Midterm 1, STATS 531/631 W26

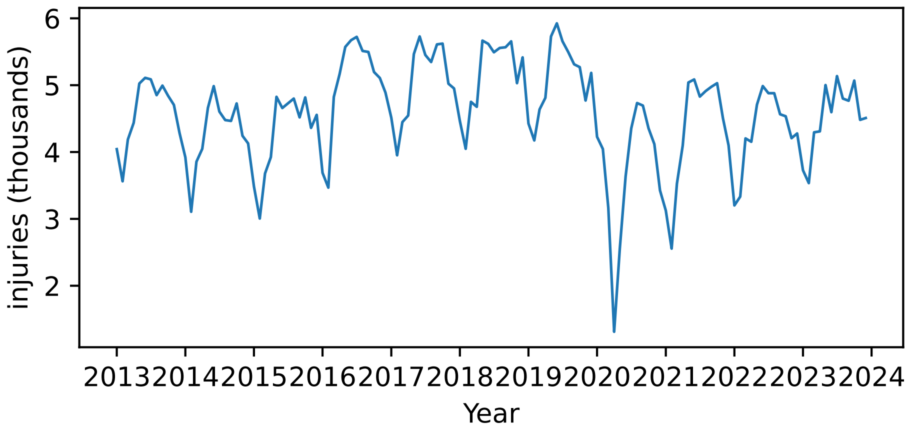
In class on 2/16

Name: **UMID:**

Instructions. The test is closed book, and you are not allowed access to any notes. Any electronic devices in your possession must be turned off and remain in a bag on the floor.

For each question, circle one letter answer and provide some supporting reasoning.

Q1. Stationarity and unit roots.



Above are monthly injuries from motor vehicle collisions in New York City. An augmented Dickey-Fuller test, `adfuller(injuries)`, gives a p-value of 0.014528. Which is the best way to proceed:

A: The time plot indicates a non-constant mean function describing a major dip due to the COVID-19 pandemic and an increasing trend at other times. The ADF test does not support or refute that model.

B: The ADF test suggests the series is stationary, supporting a decision to fit a SARMA model.

C: The ADF test suggests the series is non-stationary; it should be differenced before fitting a SARMA.

D: The ADF test indicates that the series is non-stationary, supporting the use of a non-constant mean function to describe a major dip due to the COVID-19 pandemic and an increasing trend at other times.

Q2. Calculations for ARMA models

Is it possible for an $AR(2)$ model to have a finite moving average representation, so that it is equivalent to some $MA(q)$ model for $q < \infty$?

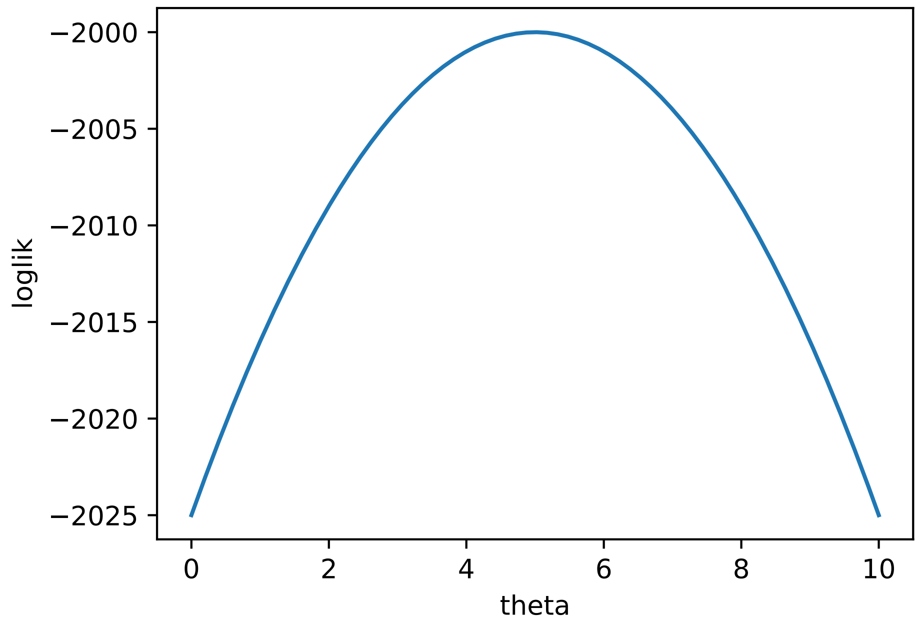
A. No. Any moving average representation of any $AR(2)$ model is $MA(\infty)$

B. Yes. Although it is not true for any $AR(2)$ process, it is possible to find particular choices of the autoregressive coefficients, p_1 and p_2, that lead to a finite $MA(q)$ representation.

