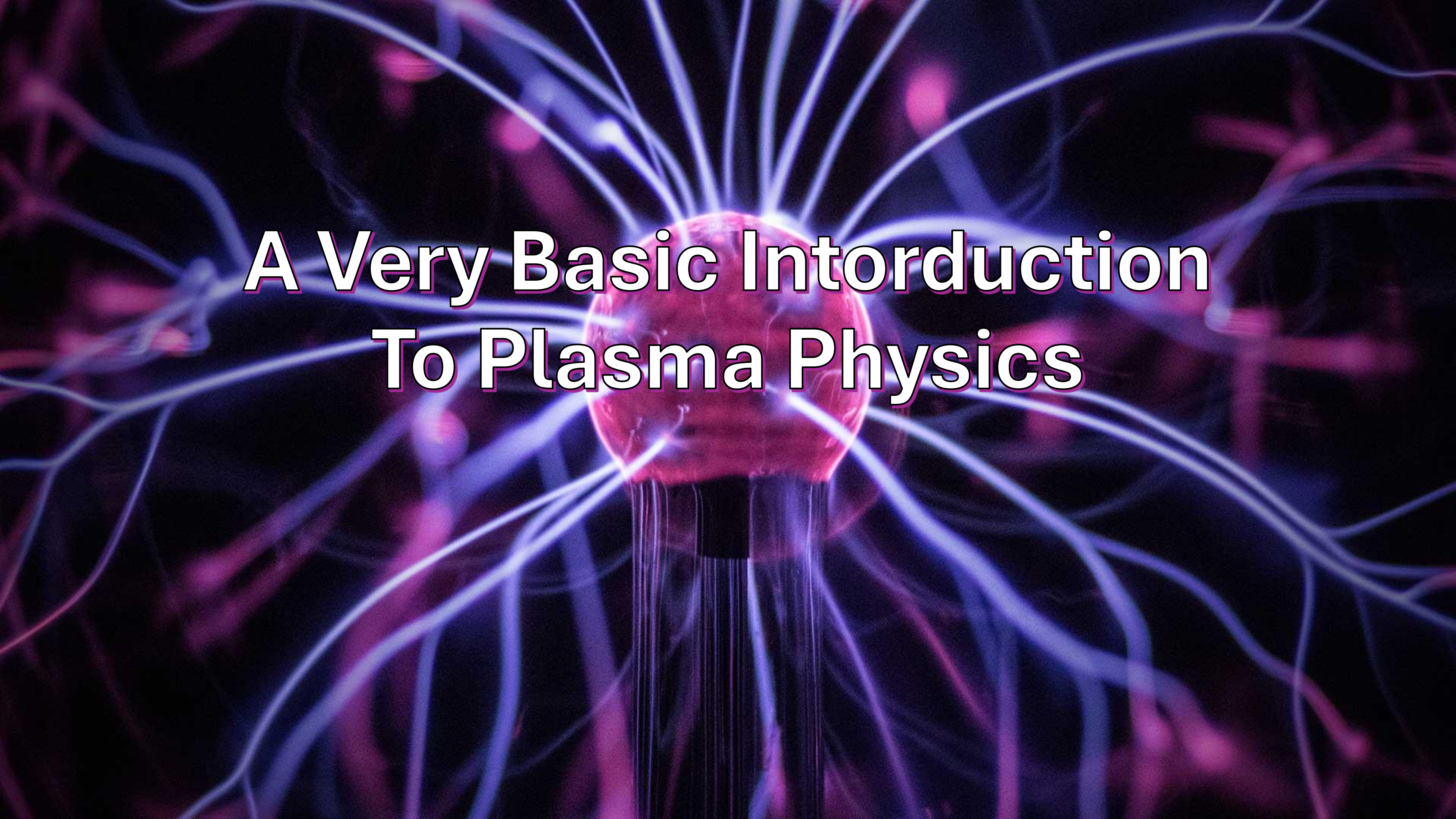
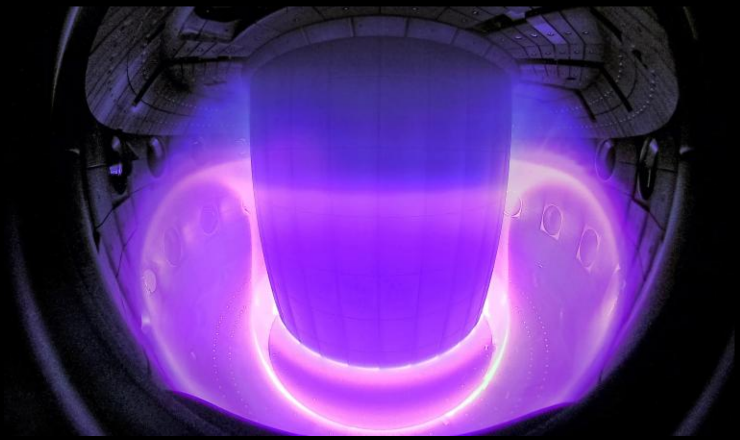
A Very Basic Intorduction To Plasma Physics



