QUANTUM ELECTRONICS

For atomic physics

REVIEW

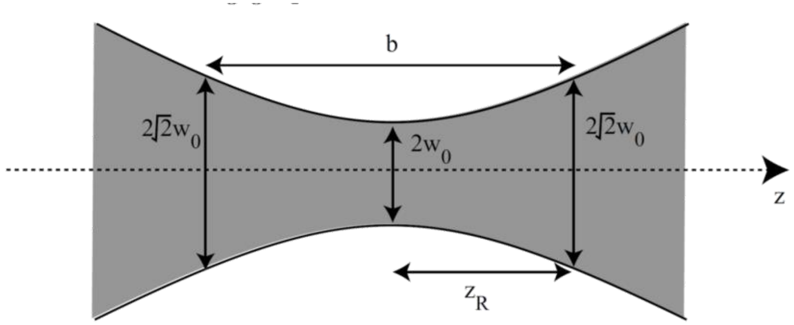
Gaussian Beam

$$\boxed{\nabla_t^2 \psi - 2ik\frac{\partial \psi}{\partial z} = 0,}$$

$$\psi(x, y, z) = \exp\left\{-i\left(P(z) + \frac{k}{2q(z)}r^2\right)\right\}$$

complex beam parameter *q*

$$\frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{n\pi\omega^2}$$



$$\omega(z) = \omega_0\left[1 + \left(\frac{z}{z_R}\right)^2\right]^{1/2}$$

$$R(z) = z + \frac{z_R^2}{z}.$$

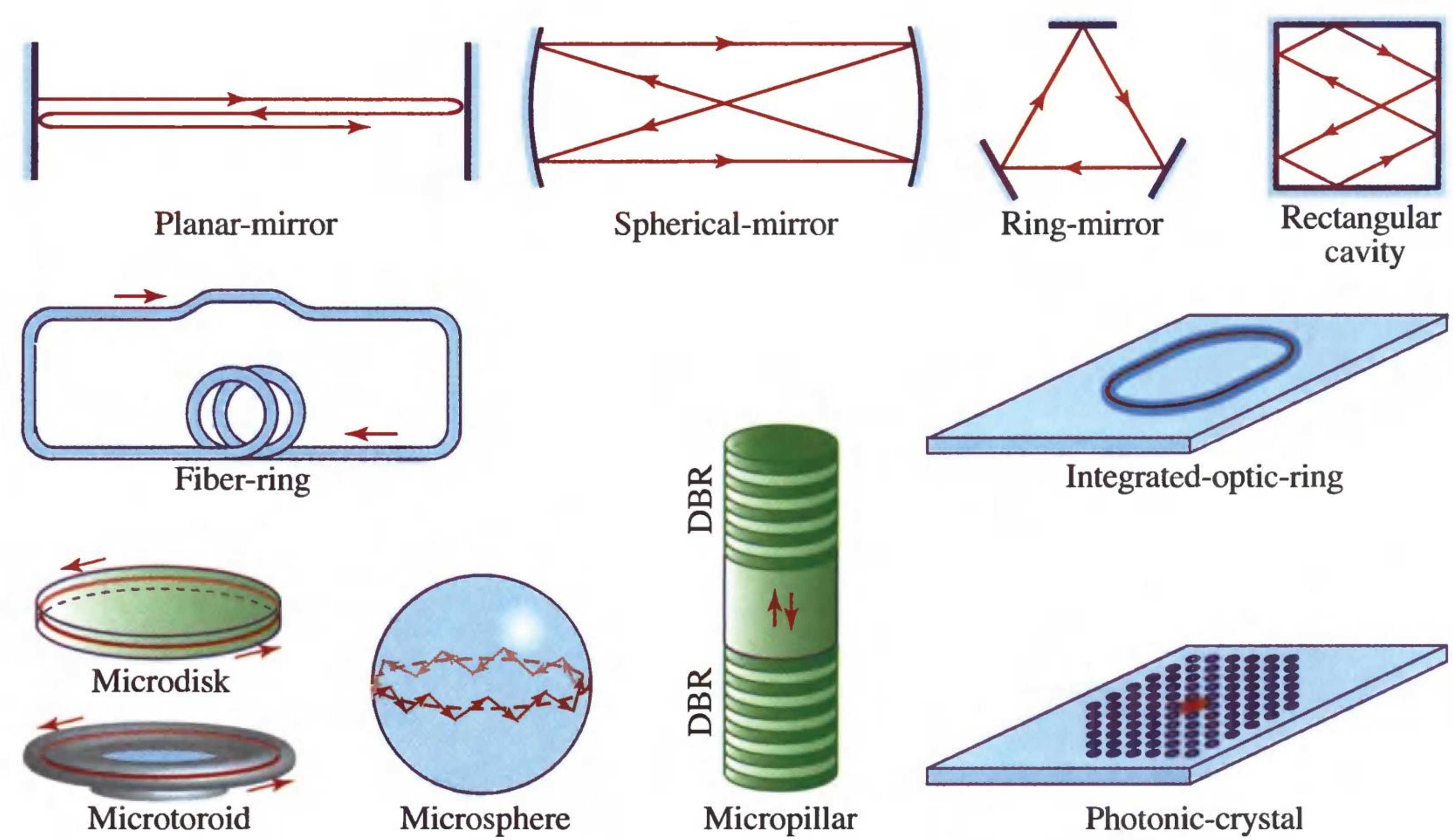
$$z_R = \frac{n\pi\omega_0^2}{\lambda}$$

Case	q-transformation	Ray matrix
Free-space	$q_1 = q_0 + d = \dfrac{(1)q_0 + (d)}{(0)q_0 + (1)}$	$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$
Thin lens	$q_1 = \dfrac{q_0}{1 - q_0/f} = \dfrac{(1)q_0 + (0)}{(-1/f)q_0 + (1)}$	$\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$
Mirror	$q_1 = \dfrac{q_0}{1 - 2q_0/R} = \dfrac{(1)q_0 + (0)}{(-2/R)q_0 + (1)}$	$\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}$
Slab	$q_1 = q_0 + d/n = \dfrac{(1)q_0 + (d/n)}{(0)q_1 + (1)}$	$\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$

