

# **An iterated block particle filter for inference on coupled dynamic systems with shared and unit-specific parameters**

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Design and Analysis of Infectious Disease Studies

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Slides are at <https://ionides.github.io/talks/mfo23.pdf>

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# Questions to be addressed

- 1 When can we carry out full-information likelihood-based inference on a general class of spatiotemporal partially observed Markov process models? (Of particular interest are metapopulation models.)
- 2 Is there an intersection between scientific problems of interest and the capabilities of the spatPomp R package?
- 3 We introduce the **iterated block particle filter** because it is currently the most powerful algorithm available in spatPomp.
- 4 Bonus question: How do we know if our model is statistically adequate, or needs more work?

# Inference challenges in population dynamics

- 1 Combining measurement noise and process noise.
- 2 Including covariates in mechanistically plausible ways.
- 3 Continuous time models.
- 4 Modeling and estimating interactions in coupled systems.
- 5 Dealing with unobserved variables.
- 6 **Modeling spatiotemporal dynamics.**
- 7 Studying population dynamics via genetic sequence data.

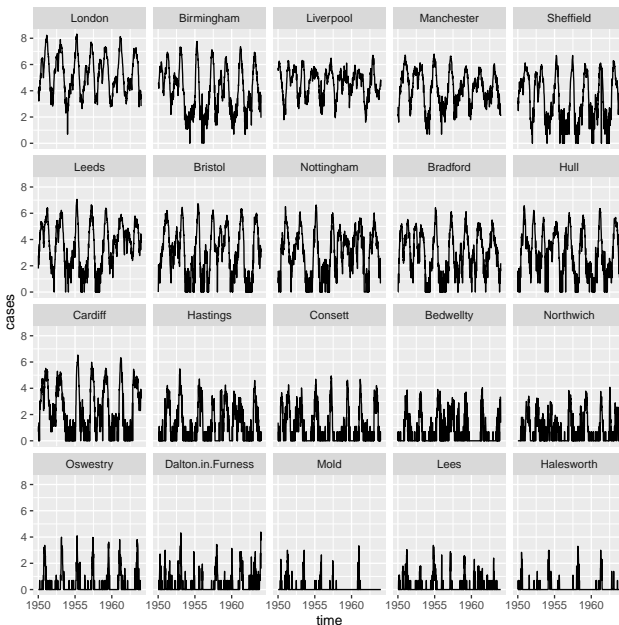
1–5 are largely solved, from a methodological perspective.

6 is our immediate topic.

7 is exciting but not the focus of this talk.

Reviews: Bjornstad & Grenfell (*Science*, 2001); Grenfell et al (*Science*, 2004)

# Pre-vaccination weakly measles in England & Wales



# Time series data, panel data & spatiotemporal data

- Looking at one unit (town) is **time series analysis**.
- Joint modeling of a few units (say, 2 or 3) is **multivariate time series analysis**.
- Analysis of many time series, without consideration of dynamic interactions, is **panel data analysis**.
- Allowing for coupling between units, we get **spatiotemporal analysis**, which in our context is **metapopulation analysis**.

- Consideration of arbitrary dynamic models. The limitations should be our scientific creativity and the information in the data.

Hence, **plug-and-play** methods which need a simulator from the model but not nice closed-form expressions for densities.

- Statistically efficient inference, to extract all the information in the data.

Hence, **likelihood-based** methods.

# Fitting mechanistic models to time series

- iterated particle filtering via `mif2` in the R package `pomp` enables Masters-level statisticians to do plug-and-play likelihood-based inference:

<https://ionides.github.io/531w22/>

- The science may be hard, but the statistics is becoming routine.

# Fitting mechanistic models to panel data

- To investigate epidemiological dynamics in multiple cities, one can consider each city independently, perhaps modeling a background immigration rate of infections for each city.
- **Decoupling** leads to panel data analysis, by assumption.
- Iterated filtering methods extend to panel data (`panelPomp`, Breto et al, *JASA*, 2019).
- We must decide which parameters should be modeled as **shared** (i.e., the same for each unit) vs **unit-specific**.



# The curse of dimensionality

- Particle filter (PF) methods fail for high-dimensional systems. They scale exponentially badly.

