Analysis of Time Series

Chapter 9: Case study: An association between unemployment and mortality?

Edward L. Ionides

Outline

Historical investigations linking business cycles to mortality

- Ogburn and Thomas (1922) were among the first to report **pro-cyclical** mortality: death rates are statistically above trend when economic activity is above trend.

- Procyclical mortality, if it exists, shows that a key measure of population health is worse in economic booms than in recessions.

- Both the economy and life expectancy have grown over the last century. However, these phenomena have not always occurred simultaneously. For example, 1950–1980 saw rapid growth in life expectancy in India and China, combined with relatively slow economic growth. Improvement in life expectancy has slowed during their recent economic surges.

- The link between economic growth and health improvement is controversial, since it has political implications. Economists and epidemiologists have argued both sides of this debate, using time series methods.

Implications of pro-cyclical mortality

- If our goal is population health and happiness, how much should our policies focus on gross domestic product (GDP) growth?
- Evidence supporting the view that economic growth is the critical engine for other improvements in living conditions would make a moral argument in favor of economic growth.
- Evidence that there are other major factors involved in improving living conditions suggest that economic growth should be only one political consideration, among others.

A time series of life expectancy in the USA

```
e_data <- read.table(file="life_expectancy_usa.csv",header=TRUE)
head(e_data,n=4)

 Year   e0F   e0M    e0
 1933 62.80 59.19 60.90
 1934 62.30 58.29 60.19
 1935 63.05 58.98 60.91
 1936 62.60 58.35 60.35
```

- Data are from the Human Mortality Database.
- e0 is **life expectancy at birth (LEB)** for civilians,
- e0F and e0M are LEB for females and males, but we focus on e0.
- LEB is an actuarial calculation based on a fictitious individual having mortality rates at each age matching census age-specific mortality rates for the current year.
- LEB is a standard way to combine all the age-specific mortality rates into a single number.

```
u_data <- read.table(file="unadjusted_unemployment.csv",
  sep=",",header=TRUE)
head(u_data,4)

 Year Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec
 1948 4.0 4.7 4.5 4.0 3.4 3.9 3.9 3.6 3.4 2.9 3.3 3.6
 1949 5.0 5.8 5.6 5.4 5.7 6.4 7.0 6.3 5.9 6.1 5.7 6.0
 1950 7.6 7.9 7.1 6.0 5.3 5.6 5.3 4.1 4.0 3.3 3.8 3.9
 1951 4.4 4.2 3.8 3.2 2.9 3.4 3.3 2.9 3.0 2.8 3.2 2.9
```

- We consider unadjusted unemployment from Bureau of Labor Statistics.
- Unemployment is just one component of the state of the economy. One could consider other measurements.
- Write e_n for life expectancy in year $t_n = 1947 + n$.
- Write u_n for mean unemployment in year t_n.

A time plot of the raw data

```
t <- intersect(e_data$Year,u_data$Year)
e <- e_data$e0[e_data$Year %in% t]
u <- apply(u_data[u_data$Year %in% t, 2:13],1,mean)
```
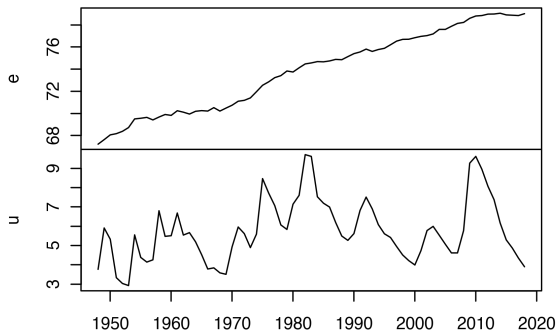

- We are interested in changes over business cycle timescales, once trends have been removed.

The Hodrick-Prescott filter

- To extract the cyclical component, we use an econometric method: the **Hodrick-Prescott (HP) filter** (Hodrick and Prescott, 1997).
- Specifically, for a time series $y_{1:N}^*$, the HP filter is the time series $s_{1:N}^*$ constructed as

$$s_{1:N}^* = \underset{s_{1:N}}{\operatorname{argmin}} \left\{ \sum_{n=1}^{N} \left(y_n^* - s_n \right)^2 + \lambda \sum_{n=2}^{N-1} \left(s_{n+1} - 2s_n + s_{n-1} \right)^2 \right\}. \quad (1)$$

- The HP filter is a **smoothing spline**. Later, we see it can also be viewed as a state space model.
- A standard econometric choice of λ for removing nonlinear trend, for extracting the business cycle component, in annual data is $\lambda = 100$.
- An R implementation of the Hodrick-Prescott filter is `hpfilter` in the R package `mFilter`.