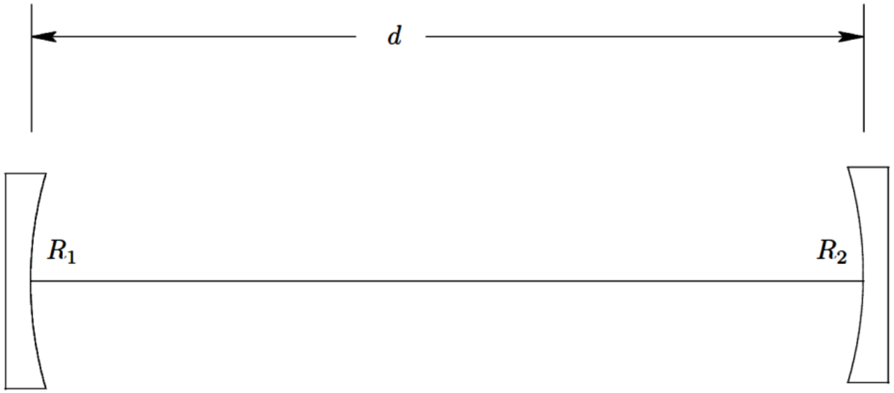
$$q_1 = \frac{Aq_0 + B}{Cq_0 + D} \Longleftrightarrow \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$q_0 \xrightarrow{ABCD} q_1 \xrightarrow{A'B'C'D'} q_2$$

