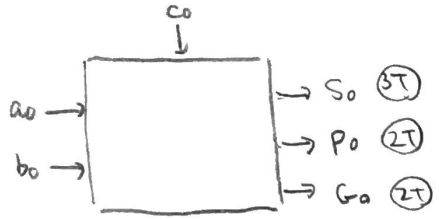
CS M151B HW3 Junhong Wang (504941113)

4-bit CLA



$S_0 = a_0 \oplus b_0 \oplus c_0$ (3T)

$G_0 = a_0 \cdot b_0$ (2T)

$P_0 = a_0 \oplus b_0$ (2T)

$C_1 = G_0 + P_0 \cdot C_0$

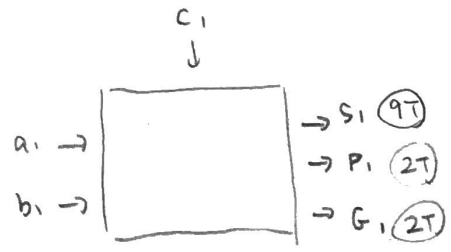
G_0: 2T, $P_0 \cdot C_0$: 2T, 0 → 4T

(6T)

$S_1 = a_1 \oplus b_1 \oplus C_1$

a_1: 0, b_1: 0, C_1: 6T

(9T)



$C_2 = G_1 + G_0 \cdot P_1 + C_0 \cdot P_0 \cdot P_1$

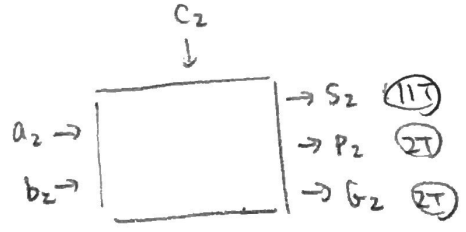
G_1: 2T, $G_0 \cdot P_1$: 2T 2T → 4T, $C_0 \cdot P_0 \cdot P_1$: 0 2T 2T → 5T

(8T)

$S_2 = a_2 \oplus b_2 \oplus C_2$

a_2: 0, b_2: 0, C_2: 8T

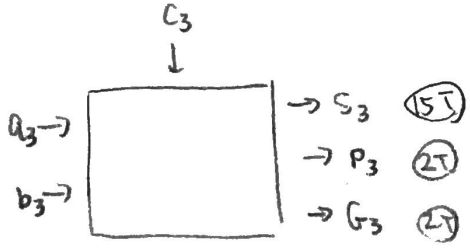
(11T)



$C_3 = G_2 + G_1 \cdot P_2 + G_0 \cdot P_1 \cdot P_2 + C_0 \cdot P_0 \cdot P_1 \cdot P_2$

G_2: 2T, $G_1 \cdot P_2$: 2T 2T → 4T, $G_0 \cdot P_1 \cdot P_2$: 2T 2T 2T → 5T, $C_0 \cdot P_0 \cdot P_1 \cdot P_2$: 0 2T 2T 2T → 7T

(12T)



$S_3 = a_3 \oplus b_3 \oplus C_3$

a_3: 0, b_3: 0, C_3: 12T

(15T)

$C_4 = G_3 + P_3 \cdot C_3$

$= G_3 + G_2 P_3 + G_1 \cdot P_2 \cdot P_3 + G_0 \cdot P_1 \cdot P_2 \cdot P_3 + C_0 \cdot P_0 \cdot P_1 \cdot P_2 \cdot P_3$

G_3: 2T, $G_2 P_3$: 2T 2T → 4T, $G_1 \cdot P_2 \cdot P_3$: 2T 2T 2T → 5T, $G_0 \cdot P_1 \cdot P_2 \cdot P_3$: 2T 2T 2T 2T → 7T, $C_0 \cdot P_0 \cdot P_1 \cdot P_2 \cdot P_3$: 0 2T 2T 2T 2T → 9T