- We use the Hodrick-Prescott filter to define the HP-detrended life expectancy, $e_{1:N}^{HP}$, and unemployment, $u_{1:N}^{HP}$.

```
library(mFilter)
e_hp <- hpfilter(e, freq=100,type="lambda",drift=F)$cycle
u_hp <- hpfilter(u, freq=100,type="lambda",drift=F)$cycle
```

- Plotting two time series on a single graph is not always advisable, but here it is helpful.

```
plot(t,u_hp,type="l",xlab="",ylab="detrended unemployment")
par(new=TRUE)
plot(t,e_hp,col="red",type="l",axes=FALSE,xlab="",ylab="")
axis(side=4, col="red",col.ticks="red",col.axis="red")
mtext("detrended e0",side=4,col="red",line=3)
```
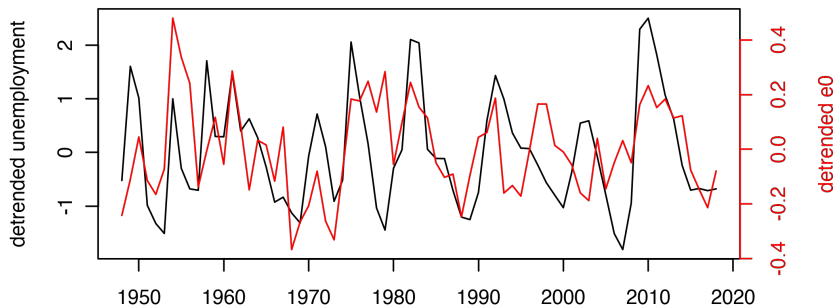
Detrended unemployment (black; left axis) and detrended life expectancy at birth (red; right axis).

- Looking at this figure may suggest that detrended life expectancy and detrended unemployment cycle together.
- We make a formal statistical test to check our eyes are not deceiving us.

A regression with ARMA errors model

- We can investigate the dependence of $e_{1:N}^{HP}$ on $u_{1:N}^{HP}$ using a regression with ARMA errors model,

$$E_n^{HP} = \alpha + \beta u_n^{HP} + \epsilon_n, \qquad (2)$$

where $\{\epsilon_n\}$ is a Gaussian ARMA process. We use an ARMA(1,0) model, as discussed in the supplementary analysis.

```
a0 <- arima(e_hp,xreg=u_hp,order=c(1,0,0))

Coefficients:
         ar1   intercept    u_hp
      0.4798    -0.0030   0.0713
s.e.  0.1042     0.0304   0.0181

sigma^2 estimated as  0.0182
log likelihood = 41.26,  aic = -74.52
```

- The standard error (computed via observed Fisher information) gives a z-statistic of $0.0713/0.0181 = 3.94$ for the coefficient of detrended unemployment.

- We can also compute a p-value from a likelihood ratio test,

```
log_lik_ratio <- as.numeric(
    logLik(arima(e_hp,xreg=u_hp,order=c(1,0,0))) -
    logLik(arima(e_hp,order=c(1,0,0)))
)
LRT_pval <- 1-pchisq(2*log_lik_ratio,df=1)
```

- This gives a p-value of 0.00018.

- We have clear statistical evidence for a positive association between detrended unemployment and detrended life expectancy.

- For all observational studies, interpretation of association needs care.

Association and causation

- We have been careful to talk about **association**, since observational data giving firm statistical evidence of an assocation between X and Y cannot readily distinguish between three possibilities:

1. X causes Y.
2. Y causes X.
3. Both X and Y are caused by a third variable Z that is unmeasured or has been omitted from the analysis. In this case, Z is called a **confounding variable**.

- Here, it is not considered plausible that mortality fluctuations drive economic fluctuations (the **reverse causation** possibility).
- Unemployment is a **proxy variable** for economic fluctuations. Increased unemployment itself is necessarily directly causing reduced mortality: all proxy variables for economic activity are confounded in this analysis.

Potential lagged relationships

- A potential confounding variable is lagged economic activity. Theoretically, reduction in mortality for an current economic down-turn could result from delayed health progress caused by the previous economic boom.

