

Introduction to Cryptography

CS 136

Computer Security

Peter Reiher

January 12, 2021

Outline

- What is data encryption?
- Cryptanalysis
- Basic encryption methods
 - Substitution ciphers
 - Permutation ciphers

Introduction to Encryption

- Much of computer security is about keeping secrets
- One method is to make the secret hard for others to read
- While (usually) making it simple for authorized parties to read

Encryption

- Encryption is the process of hiding information in plain sight
- Transform the secret data into something else
- Even if the attacker can see the transformed data, he can't understand the underlying secret

Encryption and Data Transformations

- Encryption is all about transforming the data
- One bit or byte pattern is transformed to another bit or byte pattern
- Usually in a reversible way

Encryption Terminology

- Encryption is typically described in terms of sending a message
 - Though it's used for many other purposes
- The sender is S
- The receiver is R
- And the attacker is O

More Terminology

- *Encryption* is the process of making message unreadable/unalterable by O
- *Decryption* is the process of making the encrypted message readable by R
- A system performing these transformations is a *cryptosystem*
 - Rules for transformation sometimes called a *cipher*

Plaintext and Ciphertext

- *Plaintext* is the original form of the message (often referred to as P)

```
Transfer  
$100 to my  
savings  
account
```

- *Ciphertext* is the encrypted form of the message (often referred to as C)

```
Sqzmredq  
#099 sn lx  
rzuhmfr  
zbbntms
```


Very Basics of Encryption Algorithms

- Most algorithms use a *key* to perform encryption and decryption
 - Referred to as K
- The key is a secret
- Without the key, decryption is hard
- With the key, decryption is easy

Terminology for Encryption Algorithms

- The encryption algorithm is referred to as $E()$
- $C = E(K, P)$
- The decryption algorithm is referred to as $D()$
 - Sometimes the same algorithm as $E()$
- The decryption algorithm also has a key

Symmetric and Asymmetric Encryption Systems

- Symmetric systems use the same keys for E and D :

$$P = D(K, C)$$

Expanding, $P = D(K, E(K, P))$

- Asymmetric systems use different keys for E and D:

$$C = E(K_E, P)$$

$$P = D(K_D, C)$$

Characteristics of Keyed Encryption Systems

- If you change only the key, a given plaintext encrypts to a different ciphertext
 - Same applies to decryption
- Decryption should be hard without knowing the key

Cryptanalysis

- The process of trying to break a cryptosystem
- Finding the meaning of an encrypted message without being given the key
- To build a strong cryptosystem, you must understand cryptanalysis

Forms of Cryptanalysis

- Analyze an encrypted message and deduce its contents
- Analyze one or more encrypted messages to find a common key
- Analyze a cryptosystem to find a fundamental flaw

Breaking Cryptosystems

- Most cryptosystems are breakable
- Some just cost more to break than others
- The job of the cryptosystem designer is to make the cost infeasible
 - Or incommensurate with the benefit extracted

Types of Attacks on Cryptosystems

- Ciphertext only
- Known plaintext
- Chosen plaintext
 - Differential cryptanalysis
- Algorithm and ciphertext
 - Timing attacks
- In many cases, the intent is to guess the key

Ciphertext Only

- No *a priore* knowledge of plaintext
- Or details of algorithm
- Must work with probability distributions, patterns of common characters, etc.
- Hardest type of attack

Known Plaintext

- Full or partial
- Cryptanalyst has matching sample of ciphertext and plaintext
- Or may know something about what ciphertext represents
 - E.g., an IP packet with its headers

Chosen Plaintext

- Cryptanalyst can submit chosen samples of plaintext to the cryptosystem
- And recover the resulting ciphertext
- Clever choices of plaintext may reveal many details
- Differential cryptanalysis iteratively uses varying plaintexts to break the cryptosystem
 - By observing effects of controlled changes in the offered plaintext