2. OPTICAL RESONATORS –
GEOMETRICAL PROPERTIES

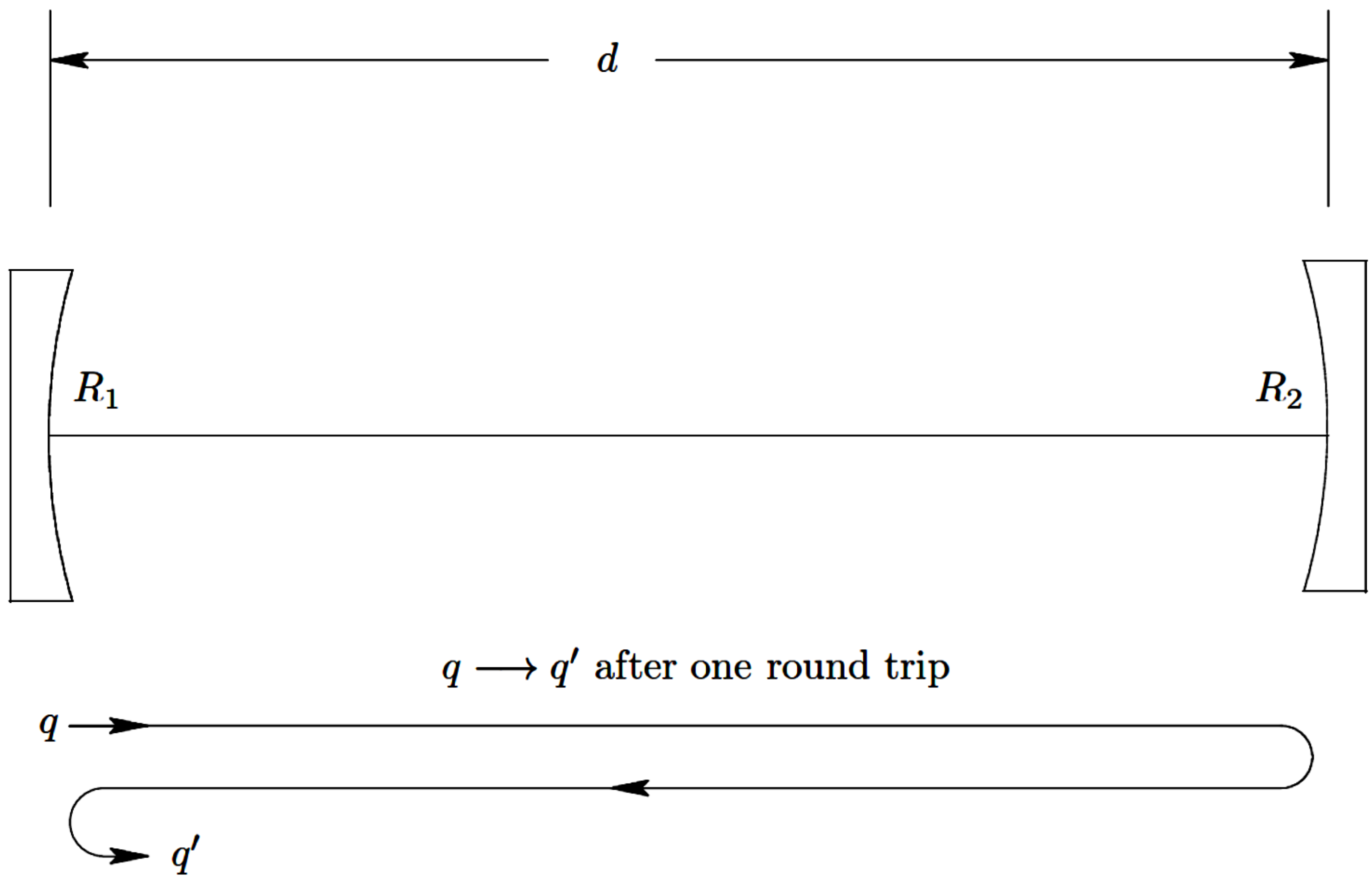


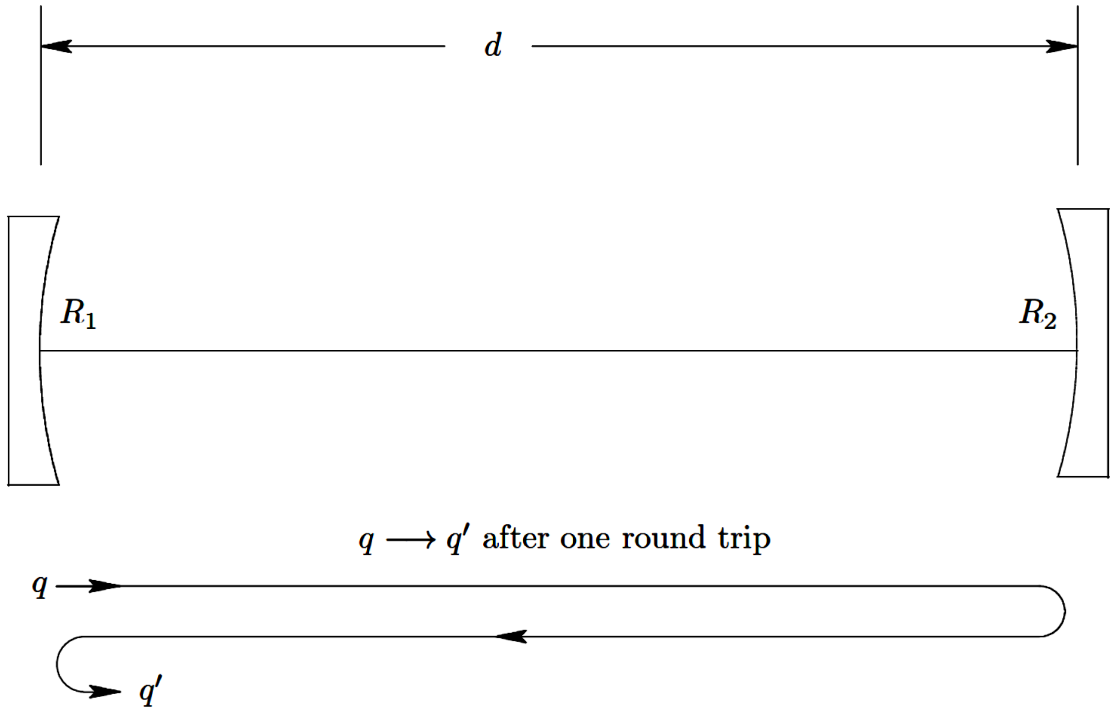
2.1 INTRODUCTION
APPLICATIONS OF OPTICAL CAVITIES



- Feedback mechanism in laser oscillators
- Optical spectrum Analyzer
- Stable frequency reference for laser stabilization
- Enhancement for the second harmonic generation

2.2 THE TWO – MIRROR STANDING WAVE CAVITY





- Self-consistency requires

$$q = \frac{Aq + B}{Cq + D} \quad \text{or} \quad \frac{1}{q} = \frac{C + D\left(\frac{1}{q}\right)}{A + B\left(\frac{1}{q}\right)}$$

$$B\left(\frac{1}{q}\right)^2 + (A - D)\left(\frac{1}{q}\right) - C = 0,$$

$$q_1 = \frac{1}{q} = \frac{D - A}{2B} \pm \frac{1}{2B}\sqrt{(A - D)^2 + 4BC}$$

2.3 STABILITY

$$q_1 = \frac{1}{q} = \frac{D-A}{2B} \pm \frac{1}{2B}\sqrt{(A-D)^2 + 4BC}$$

$$\frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{n\pi\omega^2}$$

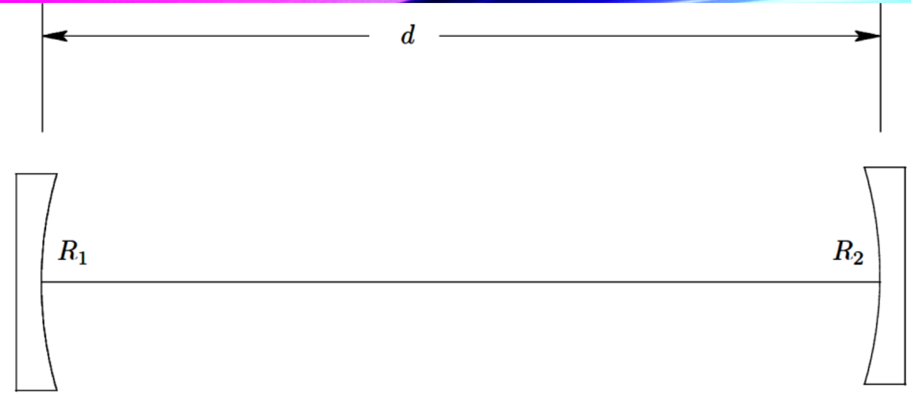
$$R = \frac{2B}{D-A}$$

$$\omega = \sqrt{\frac{2\lambda|B|}{n\pi\sqrt{4-(A+D)^2}}}$$

Here we use the fact, the determinant of the ABCD matrix is unity,

$$AD - BC = 1$$

$$\text{Stability criterion:} \quad |A+D| \le 2$$



$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{2}{R_1} & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{2}{R_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

