Lab 5
Hidden Markov Models (HMMs)

Ex. 1 (2p) A professor gives tests that can be difficult, medium, or easy. The probability of the difficulty of the first test is equal for all three. If, at some point, a difficult test is given, the next test can only be medium or easy, with equal probability. However, if a medium or easy test is given, then the next test will be difficult with probability 0.5, or medium or easy with equal probability 0.25.

A student's grade on the test — FB (very good), B (good), S (satisfactory), or NS (unsatisfactory) — depends on the difficulty of the test. The conditional probabilities of the grade given the test difficulty are:

- Difficult test: $[0.1,\ 0.2,\ 0.4,\ 0.3]$;
- Medium test: $[0.15,\ 0.25,\ 0.5,\ 0.1]$;
- Easy test: $[0.2,\ 0.3,\ 0.4,\ 0.1]$.

For the following assignments (except Bonus), you should use the hmmlearn library.

a) (0.5p.) Use the hmmlearn library to define the Hidden Markov Model and draw the state diagram

Suppose now that you have observed the following sequence of grades: FB, FB, S, B, B, S, B, B, NS, B, B.

b) (0.5p.) Determine the probability of obtaining the above observations.

c) (1p.) Determine the most probable sequence of test difficulties corresponding to these grades and the probability of that sequence.

Bonus: (1p.) Implement the *Viterbi* algorithm from Lecture 4 directly (without using a dedicated library).