Beam Load Testing
(B-Project Final Report)

ME 406 Experimental Design
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1 Abstract

We are interested in conducting an experiment that characterizes the modulus of elasticity of three different grades of steels manufactured in three different ways. The different grades of steel to be tested are ASTM A36, Tool Steel H13 and AISI 4140. Each grade will be evaluated using different production methods: casting, milling and drawing. Samples are tested in fixed position within a compression machine that subjects the rods to an incremental 5 lb load up to 640 lb. During load application, deflection of the rods is measured and recorded through a Data Acquisition Unit with respective software. Modulus of elasticity is calculated through a cantilever beam equation which uses deflection as a variable and the moment of inertia of the sample's cross section. Results from experimentation showed the direct relationship between deflection and the modulus of elasticity; as deflection increased, the modulus of elasticity increased and plateaued. In terms of the experimental variables, the milled Tool Steel sample had the largest modulus of elasticity whereas cast Tool Steel had the lowest with drawn Tool Steel in the middle. Amongst all types of steel, Tool Steel had the largest modulus of elasticity where AISI 4140 had the lowest with ASTM A36 in the middle. From these we can understand the properties of these three steel alloys and the way samples are manufactured through comparison of their respective modulus of elasticities.

2 Table of Contents

3 Methodology

3.1 Equipment and Instrumentation

Apparatus and measurement equipment to conduct the experiment and collect data:

- Compression Machine
- Load Cell
- Rod Fixation Apparatus + Sample Rod
- Agilent LXI Data Acquisition Unit (34972A)

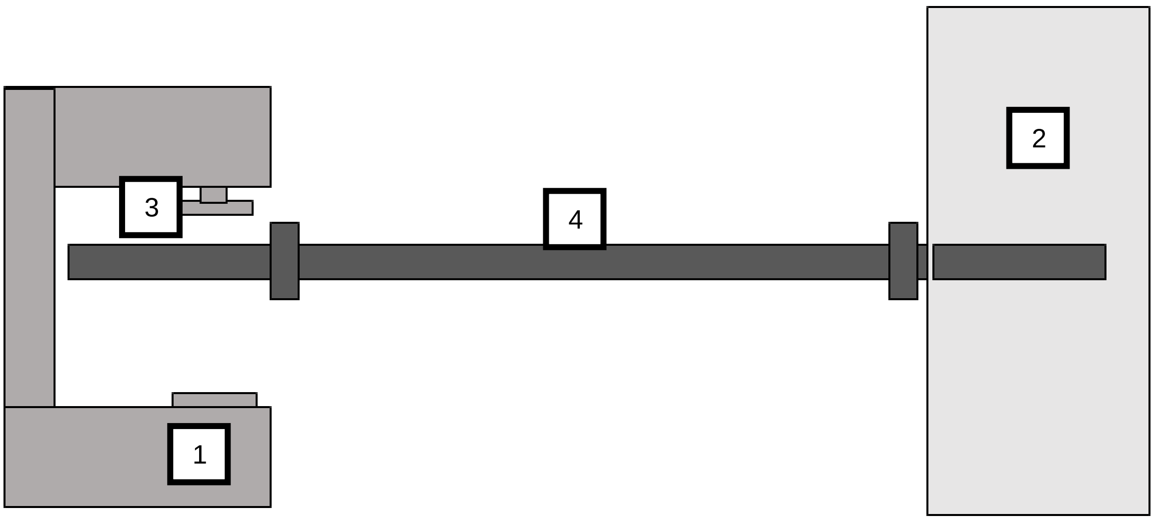

Figure 1. Experimentation setup for testing the steel rod. 1 denotes the compression machine, 2 is a fixation apparatus, 3 is the load cell connected to the compression machine, and 4 is the sample rod.

The data acquisition unit (DAQ) reads analog signals from the load cell. The values received from the load cell are read as voltages. To deliver readable data, the DAQ converts the voltages through an amplifier which translates values into through equations in the software. The output are voltages which are scaled to be more representative of the actual experiment.

3.2 Theory

The compression machine operates simply by moving its attachment arm using a stepper motor. To test beam deflection, the attachment arm has a load cell equipped to the arm in order to receive data from pushing our sample rods downward. In order to have the rod stationary during testing, we have samples fixed and locked in place with the help of a rod fixation apparatus.

