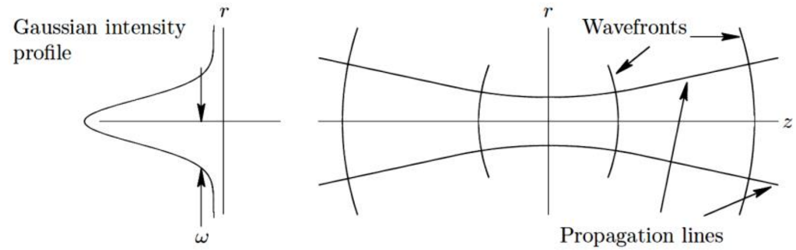
QUANTUM ELECTRONICS

For atomic physics

REVIEW

Gaussian Beam



$$\boxed{\nabla_t^2 \psi - 2ik \frac{\partial \psi}{\partial z} = 0,}$$

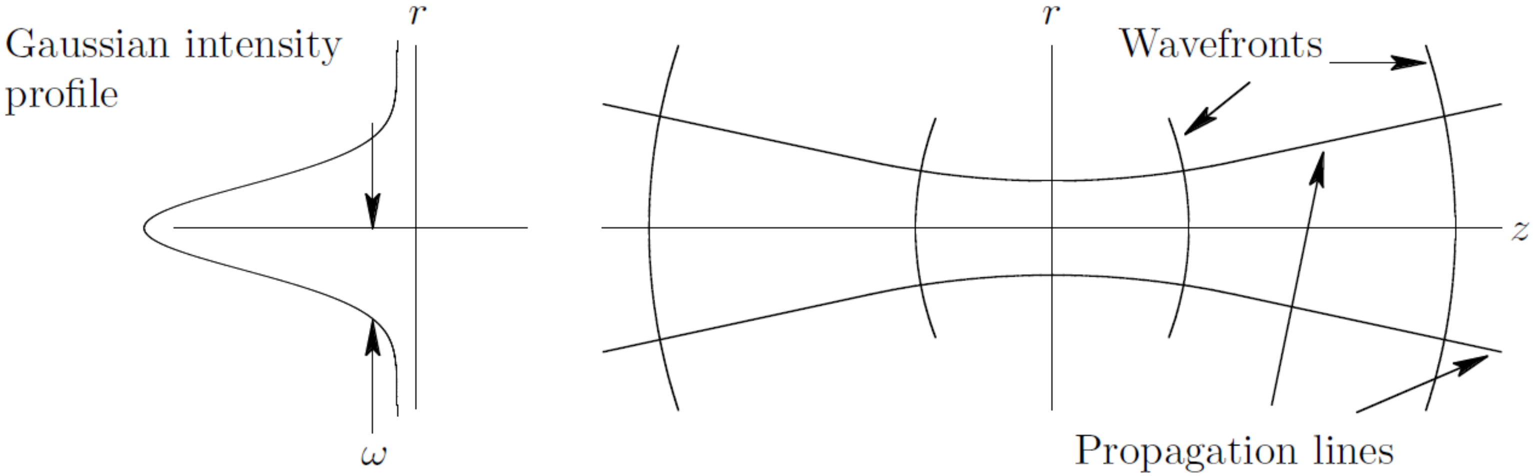
$$\psi(x, y, z) = \exp\left\{-i\left(P(z) + \frac{k}{2q(z)} r^2\right)\right\}$$

complex beam parameter q

$$\frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{n\pi\omega^2}$$

$$\omega(z) = \omega_0 \left[1 + \left(\frac{\lambda z}{n\pi\omega_0^2}\right)^2\right]^{1/2}$$

$$R(z) = z\left[1 + \left(\frac{n\pi\omega_0^2}{\lambda z}\right)^2\right].$$



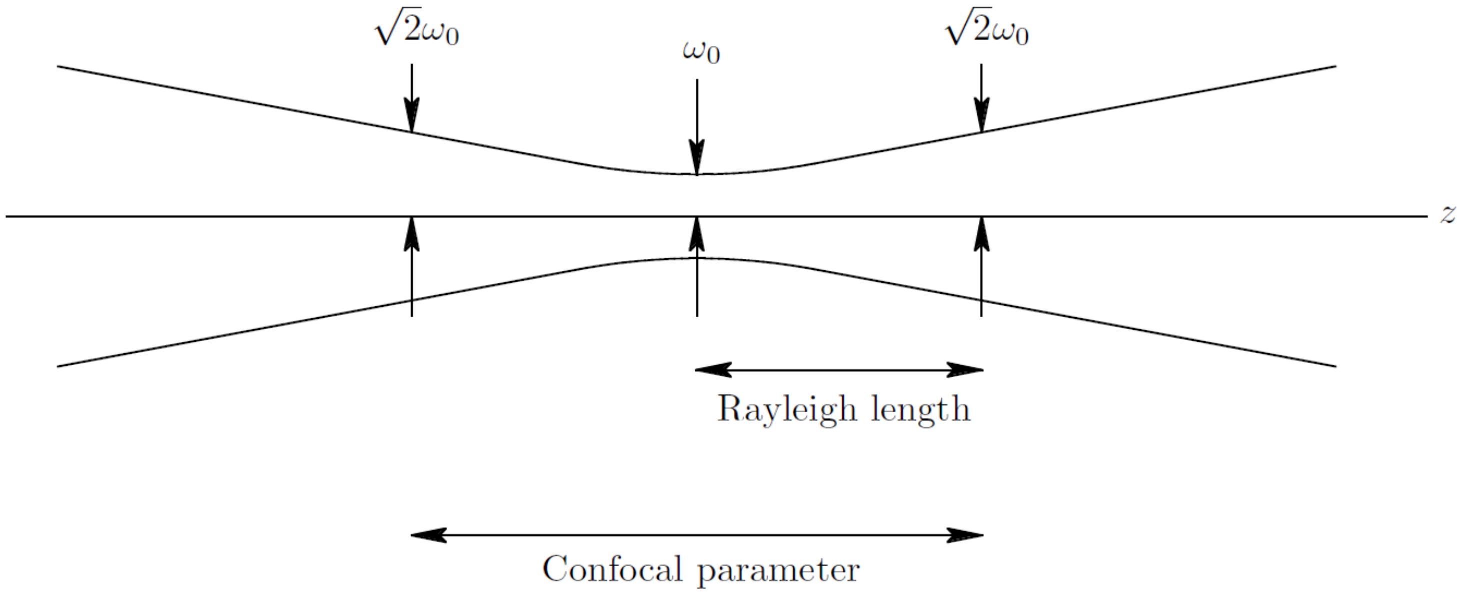
$$q(z) = q_0 + z = i\frac{n\pi\omega_0^2}{\lambda} + z \qquad \frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{n\pi\omega^2}$$

