Lab 2
Simulation of Random Variables

Ex. 1 (1p) We consider an urn containing 3 red balls, 4 blue balls, and 2 black balls. We roll a die; if we get a prime number, we add a black ball to the urn, if we get a 6, we add a red ball, and in all other cases, we add a blue ball. Then we draw one ball from the urn.

a) (0.5p) Simulate the above experiment in Python.
b) (0.5p) Using the simulation, estimate the probability of drawing a red ball.
c) **Bonus.** (0.5p) Calculate the theoretical probability of the above event and compare it with the estimated one.

Ex. 2 (2p) A call center receives incoming calls that follow a Poisson process. Depending on the day, the average number of calls per hour varies due to factors such as promotions, holidays, or technical issues. The average rate is typically one of the following values:

$$\lambda \in \{1, 2, 5, 10\}$$

1. (0.5p) Simulate 1,000 values from each of the following Poisson distributions with fixed parameters:

$$X_1 \sim \text{Poisson}(1), \quad X_2 \sim \text{Poisson}(2), \quad X_3 \sim \text{Poisson}(5), \quad X_4 \sim \text{Poisson}(10)$$

2. (0.5p) Simulate 1,000 values from a **randomized Poisson distribution**, where for each value:
   - a parameter λ is randomly selected from the set $\{1, 2, 5, 10\}$ with equal probability.
   - a Poisson random variable is generated using that λ.

a) (0.5p) Plot the histograms or empirical density plots of all five datasets (the four fixed-parameter distributions and the randomized one).
b) (0.5p) Compare and discuss:
   - How does the shape of the randomized distribution differ from the fixed ones?
   - What does this tell you about the effect of parameter uncertainty or variability in modeling real-world processes?
c) **Bonus.** (0.5p) Try changing the probabilities of selecting each λ (e.g., make $\lambda = 5$ more likely) and observe how the distribution changes.