What is a plasma

- A fully or partially ionised quasi-neutral state of matter
- Looking at a plasma on a macro scale it is neutral (quasi-neutrality)
- Look at it on a micro scale then you would see a soup of ions, electrons and maybe neutrals



Ionisation and Plasma Formation

- Ionisation of a gas occurs when a threshold energy is met for a given element (Xe: 12.1eV, Kr: 14.0eV, Ar: 15.8eV, Zn: 9.4eV)

- Several methods to cause ionisation for a plasma
  - Direct impact ionisation:
$$e^- + I \to 2e^- + I^+$$
  - Photo ionisation:
$$hv + I \to e^- + I^+$$
  - Thermal ionisation:
$$I \rightleftharpoons e^- + I^+, \mathrm{kT} > \epsilon$$

Ionisation and Plasma Formation

Radiative Recombination:

$$e^- + I^+ \rightarrow hv + I$$

Three Body Recombination:

$$e^- + e^- + I^+ \rightarrow e^- + I$$

- Ionisation fraction [χ] and equilibrium condition:

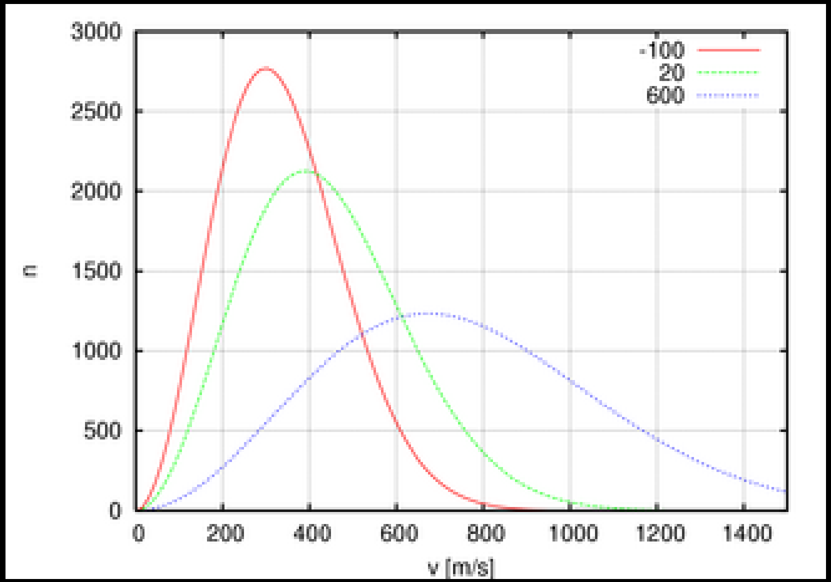
$$\chi = \frac{n_i}{n_i + n_n}$$

$$n_i \langle \sigma_r v \rangle = n_n \langle \sigma_i v \rangle, n_i \approx n_e$$