$$g_1 \equiv 1 - \frac{d}{R_1} \qquad g_2 \equiv 1 - \frac{d}{R_2}$$

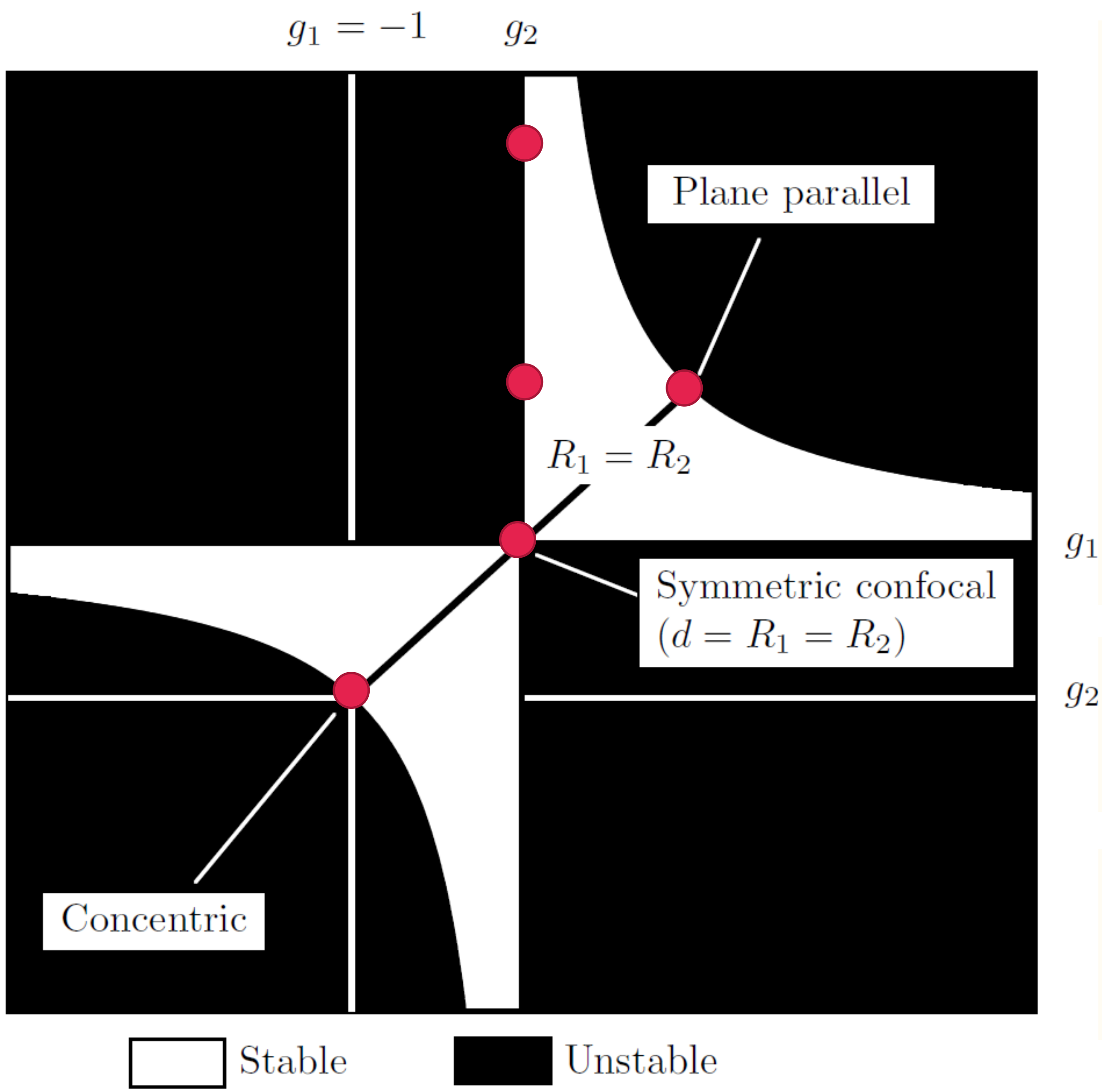
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{2}{d}(g_1 - 1) & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{2}{d}(g_2 - 1) & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

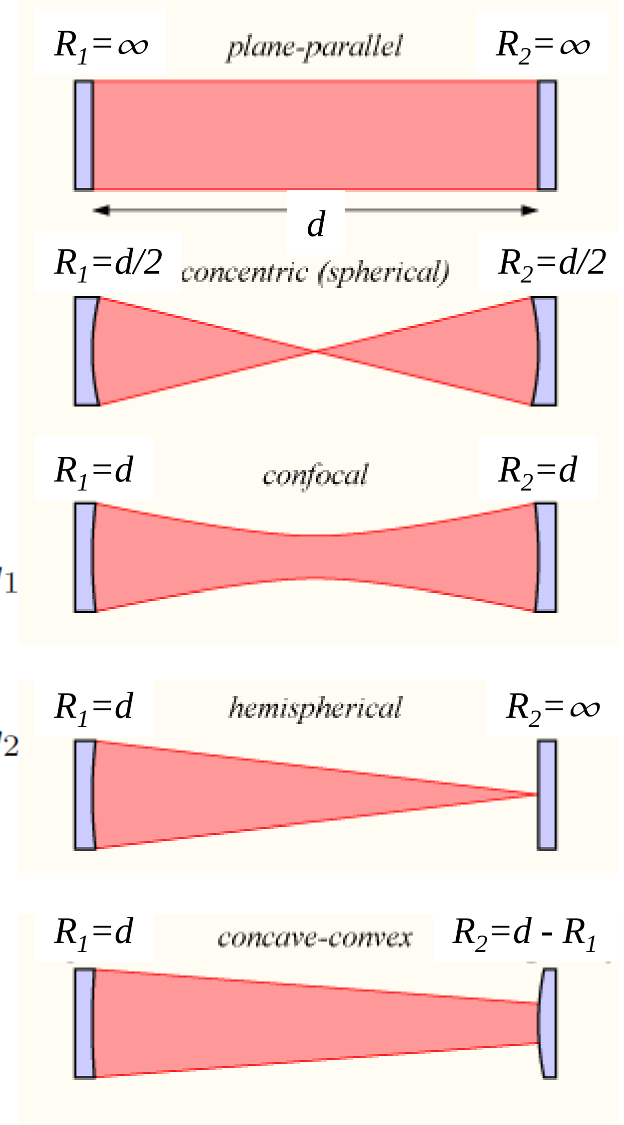
$$= \begin{pmatrix} 2g_2 - 1 & 2g_2 d \\ \frac{2}{d}(2g_1 g_2 - g_1 - g_2) & 4g_1 g_2 - 2g_2 - 1 \end{pmatrix}.$$

$$|A + D| \leq 2 \Longrightarrow |4g_1 g_2 - 2| \leq 2$$

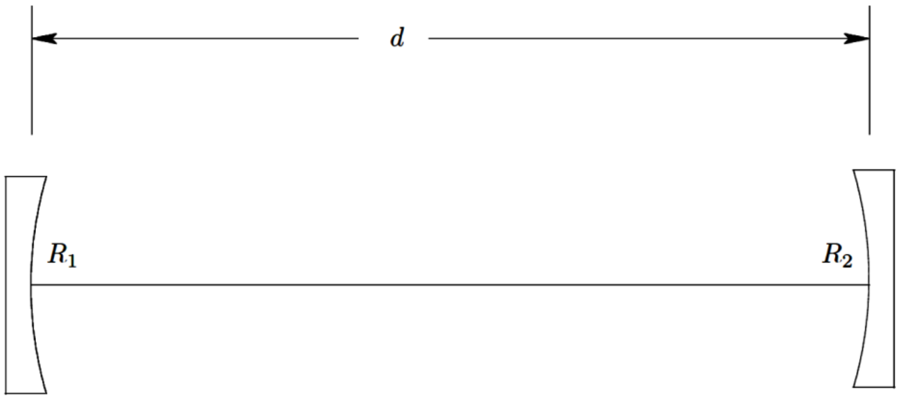
$$\text{Stability criterion:} \qquad 0 \leq g_1 g_2 \leq 1$$