C. It is not possible for any real-valued p_1 and p_2, but it is possible if you permit p_1 and p_2 to be complex-valued.

Q3. Likelihood-based inference for ARMA models

The Python function `statsmodels.tsa.arima.model.ARIMA.fit` and the R function `arima()` provide standard errors calculated by observed Fisher information. This question tests your understanding of what that means. Suppose a parametric model has a single parameter, θ, and the log-likelihood function when fitting this model to dataset is as follows:



What is the observed Fisher information (I_{obs}) for θ?

Hint 1. The observed Fisher information is accumulated over the whole dataset, not calculated per observation, so we don't have to know the number of observations, N.

Hint 2. Observations in time series models are usually not independent, so the log-likelihood is not the sum of the log-likelihoods for each observation. Its calculation will involve consideration of the dependence, and usually the job of calculating the log-likelihood is left to a computer.

Hint 3. The usual variance estimate for the maximum likelihood estimate, $\hat{\theta}$, is $\mathrm{Var}(\hat{\theta}) \approx 1/I_{obs}$.

A: $I_{\text{obs}} = 2$
B: $I_{\text{obs}} = 1$
C: $I_{\text{obs}} = 1/2$
D: $I_{\text{obs}} = 1/4$
E: None of the above

Q4. Interpreting diagnostics

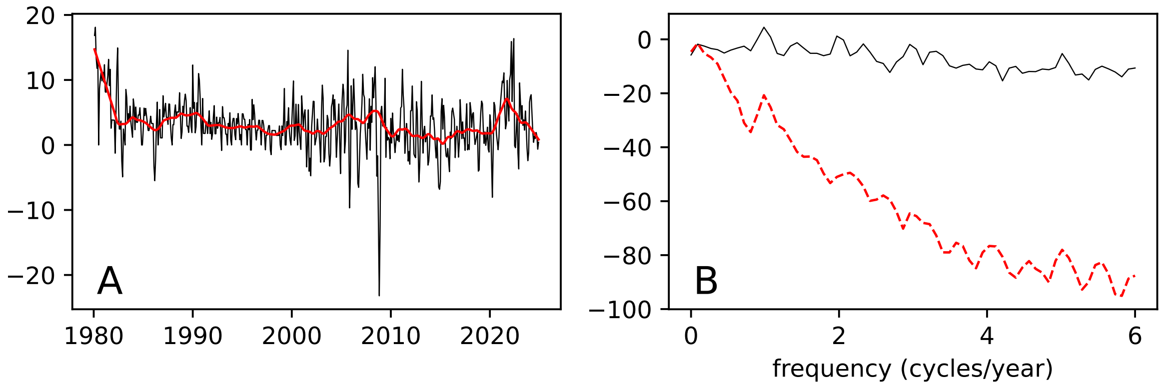
	AIC MA0	AIC MA1	AIC MA2	AIC MA3	AIC MA4	LBT MA0	LBT MA1	LBT MA2	LBT MA3	LBT MA4
AR0	174.82	48.81	9.61	-14.62	-17.98	7.6e-63	2.84e-26	2.28e-11	0.002	0.0409
AR1	-33.05	-34.44	-32.68	-30.69	-30.31	0.567	0.95	0.925	0.918	0.999
AR2	-34.07	-32.84	-33.25	-29.27	-28.51	0.951	0.926	0.98	0.954	0.996
AR3	-32.67	-33.2	-29.21	-29.91	-27.38	0.922	0.985	0.967	0.981	0.95
AR4	-30.77	-31.36	-30.11	-28.94	-24.92	0.92	0.967	0.989	0.978	0.994

The Ljung-Box test (LBT) provides an alternative approach to comparison of AIC values for selecting ARMA models. Whereas the standard sample autocorrelation function (ACF) residual plot tests each ACF component $\hat{\rho}_k$ under a null hypothesis of white noise, LBT tests $\sum_{k=1}^{h} \hat{\rho}_k^2$. Here, we present an AIC table and an LBT table (for $h = 5$). This course have favored AIC, with visual inspection of ACF and checking whether residual patterns appear in the frequency domain. There may be reasons to prefer LBT. Which of the following are good reasons to use LBT?

