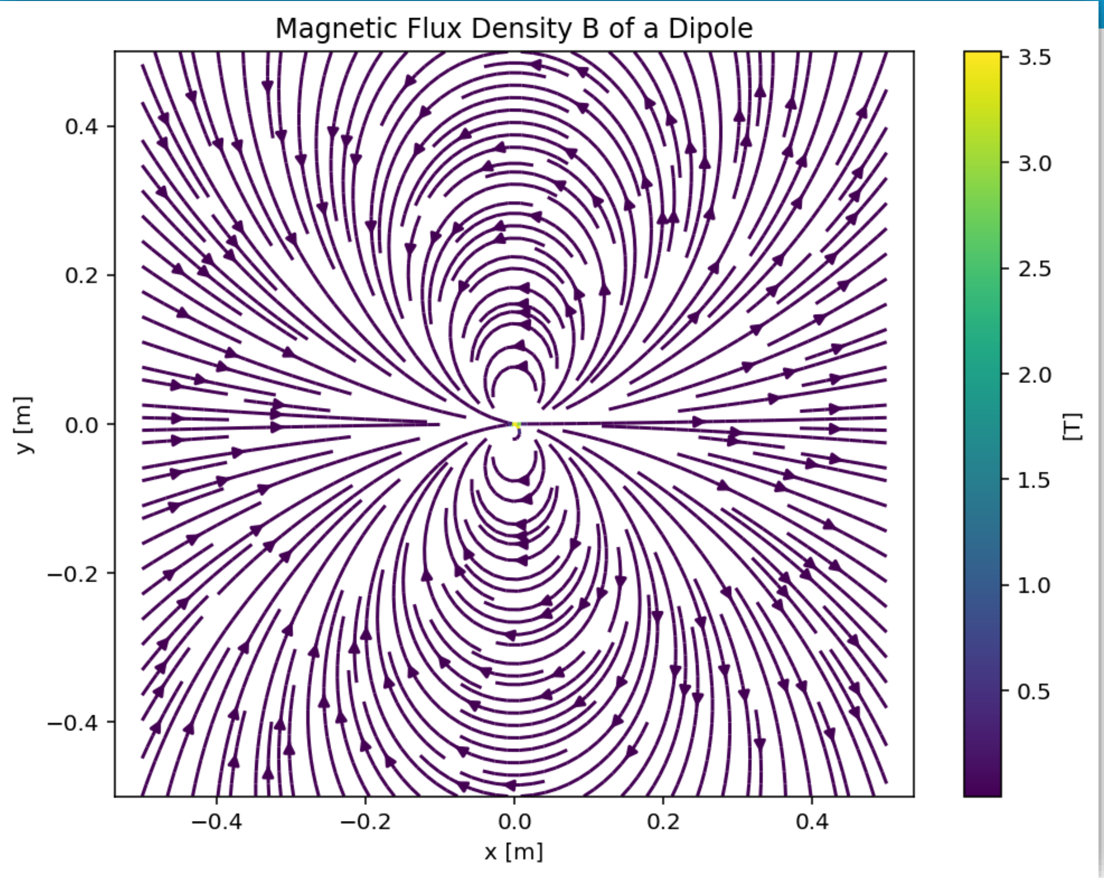


Maxwells Equations – Gauss' Law for Magnetism

- There is no source or sink for a magnetic field
- Example: There exists no observable monopole. All observable magnetic fields are created from a north and south pole

Mathematically this is expressed as:

