To check that $\mathbb{E}\left[\sum_{n=1}^{\infty} X_n 1_{\{N \geq n\}}\right] = \sum_{n=1}^{\infty} \mathbb{E}\left[X_n 1_{\{N \geq n\}}\right]$ in the proof of Wald's equation:

Recall that a sufficient condition is

$$\sum_{n=1}^{\infty} \mathbb{E}\left[\,\left|X_n 1_{\{N \geq n\}}\right|\,\right] < \infty$$

(this can be thought of as Fubini's theorem, but more fundamentally it is a consequence of the dominated convergence theorem).

Now, $\sum_{n=1}^{\infty} \mathbb{E}\left[\left|X_n 1_{\{N \geq n\}}\right|\right]$

$$= \sum_{n=1}^{\infty} \mathbb{E}\left[|X_n|\, 1_{\{N \geq n\}}\right]$$

$$= \sum_{n=1}^{\infty} \mathbb{E}\left[|X_n|\right] \mathbb{E}\left[1_{\{N \geq n\}}\right] \text{ by independence}$$

$$= \mathbb{E}\left[|X_1|\right] \sum_{n=1}^{\infty} \mathbb{P}\left[N \geq n\right]$$

$$= \mathbb{E}\left[|X_1|\right] \mathbb{E}\left[N\right] < \infty \text{ by assumption}$$

(formally, this is where we use the requirement that $\mathbb{E}[N] < \infty$)