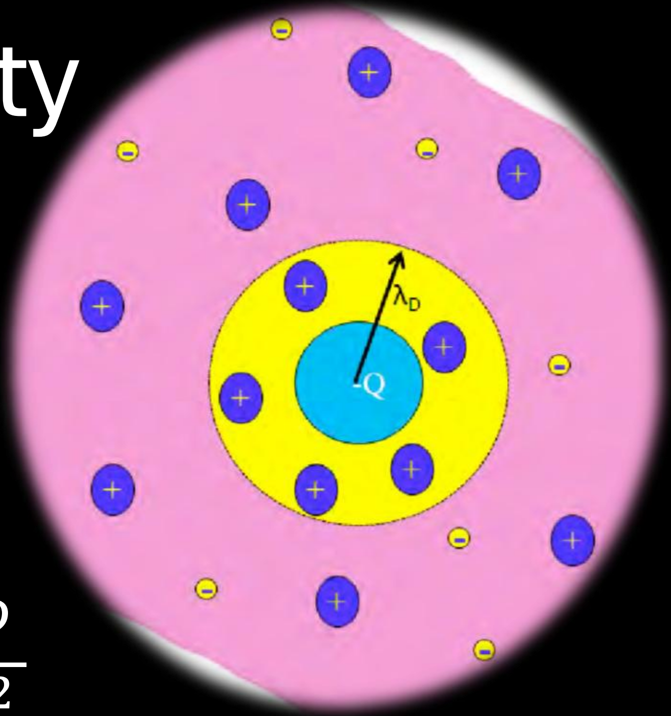
Debye Shielding and Quasi-Neutrality

- Boltzmann distributions: describes number of particles in a system at a given speed at thermal equilibrium

$$n_e = n_0 exp\left(\frac{e\phi}{k_B T_e}\right) \approx n_0\left(1 + \frac{e\phi}{k_B T_e}\right)$$



Debye Shielding and Quasi-Neutrality



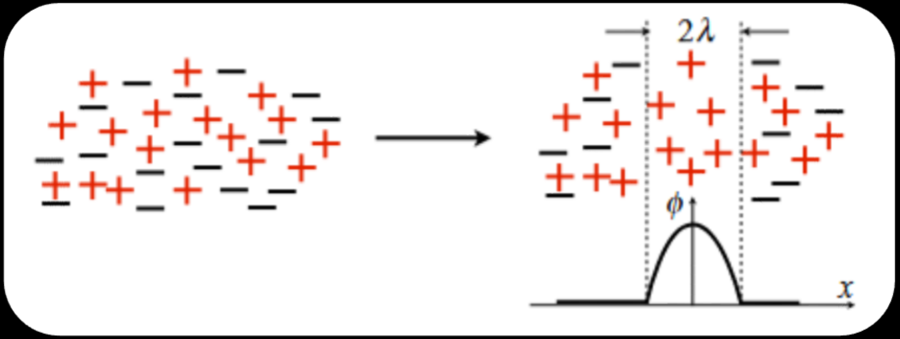
- Consider Gauss' law:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}, \vec{E} = \nabla V \Rightarrow \nabla^2 V = \frac{\rho}{\varepsilon_0}$$

- Assume that ions are fixed with density $n_i \approx n_0$

$$\rho = -\frac{n_0 e^2}{k_B T_e}\phi \Rightarrow \nabla^2 V = \frac{1}{\varepsilon_0} \cdot -\frac{n_0 e^2}{k_B T_e}\phi = \frac{\phi}{\lambda_D^2}$$

- Debye Length: The characteristic length over which electric potentials are screened in a plasma



$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{n_e e^2}}$$

Plasma Frequency and Timescales

Consider a displaced cloud of electrons:

Restoring force (per unit volume):

$$f = -en_eE$$

Equation of Motion for the cloud:

$$m_e n_e \frac{d^2x}{dt^2} = -en_eE \Rightarrow m_e \frac{d^2x}{dt^2} + eE = 0$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} = E = \frac{\rho}{\varepsilon_0} x \text{ in 1D}$$

$$m_e \frac{d^2x}{dt^2} + e\frac{\rho}{\varepsilon_0} x = \frac{d^2x}{dt^2} + e\frac{en_e}{m_e\varepsilon_0} x \Rightarrow \omega^2 = \frac{e^2n_e}{m_e\varepsilon_0}$$

Plasma Frequency and Timescales

- Assume a quasi-neutral plasma $n_e = n_i = 10^{10}$
- Mass of ion and electron: $10^{-25}, 10^{-31}$kg
- Assume ion is singularly charged

$$\omega_i = \sqrt{\frac{e^2 n_\mathrm{i}}{m_\mathrm{i}\varepsilon_0}} = 17\mathrm{kHz}, \omega_\mathrm{e} = \sqrt{\frac{e^2 n_\mathrm{e}}{m_\mathrm{e}\varepsilon_0}} = 17\mathrm{MHz}$$

$$\lambda_D \cdot \omega = v_{th} = \sqrt{\frac{k_B T_e}{m_e}}$$

Collisionality and Mean Free Path

- Mean free path: average distance a particle travels before colliding with another particle

$$\lambda = \frac{1}{n\sigma}$$

- Neutron-electron collisions:

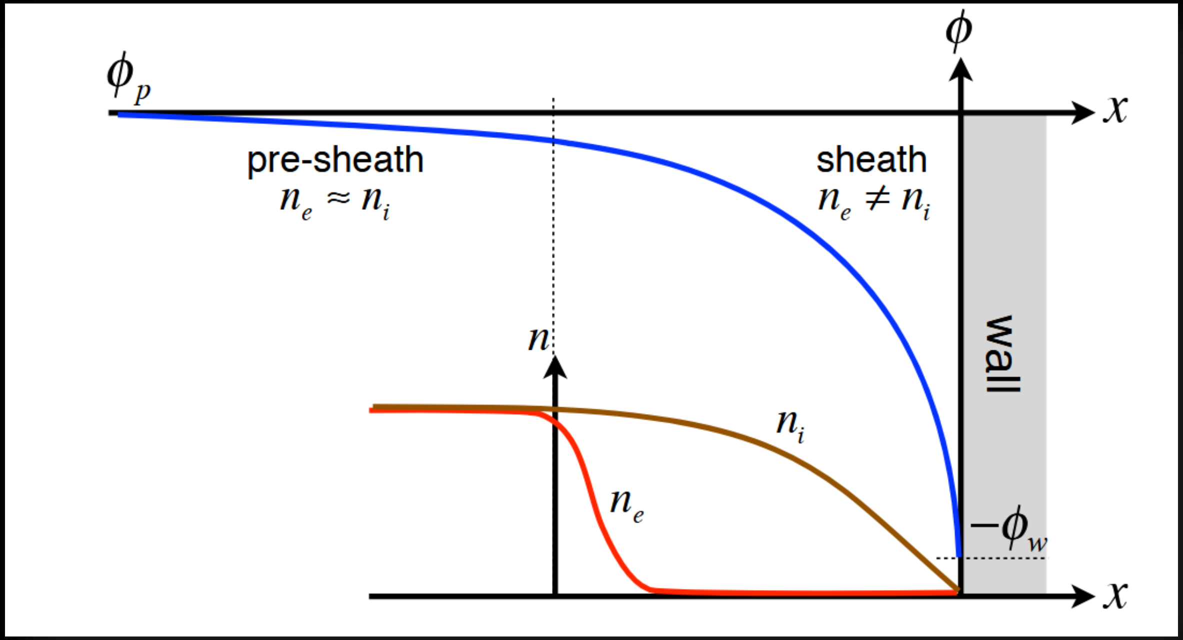
$$\lambda = \frac{v_n}{n_e\langle\sigma v_e\rangle}$$

- Describes the distance a neutral moves before it become ionised forming a plasma
- Collision frequencies: rate of collisions per unit time

$$\nu = \frac{\mathrm{v}_{th}}{\lambda}$$

Plasma Sheaths

- Materials in contact with the plasma will become negatively charged with respect to the bulk plasma
- Ions directed at the wall will eventually hit it
- Electrons will lower energy than ϕ_w will be reflected away from it