3.3 Constants

Constant	Value	Unit
Sample Length (L)	2	m
Sample Diameter (D)	0.06	m
Sample Area (A)	$2.82 \cdot 10^{-3}$	m^2
Moment of Inertia (I)	$6.36 \cdot 10^{-7}$	m^4
Gravitational Constant (g)	9.81	$\frac{m}{s^2}$

Table 1. Constants used in our equations to deliver moment of inertia, force, and modulus of elasticity

3.4 Equations

The main equation in this experiment is for a cantilevered beam that is under load. As per our testing equipment, we have one end of our sample being pushed down while the other end is fixed in place. The deflection data gathered is represented by the following equation for a cantilever beam with concentrated load at a free end.

$$\delta_{max} = \frac{PL^3}{3EI} \tag{1}$$

Force experienced by the beam is the load applied times gravity.

$$P = mg \tag{2}$$

Moment of Inertia is calculated using the cross-sectional area of the beam.

$$I = \frac{\pi D^4}{64} \tag{3}$$

With all unknowns solved for, the equation for deflection is then rearranged to solve for the Modulus of Elasticity.

$$\delta_{max} = \frac{PL^3}{3EI} \quad \rightarrow \quad E = \frac{PL^3}{3\delta_{max} I} \tag{4}$$

Variable	Unit
Load (m)	kg
Force (P)	kN
Modulus of Elasticity (E)	GPa
Maximum Deflection (δ_{max})	m
Moment of Inertia (I)	m^4

Table 2. Variables used in our equations

3.5 Procedure

Central composite testing method is used to conduct this experiment. This method utilizes values at the minimum value of loading, several values up to the median, and specifically the maximum value. This method of testing allowed us to examine the rods before the eventual breaking point due to loading.

Our independent variables are the loading on the rod and the material (ASTM A36, Tool Steel H13, AISI 4140) and manufacturing of that rod (milled, casted, drawn). The dependent variables are deflection of the rod and the modulus of elasticity, found through equations in Section 3.4.

To obtain sample points, we set one end of the rod with the starting material in the fixed end, while the other was set to be put under load by the compression machine. We began experimentation with a load of 0 lbs, up to a maximum load of 640 lbs in increments of 5 lbs. Once this process was completed, we would remove the rod and begin the same testing with the second manufacturing process of that material, then followed by the third process of the same material. Once all processes for one material were tested, the same series of experimentation would begin with the second material, then followed by the third.

Step-by-step procedure:

1) Insert and lock end of rod into fixation apparatus
2) Set loading range to 0-640 lbs on compression machine
3) Load rod with compression machine in increments of 5 lbs while collecting data via DAQ software
4) Repeat steps 1-3 with three different manufacturing methods (cast/milled/drawn)
5) Repeat steps 1-4 for three different materials (ASTM A36/Tool Steel/AISI 4140)

3.6 Uncertainties

Uncertainties were found for each sensor we used to propagate the error encountered in our data sets. This was calculated by combining the random and bias errors to determine the total error for the sensor. Full uncertainties are tabulated in the Appendix (7.1).

Bias error, B_P, was provided by the sensor manufacturer, where repeatable values for instrumentation error were given.

$$B_P = \pm\sqrt{\left(\frac{\partial P}{\partial X}\sigma_X\right)^2 + \left(\frac{\partial P}{\partial Y}\sigma_Y\right)^2 + \left(\frac{\partial P}{\partial Z}\sigma_Z\right)^2 + \cdots} \tag{5}$$

$$where\ X, Y, Z = measurement$$

Total bias error, B_{total}, is determined by combining all bias error terms.

$$B_{total} = \pm\sqrt{B_1^2 + B_2^2 + B_3^2 + \cdots} \tag{6}$$

Different from bias, random error is calculated from a tabulated t-value and the standard deviation of measurements.

Standard deviation, S_P, is derived from the variance in each measurement sample.

$$S_P = \pm\sqrt{\left(\frac{\partial P}{\partial X}\sigma_X\right)^2 + \left(\frac{\partial P}{\partial Y}\sigma_Y\right)^2 + \left(\frac{\partial P}{\partial Z}\sigma_Z\right)^2 + \cdots} \tag{7}$$
$$where\ X, Y, Z = measurement$$

Random error, P_P, is calculated from a tabulated t-value, t, and standard deviation. A t-value value of 2.571 was determined from a confidence level of 95% and full sample count.

$$P_P = t \cdot S_P \tag{8}$$

Total random error, P_{total}, is determined by combining all random error terms.

$$P_{total} = \pm\sqrt{P_1^2 + P_2^2 + P_3^2 + \cdots} \tag{9}$$

Total error, U_{total}, combines the values procured from total bias and random errors.

$$U_{total} = \pm\sqrt{B_{total}^2 + P_{total}^2} \tag{10}$$

3.7 Hypothesis

As beam deflection increases, modulus of elasticity will increase to a point where we can see it plateau and give us an estimate of what the modulus of elasticity will be. We should also see variation in modulus of elasticity among samples as we have three types of steels and three different manufacturing methods.

