CSOR W4246 9-28

Ian Johnson HW1

1. a. $f_1(2n)$ is $O(f_1(n)) \Rightarrow \boxed{f(x) = x}$

$f_1(n) = n \quad f_1(2n) = 2n$

$f_1(2n)$ is $O(f_1(n))$

$f_1(2n) \leq c\, f_1(n)$

$c = 3 \Rightarrow f_1(2n) < 3 f_1(n)$

$2n < 3n$ ✓

1. b. $\boxed{f(x) = 2^x}$

$f(n) = 2^n$

$f(2n) = 2^{2n}$

$2^{2n} \leq c\, 2^n$? $\Rightarrow$ no

2. a. $a=4 \quad b=2 \quad f(n)=n^2 \quad k=2$

$a = b^k \rightarrow \Theta(n^2 \log n)$

$\Theta(n^{\log_b a})$ if $a > b^k$

$\Theta(n^k \log n)$ if $a = b^k$

$\Theta(n^k)$ if $a < b^k$

b. $a=8 \quad b=2 \quad k=3$

$a = b^k \rightarrow \Theta(n^3 \log n)$

c. $a=11 \quad b=4 \quad k=2$

$a < b^k \rightarrow \Theta(n^2)$

d. $a=7 \quad b=3 \quad k=1$

$a > b^k \rightarrow \Theta(n^{\log_3 7}) \approx \Theta(n^{1.77})$

3. $T(n) = T(\frac{2}{3}n) + T(\frac{2}{3}n) + T(\frac{2}{3}n) + C = \boxed{3T(\frac{2}{3}n) + C}$

$a = 3 \quad b = \frac{3}{2} \quad k = 0$

$a > b^k \rightarrow \Theta(n^{\log_{3/2} 3}) \approx \boxed{\Theta(n^{2.7})}$

Mergesort is $\Theta(n \log n)$, which is faster.
No, I would not use this algorithm, unless there were space constraints, and this algorithm was able to sort in-place.

5. a) fib(n)

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fib(n)
    if n=0 or n=1
        return n
    return fib(n-1) + fib(n-2)
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4 5 1 3 2

5 operations

$T(n) = 2T(n-2) + C \quad k=1$

$= 2(2T(n-4) + C)$ // $t(n-1) \approx t(n-2)$

$= 4T(n-4) + 3C \quad k=2$ // Use $t(n-2)$ to get a lower bound

$= 4(2T(n-6) + C) + 3C$

$= 8T(n-6) + 7C \quad k=3$ // $t(0)=1, t(1)=1$

$\Rightarrow 2^k T(n-2k) + (2^k - 1)C$

to get in terms of $t(0)$:

$n - 2k = 0 \rightarrow k = \frac{n}{2}$

$2^{n/2} T(n-n) + (2^{n/2} - 1)C$

$= \underline{2^{n/2} + (2^{n/2} - 1)C}$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} \begin{cases} =0 \rightarrow f(x)\ O(g(x)) \\ 0<x<\infty \rightarrow f(x)\ \Theta(g(x)) \\ =\infty \rightarrow g(x)\ O(f(x)) \end{cases}$$

4. (30 points) In the table below, indicate the relationship between functions f and g for each pair (f, g) by writing **yes** or **no** in each box. For example, if $f = O(g)$ then write **yes** in the first box.

$\log_2$?

$n(\log 2 + \log n) \leftrightarrow n \log n$

$n \log 2 + n \log n$

$10^x = 2$

$n\log(2n) = n(\log 2 + \log n)$

$\log n,\ n^{\frac{1}{2}}$

$\boxed{\frac{n \log n}{\sqrt{n}} \rightarrow \infty \quad \frac{\log n}{\sqrt{n}} \rightarrow 0}$ ★

f	g	O	o	Ω	ω	? Θ
$10 \log n$ <	$\log^3 n$	yes ~~no~~	Yes ~~no~~	no ~~Yes~~	no	no
$n \log(2n)$ >	$n \log n$	yes ~~no~~	no	Yes	no	yes ~~no~~
$\sqrt{\log n}$ >	$\log \log n$	no	no	Yes	Yes	no
$\cancel{10}n^2 + \log n$ >	$n^2 + \cancel{11} \log^3 n$	no	no	Yes	no	no
$\sqrt{n} + \log n$ <	$n^{2/3} + 10$	Yes	yes	no	no	no
$n^2 2^n$ <	3^n	Yes	Yes	no	no	no
$n^{1/3}$ >	$(\log n)^2$	no	no	Yes	Yes	no
$n \log n$ <	$\frac{n^2}{\log n}$	Yes	Yes	no	no	no
$n!$ <	n^n	yes	yes	no	no	no
$\log n!$	$\log n^n$ $n \log n$	yes	no	Yes	no	yes

★ (beside the $\sqrt{\log n}$ row) ★ (beside the $n!$ row)

$\log(n)!$ or $\log(n!)$ ✓ ?

$\log(n!) = \log(n) + \log(n-1) + \ldots + \log(2) + \log(1)$

$n \log(n) = \log(n) + \log(n) + \ldots + \log(n) + \log(n)$