Stability criterion: $0 \le g_1 g_2 \le 1$ $g_1 \equiv 1 - \frac{d}{R_1}$ $g_2 \equiv 1 - \frac{d}{R_2}$





2.4 SOLUTION FOR AN ARBITRARY TWO – MIRROR STABLE CAVITY



$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$= \begin{pmatrix} 2g_2 - 1 & 2g_2 d \\ \frac{2}{d}(2g_1g_2 - g_1 - g_2) & 4g_1g_2 - 2g_2 - 1 \end{pmatrix}.$$

$$\frac{1}{q} = \frac{D-A}{2B} \pm \frac{1}{2B}\sqrt{(A-D)^2 + 4BC} \qquad \sqrt{(A+D)^2 - 4}$$

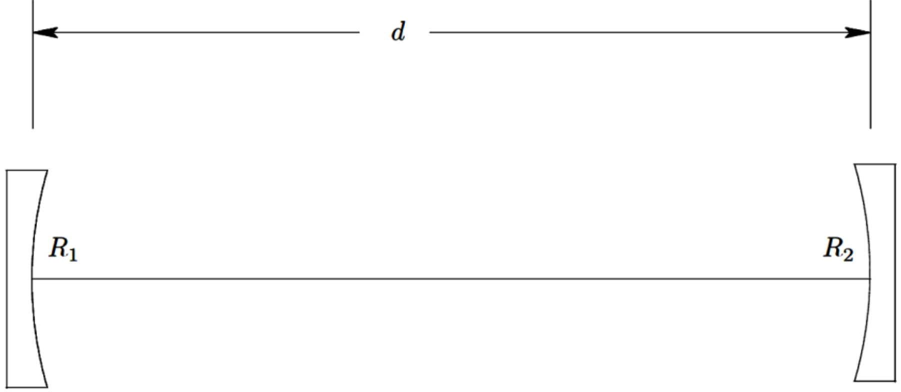
$$\frac{1}{q} = \frac{g_1 - 1}{d} \pm \frac{1}{g_2 d}\sqrt{g_1g_2(g_1g_2 - 1)}$$

$$\omega^2 = \left(\frac{\lambda d}{n\pi}\right)\sqrt{\frac{g_2}{g_1(1 - g_1g_2)}}$$

R=?

$$g_1 \equiv 1 - \frac{d}{R_1} \quad g_2 \equiv 1 - \frac{d}{R_2}$$

Homework #1: find the radius of curvature and with in terms of z and draw both with certain values of R1,R2, d, and λ.



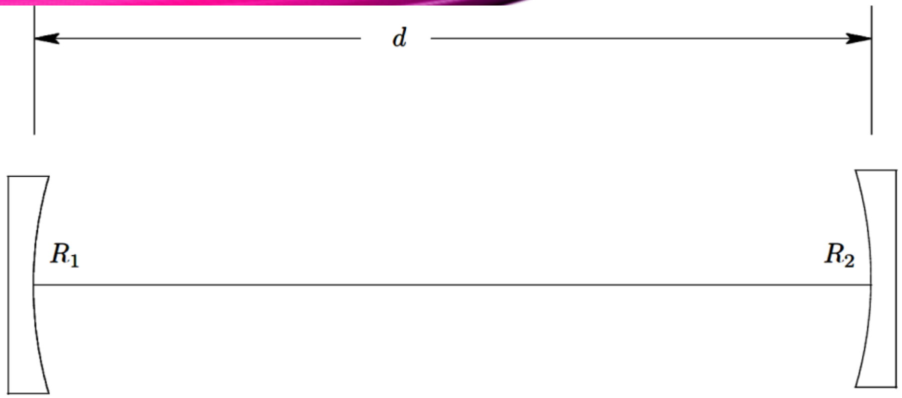
$$\frac{1}{q} = \frac{g_1 - 1}{d} \pm \frac{1}{g_2 d}\sqrt{g_1 g_2 (g_1 g_2 - 1)}$$
$$= \frac{g_1 - 1}{d} \pm i\,\frac{1}{g_2 d}\sqrt{g_1 g_2 (1 - g_1 g_2)}$$

$$q(z) = q_0 + z = i\frac{n\pi\omega_0^2}{\lambda} + z \qquad \frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{n\pi\omega^2}$$

$$\text{Rayleigh length:} \quad z_R = -\frac{Im\{q_1\}}{|q_1|^2} \qquad \left(q_1 \equiv \frac{1}{q}\right)$$

$$\text{Waist size:} \quad \omega_0^2 = \left(\frac{\lambda}{n\pi}\right) z_R = -\left(\frac{\lambda}{n\pi}\right)\frac{Im\{q_1\}}{|q_1|^2}$$

$$z_R = d\frac{\sqrt{g_1 g_2 (1 - g_1 g_2)}}{g_1 + g_2 - 2g_1 g_2} \qquad \omega_0^2 = \left(\frac{\lambda d}{n\pi}\right)\frac{\sqrt{g_1 g_2 (1 - g_1 g_2)}}{g_1 + g_2 - 2g_1 g_2}$$



$$\frac{1}{q} = \frac{g_1 - 1}{d} \pm \frac{1}{g_2 d}\sqrt{g_1 g_2 (g_1 g_2 - 1)}$$
$$= \frac{g_1 - 1}{d} \pm i\frac{1}{g_2 d}\sqrt{g_1 g_2 (1 - g_1 g_2)}$$

