An iterated block particle filter for inference on coupled dynamic systems

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Slides are at `https://ionides.github.io/talks/aprm24.pdf`

Joint work with Patricia Ning, Jesse Wheeler, Kidus Asfaw, Jifan Li, Joonha Park, Aaron King, Mercedes Pascual

Questions to be addressed

1. When can we carry out full-information likelihood-based inference on a general class of spatiotemporal partially observed Markov process models? In particular, models for networks of interacting biological population dynamics.
2. Is there an intersection between scientific problems of interest and the capabilities of the `spatPomp` R package?
3. We introduce the **iterated block particle filter**, currently the most powerful algorithm available in `spatPomp`.
4. Bonus question: How do we know if our model is statistically adequate, or needs more work?

Inference challenges in population dynamics

1. Combining measurement noise and process noise.
2. Including covariates in mechanistically plausible ways.
3. Continuous time models.
4. Modeling and estimating interactions in coupled systems.
5. Dealing with unobserved variables.
6. **Modeling spatiotemporal dynamics**.
7. Studying population dynamics via genetic sequence data.

1–5 are largely solved, from a methodological perspective.
6 is our immediate topic.
7 is exciting but not the focus of this talk.

Reviews: Bjornstad & Grenfell (*Science*, 2001); Grenfell et al (*Science*, 2004)

Desiderata

- Consideration of arbitrary dynamic models. The limitations should be our scientific creativity and the information in the data.

  Hence, **plug-and-play** methods which need a simulator from the model but not nice closed-form expressions for densities.

- Statistically efficient inference, to extract all the information in the data.

  Hence, **likelihood-based** methods.

Fitting mechanistic models to time series

- Iterated particle filtering via `mif2` in the R package `pomp` enables Masters-level statisticians to do plug-and-play likelihood-based inference:

  `https://ionides.github.io/531w22/`

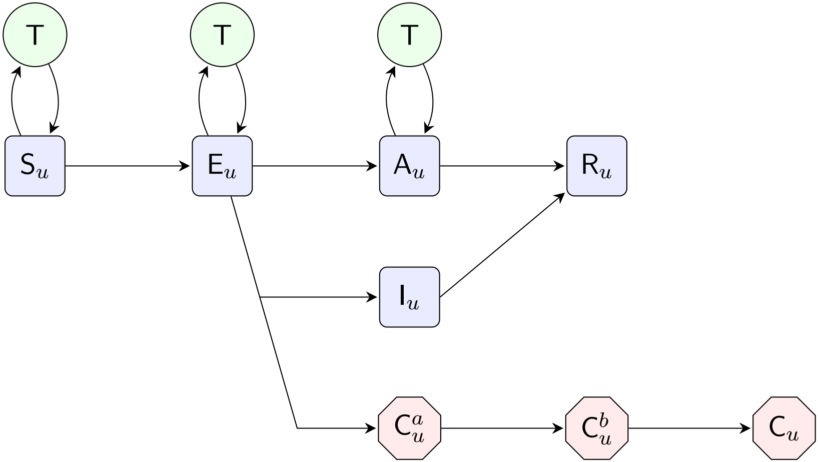
- The science may be hard, but the statistics is becoming routine.

The curse of dimensionality

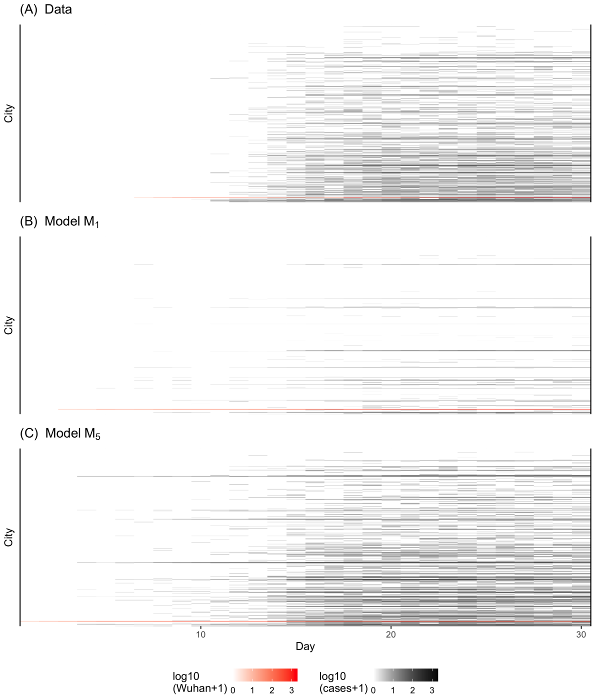
- Particle filter (PF) methods fail for high-dimensional systems. They scale exponentially badly.
- Algorithms with improved scalability include:
  Bagged filters (BF, IBF)
  Ensemble Kalman filter (EnKF, IEnKF)
  Guided intermediate resampling filter (GIRF, IGIRF)
  Block particle filter (BPF, IBPF)
- Filters estimate latent states and evaluate the likelihood.
- Iterated filters estimate parameters using stochastic parameter perturbations.
- These algorithms are all implemented in the `spatPomp` R package.