$$\text{Distance to waist } = -Re\{q(z)\} \quad \text{and,}$$

$$\text{Radius of waist } = \sqrt{\frac{\lambda}{n\pi} Im\{q(z)\}}.$$

$$\omega(z) = \omega_0 \left[1 + \left(\frac{\lambda z}{n\pi\omega_0^2}\right)^2\right]^{1/2}$$

$$R(z) = z\left[1 + \left(\frac{n\pi\omega_0^2}{\lambda z}\right)^2\right].$$



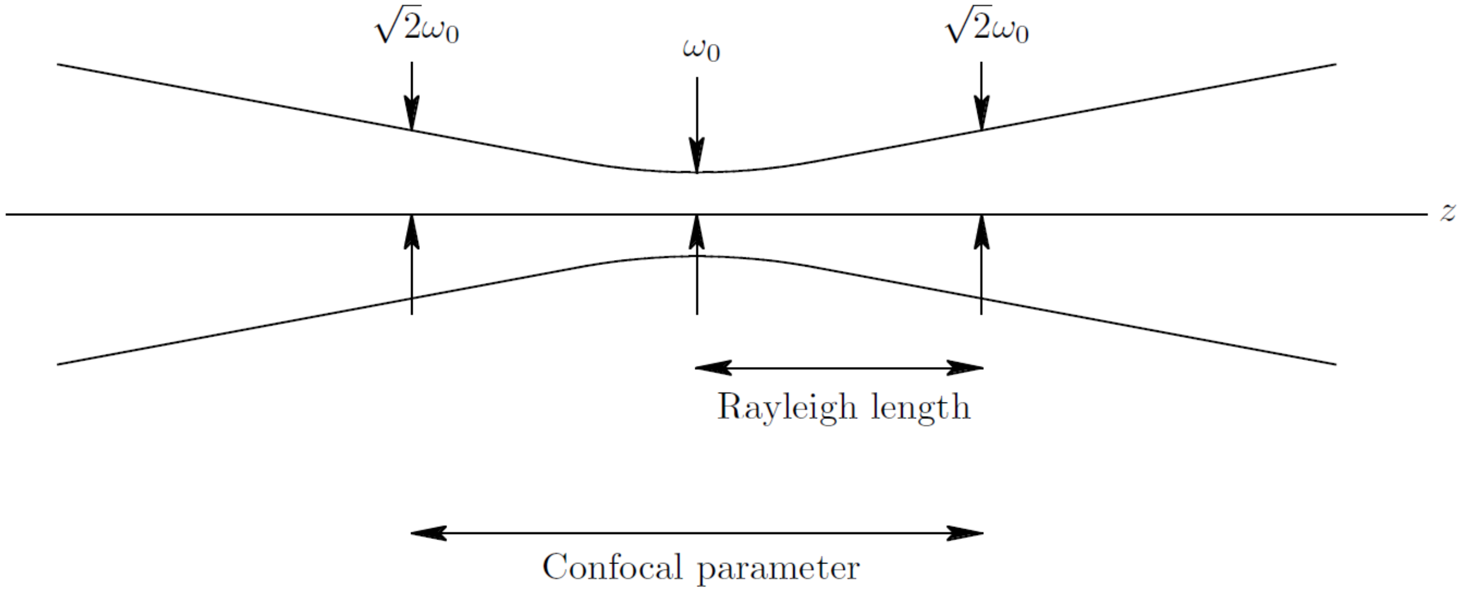
$$\text{Rayleigh length} \equiv z_R = \frac{n\pi\omega_0^2}{\lambda}$$

$$\text{Confocal parameter} \equiv b = \frac{2n\pi\omega_0^2}{\lambda}.$$

$$q = z + i\frac{n\pi\omega_0^2}{\lambda}$$
$$= z + iz_R$$

$$\omega(z) = \omega_0\left[1 + \left(\frac{z}{z_R}\right)^2\right]^{1/2}$$

$$R(z) = z + \frac{z_R^2}{z}.$$



$$\text{Distance to waist:}\quad z = -\frac{Re\{q_1\}}{|q_1|^2} \qquad \left(q_1 \equiv \frac{1}{q}\right)$$

$$\text{Rayleigh length:}\quad z_R = -\frac{Im\{q_1\}}{|q_1|^2}$$

$$\text{Waist size:}\quad \omega_0^2 = \left(\frac{\lambda}{n\pi}\right) z_R = -\left(\frac{\lambda}{n\pi}\right)\frac{Im\{q_1\}}{|q_1|^2}$$

$$\text{Spot size:}\quad \omega^2 = -\frac{\lambda}{n\pi Im\{q_1\}}.$$

$$\omega_0 = \frac{\lambda R \omega}{\sqrt{(\pi n \omega^2)^2 + (\lambda R)^2}}$$