- Algorithms with improved scalability include:

**Bagged filters (BF, IBF)**

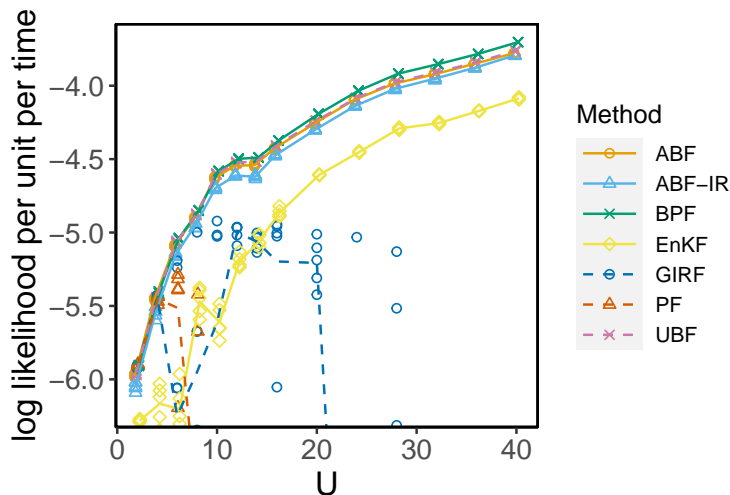
**Ensemble Kalman filter (EnKF, IEnKF)**

**Guided intermediate resampling filter (GIRF, IGIRF)**

**Block particle filter (BPF, IBPF)**

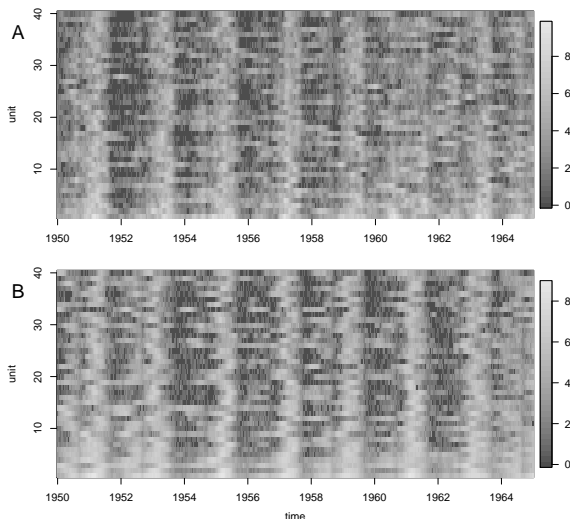
- Filters estimate latent states and evaluate the likelihood.
- Iterated filters estimate parameters using stochastic parameter perturbations.
- These algorithms are all implemented in the `spatPomp` R package.

# Filtering $U$ units of a coupled measles SEIR model



Simulated data using a gravity model with geography, demography and transmission parameters corresponding to UK pre-vaccination measles (Ionides et al, JASA, 2021).

# $U = 40$ units for a coupled measles SEIR model



**A.** Simulated Susceptible-Exposed-Infected-Recovered dynamics coupled with a gravity model (log of biweekly reported cases).

**B.** Measles UK pre-vaccination case reports for the 40 largest cities.

# Parameters for the measles model

- Seasonal transmission: mean and amplitude, using school term for contact rate.
- Durations of latency and infectious period.
- Initial values: fraction susceptible, latent and infectious.
- Cohort effect: all births in an age cohort start school in September.
- Inhomogenous mixing exponent,  $I^\alpha$ .
- Measurement fraction.
- Population movement model: gravity constant.
- Dynamic noise: process overdispersion.
- Measurement overdispersion.

(Adding gravity coupling to the model of He et al, *JRSI*, 2010)

# More on the block particle filter (BPF)

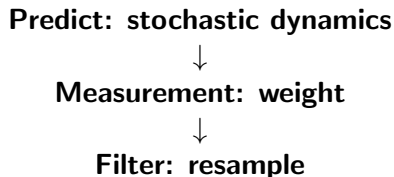
- BPF worked quickly, easily and reliably on our measles model filtering experiments.
- BPF has theoretical support in some situations (Rebeschini & Van Handel, *Annals of Applied Probability*, 2015).
- This motivated us to develop an IBPF for parameter estimation.
- IBPF has theoretical guarantees similar to BPF (Ning & Ionides, *arXiv*, 2023).
- BPF was independently proposed as the “factored particle filter” by Ng et al (2002).

