

Consider a model  $Y_{1:N}$  for data  $y_{1:N}^*$ , with a latent variable  $X_{0:N}$ , and a statistical model defined by a joint density  $f_{X_{0:N}, Y_{1:N}}(x_{0:N}, y_{1:N}; \theta)$ . The likelihood function is

$$L(\theta) = f_{Y_{1:N}}(y_{1:N}^*; \theta).$$

Are the following identities (A) true for all statistical models; (B) true for general POMP models but not all models; (C) true for linear Gaussian POMP models but not general POMP models; (D) generally false? Give a letter answer for each identity.

$$L(\theta) = \int f_{Y_{1:N}|X_{0:N}}(y_{1:N}^*|x_{0:N}; \theta) f_{X_{0:N}}(x_{0:N}; \theta) dx_{0:N} \quad (1)$$

$$L(\theta) = \prod_{n=1}^N f_{Y_n|Y_{1:n-1}}(y_n^*|y_{1:n-1}^*; \theta) \quad (2)$$

$$\text{Var}\{X_{n+1} | Y_{1:n}\} = E[\text{Var}\{X_{n+1} | X_n\} | Y_{1:n}] + \text{Var}\{E[X_{n+1} | X_n] | Y_{1:n}\} \quad (3)$$

$$L(\theta) = \int \left[ \prod_{n=1}^N f_{Y_n|X_n}(y_n^*|x_n; \theta) \right] f_{X_{0:N}}(x_{0:N}; \theta) dx_{0:N} \quad (4)$$