- A lag relationship between two time series $x_{1:N}$ and $y_{1:N}$ can be identified from the **sample cross-correlation function (CCF)**

$$\hat{\rho}_{xy}(h) = \frac{\sum_{n=1}^{N-h}(x_{n+h} - \bar{x})(y_n - \bar{y})}{\sqrt{\sum_{n=1}^{N}(x_n - \bar{x})^2 \sum_{n=1}^{N}(y_n - \bar{y})^2}} \tag{3}$$

- $\hat{\rho}_{xy}(h)$ estimates $\rho_{XY}(h) = \text{Cor}(X_{n+h}, Y_n)$, the cross-correlation at lag h for a bivariate stationary time series model $(X_{1:N}, Y_{1:N})$.

```
ccf(e_hp,u_hp)
```

e_hp & u_hp
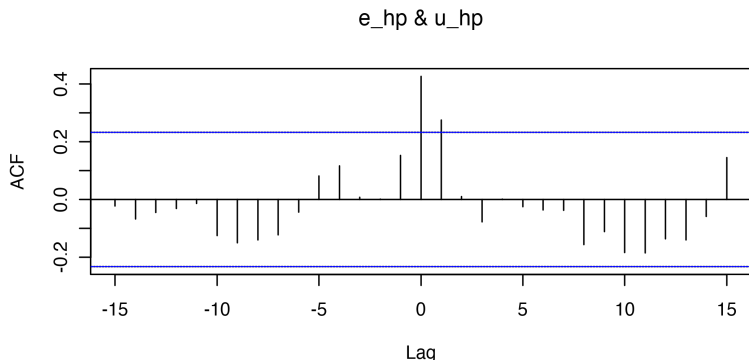
- The strong positive cross-correlation at lag zero supports pro-cyclical mortality.
- The oscillatory pattern is not significant pointwise but might be more evident in the frequency domain.

Coherency, coherence and phase

- The Fourier transform of the sample cross-correlation is called the **coherency**.

- Coherency measures correlation between frequency components of two time series at each frequency ω. It is complex-valued.

- The magnitude of the coherency is called the **coherence**. It measures whether a large amplitude at frequency ω for $x_{1:N}$ is associated with a large amplitude at ω for $y_{1:N}$.

- The angle of the coherency (in the complex plane) is called the **phase**. A phase of 0 means that peaks at frequency ω tend to occur simultaneously for $x_{1:N}$ and $y_{1:N}$. A phase of $\pm\pi$ at frequency ω means that peaks for the frequency component of $x_{1:N}$ coincide with troughs for $y_{1:N}$
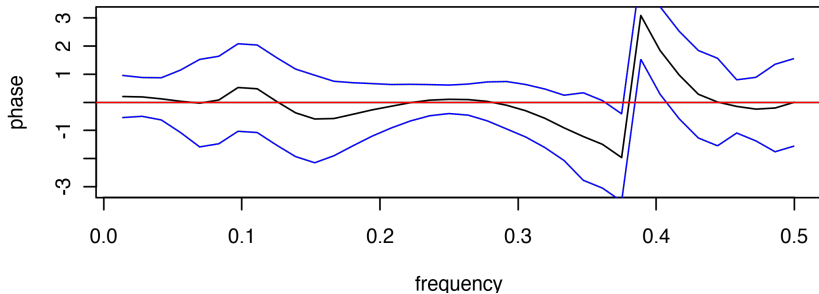
```
s <- spectrum(cbind(e_hp,u_hp),spans=c(3,5,3),plot=F)
plot(s,plot.type="coherency",main="")
```
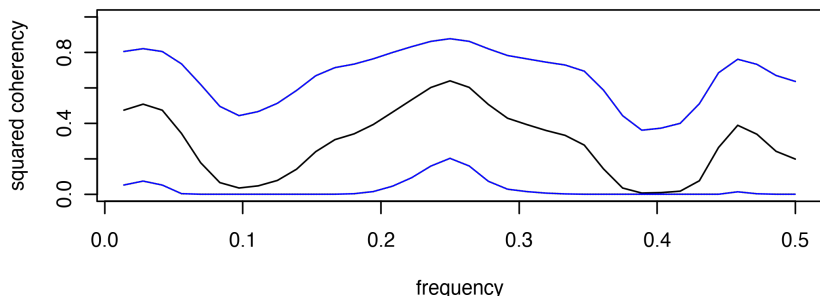

- R calculates squared coherence, the squared absolute coherency. This is like reporting r^2 for regression, rather than $|r|$.

Question 9.1. Interpret the squared coherence plot.

```
plot(s,plot.type="phase",main="")
abline(h=0,col="red")
```


Question 9.2. Interpret the phase plot.

Conclusions

- There is strong evidence of pro-cyclical mortality at a national level in the USA from 1948 to 2018. For example, the Great Recession of 2009-2010 led to high unemployment, but these two years had above-trend values of life expectancy at birth.