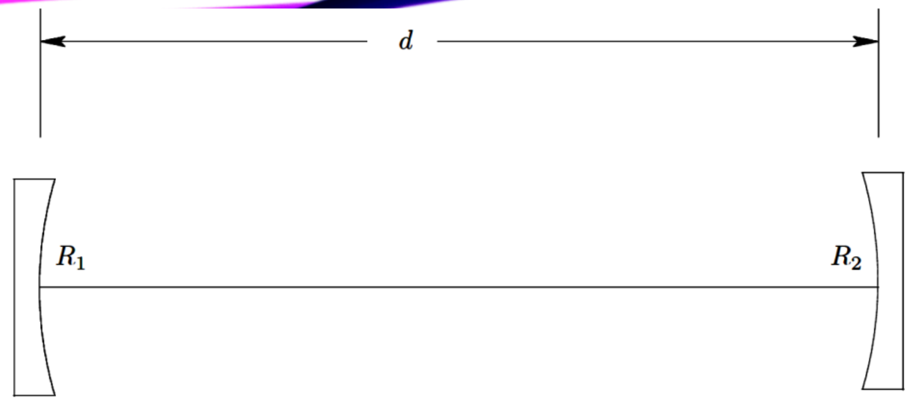
$$q(z) = q_0 + z = i\frac{n\pi\omega_0^2}{\lambda} + z \qquad \frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{n\pi\omega^2}$$

Distance to waist $= -Re\{q(z)\}$, Distance from waist = $Re\{q(z)\}$

Distance to waist: $z = -\dfrac{Re\{q_1\}}{|q_1|^2}$ $\qquad \left(q_1 \equiv \dfrac{1}{q}\right)$

$$z_1 = d\frac{g_2(g_1 - 1)}{g_1 + g_2 - 2g_1 g_2} \qquad \because z_2 - z_1 = d$$
$$z_2 = d\frac{g_1(1 - g_2)}{g_1 + g_2 - 2g_1 g_2}$$

$$g_1 \equiv 1 - \frac{d}{R_1} \quad g_2 \equiv 1 - \frac{d}{R_2}$$

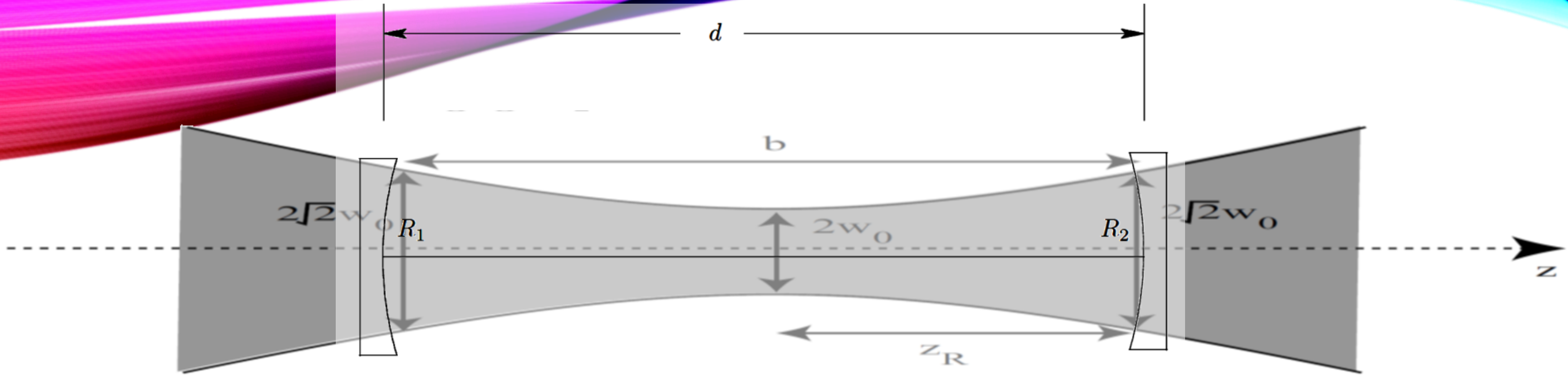


$$\omega^2 = \left(\frac{\lambda d}{n\pi}\right)\sqrt{\frac{g_2}{g_1(1-g_1g_2)}} \qquad \omega_0^2 = \left(\frac{\lambda d}{n\pi}\right)\frac{\sqrt{g_1g_2(1-g_1g_2)}}{g_1+g_2-2g_1g_2}$$

$$R_1 = R_2 = R$$
$$g_1 = g_2 = g = 1 - d/R$$

$$\text{At mirror:} \quad \omega^2 = \left(\frac{\lambda d}{n\pi}\right)\sqrt{\frac{1}{1-g^2}} = \left(\frac{\lambda R}{n\pi}\right)\sqrt{\frac{d}{2R-d}}$$