(16T)

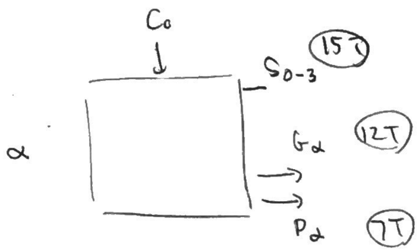
$G_4 = G_3 + G_2 P_3 + G_1 \cdot P_2 \cdot P_3 + G_0 \cdot P_1 \cdot P_2 \cdot P_3$

G_3: 2T, $G_2 P_3$: 4T, $G_1 \cdot P_2 \cdot P_3$: 5T, $G_0 \cdot P_1 \cdot P_2 \cdot P_3$: 7T

(12T)

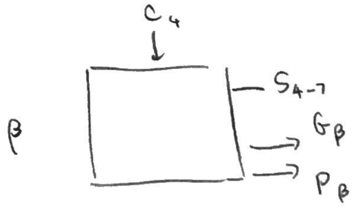
$P_4 = P_0 \cdot P_1 \cdot P_2 \cdot P_3$

(7T)



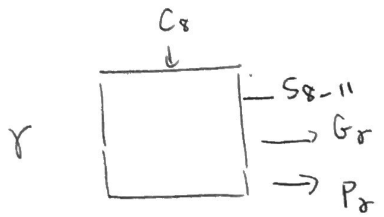
$$C_4 = \underset{12T}{G_\alpha} + \underbrace{\underset{0}{C_0} \cdot \underset{7T}{P_\alpha}}_{9T}$$

$$= 14T$$



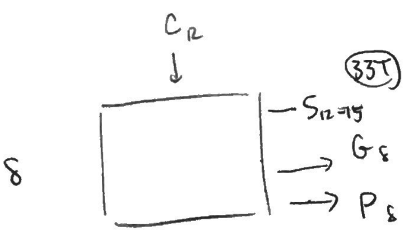
$$C_8 = \underset{12T}{G_\beta} + \underbrace{\underset{12T}{G_\alpha} \cdot \underset{7T}{P_\beta}}_{14T} + \underbrace{\underset{0}{C_0} \cdot \underset{7T}{P_\alpha} \cdot \underset{7T}{P_\beta}}_{10T}$$

$$= 17T$$



$$C_{12} = \underset{12T}{G_\gamma} + \underbrace{\underset{12T}{G_\beta} \cdot \underset{7T}{P_\gamma}}_{14T} + \underbrace{\underset{12T}{G_\alpha} \underset{7T}{P_\beta} \cdot \underset{7T}{P_\gamma}}_{15T} + \underbrace{\underset{0}{C_0} \cdot \underset{7T}{P_\alpha} \cdot \underset{7T}{P_\beta} \cdot \underset{7T}{P_\gamma}}_{12T}$$

$$= 20T$$



$$C_{15} = \underset{2T}{G_{\delta 2}} + \underbrace{\underset{2T}{G_{\delta 1}} \cdot \underset{2T}{P_{\delta 2}}}_{4T} + \underbrace{\underset{2T}{G_{\delta 0}} \cdot \underset{2T}{P_{\delta 1}} \cdot \underset{2T}{P_{\delta 2}}}_{5T} + \underbrace{\underset{20T}{C_{12}} \cdot \underset{2T}{P_{\delta 0}} \cdot \underset{2T}{P_{\delta 1}} \cdot \underset{2T}{P_{\delta 2}}}_{25T}$$

$$= 30T$$

$$S_{15} = \underset{0}{a_{15}} \oplus \underset{0}{b_{15}} \oplus \underset{30T}{C_{15}}$$

