The Heston model for volatility, V_n , is a stochastic volatility (SV) model with

$$V_n = (1 - \phi)\theta + \phi V_{n-1} + \sqrt{V_{n-1}} \omega_n,$$

for $\omega_n \sim N[0,\sigma_\omega^2]$. The log return is $Y_n \sim N[0,V_n]$, conditional on V_n . A previous 531 project (W22, #14) fitted the Heston model to investment in Ethereum, a crypto currency. They obtained a log-likelihood of 34975.3, compared to 28587.4 for GARCH and 28977 for the SV model with leverage presented in class. Their iterated filtering convergence diagnostics are shown in figure 1. What is the best conclusion from this information?

- **A**. The high likelihood shows this is a promising model despite the convergence problems identified in the figure. Attention to the diagnostics may lead to additional improvements.
- **B**. The most important diagnostic feature is the observation that the log-likelihood trace plot peaks and then declines. From the y-axis scale we see the decline is of order 1000 log units. This is evidence of substantial model misspecification which should be addressed.
- C. The most important dignostic feature is that the theta traces all drop quickly to zero. Since that is not a scientifically plausible value for the parameter, we can deduce that the model is unsuccessful despite its high likelihood.
- **D**. The most important diagnostic feature is that **phi** is close to zero and well identified. This shows that the volatility is close to constant, and is supported by the high likelihood.
- **E**. The decreasing likelihood and other convergence diagnostics problems show there is a problem with the model. Likely, there is a bug and the high likelihood obtained is simply an error.

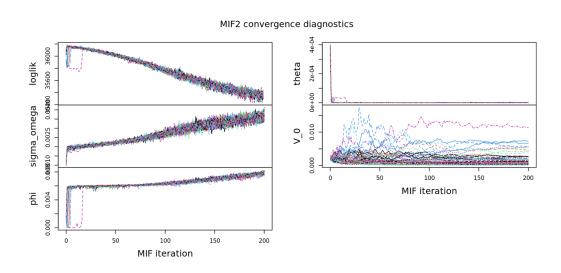


Figure 1: Diagnostic plot for fitting the Heston model