$$\nabla \cdot \vec{B} = 0$$



Maxwells Equations - Faradays Law

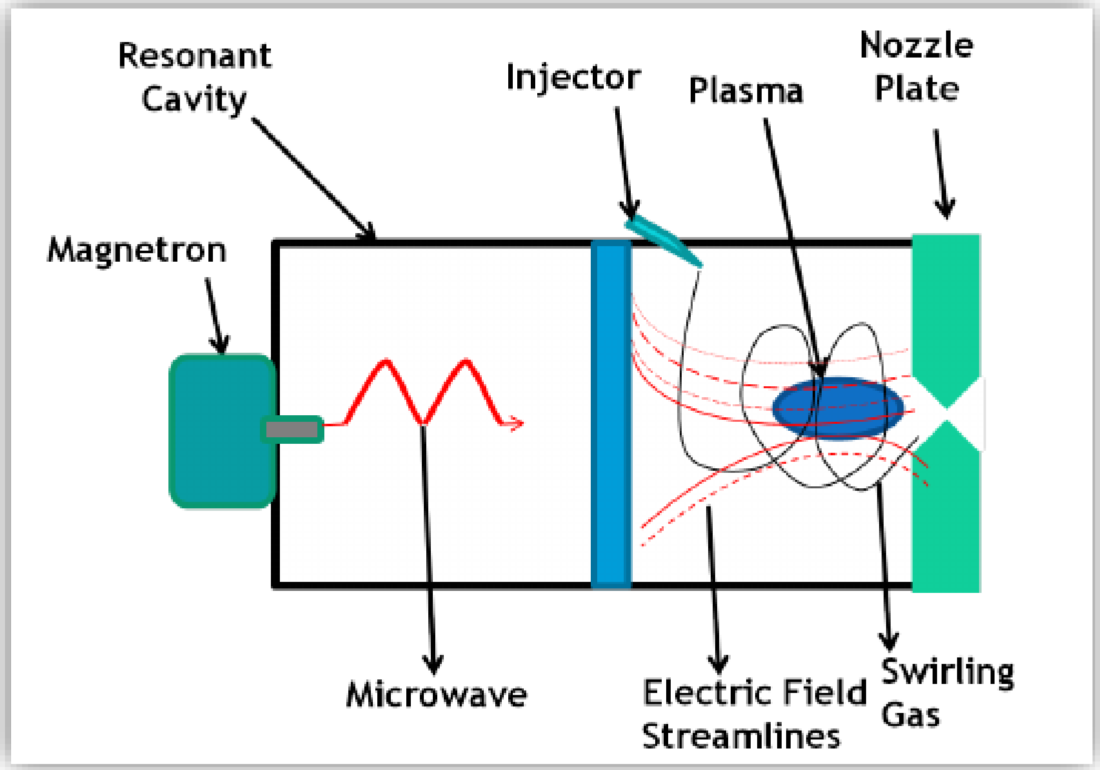
Induced electromotive force is proportional to the rate of change of magnetic flux density (A-Level Physics)

Mathematically:

$$\nabla \times \vec{V} = -\frac{\partial \vec{B}}{\partial t}$$

Electric propulsion applications:

- Used to induce radio/microwaves which can be used to ionise propellant



Maxwells Equations – Ampere's Law

- Ampere's law is the mirror of faradays law + extra bits.
- It says a flowing electric current produces an encircling magnetic field, also a time changing electric flux density produces an encircling magnetic field. Mathematically represented by:

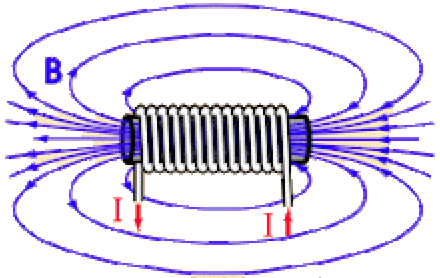
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

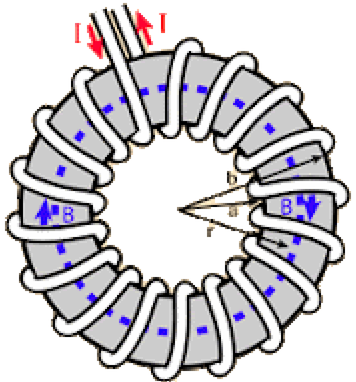
In case you don't know H is the magnetic field intensity, μ is the permeability of free space, the quantities get related by:

$$\vec{B} = \mu_0 \vec{H}$$

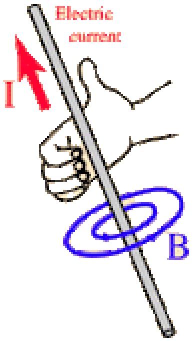
$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$



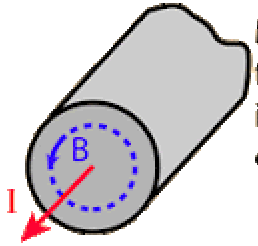
Magnetic field inside a long solenoic.



Magnetic field inside a toroidal coil.



