

answer The sum of probabilities for y given x is 1  $\int p(y | x; \theta) dy = 1$   
 Substituting for exponential family  $p(y | x; \theta) = \frac{1}{Z(\theta)} \exp(\eta^T y + a(\eta)) b(y)$   
 $\int \exp(\eta^T y + a(\eta)) b(y) dy = \exp(a(\eta)) \int b(y) \exp(\eta^T y) dy = 1$   
 Rearranging equation  $\int b(y) \exp(\eta^T y) dy = \exp(-a(\eta))$   
 Taking derivative w.r.t  $\eta$   $\frac{\partial}{\partial \eta} \int b(y) \exp(\eta^T y) dy = -\frac{\partial a(\eta)}{\partial \eta}$   
 $\int y b(y) \exp(\eta^T y) dy = -\frac{\partial}{\partial \eta} \exp(-a(\eta)) = \exp(-a(\eta)) \frac{\partial a(\eta)}{\partial \eta}$   
 Rearranging equation  $\int y b(y) \exp(\eta^T y) dy = \frac{\partial a(\eta)}{\partial \eta}$   
 Multiplying by  $\exp(a(\eta))$   $\int y \exp(\eta^T y) b(y) \exp(a(\eta)) dy = \frac{\partial a(\eta)}{\partial \eta}$   
 Additional term  $\int y p(y | x; \theta) dy = \frac{\partial a(\eta)}{\partial \eta} = E[y | x; \theta]$