# Particle filter (PF)

## Evolutionary analogy



## Particle filter algorithm

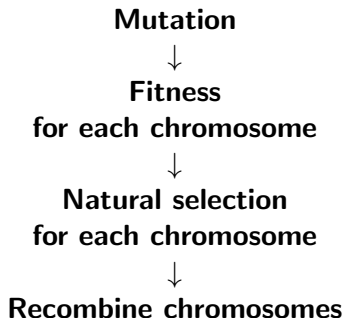


- PF is an evolutionary algorithm with good mathematical properties: an unbiased likelihood estimate and consistent latent state distribution.

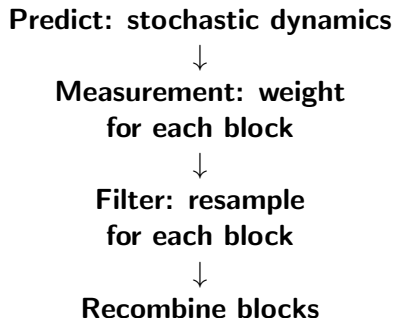
# Block particle filter (BPF)

- Blocks are a partition of the metapopulation units.
- For measles, we use each city as a block.

## Evolutionary analogy

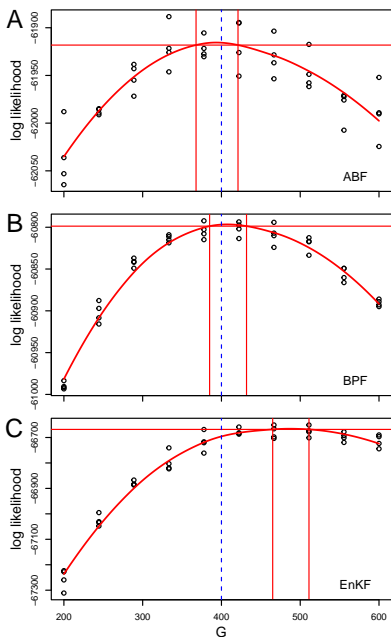


## Block particle filter



- Blocks are segments of the full state which can be reassorted between particles at the resampling step.

# Measles likelihood slices for coupling parameter, $G$



Simulating 15 year of data from  $U = 40$  cities. Slice likelihood, varying  $G$  with other parameters fixed. (Ionides et al, *JASA*, 2021).

**A.** Evaluation using adapted bagged filter (ABF).

**B.** Evaluation using block particle filter (BPF).

**C.** Evaluation using EnKF.

A much easier task than computing a profile likelihood for real data.



# Comments on the Ensemble Kalman Filter (EnKF)

- EnKF is more dependent on approximate Gaussianity than is sometimes supposed.
- The Gaussian-inspired update rule is similar to the extended Kalman filter (EKF), which has largely been superseded by particle filter methods for low-dimensional nonlinear biological dynamics.
- Simple systems can defeat EnKF: the linear Gaussian update is helpless when data inform the conditional variance rather than the conditional mean.
- Big systems need computationally tractable analysis. EnKF may sometimes be the best solution available, but be aware of its limitations.