$$= 33T$$

$$C_{16} = G_\delta + P_\delta \cdot C_{12}$$

$$= \underset{12T}{G_\delta} + \underbrace{\underset{12T}{G_\gamma} \cdot \underset{7T}{P_\delta}}_{14T} + \underbrace{\underset{12T}{G_\beta} \cdot \underset{7T}{P_\gamma} \cdot \underset{7T}{P_\delta}}_{15T} + \underbrace{\underset{12T}{G_\alpha} \cdot \underset{7T}{P_\beta} \cdot \underset{7T}{P_\gamma} \cdot \underset{7T}{P_\delta}}_{17T} + \underbrace{\underset{0}{C_0} \cdot \underset{7T}{P_\alpha} \cdot \underset{7T}{P_\beta} \cdot \underset{7T}{P_\gamma} \cdot \underset{7T}{P_\delta}}_{14T}$$

$$= 24T$$

$C'_{19} = G_2 + G_1 P_2 + G_0 P_1 P_2 + C'_{16} P_0 P_1 P_2$

$2T \quad 4T \quad 5T \quad 7T$

$(12T)$

$S'_{19} = a_{19} \oplus b_{19} \oplus C_{19}$

$0 \quad 0 \quad 12T$

$(15T)$

$C'_{16} = 1$

S_{16-19} 4-bit CLA $\rightarrow C_{20}$

$C'_{20} = G_3 + G_2 P_3 + G_1 P_2 P_3 + G_0 P_1 P_2 P_3 + C'_{16} P_0 P_1 P_2 P_3$

$2T \quad 4T \quad 5T \quad 7T \quad 9T$

$(16T)$

S_{20-23} 4-bit CLA $\rightarrow C_{24}$

$C'_{24} = G_3 + G_2 P_3 + G_1 P_2 P_3 + G_0 P_1 P_2 P_3 + C_{20} P_0 P_1 P_2 P_3$

$2T \quad 4T \quad 5T \quad 7T \quad 16T \quad 23T$

$(30T)$

S_{24-27} 4-bit CLA $\rightarrow C_{28}$

$C'_{28} = G_3 + G_2 P_3 + G_1 P_2 P_3 + G_0 P_1 P_2 P_3 + C_{24} P_0 P_1 P_2 P_3$

$2T \quad 4T \quad 5T \quad 7T \quad 30T \quad 37T$

$(44T)$

S'_{28-31} $(57T)$ 4-bit CLA $\rightarrow C'_{32}$

$C'_{32} = G_3 + G_2 P_3 + G_1 P_2 P_3 + G_0 P_1 P_2 P_3 + C_{28} P_0 P_1 P_2 P_3$

$2T \quad 4T \quad 5T \quad 7T \quad 44T \quad 51T$

$(58T)$

$(0\ 1)$ $4T$ $\rightarrow S_{16-31}$

$(61T)$ $(\because 57T + 4T = 61T)$

$(1\ 0)$ $4T$ $\rightarrow C_{32}$

$(62T)$ $(\because 58T + 4T = 62T)$

$C_{31} = G_2 + G_1 P_2 + G_0 P_1 P_2 + C_{28} P_0 P_1 P_2$

$2T \quad 4T \quad 5T \quad 44T \quad 49T$

$(54T)$

$S'_{31} = a_{31} \oplus b_{31} \oplus C_{31}$

$0 \quad 0 \quad 54T$

$(57T)$

Therefore,

Output	Delay
G_0	2T
P_0	2T
G_2	12T
P_2	7T
C_{R2}	20T
C_{15}	30T
C_{16}	24T
S_{15}	33T
C_{20}	16T
S_{19}	15T
C_{24}	30T
C_{31}	54T
C_{32} (after mux)	62T
S_{31} (after mux)	61T

Maximal Delay : 62T

DISCLAIMER : Delays are computed as sum of products. For example, delay of $a_1 + a_2a_3 + a_4a_5a_6$ is computed as the sum of the delay of a_1, delay of a_2a_3 as a whole, delay of $a_4a_5a_6$ as a whole.