Algorithm and Ciphertext

- Cryptanalyst knows the algorithm and has a sample of ciphertext
- But not the key, and cannot get any more similar ciphertext
- Can use “exhaustive” runs of algorithm against guesses at plaintext
- Password guessers often work this way
- *Brute force attacks* – try every possible key to see which one works

Timing Attacks

- Usually assume knowledge of algorithm
- And ability to watch algorithm encrypting/decrypting
- Some algorithms perform different operations based on key values
- Watch timing to try to deduce keys
- Successful against some smart card crypto
- Similarly, observe power use by hardware while it is performing cryptography

Basic Encryption Methods

- Substitutions
 - Monoalphabetic
 - Polyalphabetic
- Permutations

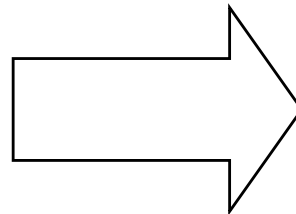
Substitution Ciphers

- Substitute one or more characters in a message with one or more different characters
- Using some set of rules
- Decryption is performed by reversing the substitutions

Example of a Simple Substitution Cipher

How did this transformation happen?

```
Sqzmredq  
#099 sn lx  
rzuhmfr  
zbbntms
```



```
Sqzmredq  
#099 sn lx  
rzuhmfr  
zbbntms
```

Every letter was changed to the “next lower” letter

Caesar Ciphers

- A simple substitution cipher like the previous example
 - Supposedly invented by Julius Caesar
- Translate each letter a fixed number of positions in the alphabet
- Reverse by translating in opposite direction

Is the Caesar Cipher a Good Cipher?

- Well, it worked great 2000 years ago
- It's simple, but
- It's simple
- Fails to conceal many important characteristics of the message
- Which makes cryptanalysis easier
- Limited number of useful keys

How Would Cryptanalysis Attack a Caesar Cipher?

- Letter frequencies
- In English (and other alphabetic languages), some letters occur more frequently than others
- Caesar ciphers translate all occurrences of a given plaintext letter into the same ciphertext letter
- All you need is the offset

More On Frequency Distributions

- In most languages, some letters used more than others
 - In English, “e,” “t,” and “s” are common
- True even in non-natural languages
 - Certain characters appear frequently in C code
 - Zero appears often in numeric data

Cryptanalysis and Frequency Distribution

- If you know what kind of data was encrypted, frequency distributions offer a path to breaking it
- Especially for Caesar ciphers
 - And other simple substitution-based encryption algorithms

Breaking Caesar Ciphers

- Identify (or guess) the kind of data
- Count frequency of each encrypted symbol
- Match to observed frequencies of unencrypted symbols in similar plaintext
- Provides probable mapping of cipher
- The more ciphertext available, the more reliable this technique

Example

- With ciphertext “Sqzmredq #099 sn lx rzuhmfr zbbntms”
- Frequencies -

a	0		b	2		c	0		d	1		e	1
f	1		g	0		h	1		i	0		j	0
k	0		l	1		m	3		n	2		o	0
p	0		q	2		r	3		s	3		t	1
u	1		v	0		w	0		x	1		y	0
z	3												

Applying Frequencies To Our Example

a	0		b	2		c	0		d	1		e	1
f	1		g	0		h	1		i	0		j	0
k	0		l	1		m	3		n	2		o	0
p	0		q	2		r	3		s	3		t	1
u	1		v	0		w	0		x	1		y	0
z	3												

- The most common English letters are typically “e,” “t,” “a,” “o,” and “s”
- Four out of five of the common English letters in the plaintext map to these letters

Cracking the Caesar Cipher

- Since all substitutions are offset by the same amount, just need to figure out how much
- How about +1?
 - That would only work for $a \Rightarrow b$
- How about -1?
 - That would work for $t \Rightarrow s$, $a \Rightarrow z$, $o \Rightarrow n$, and $s \Rightarrow r$
 - Try it on the whole message and see if it looks good

More Complex Substitutions

- Monoalphabetic substitutions
 - Each plaintext letter maps to a single, unique ciphertext letter
- Any mapping is permitted
- Key can provide method of determining the mapping
 - Key could be the mapping

Are These Monoalphabetic Ciphers Better?

