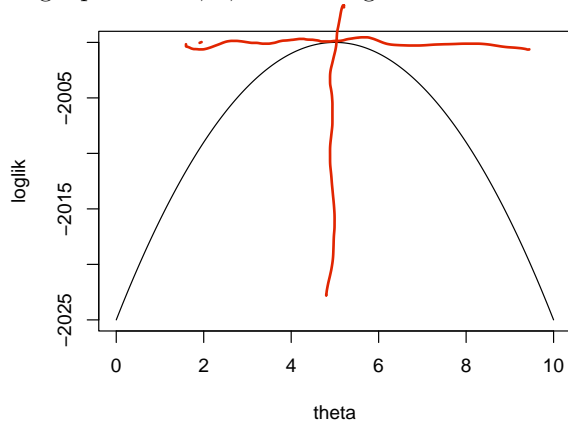


The Python function `statsmodels.tsa.arima.model.ARIMA.fit` and the R function `arima()` provide standard errors calculated by observed Fisher information. This question tests your understanding of what that means. Suppose a parametric model has a single parameter, θ , and the log-likelihood function when fitting this model to dataset is as follows:



$$\begin{aligned} \mathcal{I}_{obs} &= -\frac{d^2}{d\theta^2} \log f(y_{1:N}^*; \theta) \\ &\text{Observed Fisher information, at } \theta = \hat{\theta}_{MLE} \\ \mathcal{I} &= \mathbb{E} \left[\left(\frac{\partial}{\partial \theta} \log f_{Y_{1:N}}(Y_{1:N}; \theta) \right)^2 \right] \\ &= -\mathbb{E} \left[\frac{\partial^2}{\partial \theta^2} \log f_{Y_{1:N}}(Y_{1:N}; \theta) \right] \end{aligned}$$

What is the observed Fisher information (\mathcal{I}_{obs}) for θ ?

Hint 1. The observed Fisher information is accumulated over the whole dataset, not calculated per observation, so we don't have to know the number of observations, N .

Hint 2. Observations in time series models are usually not independent. Thus, the log-likelihood is not the sum of the log-likelihood for each observation. Its calculation will involve consideration of the dependence, and usually the job of calculating the log-likelihood is left to a computer.

Hint 3. The usual variance estimate for the maximum likelihood estimate, $\hat{\theta}$, is $\text{Var}(\hat{\theta}) \approx 1/\mathcal{I}_{obs}$.

A: $\mathcal{I}_{obs} = 2$

B: $\mathcal{I}_{obs} = 1$

C: $\mathcal{I}_{obs} = 1/2$

D: $\mathcal{I}_{obs} = 1/4$

E: None of the above