# Scalability needed for practical inference

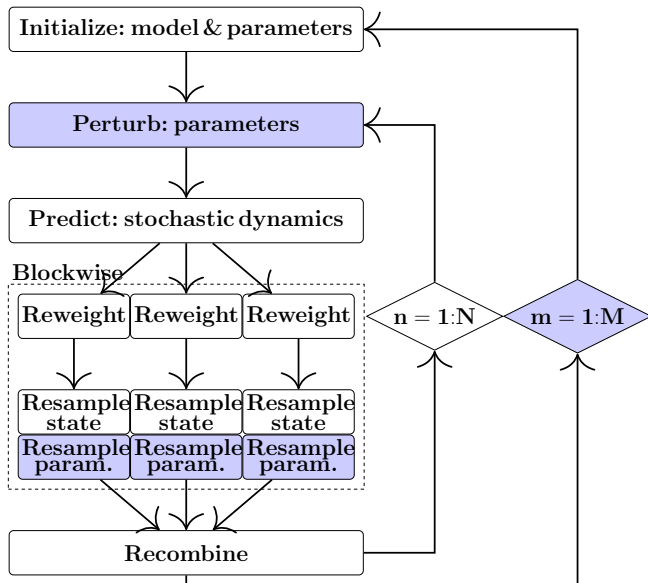
## Large numbers of parameters

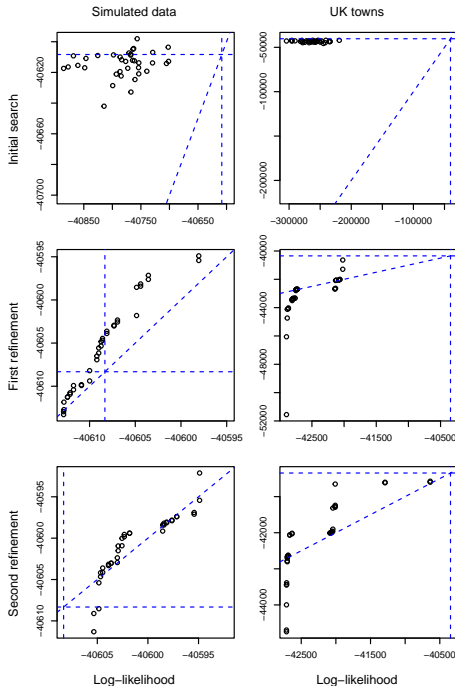
- Initial conditions will typically have to be estimated for each unit.
- Various dynamic parameters and measurement parameters (e.g., reporting rate) may also need to be unit-specific to obtain a statistical fit to the data.
- Working with hundreds of estimated parameters raises additional challenges on top of the high-dimensional coupled dynamics.

## A moderate numbers of spatial units is enough to open new possibilities.

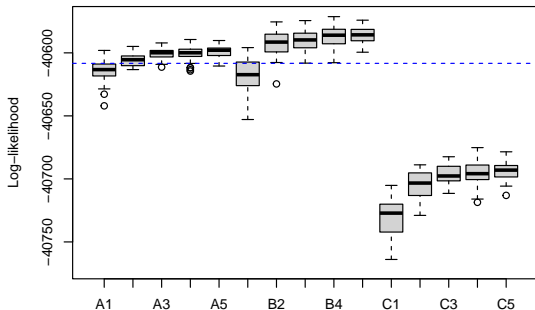
- As soon as dimension exceeds capabilities of a particle filter (say,  $U = 5$ ) we are in situations where likelihood-based inference for general models has been inaccessible.
- 5-500 coupled units is our target problem size.
- Larger problems will need numerical approximations (e.g., EnKF).

# An iterated block particle filter for parameter estimation





- IBPF applied to simulations and data.
- Multiple searches from random starting points.
- Top 25% of searches are subsequently refined.



## IBPF for measles, 20 cities

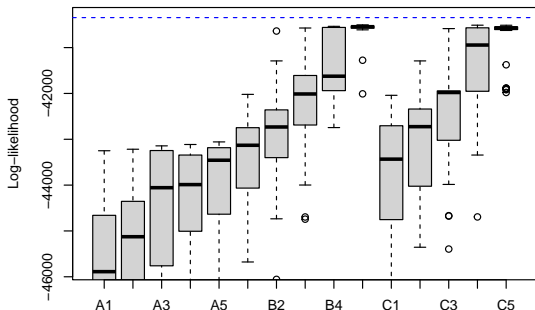
- simulations (top panel)
- dash line: likelihood at truth

- data (bottom panel)
- dash line: MLE for case C.

(A) 9 shared parameters,  
 $4 \times 20$  unit-specific  
parameters

(B) all parameters  
unit-specific

(C) all unit-specific, with  
constant immigration instead  
of coupling



## Inferences from the previous slide

- IBPF can work for simulated data, since it finds likelihoods higher than the truth when the model is well-specified
- When coupled by a gravity model, simulated data show evidence against a constant immigration model.
- Data do not favor this gravity model over a constant immigration model.

