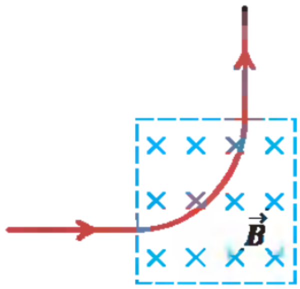
MAGNETOSTATICS

**Tutorial n°2**

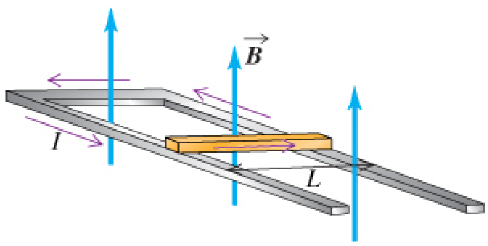
LORENTZ AND LAPLACE FORCES

## Beam of protons

A beam of protons traveling at $v = 1.2$ km·s$^{-1}$ enters a uniform magnetic field, traveling perpendicular to the field. The beam exits the magnetic field, leaving the field in a direction perpendicular to its original direction. The beam travels a distance $\ell = 1.18$ cm *while in the field*. What is the magnitude of the magnetic field ?

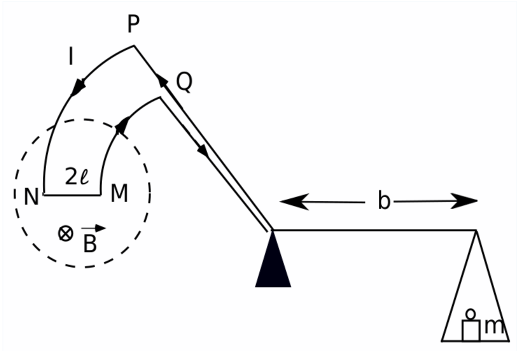



## Electromagnetic gun




A conducting bar with mass $m$ and length $L$ slides over horizontal rails that are connected to a voltage source. The voltage source maintains a constant current $I$ in the rails and bar, and a constant, uniform, vertical magnetic field $B$ fills the region between the rails.

1. Find the magnitude of the net force $\vec{F}$ on the conducting bar. Ignore friction, air resistance, and electrical resistance.
2. Find the distance $d$ that the bar must move along the rails from rest to attain speed $v$.
3. It has been suggested that rail guns based on this principle could accelerate payloads into earth orbit or beyond. Find the distance the bar must travel along the rails if it is to reach the escape speed for the earth (11.2 km/s). Let $B$ =0.50 T, $I = 2 \cdot 10^3$ A, $m = 25$ kg and $L = 50$ cm. For simplicity assume the net force on the object is equal to the magnetic force only.
4. The circuit has a constant resistance per unit length $\lambda$, and the voltage source provides a constant emf (electromotive force) or voltage. Find the equation of motion.

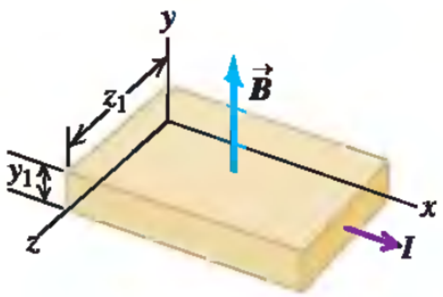



## Cotton’s scales

A Cotton’s scales (named after the physicist Aimé Cotton) is depicted in the above figure. It is made of an horizontal knife (black triangle) around which an curved beam can rotate. To one arm of length $b$ is a pan where masses can be heaped. The other arm has two parallel circular parts where an electric wire carries an intensity $I$. The common center of the arcs is on the top of the triangular knife. A magnetic field, parallel to the knife edge is almost uniform is the dashed circular region enclosing $MN$.

1. Consider the different magnetic forces acting on the device. Write down the equilibrium condition to equilibrate the beam.
2. Which mass $m$ is required in the pan to get the equilibrium, if $I = 5$ A, $a = b = 50$ cm, $\ell = 1$ cm, $g = 10$ m·s$^{-2}$, and $B = 5000$ Gauss (1 G=$10^{-4}$ T) ?

## Hall effect




The figure shows a portion of a long silver ribbon with of width $z_1 = 11.8$ mm and thickness $y_1 = 0.23$ mm, carrying a current $I = 120$ A in the +$x$-direction. The ribbon lies in a uniform magnetic field $\vec{B} = B\vec{e}_y$, with magnitude $B$ =0.95 T.

1. What is qualitatively the behaviour of the charge carriers (mobile electrons) inside the conducting ribbon ?
2. Show qualitatively that this leads to excess charges on the faces $z = z_1$ and $z = 0$. Where is the negative excess charge ?
3. Assuming that these two faces are equivalent to a capacitor, find qualitatively how is the electric field $\vec{E}$ within the ribbon.
4. We assume that the ribbon is long in either directions parallel to $x$, so that a *stationary regime* is set, and electrons move parallel to $\vec{e}_x$. Show that this stationarity yields a relation between $E_z$ and $|\vec{B}|$ and $I$.

5. Find the relation linking the potential difference $\Delta V$ between the faces $z = 0$ and $z = z_1$ to the magnetic field and $I$.
6. With one free electron per atom, and a molar volume $v_m = 7 \cdot 10^{-6}$ m$^3$·mol$^{-1}$, find the numerical value of $\Delta V$ for copper.

## Force on a square loop

The total force on a current loop produced by a *uniform* magnetic field is zero. But what if $\vec{B}$ is not uniform ?

A square metallic loop with vertices at $(x = -L/2, y = 0), (L/2, 0), (-L/2, L/2), (L/2, L/2)$ in the plane $z = 0$ carries a constant current $I$ in the clockwise direction. The magnetic field has no $x$-component but has both $y$- and $z$-components : $\vec{B} = (B_o z/L)\vec{\jmath} + (B_o y/L)\vec{k}$. where $B_o$ is a positive constant.

1. Sketch the magnetic field lines in the $yz$-plane.
2. Find the magnitude and direction of the magnetic force exerted on each of the sides of the loop
3. Find the magnitude and direction of the net magnetic force on the loop.

## Torque on a square loop

The same square current loop can rotate around the axis $Oy$, and feels a uniform magnetic field $\vec{B} = B_0 \cos\alpha \vec{\imath} + B_0 \sin\alpha \vec{k}$. What is the motion of the square ?

## Nail near a magnet

An iron nail is attracted by either side of a permanent magnet. Why ?

1. The iron is paramagnetic, namely, it is composed of atoms with permanent magnetic dipoles, whose orientations are completely at random if no external magnetic field is imposed. Knowing that the energy of a magnetic dipole with magnetic moment $\vec{\mathcal{M}}$ in a magnetic field $\vec{B}$ is $-\vec{\mathcal{M}} \cdot \vec{B}$, explain qualitatively why the nail develops an induced macroscopic magnetization $\vec{\mathcal{M}}_{\text{nail}} = \chi \vec{B}$.
2. Explain why the nail is attracted by either side of a permanent magnet.