- Only a little
- Finding the mapping for one character doesn't give you all mappings
- But the same simple techniques can be used to find the other mappings
- Generally insufficient for anything serious

Polyalphabetic Ciphers

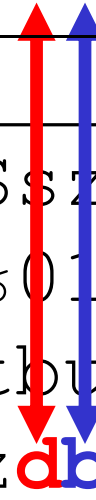
- Ciphers that don't always translate a given plaintext character into the same ciphertext character
- For example, use different substitutions for odd and even positions

Example of Simple Polyalphabetic Cipher

- Move one character “up” in even positions, one character “down” in odd positions
- Note that same character translates to different characters in some cases

Transfer
\$100 to my
savings
a**c**count

Sszorgds
%019 sp nx
tbujmhr
z**d**ptos



Are Polyalphabetic Ciphers Better?

- Depends on how easy it is to determine the pattern of substitutions
- If it's easy, then you've gained little

Cryptanalysis of Our Example

- Consider all even characters as one set
- And all odd characters as another set
- Apply basic cryptanalysis to each set
- The transformations fall out easily
- How did you know to do that?
 - You guessed
 - Might require several guesses to find the right pattern

How About For More Complex Patterns?

- Good if the attacker doesn't know the choices of which characters get transformed which way
- Attempt to hide patterns well
- But known methods still exist for breaking them

Methods of Attacking Polyalphabetic Ciphers

- Kasiski method tries to find repetitions of the encryption pattern
- Index of coincidence predicts the number of alphabets used to perform the encryption
- Both require lots of ciphertext

How Does the Cryptanalyst “Know” When He’s Succeeded?

- Every key translates a message into something
- If a cryptanalyst thinks he’s got the right key, how can he be sure?
- Usually because he doesn’t get garbage when he tries it
- He almost certainly will get garbage from any other key
- Why?

Consider A Caesar Cipher

- There are 25 useful keys (in English)
- The right one will clearly yield meaningful text
- What's the chances that any of the other 24 will?
 - Pretty poor
- So if the decrypted text makes sense, you've got the key

The More General Case

- Let's say the message is N bits long
 - So there are 2^N possible messages
 - But many of those make no sense
- Let's say the key is m bits long ($m \ll N$)
 - So there are 2^m keys
- What if only only 2^k of the possible messages make sense?
 - $2^k \ll 2^N$
 - For example, if the message was English text
- Assuming everything is random (and a good encryption algorithm tries to be)
 - The chance any wrong key decrypts to something sensible is around $1/2^N$ (k and $m \ll N$)



yes

A Simple Example

~1000 3
letter words

Let's encrypt a 3 letter English message

In ASCII, that's 24 bits

0	1	1	1	1	0	0	1	0	1	1	0	0	1	0	1	0	1	1	1	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Let's use a 4 bit key

1	1	0	1
---	---	---	---

1	0	1	0	0	1	0	0	1	0	1	1	1	0	0	0	1	0	1	0	1	1	1	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

If we don't know the key, there are 16 possible decryptions

What are the chances one of the other 15 keys
decrypts this ciphertext to an English language word?

$$15 * 1000 / 16777216$$

Around .1% For longer messages, lower odds

The Unbreakable Cipher

- There is a “perfect” substitution cipher
- One that is theoretically (and practically) unbreakable without the key
- And you can’t guess the key
 - If the key was chosen in the right way . . .

