A Planar Continuous Rock-Paper-Scissors Game

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1 Introduction

There exists a simulation somewhere on the web, where a swarm of rocks, a swarm of papers, and a swarm of scissors move around the plain in a Brownian motion and collide with each other. And every collision works as follows

- in each collision there is a winner and a loser particle, who are determined by the rules of the standard "rock-paper-scissors" game
- the loser particle is converted into a winning particle and the simulation continues.

In other words, there exists three types of interactions:

$$\begin{aligned} \text{rock} + \text{paper} &= 2 \cdot \text{paper} \\ \text{paper} + \text{scissors} &= 2 \cdot \text{scissors} \\ \text{scissors} + \text{rock} &= 2 \cdot \text{rock} \end{aligned}$$

The simulation is mildly amusing to watch as is. But one can also wonder, how would the same process work if we were to replace the discrete particles with a smooth continous approximation. How would the same process work, if instead of three swarms we had three "liquids" on a plane?

2 Mathematical model

To work with the continous approximation we need to represent the distribution of particles on a plane with a smooth density function. Since we have three kinds of particles, we need to introduce three different density functions, namely:

- $r(x, y; t)$ for rocks
- $p(x, y; t)$ for paper

- $s(x, y; t)$ for scissors.

Now since every particle in a liquid is a constant state of Brownian motion, we can say that in a collision-free environment each density function should behave according to the diffusion equation

$$\partial_t f + \frac{1}{2}\Delta f(x, y; t) = 0,$$

where Δ is the Laplacian $\partial^2_{xx} + \partial^2_{yy}$. Taking the collisions into account we can write the following system of equations

$$\partial_t r + \frac{1}{2}\Delta r = -p\,r + s\,r \tag{1a}$$

$$\partial_t p + \frac{1}{2}\Delta p = -s\,p + r\,p \tag{1b}$$

$$\partial_t s + \frac{1}{2}\Delta s = -r\,s + p\,s. \tag{1c}$$

3 Numerical methods

(1) is a system of nonlinear partial differential equations. They are notoriously hard to reason about. Nevertheless we will try in the sections that follow. But first let us try and propose a numerical approach the equations.