Name(last, first): _____

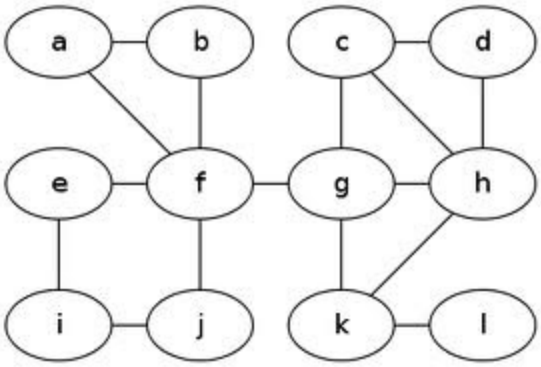
CS 180 Algorithms & Complexity

ID (last 4 digit): _ _ _ _

(5 problems; 6 pages; each problem has 20 points)

1. **a.** Describe Prim's MST algorithm (bullet by bullet). **b.** Analyze its time complexity.

2. Apply depth first search (DFS) to the following graph starting at node **a** and show the corresponding DFS tree.



3. A majority in a list of n integers is a number that appears more than n/2 times.
a. How many majorities can there be in a list?

b. Design an algorithm that finds a majority in a list (bullet by bullet).

c. Analyze the time complexity of your algorithm.

4. You have been hired to manage the translation process for some documentation. Unfortunately, different sections of the documentation were written in different languages: n languages in total. Your boss wants the entire documentation to be available in all n languages. There are m different translators for hire. Some of those translators are volunteers that do not get any money for their services. Each translator knows exactly two different languages and can translate back and forth between them. Each translator has a non-negative hiring cost (some may work for free). Unfortunately, your budget is too small to hire one translator for each pair of languages. Instead, you must rely on chains of translators: an English-Spanish translator and a Spanish-French translator, working together, can translate between English and French. Your goal is to find a minimum-cost set of translators that will let you translate between every pair of languages.

We may formulate this problem as a connected undirected graph G = (V, E) with non-negative (i.e., zero or positive) edge weights w. The vertices V are the languages for which you wish to generate translations. The edges E are the translators. The edge weight w(e) for a translator e gives the cost for hiring the translator e. A subset $S \subseteq E$ of translators can translate between a, $b \in V$ if and only if the subgraph G_S = (V, S) contains a path between a and b. The set $S \subseteq E$ is a **translation network** if and only if S can be used to translate between all pairs a, $b \in V$.

Design an algorithm for finding a translation network (hint: it is similar to a well known problem that we have studied in class). It should find the cheapest (minimum cost) possible network.

5. A Hamiltonian path in a graph is a path that has no repeated vertices and covers all vertices. Design an efficient algorithm for finding a Hamiltonian path in a directed acyclic graph (DAG). Describe your algorithm in English, bullet by bullet.