1.5 THE PHASE TERM: GOUY PHASE

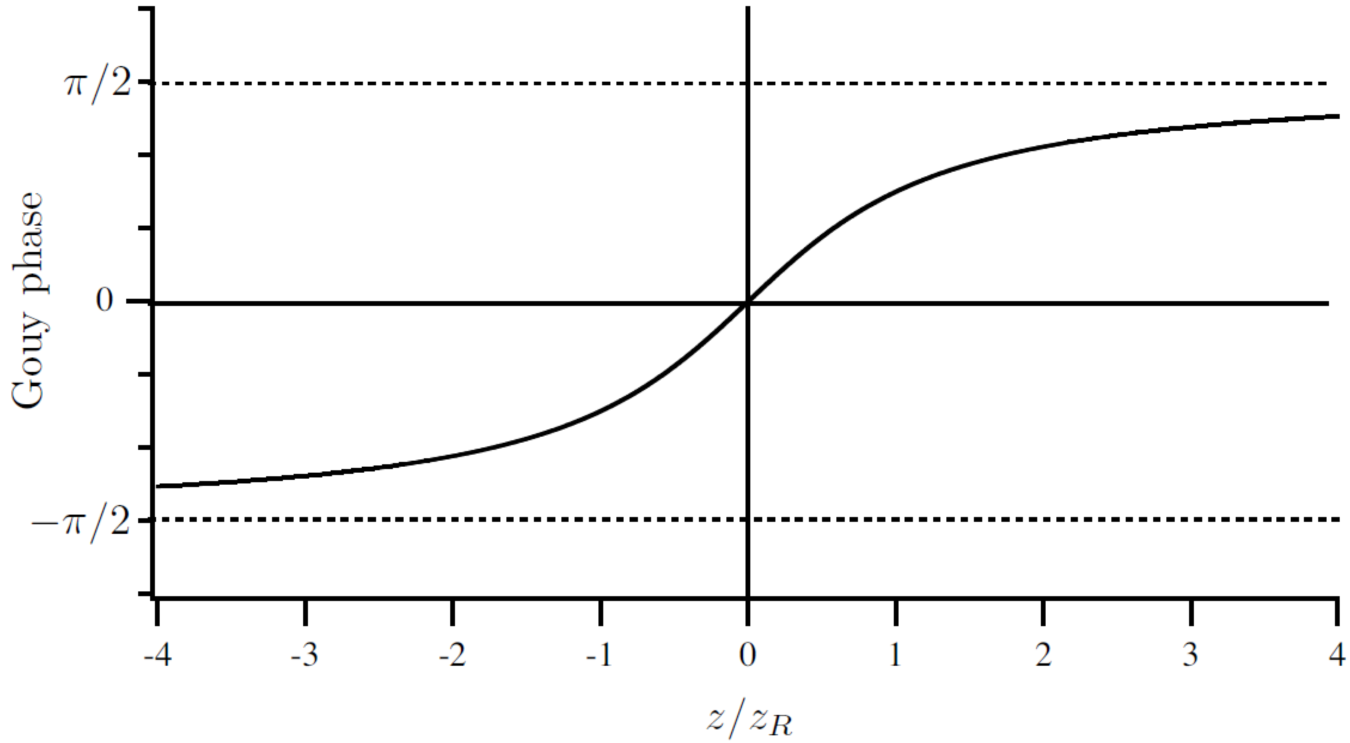
$$\psi(x,y,z) = \exp\left\{-i\left(P(z) + \frac{k}{2q(z)}r^2\right)\right\}$$

$$\frac{dP(z)}{dz} = -\frac{i}{q(z)} = -\frac{i}{z + i(n\pi\omega_0^2/\lambda)}.$$

$$iP(z) = \ln[1 - i(\lambda z/n\pi\omega_0^2)] = \ln\sqrt{1 + \left(\frac{\lambda z}{n\pi\omega_0^2}\right)^2} - i\tan^{-1}\left(\frac{\lambda z}{n\pi\omega_0^2}\right)$$

$$u(x,y,z) = \psi(x,y,z)e^{-ikz}$$

$$u = \frac{1}{\sqrt{1 + \left(\frac{z}{z_R}\right)^2}}\exp\left\{i\tan^{-1}\left(\frac{z}{z_R}\right) - ik\left(z + \frac{r^2}{2R}\right) - \frac{r^2}{\omega^2}\right\}$$



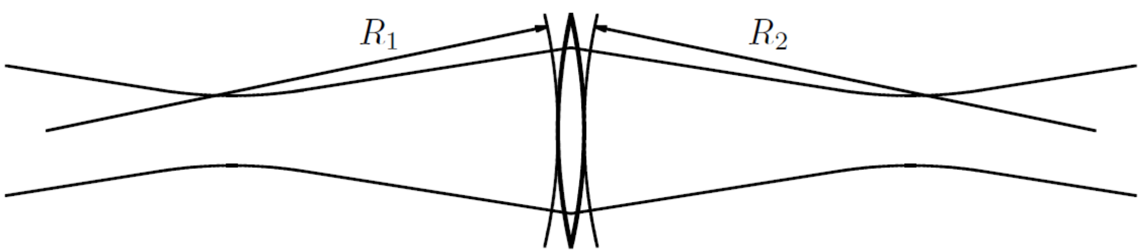
$$u = \frac{1}{\sqrt{1+\left(\frac{z}{z_R}\right)^2}} \exp\left\{ i\tan^{-1}\left(\frac{z}{z_R}\right) - ik\left(z + \frac{r^2}{2R}\right) - \frac{r^2}{\omega^2} \right\}$$

$$\text{Phase shift of Gaussian beam} = i\phi = \overbrace{ikz}^{\text{normal}} \overbrace{- i\tan^{-1}\frac{z}{z_R}}^{\text{Gouy}}$$