Plasma Sheaths

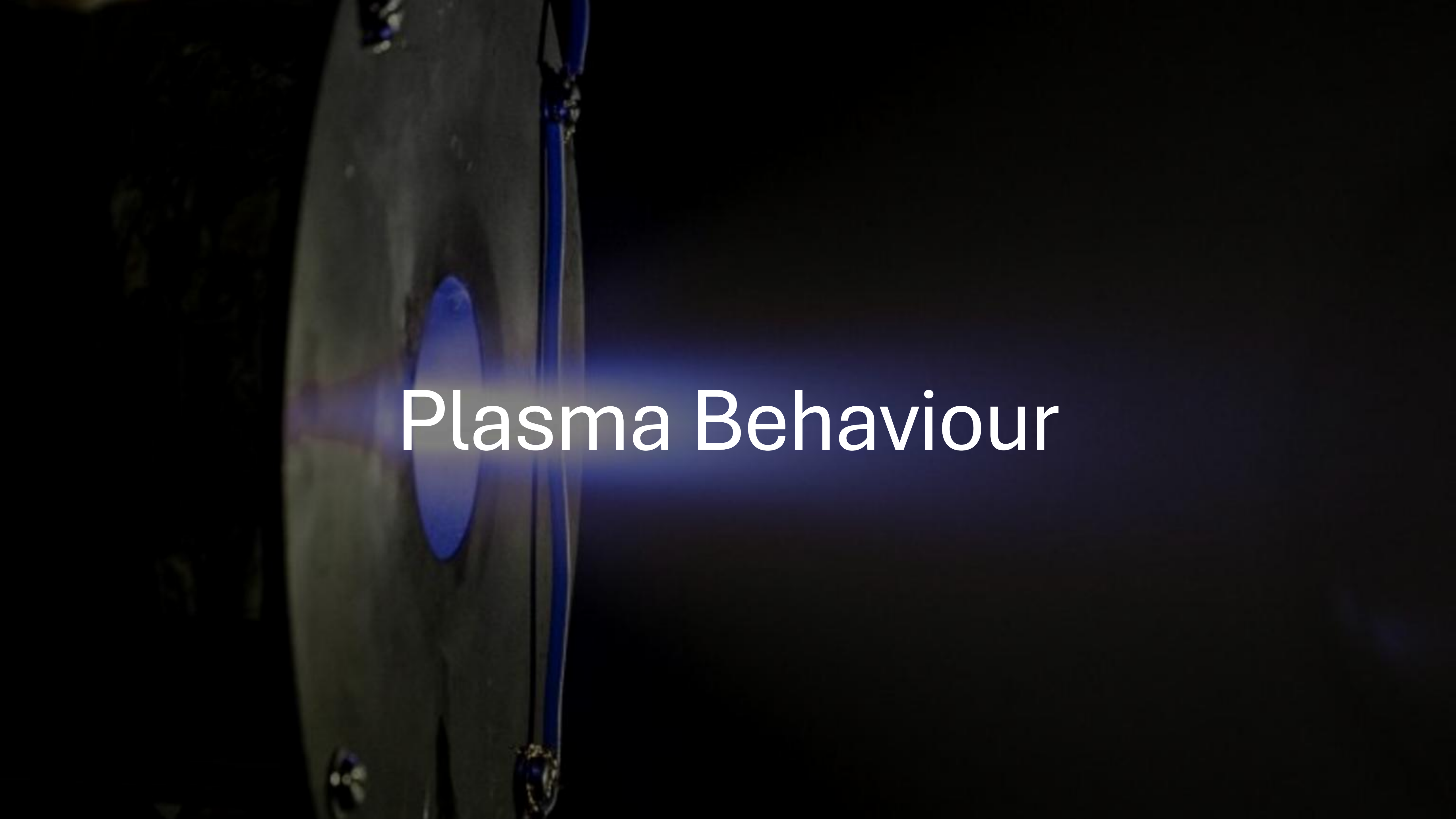
Bohm Criterion

- For a sheath to form and be stable ions must enter with sufficient speed

$$v_i \geq c_s = \sqrt{\frac{k_B T_e}{m_i}}$$

To describe the potential at which the plasma self biases (no current flow between plasma and walls)

$$\phi = \frac{k_B T_e}{e} ln \sqrt{\frac{2m_i}{\pi m_e}}$$


Plasma Behaviour

Kinetic Description

- For modelling individual particle behaviour this method is often employed
- It is frequently used in collision less situations

- The distribution function:
  $f_s(r, v, t)$ $\mathrm{r} = \mathrm{position\ (3D)}$, $\mathrm{v} = \mathrm{velocity\ (3D)}, \mathrm{t} \ = \ \mathrm{time\ (1D)}$

Where s can be e, i, or n

Kinetic Description

- The following macroscopic quantities can be extracted
  - Number density: $n_s(r,t)$
  - Mean velocity: $\mathrm{u}_s(r,t)$
  - Temperature: $n_s(r,t)$
  - Pressure: P_s