He et al (2010) estimates.  $r$  is rank correlation with  $N_{1950}$

	$N_{1950}$	$R_0$	IP	LP	$\alpha$	$a$	$\iota$	$\psi$	$\rho$	$\sigma_{SE}$
Halesworth	2200	33.00	2.30	7.90	0.95	0.38	0.0091	0.64	0.75	0.075
Lees	4200	30.00	2.10	8.50	0.97	0.15	0.0310	0.68	0.61	0.080
Mold	6400	21.00	1.80	5.90	1.00	0.27	0.0140	2.90	0.13	0.054
Dalton in Furness	11000	28.00	2.00	5.50	0.99	0.20	0.0390	0.82	0.46	0.078
Oswestry	11000	53.00	2.70	10.00	1.00	0.34	0.0300	0.48	0.63	0.070
Northwich	18000	30.00	3.00	8.50	0.95	0.42	0.0600	0.40	0.80	0.086
Bedwellty	29000	25.00	3.00	6.80	0.94	0.16	0.0400	0.95	0.31	0.061
Consett	39000	36.00	2.70	9.10	1.00	0.20	0.0730	0.41	0.65	0.071
Hastings	66000	34.00	5.40	7.00	1.00	0.30	0.1900	0.40	0.70	0.096
Cardiff	240000	34.00	3.10	9.90	1.00	0.22	0.1400	0.27	0.60	0.054
Bradford	290000	32.00	3.40	8.50	0.99	0.24	0.2400	0.19	0.60	0.045
Hull	300000	39.00	5.50	9.20	0.97	0.22	0.1400	0.26	0.58	0.064
Nottingham	310000	23.00	3.70	5.70	0.98	0.16	0.1700	0.26	0.61	0.038
Bristol	440000	27.00	4.90	6.20	1.00	0.20	0.4400	0.20	0.63	0.039
Leeds	510000	48.00	11.00	9.50	1.00	0.27	1.2000	0.17	0.67	0.078
Sheffield	520000	33.00	6.40	7.20	1.00	0.31	0.8500	0.18	0.65	0.043
Manchester	700000	33.00	6.90	11.00	0.96	0.29	0.5900	0.16	0.55	0.055
Liverpool	800000	48.00	9.80	7.90	0.98	0.30	0.2600	0.14	0.49	0.053
Birmingham	1100000	43.00	12.00	8.50	1.00	0.43	0.3400	0.18	0.54	0.061
London	3400000	57.00	13.00	13.00	0.98	0.55	2.9000	0.12	0.49	0.088
$r$	1	0.46	0.95	0.32	0.11	0.30	0.9300	-0.93	-0.20	-0.330

# Cholera in Haiti, 2010-2019

- 820,000 reported cases
- nearly 10,000 deaths
- Population of 10,000,000 in 10 départements

- Lee et al (2020) developed 4 models to guide vaccination plans:

1. stochastic, national
2. ODE, spatial
3. stochastic, spatial
4. agent based

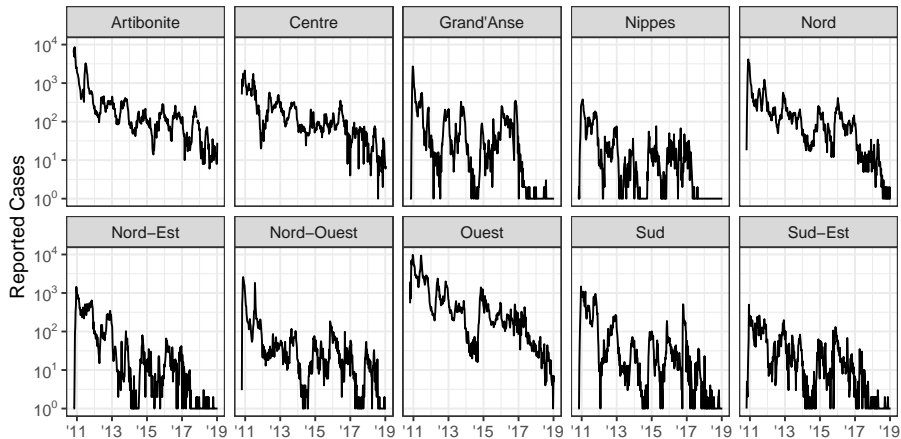
- Wheeler et al (2023) continued the data analysis



<https://doi.org/10.5194/nhess-20-471-2020>



# Haiti cholera weekly reports 2010-2019 (log scale)



# Log-likelihoods of models and an ARMA benchmark

	Model 1 (p)	Model 2 (p)	Model 3 (p)
Wheeler et al	-2731.3 (15)	-21957.3 (6)	-17850.4 (35)
Lee et al	-3030.9 (20)	-29367.4 (6)	-31840.8 (29)
Log-ARMA(2,1)	-2803.7 (4)	-18027.0 (40)	-18027.0 (40)

Model 1 (stochastic, aggregated):

We add process overdispersion, and beat the ARMA benchmark.