- We have argued that this evidence supports a claim that above-trend economic growth CAUSES above-trend mortality.

- We CANNOT infer that unemployment reduces mortality for those who lose their jobs. Adverse individual-level effects of unemployment can be reconciled with our result (Tapia Granados *et al.*, 2014).

- More data might give statistical precision to investigate sub-populations more accurately than can be done with a national-level dataset. For example, **panel data analysis** combining time series for each state (Ionides *et al.*, 2013).

Explaining the choice of an ARMA(1,0) error model

- Model selection by AIC for regression with ARMA errors follows the same approach as for ARMA models.

```
aic_table <- function(data,P,Q,xreg=NULL){
  table <- matrix(NA,(P+1),(Q+1))
  for(p in 0:P) {
    for(q in 0:Q) {
        table[p+1,q+1] <- arima(data,order=c(p,0,q),xreg=xreg)$aic
    }
  }
  dimnames(table) <- list(paste("AR",0:P, sep=""),
    paste("MA",0:Q,sep=""))
  table
}
e_aic_table <- aic_table(e_hp,4,5,xreg=u_hp)
require(knitr)
kable(e_aic_table,digits=2)
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	-58.16	-70.90	-73.08	-74.29	-73.86	-72.33
AR1	-74.52	-72.52	-72.23	-73.19	-79.41	-77.44
AR2	-72.52	-70.52	-72.33	-71.79	-77.46	-78.56
AR3	-72.19	-79.45	-70.40	-70.16	-75.69	-79.91
AR4	-74.43	-73.52	-72.35	-79.08	-76.08	-78.89

- ARMA(1,0) gives the best AIC among small models.
- Some larger models have better AIC, but notice inconsistencies in the AIC table. For example, consider ARMA(3,1) errors:

```
a3 <- arima(e_hp,xreg=u_hp,order=c(3,0,1))
```

```
          ar1      ar2      ar3      ma1  intercept     u_hp
       1.3013  -0.3034  -0.1756  -1.0000    -0.0008   0.0788
s.e.   0.1162   0.1912   0.1192   0.0377     0.0038   0.0185

sigma^2 estimated as  0.0150
log likelihood = 46.72,  aic = -79.45
```

- The estimated ARMA(3,1) is at the boundary of invertiblity, with an MA1 coefficient on the unit circle.

- This is reminiscent of our earlier analysis of the Lake Huron depth data.

- Likely the ARMA(3,1) analysis is not very stable. A simulation study might find that the Fisher confidence intervals are not reliable.

- A useful relationship should be consistent through time. We check this by repeating the analysis on temporal subsets.

```
t1 <- 1:35
a1 <- arima(e_hp[t1],xreg=u_hp[t1],order=c(1,0,0))

          ar1   intercept   u_hp[t1]
       0.5040      0.0036     0.0851
s.e.   0.1449      0.0531     0.0274


t2 <- 37:71
a2 <- arima(e_hp[t2],xreg=u_hp[t2],order=c(1,0,0))

          ar1   intercept   u_hp[t2]
       0.4277     -0.0071     0.0558
s.e.   0.1533      0.0302     0.0231
```
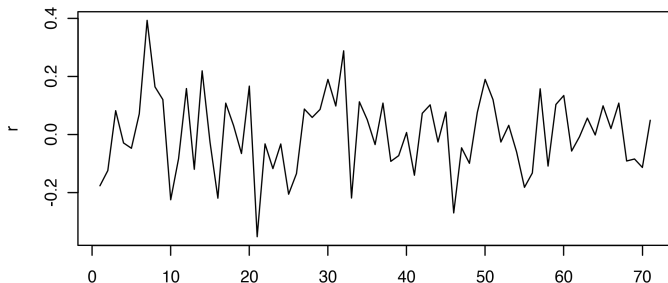
- The difference is small compared to the standard error. Overall there is consistency through this 71 year interval, though the pattern is weak in the 1990s.
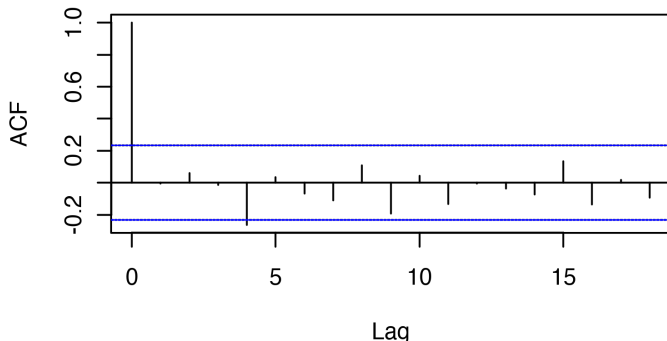
Residual analysis

- We inspect the residuals for the fitted model, and look at their sample autocorrelation.