4 Results

4.1 Measurements

Using the central composite method for taking samples, an ample amount of data was collected to contribute to the characterization of the rod. From our independent variables of rod loading and material, we were able to deliver attributes in terms of the deflection and modulus of elasticity for the rod as loading increased. Significant data collected is represented in the tables below of loading measurements 100-120 lbs. for Cast Tool Steel H13. Sample calculations used in the table are shown in the Appendix (7.2)

Load (lb)	Load (kg)	Deflection (mm)				
		Cast 1	Cast 2	Cast 3	Cast 4	Cast 5
100	45.359	1.4935	1.4814	1.4808	1.4846	1.4764
105	47.627	1.5665	1.5661	1.5526	1.5613	1.5616
110	49.895	1.6362	1.6420	1.6392	1.6220	1.6361
115	52.163	1.6960	1.7108	1.7059	1.7117	1.6988
120	54.431	1.7764	1.7799	1.7859	1.7775	1.7832

Table 3. Raw data collected through the DAQ for Cast Tool Steel H13 (converted to respective units)

Average Deflection (mm)	Force (kN)	Modulus of Elasticity (GPa)
1.4834	0.4450	1257.4
1.5616	0.4672	1254.1
1.6349	0.4895	1255.0
1.7046	0.5117	1258.3
1.7806	0.5340	1257.0

Table 4. Processed data used to characterize the rod for Cast Tool Steel H13

	Tool Steel	ASTM A36	AISI 4140
Cast	1262	1174	1116
Milled	2101	1955	1858
Drawn	1892	1760	1672

Table 5. Average modulus of elasticity (GPa) for each material and process

4.2 Data Plots

Curves of the modulus of elasticity are split up into three plots to show the relationship between deflection, manufacturing method, and material. Modulus of elasticity approaches a point as deflection increases and varies between manufacturing method and material.

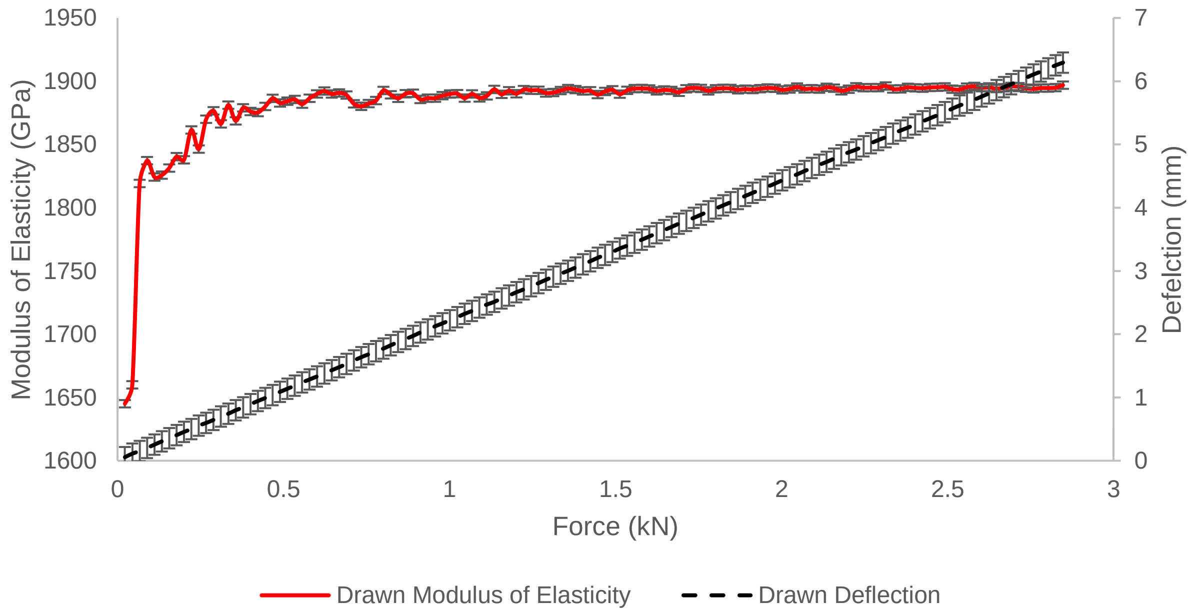

Figure 2. Modulus of elasticity and deflection for Tool Steel H13 as force increases (inclusion of error bars).