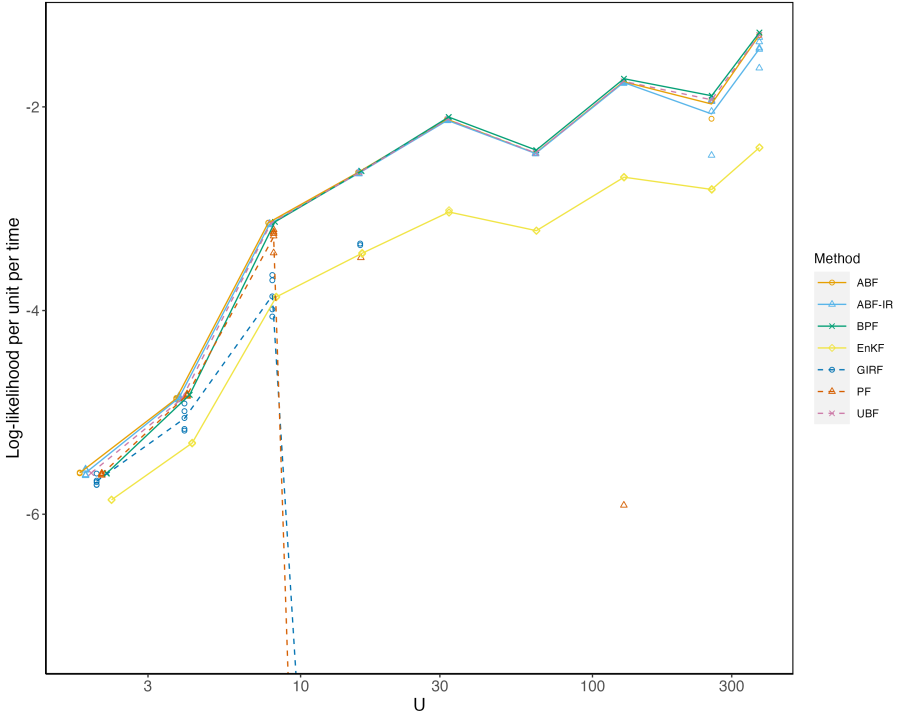
COVID in 375 cities in China, 10-23 January, 2020

- Metapopulation data were used to infer the fraction of asymptomatic cases and their contagiousness (Li et al, *Science*, May 2020).
- SEIR (susceptible-exposed-infected-removed) model with asymptomatics, reporting delay, and coupling based on cell phone data.
- Li et al (2020) used iterated EnKF for inference.
- The time interval covers the initial China lockdown.
- Here, we summarize our re-analysis of this model and data, recently posted on arXiv.