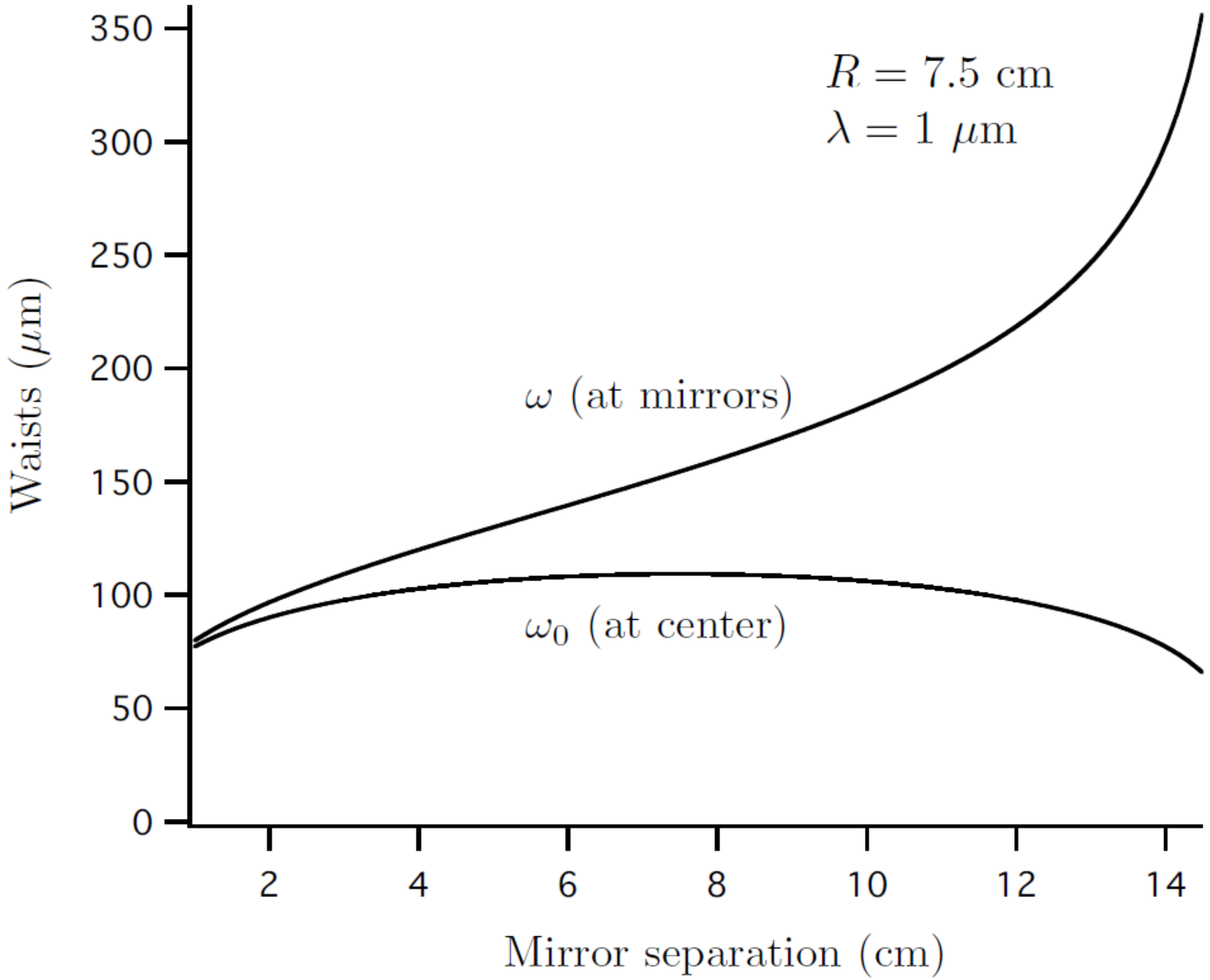
$$\text{Waist:} \quad \omega_0^2 = \left(\frac{\lambda d}{2n\pi}\right)\sqrt{\frac{1+g}{1-g}} = \left(\frac{\lambda}{n\pi}\right)\sqrt{\frac{dR}{2}-\frac{d^2}{4}}$$



Confocal Cavity $d = R$

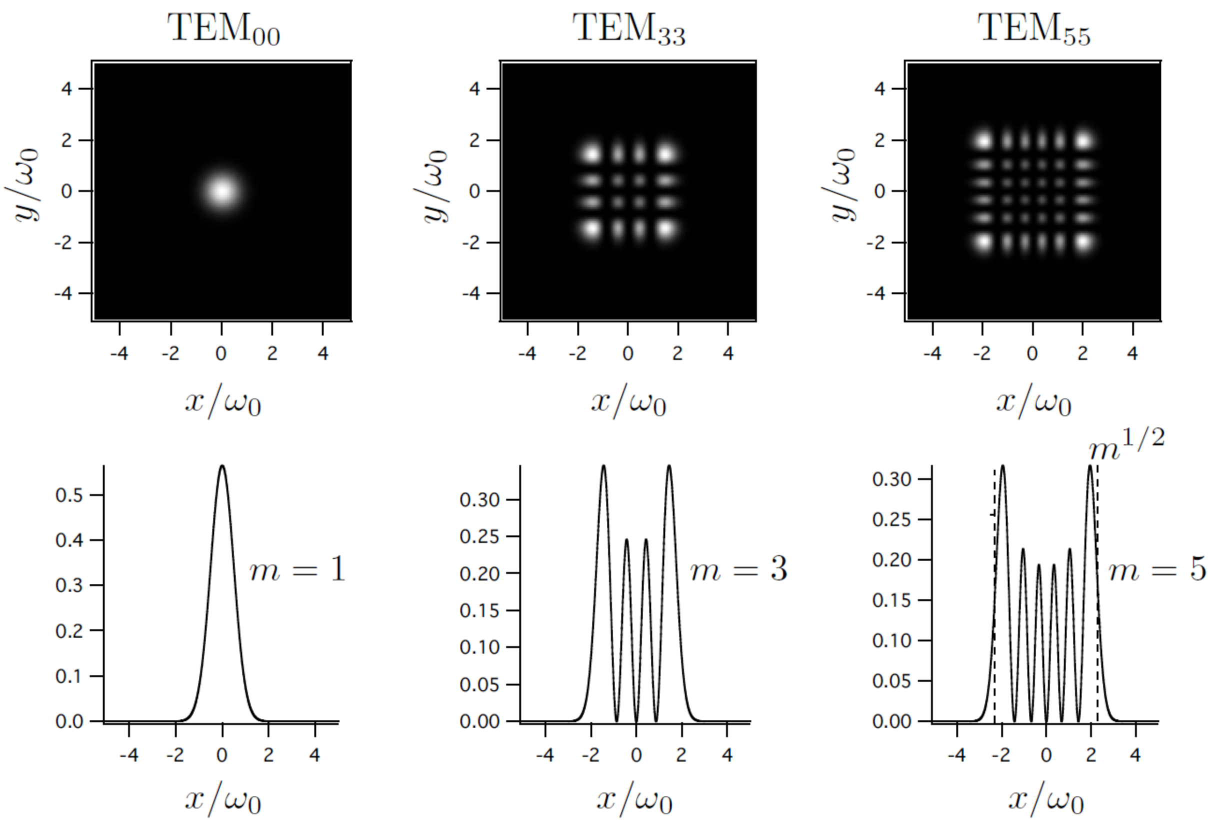
$$\omega_0 = \left(\frac{\lambda d}{2n\pi}\right)^{1/2}$$

$$\omega_{mirror} = \left(\frac{\lambda d}{n\pi}\right)^{1/2} = \sqrt{2}\omega_0$$



Homework #2: draw the curves with the parameters that you are interested in. (e.g. R = 15 cm, λ = 0.37 um or R = 20 cm, λ = 0.78 um

