Lab 6
Bayesian inference

Ex. 1. *(Bayes' Theorem)*. (1p.) Let's suppose we are investigating a disease (B), which affects only 1% of the population: ($P(B) = 0.01$). We have a diagnostic test for this disease with the following characteristics:

- **Sensitivity** (the probability that the test is positive if the person has the disease):
$$P(\text{Test} = \text{Positive}|B) = 0.95.$$
- **Specificity** (the probability that the test is negative if the person does not have the disease):
$$P(\text{Test} = \text{Negativ}|\neg B) = 0.90.$$

a) (0.5p) If a person is tested and the result is positive, what is the probability that they actually have the disease? Explain the result.
b) (0.5p) What should be the minimum specificity for the above probability to reach 50%?

Upload your argument either in Markdown/LaTeX or as a photo of your worksheet.

Ex. 2. (1p.) A telecommunications company wants to estimate the average call rate received by a call center in one hour. From historical data, the company knows that the average call rate per hour varies depending on the time of day and certain days of the week. During a specific period, the manager observes that over 10 hours, a total of 180 calls were received.

Let λ denote the average call rate per hour. Given that the number of calls per hour can be modeled as a *Poisson variable*, we have the following information:

- *Observed data*: Over 10 hours, 180 calls were received, which gives an observed average rate of 18 calls per hour.
- *Likelihood distribution*: We assume that the number of calls per hour follows a Poisson distribution with parameter λ.

By choosing a *Gamma distribution* as the *prior* for λ (since it is the *conjugate prior* of the *Poisson distribution*), determine:

a) (0.5p) the *posterior distribution* of λ;
b) (0.25p) a 94% HDI (Highest Density Interval);
c) (0.25p) the most probable value of λ.

Hints:

- The *Poisson distribution* with parameter $\lambda > 0$ is given by $P(X = k) = e^{-\lambda}\dfrac{\lambda^k}{k!}$.
- The *Gamma distribution* with parameters $\alpha, \beta > 0$ is given by $p(\lambda) = \dfrac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda}$ (with *mean* = α/β, *variance* = α/β^2, and *mode* = $[(\alpha - 1) \vee 0]/\beta$). How do we choose the parameters α and β?
- An HDI interval and the mode of a distribution can be computed using the Arviz library with plot_posterior function.