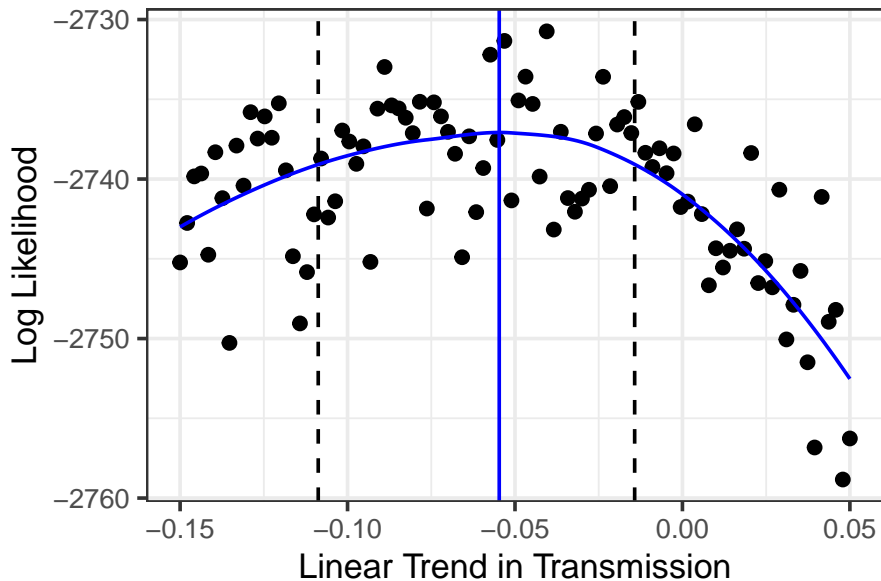
Model 2 (deterministic, spatial):

We use a log-normal measurement model. ODE models fall far short of the ARMA benchmark.

Model 3 (stochastic, spatial):

We fit the model using IBPF, and beat the ARMA benchmark.

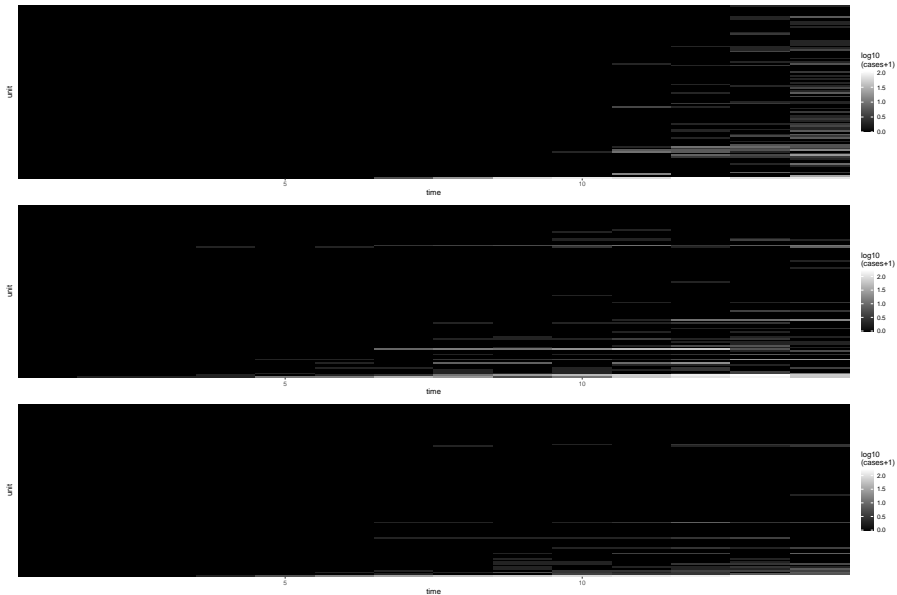
# Evidence for a decreasing transmission rate in Model 1



# COVID in 375 cities in China, 10-23 January, 2020

- Metapopulation data were used to infer the fraction of asymptomatic cases and their contagiousness (Li et al, *Science*, May 2020).
- SEIR model with asymptomatics, reporting delay, and coupling based on cell phone data.
- Inference using iterated EnKF.
- The time interval predates Wuhan travel restrictions.
- Total cases 801, Wuhan 454, Chongqing 27, Beijing 26.
- 259 cities with 0 cases, 99 with 1–5 cases, 17 with  $> 5$  cases.
- 0 cases reported for all cities during Jan 10–15.