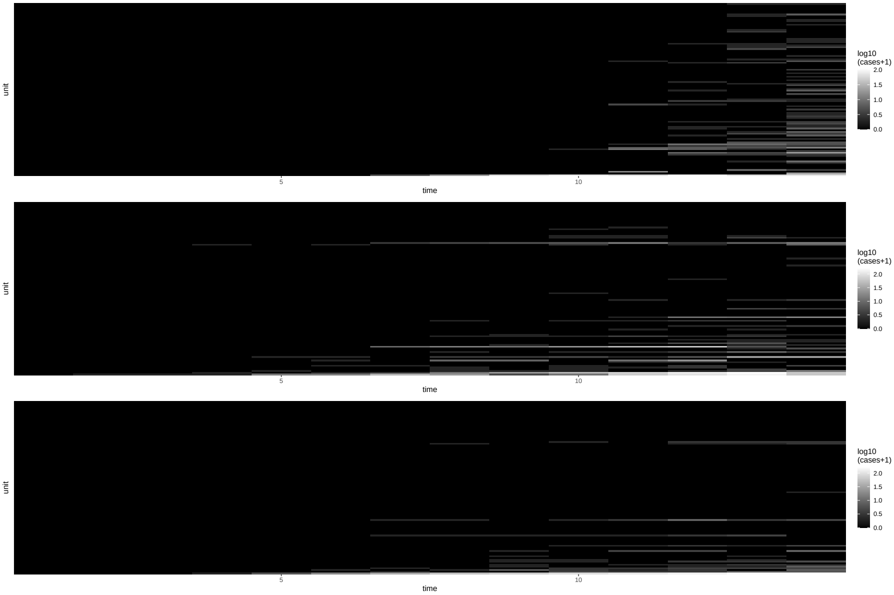


- Reportably infectious individuals, I_u for city u, are included in the delayed reporting compartment, $\mathrm{C}_\mathrm{u}^\mathrm{a}$.
- An individual arriving at C_u is a case report for city u.
- Individuals in A_u are not reportable and transmit at a reduced rate.
- Travel occurs to and from T, based on 2018 data from Tencent.


(A) Data
City
(B) Model M_1
City
(C) Model M_5
City
10
20
30
Day
log10
(Wuhan+1)
0 1 2 3
log10
(cases+1)
0 1 2 3


Log-likelihood per unit per time
-2
-4
-6
3
10
30
100
300
U
Method
ABF
ABF-IR
BPF
EnKF
GIRF
PF
UBF

Top: data. Middle: IBPF fit. Bottom: IEnKF fit



More on the block particle filter (BPF)

- BPF worked quickly, easily and reliably on a measles metapopulation (Ionides et al, *JASA*, 2023).
- BPF has theoretical support in some situations (Rebeschini & Van Handel, *Annals of Applied Probability*, 2015).
- This motivated us to develop an IBPF for parameter estimation.
- IBPF has theoretical guarantees similar to BPF (Ning & Ionides, *JMLR*, 2023).
- BPF was independently proposed as the "factored particle filter" by Ng et al (2002).

Particle filter (PF)

Evolutionary analogy

Mutation
$\downarrow$
Fitness
$\downarrow$
Natural selection

Particle filter algorithm

Predict: stochastic dynamics
$\downarrow$
Measurement: weight
$\downarrow$
Filter: resample

- PF is an evolutionary algorithm with good mathematical properties: an unbiased likelihood estimate and consistent latent state distribution.

Block particle filter (BPF)

- Blocks are a partition of the metapopulation units.
- For measles, we use each city as a block.

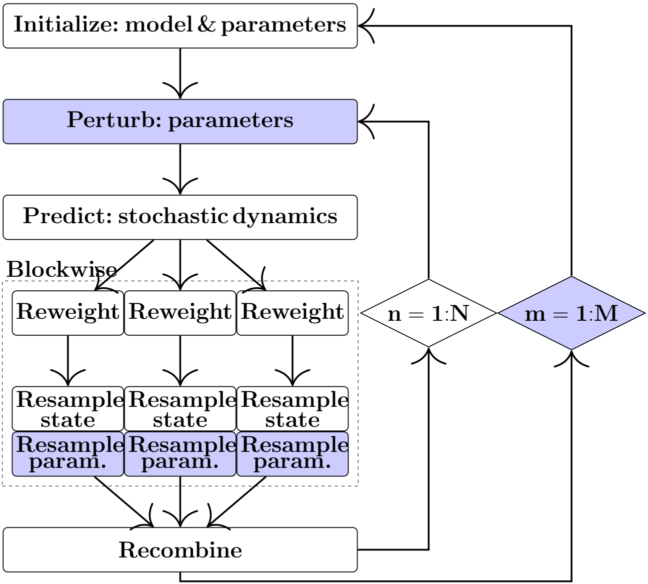
Evolutionary analogy	**Block particle filter**
Mutation	**Predict: stochastic dynamics**
$\downarrow$	$\downarrow$
Fitness for each chromosome	**Measurement: weight for each block**
$\downarrow$	$\downarrow$
Natural selection for each chromosome	**Filter: resample for each block**
$\downarrow$	$\downarrow$
Recombine chromosomes	**Recombine blocks**

- Blocks are segments of the full state which can be reassorted between particles at the resampling step.

Comments on the Ensemble Kalman Filter (EnKF)

- EnKF is more dependent on approximate Gaussianity than is sometimes supposed.
- The Gaussian-inspired update rule is similar to the extended Kalman filter (EKF), which has largely been superseded by particle filter methods for low-dimensional nonlinear biological dynamics.
- Simple systems can defeat EnKF: the linear Gaussian update is helpless when data inform the conditional variance rather than the conditional mean.
- Big systems need computationally tractable analysis. EnKF may sometimes be the best solution available, but be aware of its limitations.

