

answer

$$\begin{aligned}
& \text{Given that: } p(y|x) = p(x|y)p(y)p(x)p(y=1|x; \phi, \mu_0, \mu_1, \Sigma) = p(x|y=1; \phi, \mu_0, \mu_1, \Sigma)p(y=1; \phi, \mu_0, \mu_1, \Sigma)p(x; \phi, \mu_0, \mu_1, \Sigma) \\
& p(x|y=1)p(y=1)p(x) = p(x|y=1)p(y=1)p(x|y=0)p(y=0) + p(x|y=1)p(y=1) = \exp(-12(x-\mu_1)^T \Sigma^{-1}(x-\mu_1)) \\
& 11 + 1 - \phi \phi \exp(-12(x-\mu_0)^T \Sigma^{-1}(x-\mu_0) + 12(x-\mu_1)^T \Sigma^{-1}(x-\mu_1)) = 11 + \exp(-12((x-\mu_0)^T \Sigma^{-1}\mu_0 - \mu_1)^T \Sigma^{-1}\mu_1) - \\
& \text{rearranging terms } (x-\mu_0)^T \Sigma^{-1}(x-\mu_0) - (x-\mu_1)^T \Sigma^{-1}(x-\mu_1) = x^T \Sigma^{-1}x - \mu_0^T \Sigma^{-1}x - x^T \Sigma^{-1}\mu_0 + \\
& \mu_0^T \Sigma^{-1}\mu_0 - x^T \Sigma^{-1}x + \mu_1^T \Sigma^{-1}x + x^T \Sigma^{-1}\mu_1 - \mu_1^T \Sigma^{-1}\mu_1 = -2\mu_0^T \Sigma^{-1}x + \mu_0^T \Sigma^{-1}\mu_0 + 2\mu_1^T \Sigma^{-1}x - \mu_1^T \Sigma^{-1}\mu_1 = \\
& 2(\mu_1 - \mu_0)^T \Sigma^{-1}x + \mu_0^T \Sigma^{-1}\mu_0 - \mu_1^T \Sigma^{-1}\mu_1 \text{ Therefore : } p(y=1|x; \phi, \mu_0, \mu_1, \Sigma) = 11 + \exp(\log 1 - \phi \phi + 12\mu_1^T \Sigma^{-1}\mu_1 - 12\mu_0^T \Sigma^{-1}\mu_1) \\
& \Sigma^{-1}(\mu_1 - \mu_0)\theta_0 = 12(\mu_0^T \Sigma^{-1}\mu_0 - \mu_1^T \Sigma^{-1}\mu_1) - \log 1 - \phi \phi
\end{aligned}$$