One-Time Pads

- Essentially, use a new substitution alphabet for every character
- Substitution alphabets chosen purely at random
 - These constitute the key
- Provably unbreakable without knowing this key

Example of One Time Pads

- Usually explained with bits, not characters
- We shall use a highly complex cryptographic transformation:
 - XOR
- And a three bit message
 - 010

One Time Pads at Work

0	1	0
---	---	---

Apply our
sophisticated
cryptographic
algorithm

Flip some coins to
get random
numbers

0	0	1
---	---	---

0	1	1
---	---	---

We now have an
unbreakable
cryptographic
message

What's So Secure About That?

- Any key was equally likely
- Any plaintext could have produced this message with one of those keys
- Let's look at our example more closely

Why Is the Message Secure?

Let's say there
are only two
possible
meaningful
messages

Could the
message decrypt
to either or both
of these?

0	1	1
---	---	---

0	1	0
---	---	---

0	0	0
---	---	---

There's a key that
works for each
And they're
equally likely

0	0	1
---	---	---

0	1	1
---	---	---

Security of One-Time Pads

- If the key is truly random, provable that it can't be broken without the key
- But there are problems
- Need one bit of key per bit of message
- Key distribution is painful
- Synchronization of keys is vital
- A good random number generator is hard to find

One-Time Pads and Cryptographic Snake Oil

- Companies regularly claim they have “unbreakable” cryptography
- Usually based on one-time pads
- But typically misused
 - Pads distributed with some other crypto mechanism
 - Pads generated with non-random process
 - Pads reused

Permutation Ciphers

- Instead of substituting different characters, scramble up the existing characters
- Use algorithm based on the key to control how they're scrambled
- Decryption uses key to unscramble

Characteristics of Permutation Ciphers

- Doesn't change the characters in the message
 - Just where they occur
- Thus, character frequency analysis doesn't help cryptanalyst

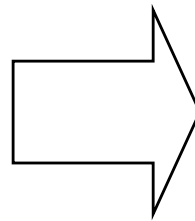
Columnar Transpositions

- Write the message characters in a series of columns
- Copy from top to bottom of first column, then second, etc.

Example of Columnar Substitution

How did this transformation happen?

T	r	a	n	s	f
e	r		\$	l	0
0		t	o		m
y		s	a	v	i
n	g	s		a	c
c	o	u	n	t	



T	e	0	y	n	c
r	r			g	o
a		t	s	s	u
n	\$	o	a		n
s	l		v	a	t
f	0	m	i	c	

Looks a lot more cryptic written this way:

Te0yncrr goa tssun\$oa ns1 vatf0mic

Attacking Columnar Transformations

- The trick is figuring out how many columns were used
- Use information about digrams, trigrams, and other patterns
- Digrams are pairs of letters that frequently occur together (“re”, “th”, “en”, e.g.)
- For each possibility, check digram frequency

For Example,

4 5 6 1 2 3 4 5 6 1 2 3 4 5 6 1 2 3
Te0yncrr goa tssun\$oa ns1 vatf0mic

\$ 1 0 0

In our case, the presence of dollar signs
and numerals in the text is suspicious

Maybe they belong together?

Umm, maybe there's 6 columns?

Double Transpositions

- Do it twice
- Using different numbers of columns
- How do you break it?
 - Find pairs of letters that probably appeared together in the plaintext
 - Figure out what transformations would put them in their positions in the ciphertext
- Can transform more than twice, if you want

Generalized Transpositions

- Any algorithm can be used to scramble the text
- Usually somehow controlled by a key
- Generality of possible transpositions makes cryptanalysis harder

Which Is Better, Transposition or Substitution?

- Well, neither, really
- Strong modern ciphers tend to use both
- Transposition scrambles text patterns
- Substitution hides underlying text characters/bits
- Combining them can achieve both effects
 - If you do it right . . .

Quantum Cryptography

- Using quantum mechanics to perform crypto
 - Mostly for key exchange
- Rely on quantum indeterminacy or quantum entanglement
- Existing implementations rely on assumptions
 - Quantum hacks have attacked those assumptions
- Not ready for real-world use, yet
- Quantum cryptanalysis even further off
 - And only helps on some ciphers