

**Discussion of Jonas Ströjby's thesis:**

**“On inference in partially observed Markov models  
using sequential Monte Carlo methods”**

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**“Everything changes and nothing remains still”**

*Plato, following Heraclitus of Ephesus, c. 500 BCE*

- A dynamic system is a collection of variables whose values change with time. A **Markov model** for a dynamic system is one where the current state of the system contains all the information appropriate for forecasting the future behavior of the system.
- The Markov property arises naturally in an ideal model of a system: any quantity which affects the future progression of the system can be included in the model.

- Thinking about the real world by imagining an ideal world (unobservable, even nonexistent, but mathematically elegant) has been central to Western thought since Plato.
- According to Plato, the physical world contains **partially observed** versions of an ideal world (a Markov model). Philosophers (scientists) strive to understand this ideal world.
- Mathematical statistics (a 20th century invention) studies systematic ways to draw **inference** about ideal systems from partial observations.
- Sweden (especially Stockholm) has been prominent in developing mathematical statistics: Harald Cramér, Carl-Gustav Esseen, Ulf Grenander, Herman Wold.

- Math stat uses probability theory (stochasticity, chance variation) for two distinct roles: to make the ideal system unpredictable, and as a formal model for the process of measurement.
- Inference for partially observed, nonlinear, stochastic dynamic systems is an important and challenging research area. It is critical to many current problems in **economics**, **finance**, **ecology**, and **engineering**.
- **sequential Monte Carlo methods** are state-of-the-art algorithms which compare large numbers of computer simulations from the dynamic system model to time series data.

## **Applications to economics**

- Over the last decade, most major central banks have implemented nonlinear stochastic models for national economies. Their structure is based on economic theory, but the models are calibrated to data. The role of the models is to encourage consistency in planning economic policies.

## Applications to finance

- Understanding the risk associated with financial agreements has motivated the development of mathematical statistics (e.g., Cramér).
- Financial markets have become complex dynamic systems. The 2008 financial crisis highlighted the need to improve management of risk in these days of rapid computer-based trading strategies.

improved models + improved inference  $\Rightarrow$  improved understanding of risk

## Applications to ecology

- Mankind is increasingly responsible for managing ecosystems. Ensuring the productivity of agriculture, forests and oceans is the key to a sustainable global economy. Ecological systems have notoriously nonlinear dynamics, with threshold effects and the potential for rapid exponential growth or decay of (desirable or undesirable) species.

## Applications to engineering

- Much of the theory of partially observed Markov models originally arose in control theory (rocket science and industrial process control).
- Robotics and automated control continue to generate new theory and applications.



## Plug-and-play inference for dynamic systems

- Nonlinear deterministic dynamic models (e.g., systems of ordinary differential equations) are routinely studied by numerical solution, since closed-form solutions are usually unavailable.
- Similar approaches are now being developed for inference on stochastic models. Methods where computer code for numerical solutions can be ‘plugged in’ to the inference procedure have been called plug-and-play.
- Four of Jonas Ströjby’s chapters develop plug-and-play procedures. Realizing the scientific potential of plug-and-play methods will be important future work for Jonas and others.
- As an application (chronologically, a motivation) Jonas will present an analysis fitting dynamic models for stock prices implied by the quoted prices of financial instruments (stock options).