Top: data. Middle: IBPF fit. Bottom: IEnKF fit

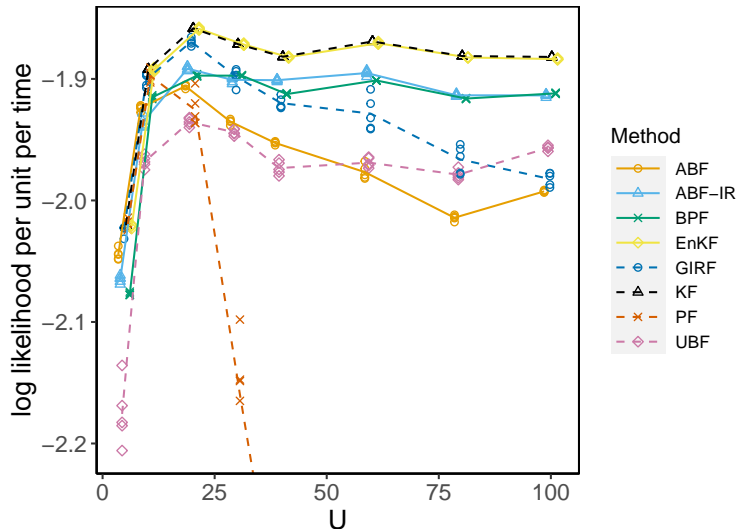


# Technical issues with EnKF

- EnKF is based on a continuous Gaussian approximation.
- Log-likelihoods with respect to counting measure are never positive, and cannot properly be compared with log-likelihoods corresponding to continuous densities.
- For data with many zeros, the unbounded EnKF likelihood can substantially bias the MLE.
- Adapting EnKF for count data is non-trivial (Katzfuss et al, *JASA*, 2019).
- Li et al avoided the issue by using a “heuristic observation error variance” within EnKF. The corresponding “log-likelihood” is not the log-likelihood of any model. In particular, it cannot validly be compared against simple statistical benchmarks.
- Data and models will never be perfect. We want to reveal, rather than conceal, limitations.

- We anticipate continued progress in modeling and inference for nonlinear spatiotemporal partially observed stochastic dynamic systems.
- The spatPomp package facilitates data analysis.
- Questions and comments?

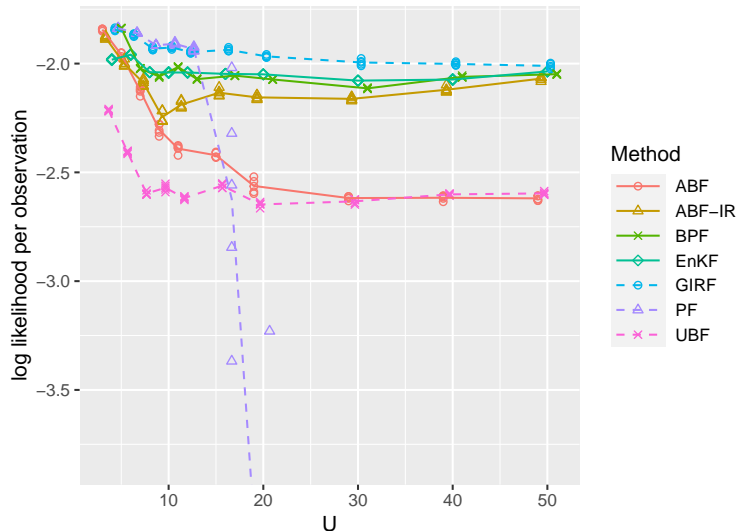
# Filtering $U$ -dimensional correlated Brownian motion



$$\text{Cov}(X_{u,n} - X_{u,n-1}, X_{\tilde{u},n} - X_{\tilde{u},n-1}) \sim 0.4^{|u-\tilde{u}|}$$



# Filtering $U$ units of Lorenz 96 toy atmospheric model



$$dX_u(t) = \{X_{u-1}(t)(X_{u+1}(t) - X_{u-2}(t)) - X_u(t) + F\}dt + \sigma dB_u(t)$$

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