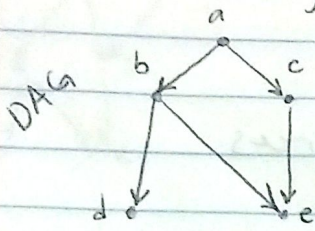


Breadth first Search will tell you the number of connected components

1/21/16

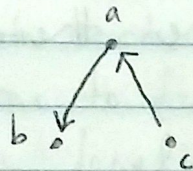
### Directionality



in a directed graph, every edge has a direction

a directed graph may or may not have cycles

BFS + DFS algorithms are similar to those for undirected graphs but the definition of neighbor is different

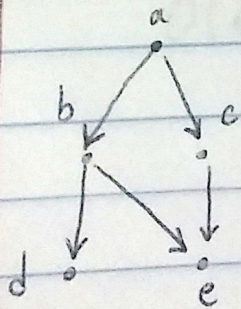


c is NOT a neighbor of a

DAG : directed acyclic graph

### Sorting Graphs

each vertex may represent a task and some tasks may have to be completed before others (precedence ordering)



possible sorting: a c b e d

b + c depend on a

e depends on b and c

Topological Sorting of the DAG



directed  
graph

{ in-degree: number of edges going to a vertex  
out-degree: number of edges leaving a vertex

if in-degree = 0 : source

if out-degree = 0 : sink

undirected graphs just have degrees

$n-1$  sources possible for a graph

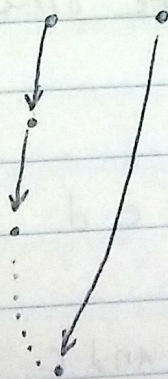
$n-1$  sinks possible for a graph

a topological sort doesn't make sense  
for a graph that isn't DAG b/c that  
would mean sorting vertices when there  
is a cycle

in a topological sort the order matters b/c it  
implies there is an edge between an earlier  
vertex to one further in the list

always check if there is a cycle using BFS

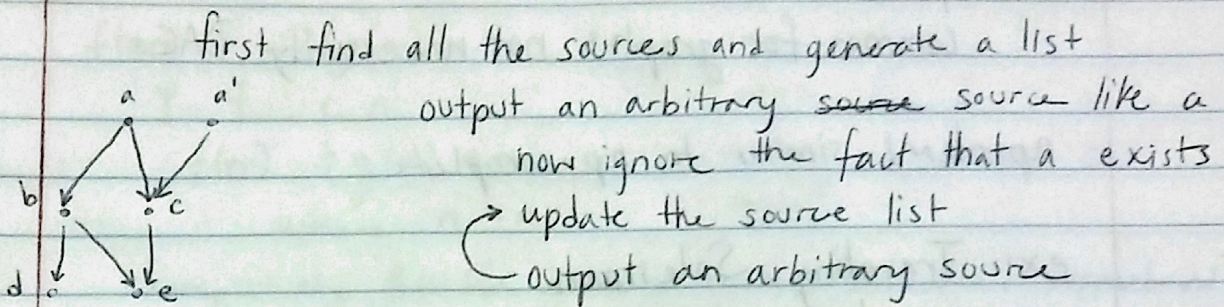
ex:



$(n-1)!$  → possible topological sort outputs  
not possible to do all of them in linear time



efficient algorithm for topological sort:



~~take an arbitrary edge~~

first assume we have a set of vertices  
with no edges (in-deg = 0, out-deg = 0)

start adding edges and update in-deg, out-deg for  
each vertex  $\Rightarrow$  for every vertex we spend  $n-1$   
time to find all in-degree, out-degree

localized analysis  $O(n^2)$  -- maybe linear in  $e$

$O(e)$ :  $e$  is number of edges

$\hookrightarrow$  always linear in  $e$

linear means linear in input size

generating list of sources  $O(n)$

output sources  $O(1) \rightarrow O(n)$

update source list  $O(n) \rightarrow O(n^2) \Rightarrow O(e)$

whenever we output a source, I look at the  
neighbors and decrement their in-degrees

$O(e + n) \rightarrow$  time complexity

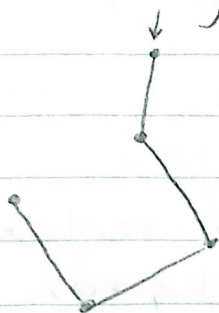


np-complete: an efficient solution has not yet been found

common for graphs not necessarily DAGs

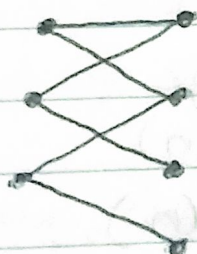
np-hard similar to np-complete

ex: Traveling Salesman



Bipartite Graph (undirected)

if vertices can be partitioned into 2 groups and the edges run between the two groups



not every graph  
is bipartite



bipartites can't have odd cycles

Algorithm to determine bipartite:

do BFS