An iterated block particle filter for parameter estimation



	Model 1	(p)	Model 2	(p)	Model 3	(p)
Wheeler et al	-2731.3	(15)	-21957.3	(6)	-17850.4	(35)
Lee et al	-3030.9	(20)	-29367.4	(6)	-31840.8	(29)
Log-ARMA(2,1)	-2803.7	(4)	-18027.0	(40)	-18027.0	(40)

Model 1 (stochastic, aggregated):
We add process overdispersion, and beat the ARMA benchmark.

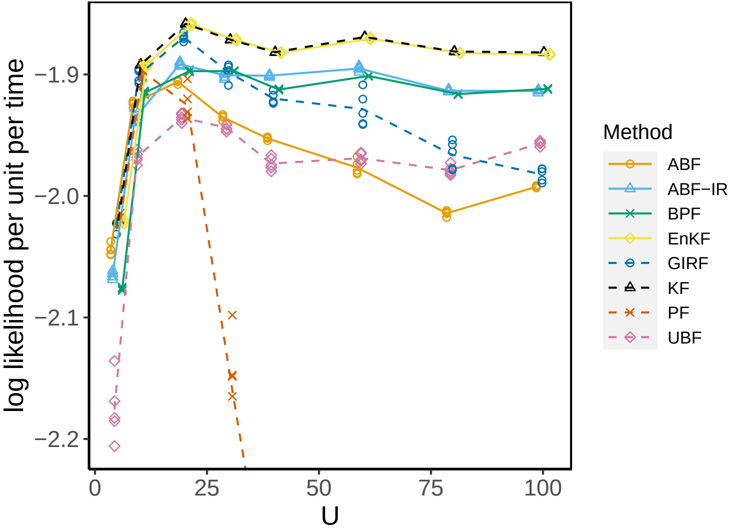
Model 2 (deterministic, spatial):
We use a log-normal measurement model. ODE models fall far short of the ARMA benchmark.

Model 3 (stochastic, spatial):
We fit the model using IBPF, and beat the ARMA benchmark.

Technical issues with EnKF

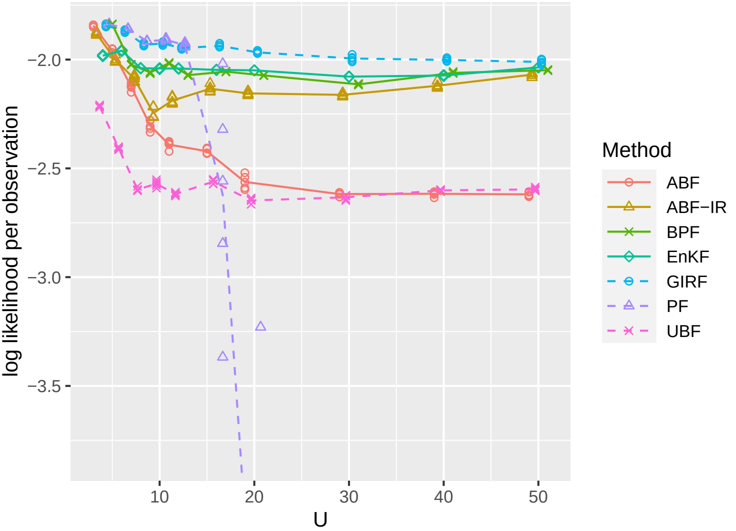
- EnKF is based on a continuous Gaussian approximation.
- Log-likelihoods with respect to counting measure are never positive, and cannot properly be compared with log-likelihoods corresponding to continuous densities.
- For data with many zeros, the unbounded EnKF likelihood can substantially bias the MLE.
- Adapting EnKF for count data is non-trivial (Katzfuss et al, *JASA*, 2019).

Filtering U-dimensional correlated Brownian motion



$$\mathrm{Cov}\Big(X_{u,n} - X_{u,n-1}, X_{\tilde{u},n} - X_{\tilde{u},n-1}\Big) \sim 0.4^{|u-\tilde{u}|}$$

Filtering U units of Lorenz 96 toy atmospheric model



$$dX_u(t) = \Big\{ X_{u-1}(t) \big(X_{u+1}(t) - X_{u-2}(t) \big) - X_u(t) + F \Big\} dt + \sigma \, dB_u(t)$$

References I