1.6 SIMPLE TRANSFORMATION PROPERTIES OF THE COMPLEX BEAM PARAMETER

Free space $$q(z_2) = q(z_1) + z_2 - z_1$$

Thin Lens



$$\frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{n\pi\omega^2} \qquad \frac{1}{i} + \frac{1}{o} = \frac{1}{f} \longrightarrow \frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{f}$$

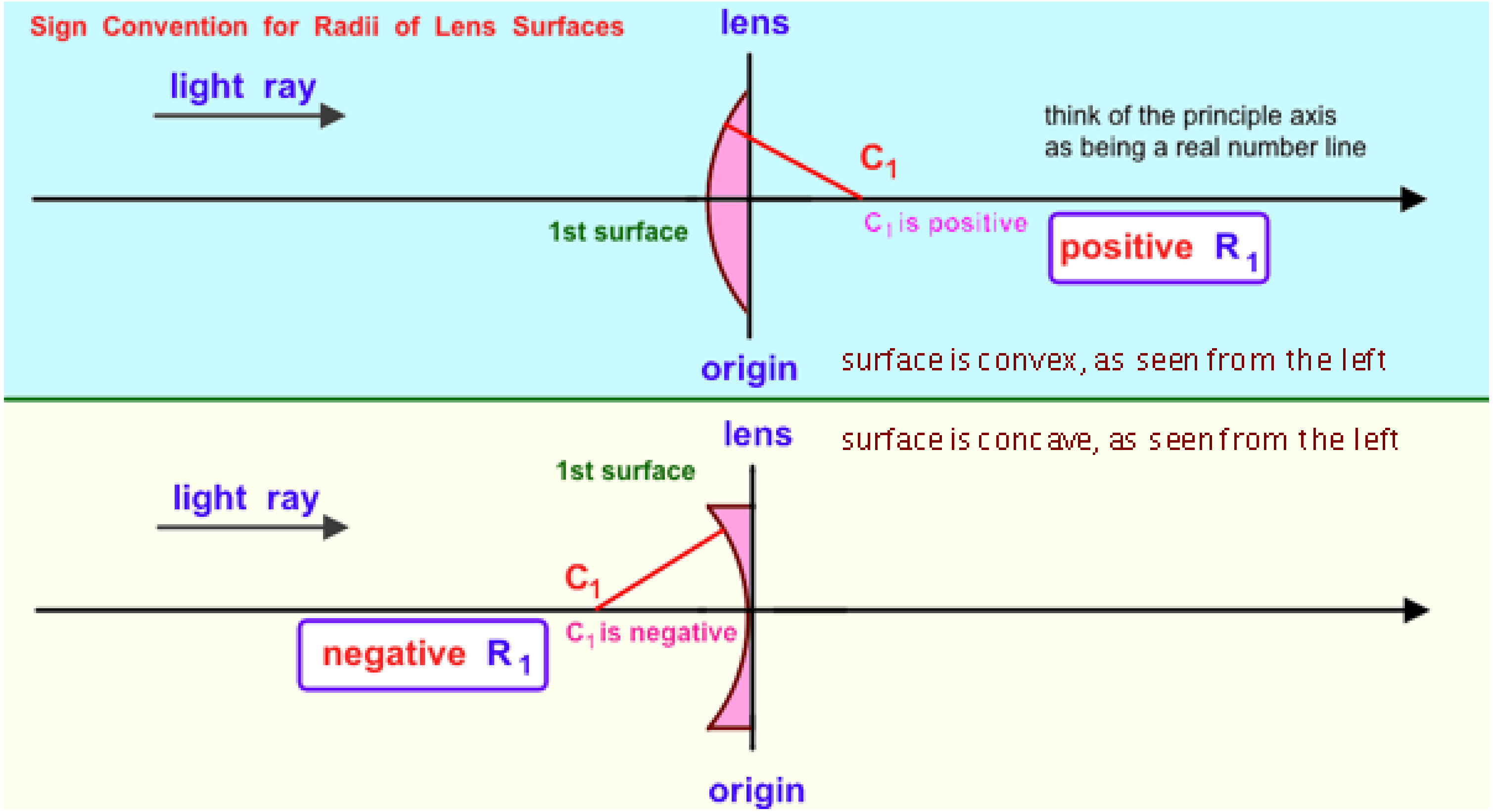
$$\text{Since } \omega_1 = \omega_2, \quad \frac{1}{q_1} - \frac{1}{q_2} = \frac{1}{f} \longrightarrow q_2 = \frac{q_1}{1 - q_1/f}$$

RULES FOR SIGN

- We are taking the frame along the propagation direction of the light.
- Positive *f* of a lens (*R* of a mirror) is related to the real image.
- Negative *f* of a lens (*R* of a mirror) is related to the virtual image.
- *R* of Gaussian beam is different from the radius of curvature of optics.

