Lab 7
Bayesian inference with PyMC

Ex. 1. (2p.) A researcher measures the ambient noise level in different locations of a city. He suspects that the noise level, denoted by X, follows a normal distribution, but both the mean μ and the standard deviation σ are unknown.

The researcher has 10 observations of the noise level (in decibels, dB):

$$56, 60, 58, 55, 57, 59, 61, 56, 58, 60.$$

The goal is to use Bayesian inference to estimate μ and σ.

  a) (0.5p) Define a model in PyMC for this problem. You may choose the prior distributions as $\mu \sim \mathcal{N}(x, 10^2)$ and $\sigma \sim \text{HalfNormal}(10)$. What should x be?

  b) (0.5p) Infer on μ and σ. What is the 95% HDI (Highest Density Interval) for μ and σ?

  c) (0.5p) Compare the Bayesian estimates of μ and σ with the frequentist estimates (sample mean and sample standard deviation). Discuss any differences and why they occur.

  d) (0.5p) Investigate the effect of a strong prior: assume $\mu \sim \mathcal{N}(50, 1^2)$ and $\sigma \sim \text{HalfNormal}(10)$. Compare the posterior estimates of μ and σ with those obtained in b) and c). Explain why the results differ.