

# What Is a Mathematical Property?

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# Examples of mathematical objects

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

$$Trg = \{(A, B, C); A, B, C \text{ points in the plane}\}$$

$$\begin{aligned}\mathbb{N} \times \mathbb{N} &= \{(m, n); m \in Nat, n \in \mathbb{N}\} \\ &= \{(0, 0), (0, 1), (0, 2), (0, 3), \dots, \\ &\quad (1, 0), (1, 1), (1, 2), (1, 3), \dots, \\ &\quad (2, 0), (2, 1), (2, 2), (2, 3), \dots, \\ &\quad \dots\}\end{aligned}$$

# Examples of mathematical properties

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

$$Trg = \{(A, B, C); A, B, C \text{ points in the plane}\}$$

$$\mathbb{N} \times \mathbb{N} = \{(m, n); m \in \mathbb{N}, n \in \mathbb{N}\}$$

$$\{(0, 0), (0, 1), \dots, (1, 0), (1, 1), \dots\}$$

- even (applies to  $\mathbb{N}$ , not  $Trg$ )
- equilateral (applies to  $Trg$ , not  $\mathbb{N}$ )
- is equal to 2 (applies to  $\mathbb{N}$ )
- first smaller than the second (applies to  $\mathbb{N} \times \mathbb{N}$ )

# Examples of non-mathematical properties

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

$$Trg = \{(A, B, C); A, B, C \text{ points in the plane}\}$$

$$\mathbb{N} \times \mathbb{N} = \{(m, n); m \in Nat, n \in \mathbb{N}\} \\ \{(0, 0), (0, 1), \dots, (1, 0), (1, 1), \dots\}$$

- is big (applied to  $\mathbb{N}$ , could also apply to  $Trg$ )
- is pretty (applied to  $Trg$ , could also apply to  $\mathbb{N}$ )
- is interesting, friendly, heavy, ...

# The extensional view

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

$$Trg = \{(A, B, C); A, B, C \text{ points in the plane}\}$$

$$\mathbb{N} \times \mathbb{N} = \{(m, n); m \in Nat, n \in \mathbb{N}\} \\ \{(0, 0), (0, 1), \dots, (1, 0), (1, 1), \dots\}$$

- $even = \{0, 2, 4, 6, 8, 10, \dots\}$
- $equilateral = \{(A, B, C); AB = AC = BC\}$
- $is\ equal\ to\ 2 = \{2\}$
- $first\ smaller\ than\ the\ second$   
 $= \{(0, 1), (0, 2), \dots, (1, 2), \dots\}$

# The intensional view

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

$$Trg = \{(A, B, C); A, B, C \text{ points in the plane}\}$$

$$\mathbb{N} \times \mathbb{N} = \{(m, n); m \in Nat, n \in \mathbb{N}\}$$

$$\{(0, 0), (0, 1), \dots, (1, 0), (1, 1), \dots\}$$

- even
- equilateral =  $\{(A, B, C); AB = AC = BC\}$
- is equal to 2 =  $\{2\}$
- first smaller than the second  
=  $\{(0, 1), (0, 2), \dots, (1, 2), \dots\}$