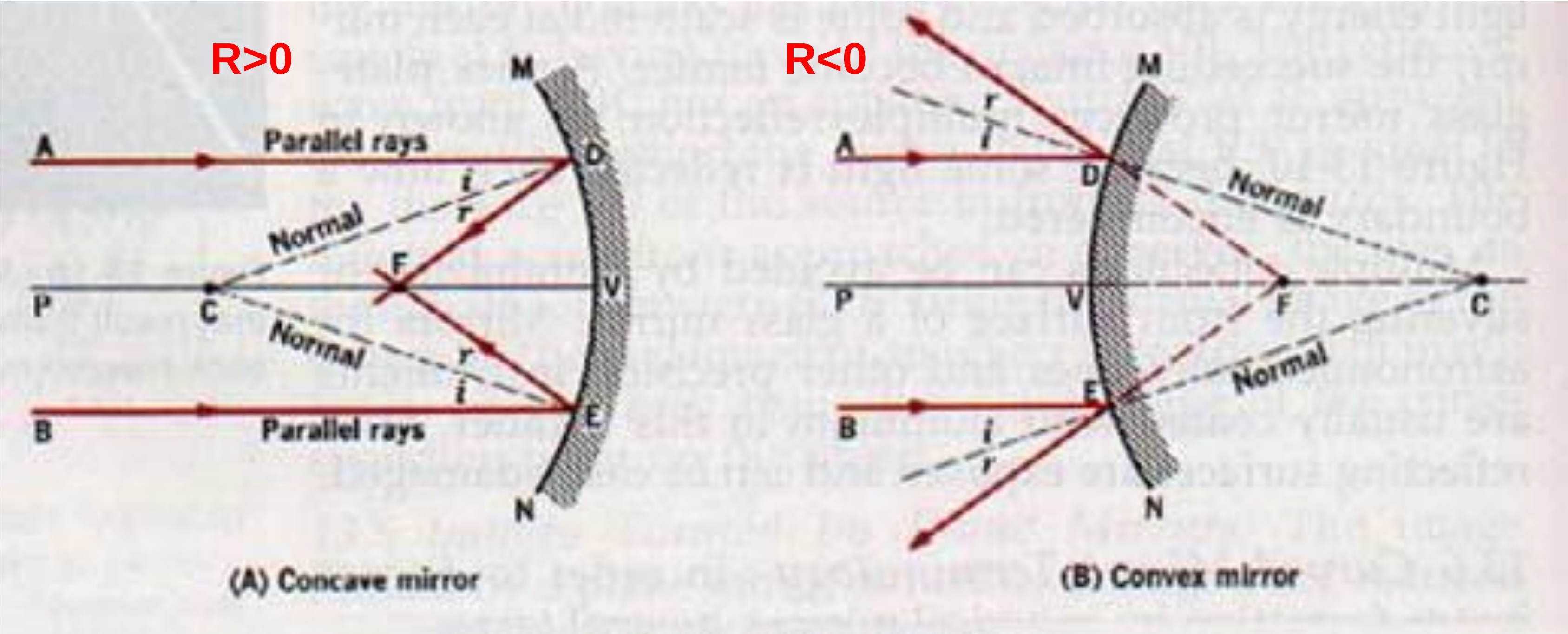
***f* >0**

***f* <0**



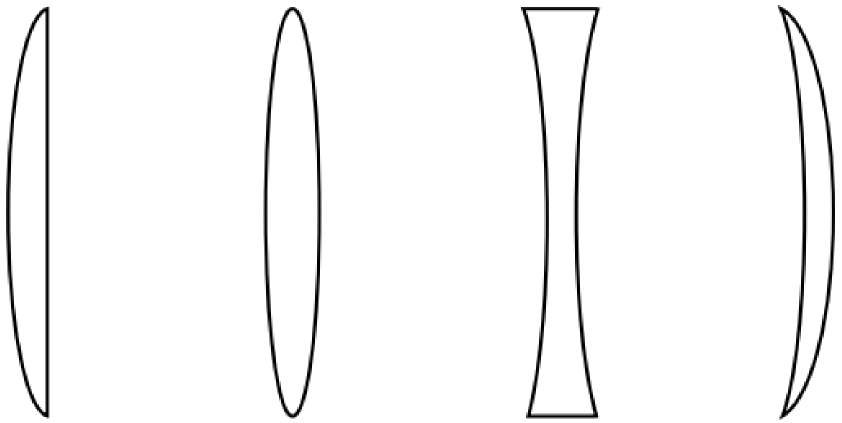
RULES FOR SIGN

- We are taking the frame along the propagation direction of the light.
- Positive *f* of a lens (*R* of a mirror) is related to the real image.
- Negative *f* of a lens (*R* of a mirror) is related to the virtual image.
- *R* of Gaussian beam is different from the radius of curvature of optics.



RULES FOR SIGN

- We are taking the frame along the propagation direction of the light.
- Positive *f* of a lens (*R* of a mirror) is related to the real image.
- Negative *f* of a lens (*R* of a mirror) is related to the virtual image.
- *R* of Gaussian beam is different from the radius of curvature of optics.



plano-convex	biconvex	biconcave	meniscus
$R_1>0$	$R_1>0$	$R_1<0$	$R_1 < 0$
$R_2=\infty$	$R_2<0$	$R_2>0$	$R_2 < 0$