As force increases, the modulus of elasticity for steel approaches a point before stabilizing whereas the deflection increases proportionally. This displays how modulus of elasticity for steel reaches a maximum point after a certain amount of deflection. Uncertainty, displayed by error bars in the figure, shows how uncertainty stays constant in deflection but decreases for modulus of elasticity as load increases.

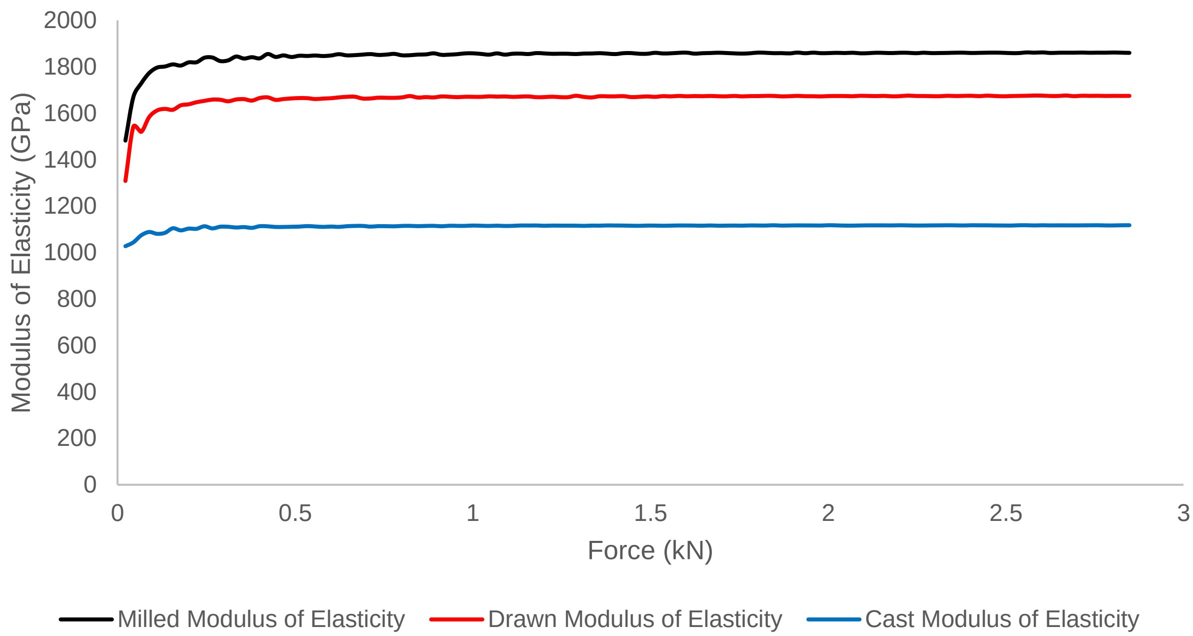

Figure 3. Modulus of elasticity comparing each manufacturing method using AISI 4140.

We have established the relationship between force/deflection and modulus of elasticity, so here is where we can see the differences between manufacturing process more clearly. Milled has the largest modulus whereas cast has the lowest, with drawn in between.