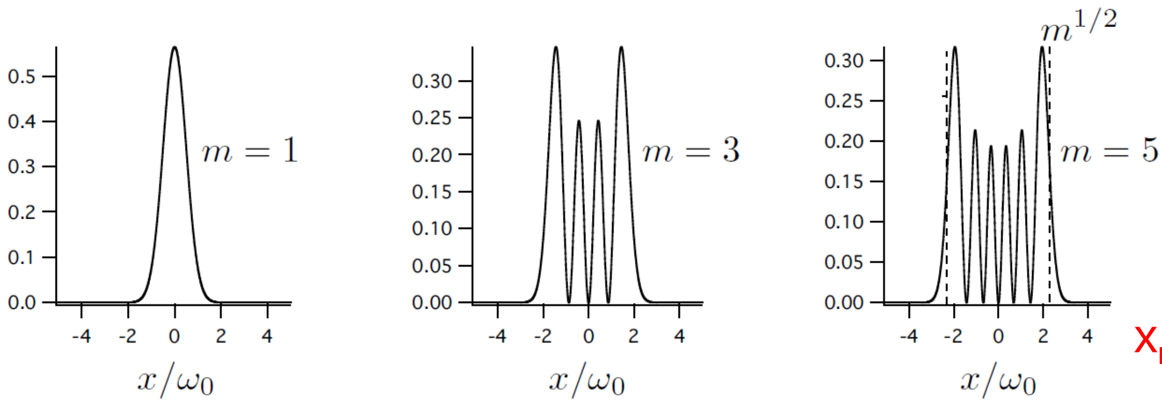
2.5 HIGHER-ORDER MODES



$$u(x, y, z)_{nm} =$$

$$\frac{\omega_0}{\omega(z)} H_n(\sqrt{2}\frac{x}{\omega}) H_m(\sqrt{2}\frac{y}{\omega})$$

$$\times \exp\left\{-i(kz - \Phi(m, n; z)) - i\frac{k}{2q}(x^2 + y^2)\right\}$$



$x_m \sim \sqrt{m}\,\omega_0$

$$u(x,y,z)_{nm} =$$
$$\frac{\omega_0}{\omega(z)} H_n(\sqrt{2}\frac{x}{\omega}) H_m(\sqrt{2}\frac{y}{\omega})$$
$$\times \exp\left\{-i(kz - \Phi(m,n;z)) - i\frac{k}{2q}(x^2+y^2)\right\}$$

same q independent of m,n
Same ABCD rule

$$\frac{\omega_0}{\omega(z)} = \frac{1}{\sqrt{1+(z/z_R)^2}}$$
$$\Phi(n,m;z) = (n+m+1)\tan^{-1}(z/z_R)$$

HERMIT GAUSSIAN MODES

- They are all characterized by the *same* complex beam parameter, q, defined by

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda}{n\pi\omega^2(z)}. \tag{2.28}$$

- They all satisfy the same ABCD rule as (lowest order) Gaussian beams. The *mode numbers*, m and n are preserved under all of the transformations discussed in this book.
- The mode *shape* is independent of z and scales with $\omega(z)$.
- The mode of index m has a half-width, x_m, (in one transverse coordinate), where

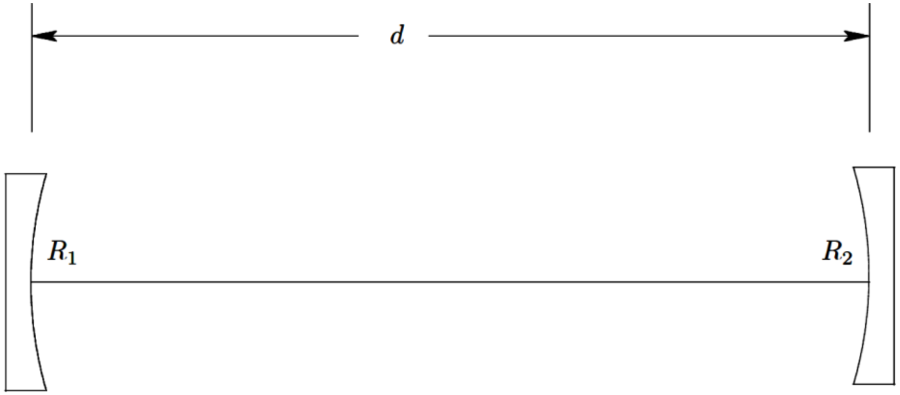
$$x_m \approx \sqrt{m} \times \omega. \tag{2.29}$$

  Thus, ω is no longer the beam size in higher-order modes. A focused spot has its size *degraded* by $\sqrt{m}$.
- The maximum mode number (m_{max}) at a waist which will "fit" into an aperture of radius a is:

$$m_{max} \approx (a/\omega_0)^2. \tag{2.30}$$

  This *spatial filtering* behavior of small apertures allows one to filter out modes whose mode number (in either coordinate) is greater than m_{max}.

2.6 RESONANT FREQUENCIES



$$\exp\left\{-i(kz-\Phi(m,n;z))-i\frac{k}{2q}(x^2+y^2)\right\}$$

$$\Phi(n,m;z)=(n+m+1)\tan^{-1}(z/z_R)$$

$$\delta=2kd-2(n+m+1)(\tan^{-1}(z_2/z_R)-\tan^{-1}(z_1/z_R))$$

$$\delta=q(2\pi)\Longrightarrow\frac{\omega d}{c}-(n+m+1)\cos^{-1}\pm\sqrt{g_1g_2}=q\pi$$

$$\nu_{nmq}=\left(q+(n+m+1)\frac{\cos^{-1}\pm\sqrt{g_1g_2}}{\pi}\right)\frac{c}{2d}$$

Free spectral range

$$\frac{\cos^{-1}\pm\sqrt{g_1g_2}}{\pi}\approx\begin{cases}0 & : \quad g_1,g_2\rightarrow 1 \quad \text{(near-planar)}\\ 1/2 & : \quad g_1,g_2\rightarrow 0 \quad \text{(near-confocal)}\\ 1 & : \quad g_1,g_2\rightarrow -1 \quad \text{(near-spherical)}\end{cases}$$