(i). LBT provides a p-value which is more formal than the comparison of AIC values.
(ii). Numerical issues involved in fitting an ARMA model may cause problems for comparing AIC values.
(iii). The LBT gives insights into what model to investigate next if the null hypothesis is rejected.
(iv). The LBT is useful in conjunction with AIC and ACF, since it provides an alternative perspective.

A. (i) only
B. (i, ii, iv)
C. (i,iii, iv)
D. (ii, iii, iv)
E. None of the above

Q5. The frequency domain



The monthly US consumer price index (CPI) combines the price of a basket of products, such as eggs and bread and gasoline. (A) Annualized monthly percent inflation, i.e., the difference of log-CPI multiplied by 12×100 (black line); a smooth estimate via local linear regression (red line). (B) The periodogram of inflation and its smooth estimate. Which best characterizes the behavior of the smoother?

A: Cycles longer than 2 months are removed
B: Cycles shorter than 2 months are removed
C: Cycles longer than 2 year are removed
D: Cycles shorter than 2 year are removed
E: Cycles longer than (1/2) year are removed
F: Cycles shorter than (1/2) year are removed

Q6. Scholarship for time series projects

Four people in a team collaborate on a project. After the project is submitted, a reader identifies that part of the project is adapted from an unreferenced source, i.e., it has been plagiarized. The team worked using git and cooperates on tracking down the issue, and the commit history clearly reveals who wrote the problematic part of the project. What is the most appropriate course of action:

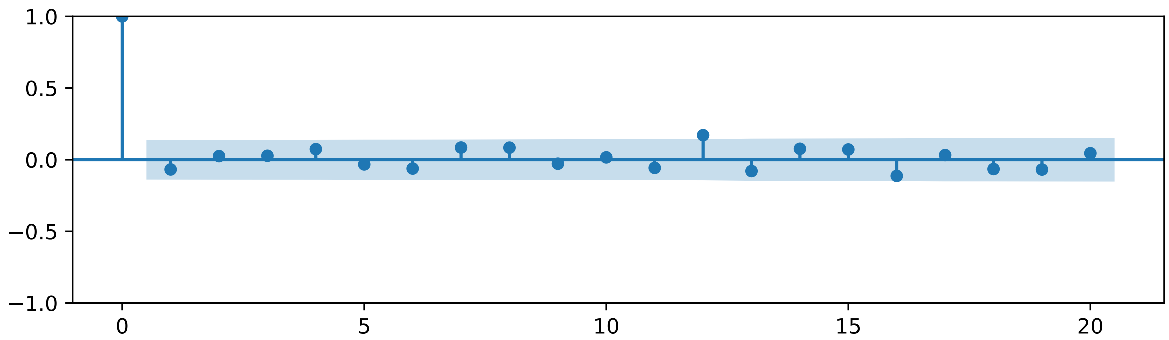
A. The guilty coauthor should be penalized heavily for poor scholarship, and the other coauthors should have a minor penalty for failing to check their colleague's work.

B. All coauthors should share the same penalty, since this is a team project and all coauthors share equal responsibility for the submitted report.

C. The guilty coauthor should be penalized heavily for poor scholarship. The other coauthors have demonstrated strong scholarship by following good transparent working practices that enabled this issue to get quickly resolved, so they should not receive any penalty.

D. It is necessary to collect more information before coming to a decision. For example, the team may argue that the source is well known to all readers so did not have to be cited.

Q7. Data analysis



The plot above is the sample autocorrelation function (ACF) for a time series $y_{1:N}$. What is the best conclusion to draw from this evidence?

A. The sample ACF is statistically consistent with an iid model. Therefore, standard statistical reasoning lets us conclude that the time series is iid.

B. The time series passed this test for being iid, but it might fail other tests. In practice, we can never make all possible tests so we cannot reliably conclude that the time series is iid.

C. It is meaningless to say that a time series is iid, since a time series is a sequence of numbers and iid is a property of a sequence of random variables.

D. The statistical evidence in the sample ACF is consistent with using an iid model to describe the data.

E. There is an oscillating pattern in the sample ACF. Even though no individual lag contradicts an iid model assumption at the 5% level, the overall pattern is clear evidence against iid.