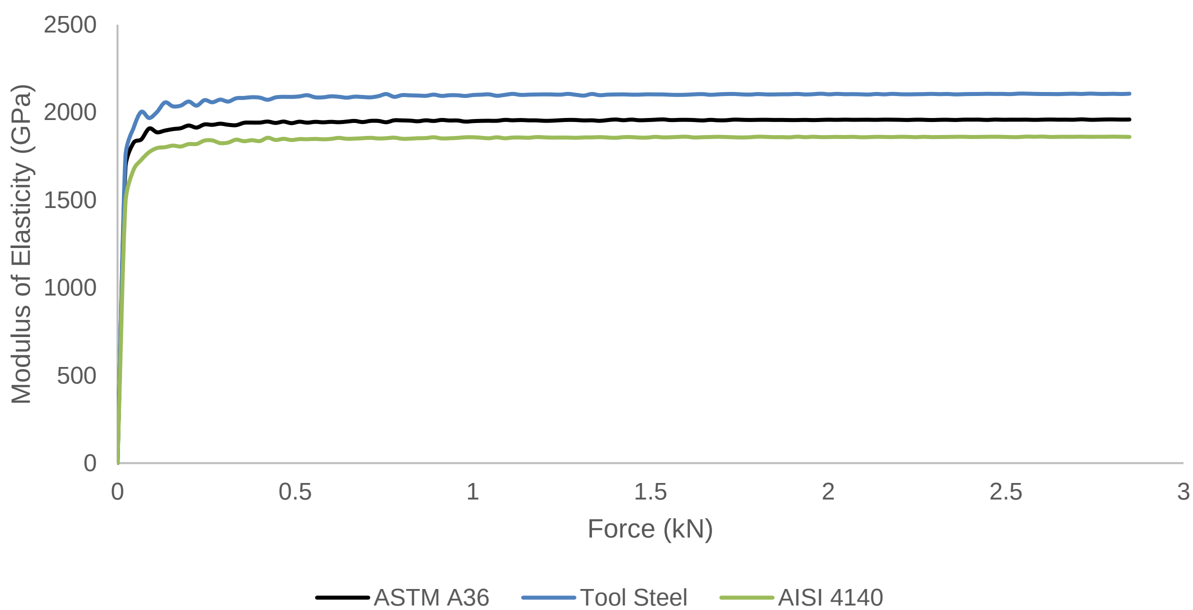

Figure 4. Modulus of elasticity comparing each material using the milling as the manufacturing method.

Here we can see the comparison of modulus elasticity in terms of material. We see that Tool Steel has the highest modulus overall, whereas AISI 4140 has the lowest, with ASTM A36 in between.

5 Conclusion

The beam loading test was able to estimate the modulus of elasticity for our tested metal samples. In addition, we were also able to determine the modulus of elasticity in relationship to the manufacturing method per metal. As per our experimental results, we have concluded that a milled sample of Tool Steel is the strongest out of all our samples. Among all types of manufacturing methods, all milled samples were strongest in their material category. Second strongest were drawn samples and the last being cast. These results are in-line with material theory, as a milled object is not subject to heat whereas drawn and cast are. In terms of metals, Tool Steel samples had the highest modulus of elasticities where AISI 4140 was least high, with ASTM A36 in the middle. From these results it is easy to categorize the various steel alloys and manufacturing methods from weak to strong based on the modulus of elasticity calculated.

6 References

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7 Appendix

7.1 Table of Uncertainties

Instrument	Uncertainty
Compression Test Machine (Load Cell)	± 0.0005 V

Table 5. Propagated uncertainties from instruments used in this experiment

7.2 Sample Calculations

Calculations taken with 2 x 0.06 m samples. Load taken at 100 lbs. (45.359 kg) where there was an average deflection of 1.4834 mm using Cast Tool Steel.

Moment of Inertia
$$I = \frac{\pi D^4}{64} = \frac{\pi (0.06\ m)^4}{64} = 6.36 \cdot 10^{-7}\ m^4 \tag{1}$$

Force
$$P = mg = 45.359 \cdot 9.81 \cdot \left(\frac{1\ kN}{1000\ N}\right) = 0.4550\ kN \tag{2}$$

Modulus of Elasticity
$$E = \frac{PL^3}{3\delta_{max} I} = \left[\frac{(0.4550\ kN) \cdot (2\ m)^3}{3 \cdot (0.0014834\ m) \cdot (6.36 \cdot 10^{-7}\ m^4)}\right] \cdot \left(\frac{1\ GPa}{10^6 kPa}\right) = 1257.4\ GPa \tag{3}$$

Uncertainty at Load Cell (Milled Tool Steel):

Bias Error
$$B_P = \sqrt{\left(\frac{0.05}{100} * 250\right)^2 + \left(\frac{0.03}{100} * 250\right)^2} \tag{4}$$
$$B_{total} = B_P = \pm\ 0.3010\ GPa$$

Standard Deviation
$$S_P = \pm\ 40.6072\ GPa \tag{5}$$
$$(Excel\ std. dev. for\ all\ data\ points)$$

Random Error
$$P_P = 2.571(40.6072) = \pm\ 104.401\ GPa \tag{6}$$

Total Error
$$U_{total} = \sqrt{(104.401)^2 + (0.30104)^2} = \pm\ 104.401\ GPa \tag{7}$$