```
r <- resid(arima(e_hp,xreg=u_hp,order=c(1,0,0)))
plot(r)
```


- There is some evidence for fluctuations decreasing in amplitude over time. This is an example of **heteroskedasticity**. It is not extreme here, but could be studied in a future analysis.

`acf(r)`

- It is not a major model violation to have one out of 18 lags narrowly outside the dashed lines showing pointwise acceptance regions at the 5% level under a null hypothesis of Gaussian white noise.
- The presence of some small amount of sample autocorrelation is consistent with the AIC table, which finds the possibility of gains by fitting larger models to the regression errors.
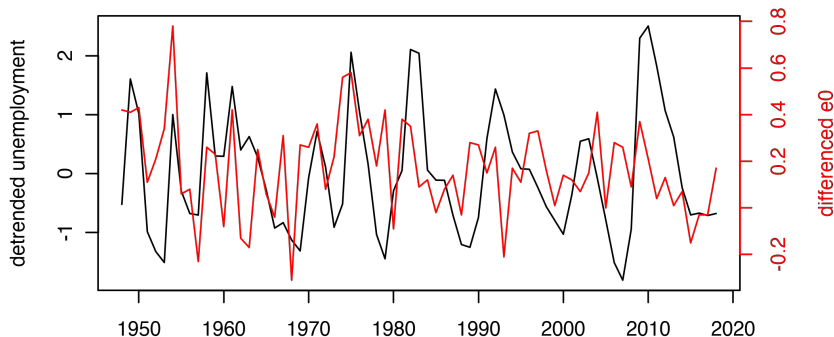
Analysis of temporal differences

- One might model annual changes in life expectancy, rather than difference from a trend. In this case, we consider the variable

$$\Delta e_n = e_n - e_{n-1}, \tag{4}$$

computed as

```
delta_e <- e - e_data$e0[e_data$Year %in% (t-1)]
```

- The relationship between unemployment and differenced life expectancy is harder to see than when HP-detrended.
- The relationship is also harder to find by statistical methods:

```
a4 <- arima(delta_e,xreg=u_hp,order=c(1,0,1))
```

```
         ar1      ma1   intercept     u_hp
      0.7482  -0.6023      0.1740   0.0496
s.e.  0.1936   0.2157      0.0351   0.0228

sigma^2 estimated as  0.0364
log likelihood = 16.84,  aic = -23.67
```

- Temporal differencing z-statistic of $0.0496/0.0228 = 2.17$ which is weaker evidence than the z-statistic of 3.94 for HP-detrended LEB.

Multiple testing considerations

- A scientific principle for interpreting experimental results is as follows: **An experiment which finds evidence of an effect is usually a better foundation for future investigations than one which fails to find evidence.**

- The experiment which found no evidence of an effect might have been a bad choice of experiment, or might have been carried out poorly.

- The principle of preference for methods giving positive results must be balanced with consideration of **multiple testing**. If we make 20 hypothesis tests, we expect one to be significant at the 5% level just by chance. There is a danger in trying many approaches and settling on one that claims statistical significance.

- The generalizability of any result is tentative until confirmed in other studies.

License, acknowledgments, and links

- Licensed under the Creative Commons Attribution-NonCommercial license. Please share and remix non-commercially, mentioning its origin.
- The materials builds on previous courses.
- Compiled on February 25, 2021 using R version 4.0.4.

Back to course homepage

References

Hodrick RJ, Prescott EC (1997). "Postwar U.S. Business Cycles: An Empirical Investigation." *Journal of Money, Credit and Banking*, **29**(1), 1–16.

Ionides EL, Wang Z, Tapia Granados JA (2013). "Macroeconomic effects on mortality revealed by panel analysis with nonlinear trends." *Annals of Applied Statistics*, **7**, 1362–1385.

Ogburn WF, Thomas DS (1922). "The Influence of the Business Cycle on Certain Social Conditions." *Journal of the American Statistical Association*, **18**(139), 324–340.

Tapia Granados JA, House JS, Ionides EL, Burgard S, Schoeni RS (2014). "Individual Joblessness, Contextual Unemployment, and Mortality Risk." *American Journal of Epidemiology*, **180**(3), 280–287.