Thin Spherical Mirror

$$q_2 = \frac{q_1}{1 - 2q_2/R}$$

Since, $f = R/2$

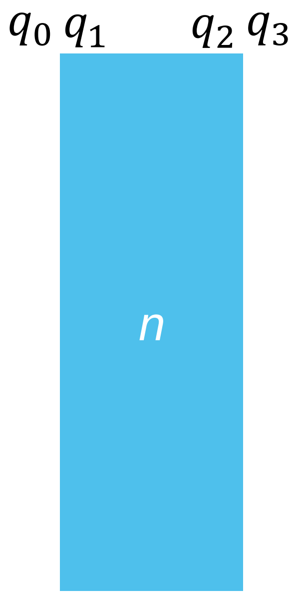
Slap of thickness, d, and index, n

At the interface, $q_1 = nq_0$

The internal travel, $q_2 = q_1 + d$

At the other interface, $q_3 = \frac{1}{n}q_2$

The total $q_3 = q_0 + \frac{d}{n}$



Case	q-transformation	Ray matrix
Free-space	$q_1 = q_0 + d = \dfrac{(1)q_0 + (d)}{(0)q_0 + (1)}$	$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$
Thin lens	$q_1 = \dfrac{q_0}{1 - q_0/f} = \dfrac{(1)q_0 + (0)}{(-1/f)q_0 + (1)}$	$\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$
Mirror	$q_1 = \dfrac{q_0}{1 - 2q_0/R} = \dfrac{(1)q_0 + (0)}{(-2/R)q_0 + (1)}$	$\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}$
Slab	$q_1 = q_0 + d/n = \dfrac{(1)q_0 + (d/n)}{(0)q_1 + (1)}$	$\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$

1.7 MATRIX FORMULATION OF PARAXIAL RAY OPTICS: ABCD RULE

Ray vector: $\begin{pmatrix} y \\ y' \end{pmatrix}$ Height y, Slope y'

$$q_1 = \frac{Aq_0 + B}{Cq_0 + D} \Longleftrightarrow \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

COMPOUND OPTICS – ABCD MATRIX

- The ray tracing technique is based on two reference planes, called the input and output planes, each perpendicular to the optical axis of the system.
- A light ray enters the system when the ray crosses the input plane at a distance y1 from the optical axis while traveling in a direction that makes an angle y’1 with the optical axis.
- Some distance further along, the ray crosses the output plane, this time at a distance y2 from the optical axis and making an angle y’2.

, where y’1, y’2 <<1

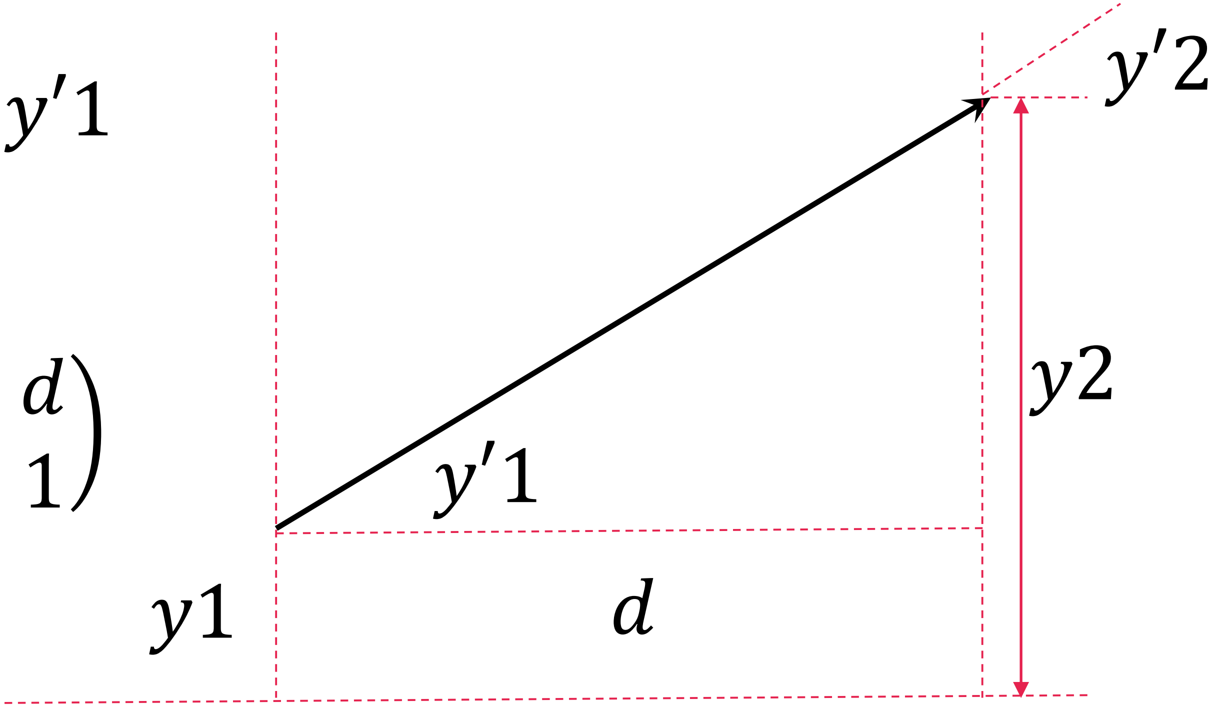
$$\begin{pmatrix} y2 \\ y'2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y1 \\ y'1 \end{pmatrix}$$

ABCD MATRIX – SPACE

$$\begin{pmatrix} y2 \\ y'2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y1 \\ y'1 \end{pmatrix}$$