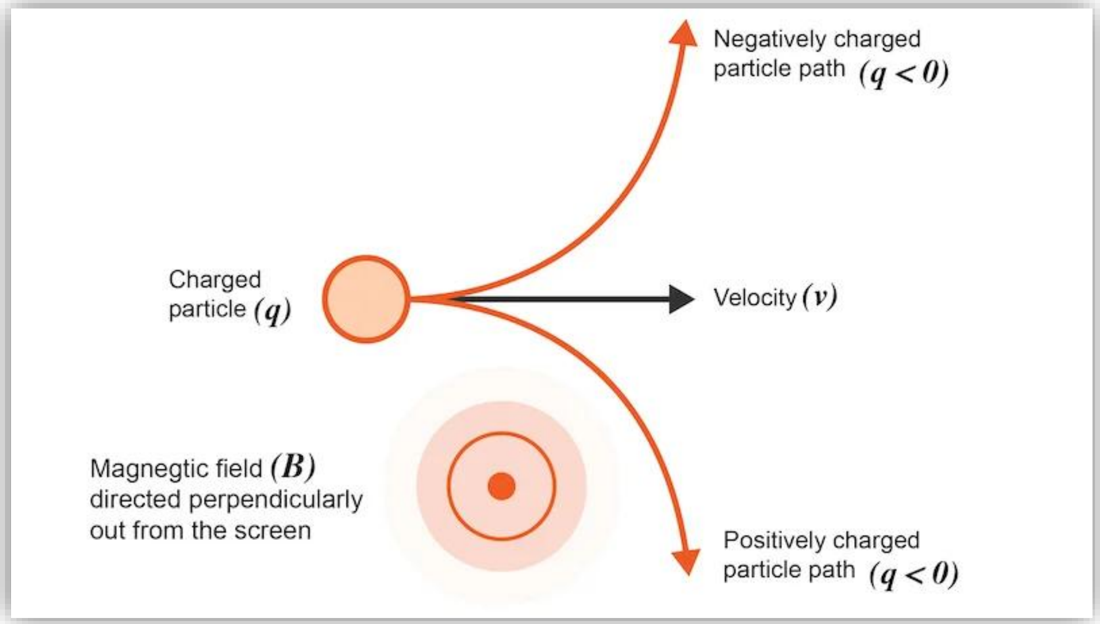
Magnetic field from a long straight wire.



Magnetic field inside a conductor.

Lorentz Force

- Charged particles inside electric and magnetic fields experience a force.
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$
- Electric force is in direction of electric field
- Magnetic force is perpendicular to magnetic field and particles velocity
- Keynote: all particles experience the same force no matter their mass



Cyclotron Motion

Ignore the electric field, only consider the magnetic field

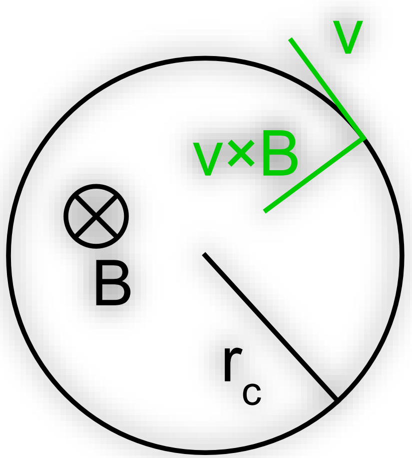
$$F = \left|q\left(\vec{v} \times \vec{B}\right)\right| = qv_{\perp}B$$

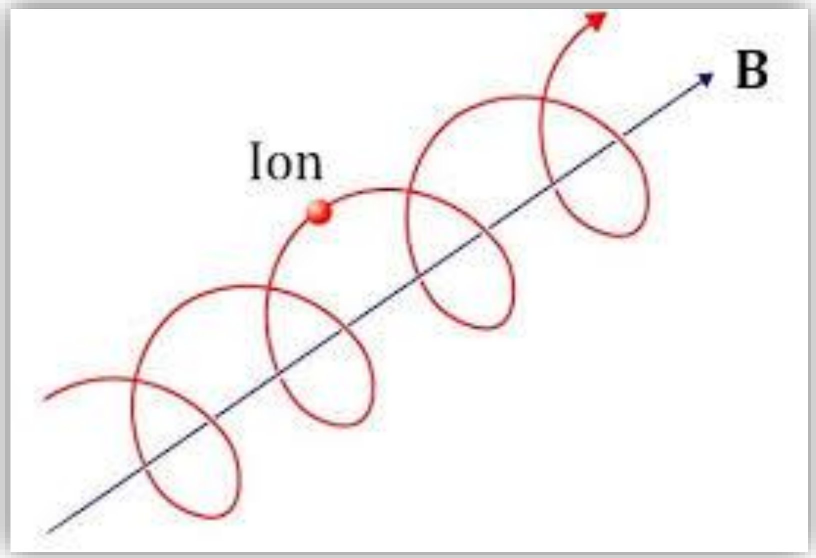
$$\frac{mv_{\perp}^2}{r_c} = qv_{\perp}B$$

$$r_c = \frac{mv_{\perp}}{qB}, v_{\perp} = \omega_c r_c$$

$$\Rightarrow \omega_c = \frac{qB}{m}$$

This variable becomes very important in hall thrusters as it is one of the key factors that defines something called the hall parameter ($\Omega_e \gg 1$)





Drift Motion

Consider a charged particle moving through a E and B field like in the image.