Kinetic Description

- The time dependent Boltzmann equation is as written:

$$\frac{\partial f_s}{\partial \mathrm{t}} + \vec{\mathrm{v}} \cdot \nabla_{\vec{\mathrm{r}}} f_s + \frac{q_s}{m_s}\left(\vec{\mathrm{E}} + \vec{\mathrm{v}} \times \vec{\mathrm{B}}\right) \cdot \nabla_{\vec{\mathrm{v}}} f_s = \mathrm{collisions}$$

  - $\frac{\partial f_s}{\partial \mathrm{t}}$
    - Is the local time rate of change of the function
  - $\vec{\mathrm{v}} \cdot \nabla_{\vec{\mathrm{r}}} f_s$
    - How the particles move due to their velocity
  - $\frac{q_s}{m_s}\left(\vec{\mathrm{E}} + \vec{\mathrm{v}} \times \vec{\mathrm{B}}\right) \cdot \nabla_{\vec{\mathrm{v}}} f_s$
    - Force term which describes how particles velocities change due to the Lorentz force

- E and B fields can be internal and external so must be calculated through maxwells equations

Fluid Models

- First consider the classical Navier-Stokes Equations
  - Continuity:

$$\frac{\partial \rho}{\partial \mathrm{t}} + \nabla \cdot (\rho \vec{\mathrm{u}}) = 0$$

  - Momentum:

$$\rho \left(\frac{\partial \vec{\mathrm{u}}}{\partial \mathrm{t}} + \vec{\mathrm{u}} \cdot \nabla \vec{\mathrm{u}} \right) = -\nabla p + \mu \nabla^2 \vec{\mathrm{u}} + \rho g$$

Fluid Models

- Similar equations can be used to model plasmas on a macro scale
  - Continuity:

$$\frac{\partial n_s}{\partial \mathrm{t}} + \nabla \cdot (n_s \overrightarrow{u_s}) = S_s$$

  - Momentum:

$$m_s n_s \left(\frac{\partial \overrightarrow{u_s}}{\partial \mathrm{t}} + \overrightarrow{u_s} \cdot \nabla \overrightarrow{u_s} \right) = -\nabla p_s - m_s n_s \nu_s (\overrightarrow{u_s} - u_{neutral}) + q_s n_s \left(\vec{\mathrm{E}} + \overrightarrow{u_s} \times \vec{\mathrm{B}} \right)$$

Fluid Models

- Further equations are required to fully describe plasmas nature
  - Energy (simplified):

$$\frac{3}{2} n_s k_B \frac{dT_s}{dt} = Q$$

  - Field Coupling:

$$\nabla \cdot \vec{\mathrm{E}} = \frac{\rho}{\epsilon_0} = \frac{e(n_i - n_e)}{\epsilon_0} \Rightarrow \nabla^2 \phi = \frac{e(n_i - n_e)}{\epsilon_0}$$

Fluid Models

Term	**Navier Stokes**	**Plasma Fluid**
Inertia	$\rho\left(\frac{\partial\vec{\mathrm{u}}}{\partial \mathrm{t}} + \vec{\mathrm{u}} \cdot \nabla\vec{\mathrm{u}}\right)$	$m_s n_s\left(\frac{\partial\overrightarrow{u_s}}{\partial \mathrm{t}} + \overrightarrow{u_s} \cdot \nabla\overrightarrow{u_s}\right)$
Pressure Gradient	$-\nabla p$	$-\nabla p_s$
Body Forces	ρg	$q_s n_s\left(\vec{\mathrm{E}} + \overrightarrow{u_s} \times \vec{\mathrm{B}}\right)$
Internal viscous forces	$\mu\nabla^2\vec{\mathrm{u}}$	$-m_s n_s \nu_s(\overrightarrow{u_s} - u_{neutral})$
Coupling	No coupling	$\nabla^2\phi$
Multiple Species	Could have multiple species if multi-phase or a mixture but usually just one	Multiple species: ions, electrons and neutrons

Model Comparison

Factor	**Kinetic Model**	**Fluid Model**
Description	Describes individual particles and collisions between particles	Describes the plasma at a more macroscopic level capturing less detail
Assumptions	Very few assumptions, tries to capture all details	Assumes an averaged temperature and a near Maxwellian distribution
Computational cost	Very high	Less than kinetic
Physics	Captures all physics	Only captures bulk motion