- $y2 = y1 + d\, y'1$
- $y'2 = y'1$

$$\therefore \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

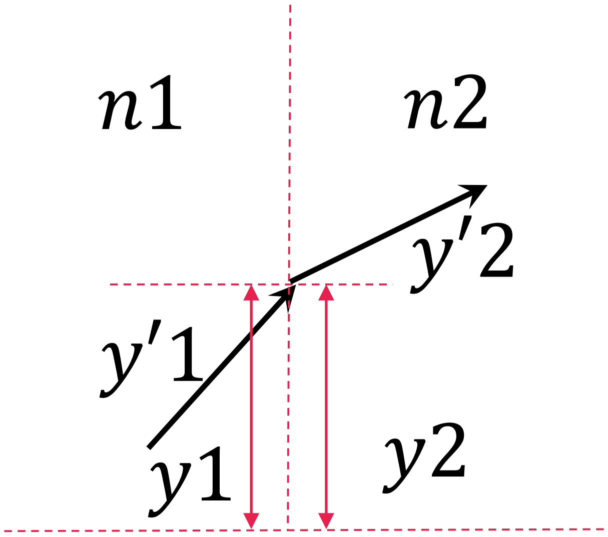


ABCD MATRIX – INTERFACE

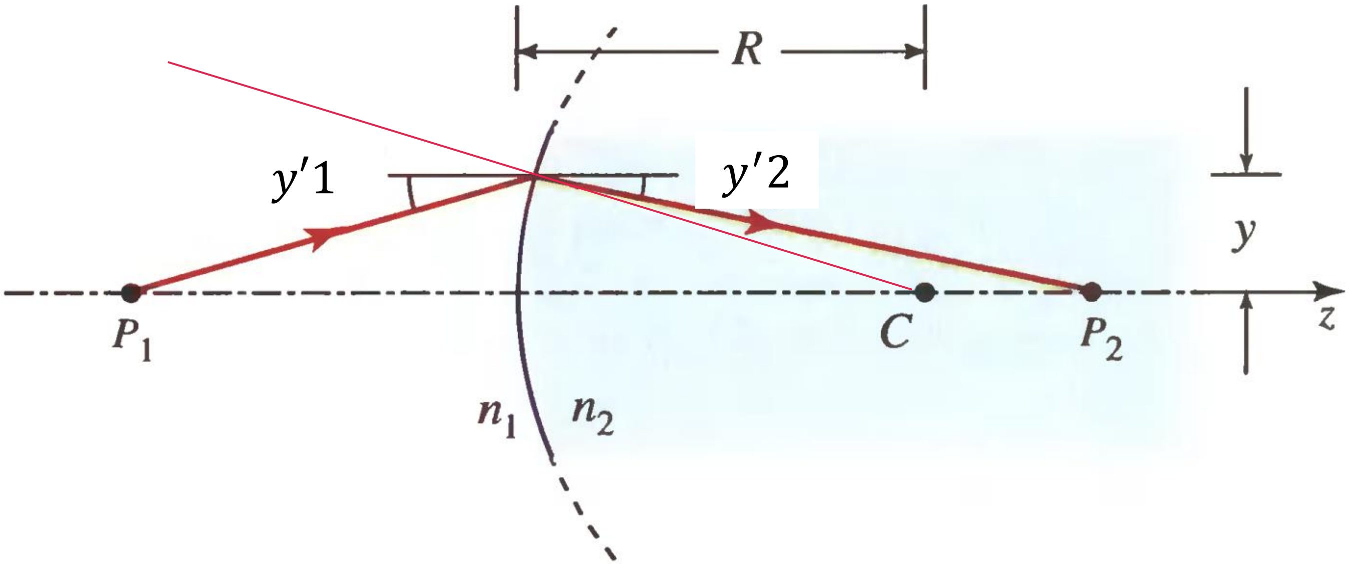
$$\begin{pmatrix} y2 \\ y'2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y1 \\ y'1 \end{pmatrix}$$

- $y2 = y1$
- $n2\, y'2 = n1\, y'1$

$$\therefore \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & n1/n2 \end{pmatrix}$$



ABCD MATRIX – CURVED INTERFACE



- $y2 = y1$
- $y'2 = \frac{n1 - n2}{R\,n2} y1 + \frac{n1}{n2}\, y'1$

$$\therefore \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \dfrac{n1 - n2}{R\,n2} & n1/n2 \end{pmatrix}$$

Element	Matrix	Remarks
Propagation in free space or in a medium of constant refractive index	$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$	d = distance
Refaction at a flat interface	$\begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$	n_1 = initial refractive index n_2 = final refractive index
Refaction at a curved interface	$\begin{pmatrix} 1 & 0 \\ \frac{n_1 - n_2}{R \cdot n_2} & \frac{n_1}{n_2} \end{pmatrix}$	R = radius of curvature, R > for convex (centre of curvature after interface) n_1 = initial refractive index n_2 = final refractive index
Reflection from a flat mirror	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	Identity matrix
Reflection from a curved mirror	$\begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix}$	R = radius of curvature, R > 0 for convex
Thin lens	$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$	f = focal length of lens where f > 0 for convex/positive (converging) lens. Valid if and only if the focal length is

Homework: derive ABCD matrix for a curved mirror, a thin lens and a thick lens

$$q_1 = \frac{Aq_0 + B}{Cq_0 + D} \iff \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

Case	***q*-transformation**	**Ray matrix**
Free-space	$q_1 = q_0 + d = \dfrac{(1)q_0 + (d)}{(0)q_0 + (1)}$	$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$
Thin lens	$q_1 = \dfrac{q_0}{1 - q_0/f} = \dfrac{(1)q_0 + (0)}{(-1/f)q_0 + (1)}$	$\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$
Mirror	$q_1 = \dfrac{q_0}{1 - 2q_0/R} = \dfrac{(1)q_0 + (0)}{(-2/R)q_0 + (1)}$	$\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}$
Slab	$q_1 = q_0 + d/n = \dfrac{(1)q_0 + (d/n)}{(0)q_1 + (1)}$	$\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$

