

answer

$$J(\theta) = -1 \sum_{i=1}^{(y^{(i)} \log(h_\theta(x^{(i)})) + (1-y^{(i)}) \log(1-h_\theta(x^{(i)})))} \partial J(\theta) \partial x_i = \Sigma - (y-h(x)x_i H_{ij} = \partial^2 J(\theta) \partial x^2 = \Sigma(\partial h(x) \partial \theta_j) x_i = \Sigma(h(x)(1-h(x))x_j x_i)$$

$z^T H z \geq 0$ is always positive - semidefinite when $H \geq 0$

$$z^T H z = z^T (\Sigma(h(x)(1-h(x))x x^T) z = \Sigma(h(x)(1-h(x))(z^T x x^T z) = \Sigma(h(x)(1-h(x))(x^T z)^2$$

$(x^T z)^2$ is positive, and $h(x)(1-h(x))$ is a bernoulli probability where $0 \leq h(x) \leq 1$ Therefore, the value of that replaces $h(x)(1-h(x))$ is positive semidefinite, or negative semidefinite. In this case the Hessian is positive semidefinite. So,

$$z^T H z \geq 0$$