Consider the summation of forces acting on a charged particle:

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$(1)\ m\frac{dv_d}{dt} = q(\vec{E} + v_d \times \vec{B}) = 0$$

$$(2)\ m\frac{dv_c}{dt} = q(\vec{E} + v_c \times \vec{B}) = F_c$$

$$\Longrightarrow \vec{E} + v_d \times \vec{B} = 0$$

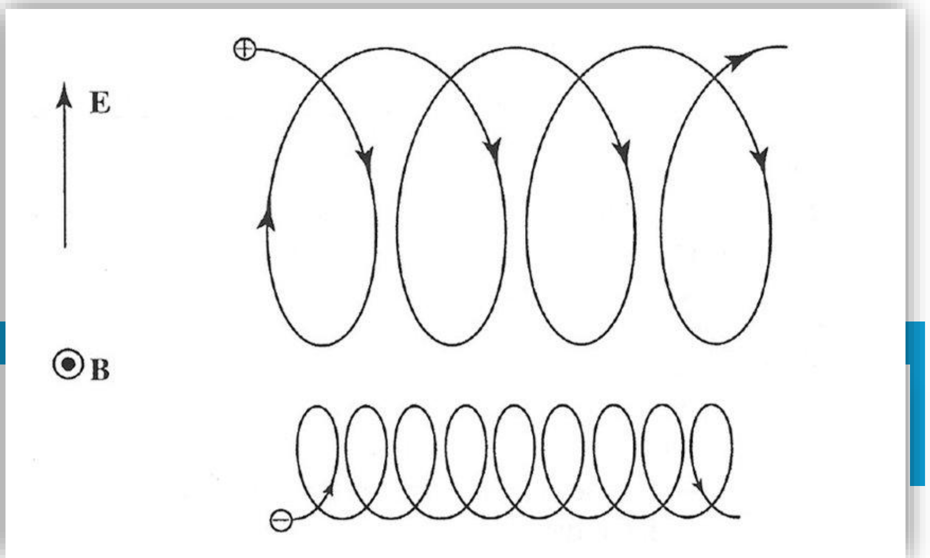
$$-\vec{E} = \vec{v}_d \times \vec{B}$$

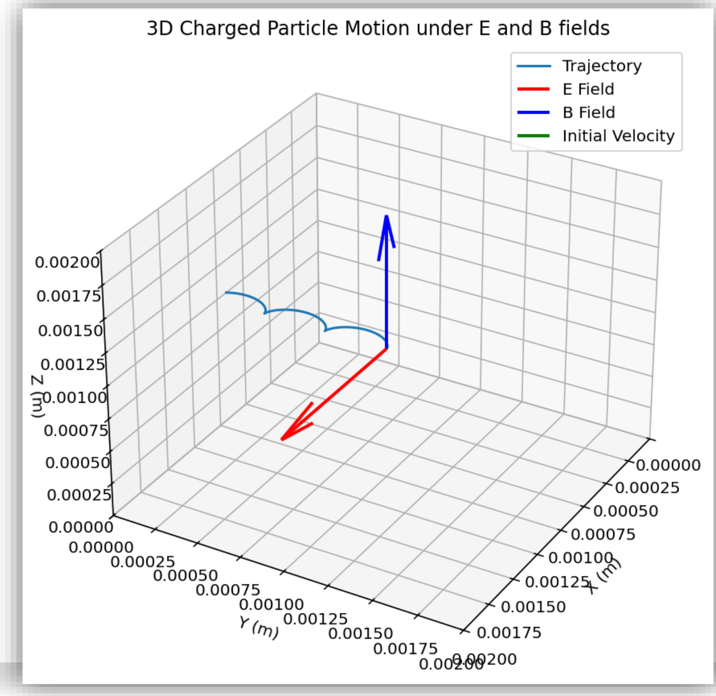
$$-\vec{E} \times \vec{B} = (\vec{v}_d \times \vec{B}) \times \vec{B}, \qquad (A \times B) \times C = B(C \cdot A) - A(B \cdot C)$$

$$-\vec{E} \times \vec{B} = \vec{B}(\vec{B} \cdot \vec{v}_d) - \vec{v}_d(\vec{B} \cdot \vec{B})$$

$$\vec{E} \times \vec{B} = \vec{v}_d B^2$$

$$\frac{\vec{E} \times \vec{B}}{B^2} = \vec{v}_d$$





Further Readings

http://hyperphysics.phy-astr.gsu.edu/hbase/electric/maxeq.html

https://maxwells-equations.com/

Demo programs can be found on SharePoint: Teaching\Demo Scripts