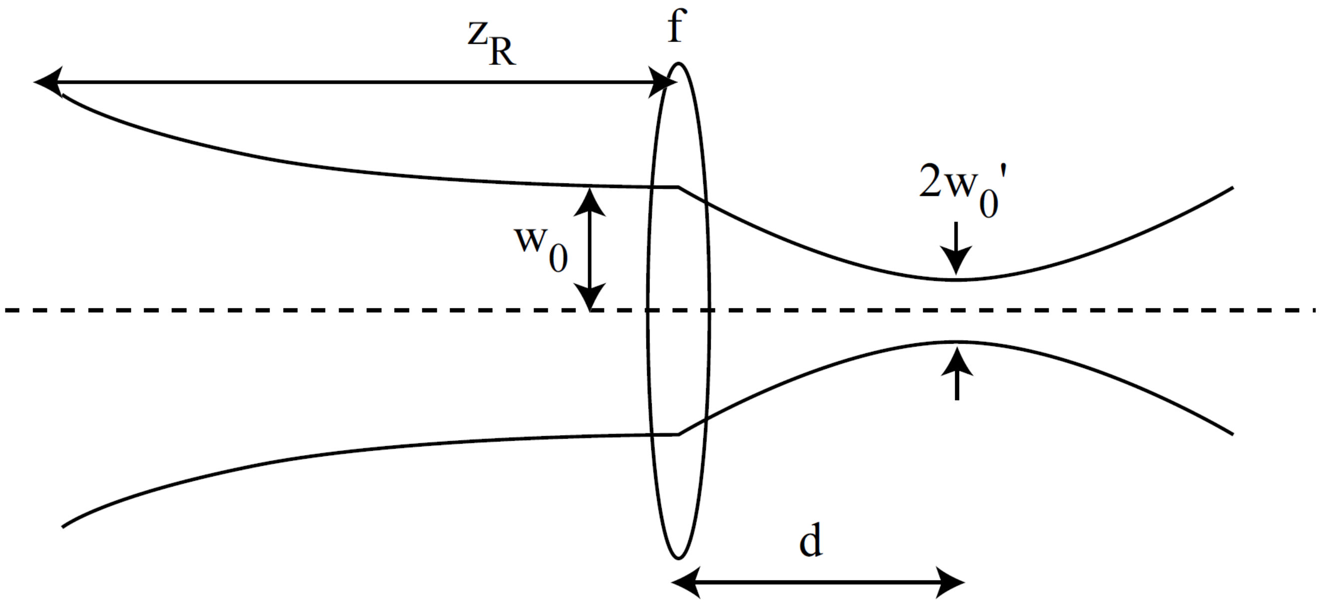
ABCD rule: The overall q-transformation for a complex system composed of thin lenses, thin mirrors, free spaces and slabs can be obtained by determining the ABCD matrices of the individual components, multiplying the matrices together and applying to the composite matrix.

$$q_0 \xrightarrow{ABCD} q_1 \xrightarrow{A'B'C'D'} q_2$$

$$q_0 \longrightarrow q_1 : \begin{pmatrix} A & B \\ C & D \end{pmatrix} \qquad q_1 \longrightarrow q_2 : \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}$$

$$q_2 = \frac{A'q_1 + B'}{C'q_1 + D'} = \frac{(A'A + B'C)q_0 + (A'B + B'D)}{(C'A + D'C)q_0 + (C'B + D'D)}$$

GAUSSIAN BEAM PROPAGATION



Homework: By using ABCD matrix, find the q-parameters after passing through the lens with focal length f depending on d. Find the new beam waist w_0' and the distance to it. When Z_R>>f, find the w_0' and distance to it in terms of f, w0, and λ.

PROPER USAGE OF PLANO-CONVEX LENS

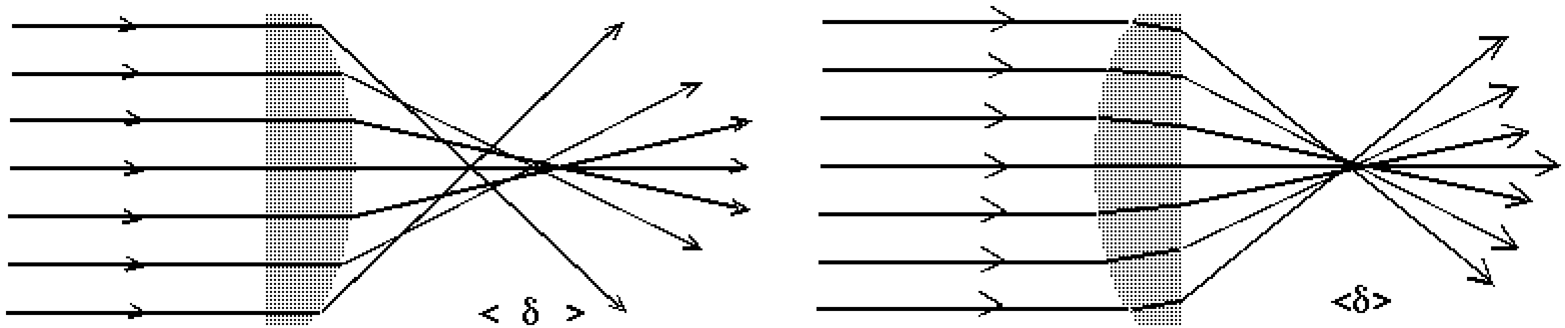


Figure 2.9: Spread in focal length for plano-convex lens with the two possible orientations. For the left orientation the spread of the focal points are larger than for the one to the right.

Homework: Show that one of them give smaller beam waist.