根据 Fig.2 推导 BPD 接收光强 I.

设 probe 光在未进入介质（阴极灯）前为偏振与 $\hat{x}$（水平）成 φ 角的偏振光

则 $\vec{E} = E_0(\hat{x}\cos\varphi + \hat{y}\sin\varphi)e^{i(\omega t-kz)} = \frac{1}{2}E_0\left(\hat{x}(e^{i\varphi}+e^{-i\varphi}) - i\hat{y}(e^{i\varphi}-e^{-i\varphi})\right)e^{i(\omega t-kz)}$

$= \vec{E}_R + \vec{E}_L$ 其中 $\vec{E}_R = \frac{1}{2}E_0(\hat{x}-i\hat{y})e^{i(\omega t-kz+\varphi)}$ $\vec{E}_L = \frac{1}{2}E_0(\hat{x}+i\hat{y})e^{i(\omega t-kz-\varphi)}$

分别为右手偏振光和左手偏振光（螺旋度分别为 +1, −1）

进入介质后 $\vec{E}_R$ 和 $\vec{E}_L$ 发生不同的变化（Pump 光的存在导致两种光被介质的吸收率不同，也反映在折射率不同）.

$$\vec{E}_R \rightarrow \vec{E}_R = \frac{1}{2}E_0(\hat{x}-i\hat{y})e^{i(\omega t-k^- z+\varphi)} \qquad k^- = \frac{\omega n^-}{c}$$

$$\vec{E}_L \rightarrow \vec{E}_L = \frac{1}{2}E_0(\hat{x}+i\hat{y})e^{i(\omega t-k^+ z+\varphi)} \qquad k^+ = \frac{\omega n^+}{c}$$

从介质出射后 $\vec{E} \rightarrow \vec{E} = \frac{E_0 e^{-\alpha^-/2\, L}}{2}(\hat{x}-i\hat{y})e^{i(\omega t-k^- L+\varphi)} + \frac{E_0 e^{-\alpha^+/2\, L}}{2}(\hat{x}+i\hat{y})$

$e^{i(\omega t-k^+ L-\varphi)}$ （L 为 probe 光与介质相互作用有效长度）.

$= E_x\hat{x} + E_y\hat{y}$ 其中 $E_x = \frac{E_0 e^{i\omega t}}{2}\left(e^{-\frac{\alpha^- L}{2}}e^{i(\varphi-\frac{\omega L}{c}n^-)} + e^{-\frac{\alpha^+ L}{2}}e^{-i(\varphi+\frac{\omega L}{c}n^+)}\right)$

$E_y = \frac{E_0 e^{i\omega t}}{2}i\left(e^{-\frac{\alpha^+ L}{2}}e^{-i(\varphi+\frac{\omega L}{c}n^+)} - e^{-\frac{\alpha^- L}{2}}e^{i(\varphi-\frac{\omega L}{c}n^-)}\right)$

经过阴极灯后的 $\frac{1}{2}$ 波片得 $\begin{pmatrix} E_x' \\ E_y' \end{pmatrix} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix}$ θ 为波片快轴和 $\hat{x}$ 夹角.

$$\Rightarrow E_x' = \cos 2\theta\left(e^{i(\varphi-\frac{\omega L}{c}n^- + i\frac{\alpha^- L}{2})} + e^{-i(\varphi+\frac{\omega L}{c}n^+ - i\frac{\alpha^+ L}{2})}\right)$$
$$+ \sin 2\theta\left(ie^{-i(\varphi+\frac{\omega L}{c}n^+ - i\frac{\alpha^+ L}{2})} - ie^{i(\varphi-\frac{\omega L}{c}n^- + i\frac{\alpha^- L}{2})}\right)$$

$$E_y' = \sin 2\theta\left(e^{i(\varphi-\frac{\omega L}{c}n^- + i\frac{\alpha^- L}{2})} + e^{-i(\varphi+\frac{\omega L}{c}n^+ - i\frac{\alpha^+ L}{2})}\right)$$
$$-\cos 2\theta\left(ie^{-i(\varphi+\frac{\omega L}{c}n^+ - i\frac{\alpha^+ L}{2})} - ie^{i(\varphi-\frac{\omega L}{c}n^- + i\frac{\alpha^- L}{2})}\right)$$

令 $\begin{cases} \frac{\omega L}{c} n^- = a^- & \frac{\omega L}{c} n^+ = a^+ \\ \frac{\alpha^- L}{2} = b^- & \frac{\alpha^+ L}{2} = b^+ \end{cases}$ 经过一番冗长的计算。

$$|E_x'|^2 = e^{-2b^-} + e^{-2b^+} + 2e^{-(b^- + b^+)} \cos(4\theta - 2\varphi - a^+ + a^-)$$

$$|E_y'|^2 = e^{-2b^-} + e^{-2b^+} - 2e^{-(b^- + b^+)} \cos(4\theta - 2\varphi - a^+ + a^-)$$

$$\Rightarrow I = I_H - I_V = |E_x'|^2 - |E_y'|^2 = 4e^{-(b^- + b^+)} \cos(4\theta - 2\varphi - a^+ + a^-) \cdot \frac{I_0}{4}$$

$= I_0 e^{-\frac{L}{2}(\alpha^+ + \alpha^-)} \cos(2\varphi + \frac{\omega L}{c} \Delta n - 4\theta)$ 其中 $\Delta n = n^+ - n^-$，若 $2\varphi - 4\theta$ 取 $\frac{\pi}{2}$

$$= I_0 e^{-\frac{L}{2}(\alpha^+ + \alpha^-)} \sin\left(-\frac{\omega L \Delta n}{c}\right) \sim -I_0 e^{-\frac{L}{2}(\alpha^+ + \alpha^-)} \frac{\omega L \Delta n}{c}$$

$= -I_0 e^{-\frac{L}{2}(\alpha^+ + \alpha^-)} \Delta\alpha_0 L \frac{x}{1+x^2}$ $\quad x = \frac{2(\omega_0 - \omega)}{\Gamma}$ $\quad \Delta\alpha_0 = \alpha^+(\omega_0) - \alpha^-(\omega_0)$

对 pump光加占空比调制最终反应在 $\Delta\alpha_0$ 上，即相当于 $\Delta\alpha_0$ 加了调制，故需用 Lock-in 解调，最终反馈给PID的误差信号正比于 $\frac{x}{1+x^2}$.