Table 6. Cantilever Beam Equations

BEAM DEFLECTION FORMULAE

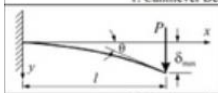
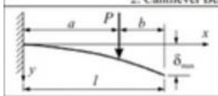
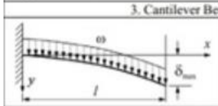
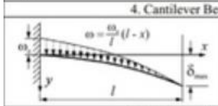
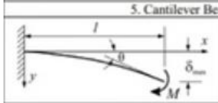
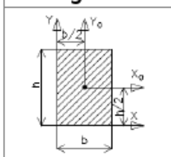
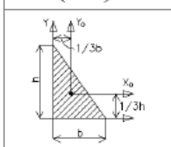
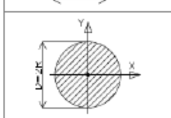
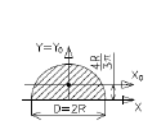
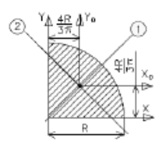
BEAM TYPE	SLOPE AT FREE END	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM DEFLECTION
1. Cantilever Beam – Concentrated load P at the free end			
	$\theta = \frac{Pl^2}{2EI}$	$y = \frac{Px^2}{6EI}(3l - x)$	$\delta_{max} = \frac{Pl^3}{3EI}$
2. Cantilever Beam – Concentrated load P at any point			
	$\theta = \frac{Pa^2}{2EI}$	$y = \frac{Px^2}{6EI}(3a - x)$ for $0 < x < a$ $y = \frac{Pa^2}{6EI}(3x - a)$ for $a < x < l$	$\delta_{max} = \frac{Pa^2}{6EI}(3l - a)$
3. Cantilever Beam – Uniformly distributed load ω (N/m)			
	$\theta = \frac{\omega l^3}{6EI}$	$y = \frac{\omega x^2}{24EI}\left(x^2 + 6l^2 - 4lx\right)$	$\delta_{max} = \frac{\omega l^4}{8EI}$
4. Cantilever Beam – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta = \frac{\omega_o l^3}{24EI}$	$y = \frac{\omega_o x^2}{120lEI}\left(10l^3 - 10l^2x + 5lx^2 - x^3\right)$	$\delta_{max} = \frac{\omega_o l^4}{30EI}$
5. Cantilever Beam – Couple moment M at the free end			
	$\theta = \frac{Ml}{EI}$	$y = \frac{Mx^2}{2EI}$	$\delta_{max} = \frac{Ml^2}{2EI}$

Table 7. Moment of Inertia Equations

Moments of Inertia of an Area

Figures	J_x	J_y	J_{xy}
	$J_{x_0} = \frac{bh^3}{12}$ $J_x = \frac{bh^3}{3}$	$J_{y_0} = \frac{hb^3}{12}$ $J_y = \frac{hb^3}{3}$	$J_{x_0y_0} = 0$ $J_{xy} = \frac{b^2h^2}{4}$
	$J_{x_0} = \frac{bh^3}{36}$ $J_x = \frac{bh^3}{12}$	$J_{x_0} = \frac{hb^3}{36}$ $J_x = \frac{hb^3}{12}$	$J_{x_0y_0} = -\frac{b^2h^2}{72}$ $J_{xy} = \frac{b^2h^2}{24}$
	$J_x = \frac{\pi D^4}{64} = \frac{\pi R^4}{4}$	$J_y = \frac{\pi D^4}{64} = \frac{\pi R^4}{4}$	$J_{xy} = 0$
	$J_{x_0} = \frac{D^4}{16}\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) = \approx 0{,}00686D^4 = \approx 0{,}1098R^4$ $J_x = \frac{\pi D^4}{128} = \frac{\pi R^4}{8}$	$J_y = \frac{\pi D^4}{128} = \frac{\pi R^4}{8}$	$J_{xy} = 0$ $J_{x_0y_0} = 0$
	$J_{x_0} = R^4\left(\frac{\pi}{16} - \frac{4}{9\pi}\right) = \approx 0{,}0549R^4$ $J_x = \frac{\pi D^4}{256} = \frac{\pi R^4}{16}$	$J_x = \frac{\pi R^4}{16}$	$J_{xy} = \frac{R^4}{8}$ $J_{x_0y_0} = \frac{R^4}{8} - \frac{4R^4}{9\pi} = = -0{,}0165R^4$