STATS 700-002 Class 3.
The Volz Coalescent

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The Volz Coalescent

- ▶ Overview: Kingman's coalescent provides a link between the observed rate of coalescence, the *effective population size* and the generation interval, in a fixed-size population.

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Volz, E. M., Kosakovsky Pond, S. L., Ward, M. J., Leigh Brown, A. J., & Frost, S. D. (2009). Phylodynamics of infectious disease epidemics. Genetics, 183(4), 1421–1430. https://doi.org/10.1534/genetics.109.106021.

▶ Overview: Kingman's coalescent provides a link between the observed rate of coalescence, the *effective population size* and the generation interval, in a fixed-size population.

▶ A time-varying population can be approximated as piecewise constant. Volz et al. (2009) noticed that this can be inappropriate for fast-changing populations and derived an alternative approach.

Cluster size distributions

▶ Why are Volz et al. (2009) interested in cluster size distribution?

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▶ What roles are played by the assumption that the population size is large?

Intuition

▶ From the introduction, "Coalescent rates are low near peak prevalence, but higher when there is a large ratio of incidence to prevalence."

How is this evident?

Definition or derivation?

▶ Consider equation (2):

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▶ Either way, once a model gives us $p_c(t,T)$, this defines a coalescent process having coalescent rate $f_{SI} \, p_c$.

▶ Note, this depends on the expected number of lineages (normalized as the large sample fraction) not the actual number.

Likelihood

▶ Equation (12) is written in terms of $A(t, T)$, which is a property of the model not of the data.

How can this be considered a likelihood?

Sufficient statistics

▶ In the Introduction, Volz et al. claim that "…branching patterns provide information about the frequency of transmissions over time."

This begs the question: What, according to Volz et al. (2009), are sufficient statistics for the frequency of transmissions over time?
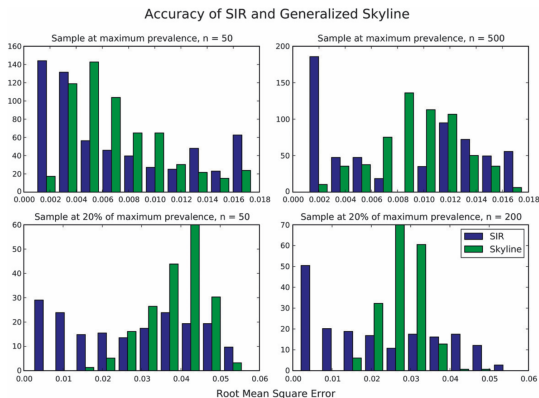
Comparison with the skyline plot

FIGURE 3.—Root mean square error of SIR and generalized skyline estimates of epidemic prevalence. Data are based on 300 simulated epidemics ($R_0 = 2$). RMSE is averaged over 100 time points.

Does it indeed "…seem surprising that the SIR model based on ODEs outperforms the generalized skyline even in the presence of stochasticity at small population sizes"? Why or why not?

References I

Volz, E. M., Kosakovsky Pond, S. L., Ward, M. J., Leigh Brown, A. J., and Frost, S. D. W. (2009). Phylodynamics of infectious disease epidemics. *Genetics*, 183(4):1421–1430.