A generalized autoregressive conditional heteroskedasticity (GARCH) model has $Y_n = \sigma_n Z_n$ where $Z_n \sim$ i.i.d.$N(0,1)$ and $\sigma_n^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i Y_{n-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{n-j}^2$. There are many extensions to GARCH implemented by various R packages. When comparing models by likelihood or AIC, care is required since packages do not always use standard definitions. What is the most reasonable interpretation of this table?

```
for (i in 1:p) {
  for (j in 1:q) {
    fit_garch <- tseries::garch(log_returns, order = c(i, j))
    garch_table[i, j] <- tseries:::logLik.garch(fit_garch)
  }
}
```

	q1	q2	q3	q4
p1	2646.277	2642.919	2620.280	2616.151
p2	2644.417	2625.417	2622.460	2616.427
p3	2641.804	2637.538	2625.953	2625.740
p4	2639.728	2629.869	2629.969	2628.345

A. The positive values of the log-likelihood are implausible. Perhaps the software actually reports the negative log-likelihood since many optimizers are designed to minimize rather than maximize.
B. The models are nested and so a larger model should mathematically have a larger likelihood. In this table, the larger model usually has lower likelihood, so optimization is problematic.
C. This table would make more sense if `logLik` in fact returns an AIC value. The preferred model is $(p, q) = (1, 4)$.
D. The preferred model is $(p, q) = (1, 1)$ since it is both the simplest model and the one with the highest log-likelihood.
E. `tseries::garch` produces something that is not the likelihood of $y_{1:N}$ or the AIC, and so we cannot readily compare it between models.