Statistics 620
Midterm takehome exam, Fall 2011

Name: ______________________________ **UMID #:** ______________________________

Midterm Exam

- THIS IS THE SAME AS THE IN-CLASS MIDTERM EXAM. IT IS DUE IN CLASS ON 10/27.
- **YOU MAY CONSULT THE TEXTBOOK AND WIKIPEDIA. PLEASE DO NOT CONSULT OTHER SOURCES OF ANY KIND, INCLUDING CLASSMATES.**
- THE OVERALL MIDTERM SCORE WILL BE AN AVERAGE OF THE IN-CLASS AND TAKE-HOME COMPONENTS.
- PLEASE PRINT THE EXAM OUT SINGLE-SIDED. WRITE ANSWERS ON THE EXAM. YOU MAY USE THE REVERSE SIDES IF YOU NEED EXTRA SPACE.
- There are 4 questions, each worth 10 points.
- Credit will be given for clear explanation and justification, as well as for getting the correct answer.

Problem	Points	Your Score
1	10	
2	10	
3	10	
4	10	
Total	40	

1. A box contains a white balls and b black balls. Balls are randomly drawn from the box, one at a time. If the drawn ball is white, it is returned to the box. If the drawn ball is black, it is painted white and then returned to the box. Find the expected number of white balls in the box after the nth draw.

2. **(a)** Let $N(t)$ be a Poisson process with rate λ. Find the probability that $N(t)$ is even.

Hint: you may or may not wish to proceed as follows. Let $e(t)$ be the probability that $N(t)$ is even; condition on $N(t-\delta)$ and construct a differential equation by taking the limit as $\delta \to 0$.

(b) The owner of a computer store hands out discount coupons to every other visitor entering the store, starting with the first arrival. Suppose that arrival of visitors follows a Poisson process with rate λ. Find the expected number of coupons given out by time t.

3. Let $N(t)$ be a renewal process with interarrival distribution F. Let W be the time at which the age of the renewal process first exceeds some constant s. In other words, writing S_n for the nth arrival time, define $W = \inf\{t : t - S_{N(t)} > s\}$. Let $V(t) = \mathbb{P}\big[W \leq t\big]$.

(a) Determine $\mathbb{E}[W]$.

(b) Establish an integral equation satisfied by $V(t)$.

4. Let $\{X_n\}$ be a homogeneous Markov chain with states $\{1, 2, 3, 4\}$ having transition probabilities given by the matrix $P = [P_{ij}]$. Let $f_{ij}(n)$ be the probability mass function of the first passage time from state i to state j, defined as

$$f_{ij}(n) = \mathbb{P}\big[X_n = j, X_{n-1} \neq j, \dots, X_2 \neq j \,|\, X_1 = i\big].$$

Evaluate $f_{12}(n)$ for

$$P = \begin{pmatrix} 0 & 1/2 & 0 & 1/2 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 1/2 & 0 & 1/2 & 0 \end{pmatrix}$$