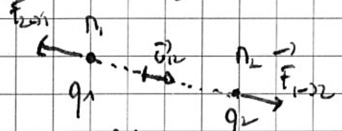
MAGNETOSTATICS

Bibliography : * Jackson : classical electrodynamics
* University Physics

I) Introduction

1) Electrostatics in a nutshell

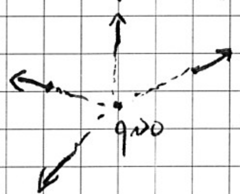
- some particles (e^-, p...) carry an electric charge $q = ne$ $\begin{cases} n \in \mathbb{Z} \\ e = 1{,}6 \cdot 10^{-19}\,C \end{cases}$
- These particles with charges exert on each other Coulomb forces



$$\vec F_{1\to2} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\,\vec u_{12}$$

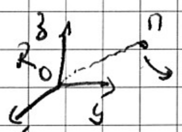
- These forces are interpreted via the concept of electric field

$$\vec F_{1\to2} = q_2\,\vec E_1(M_2)$$



q_1 creates a vectorial field $\vec E_1(M) = \dfrac{q_1}{4\pi\varepsilon_0 |M_1M|^2}\,\vec u_{M_1M}$ ← unitary vector

- Continuous description of charges



in a small volume $d^3\tau$ ($= dx\,dy\,dz$) around M there is a charge dq. The ratio $\rho(\vec r = \vec{OM}) = \dfrac{dq}{d^3\tau}$ is called the density of charges

- Elemental laws of electrostatics (Maxwell laws of electrostatics)

$\Rightarrow$ $\operatorname{div} \vec E = \rho/\varepsilon_0$ $\Leftrightarrow$ Gauss theorem $\oint_{\partial V} \vec E \cdot d\vec S = \dfrac{Q_{int}}{\varepsilon_0}$

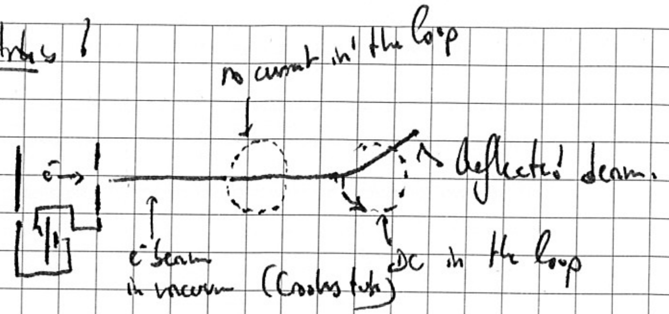
$\Rightarrow$ $\vec{\operatorname{rot}}\,\vec E = \vec 0$ $\Leftrightarrow$ $\exists$ electric potential V : $\vec E = -\vec\nabla V$

+ formulas : $\vec E(\vec r) = \iiint \dfrac{d^3\tau'\,\rho(\vec r')}{4\pi\varepsilon_0 |\vec r - \vec r'|^3}(\vec r - \vec r')$ $\qquad V(\vec r) = \iiint \dfrac{d^3\tau'\,\rho(\vec r')}{4\pi\varepsilon_0 |\vec r - \vec r'|}$

So : static charges $\Rightarrow$ electric field $\Rightarrow$ action at distance on charges

2) What is magnetostatics ?

• Experimental observation



⇒ the presence of an electric current is capable of a mechanical action on charged particles

⇒ the wire remains electrically neutral despite the current

↳ electrostatics does not allow to explain the phenomenon ($\rho(\vec{r}) = 0$ but in the beam)

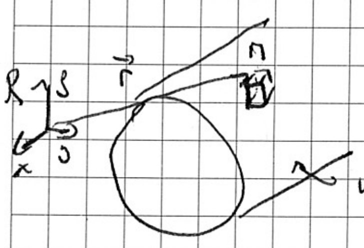
• In a nutshell

moving charges = current (in a neutral environment) ⇒ magnetic field $\vec{B}(\vec{r})$ ⇒ action at distance on charges

why magneto statics ???

Magnetism = phenomena arising from charged current, which have a mechanical action on charged particles.

3) Current density - intensity of the current



in the volume $d^3\tau$ around $\vec{r}$ $\quad \rho(\vec{r}) = \frac{\delta q}{\delta \tau} = 0$ (a result established in elstat still valid with currents)

but there is 2 types of charges in the wire

ρ_s → static (+) charges : ions of the metallic matrix partially ionized with heart electrons

ρ_m → mobile (−) electrons : these charges are mobile

$\rho_s(\vec{r}) = -\rho_m(\vec{r}) \neq 0$

$\vec{v}_m(\vec{r})$ = velocity of the charge carrier

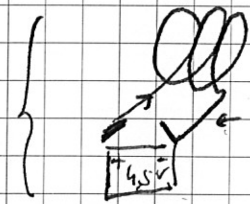
current density

$$\boxed{\vec{j}(\vec{r},t) = \rho_m(\vec{r},t)\,\vec{v}_m(\vec{r},t)}$$

- In the general case, everything is time dependent. In stationary situations, the density of current $\vec{j}(\vec{r},t)$ can be considered as time-independent : $\vec{j}(\vec{r},t) = \vec{j}(\vec{r})$ (or $\frac{\partial \vec{j}}{\partial t}(\vec{r},t) = 0 \quad \forall \vec{r}$)

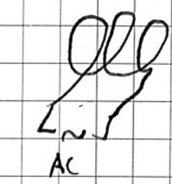
- The magnetoSTATICS is the study of the structure of magnetic fields generated by stationary currents (we don't say there is no motion in the wire, there are displacements of e^- but the fluxes are constant throughout time $\Rightarrow$ stationarity $\neq$ equilibrium)

OK for magnetostatics



← the intensity is constant

not OK $i = i(t)$



Intensity

To quantify the motion of electric carriers in wires one knows the notion of intensity I measured in Ampers (A). What is the link between $\vec{j}$ and I?

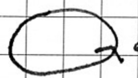
Def: Draw a closed curve at the surface of the wire enclosing the wire

Consider a surface $\mathcal{S}$ which has $\mathcal{C}$ as a boundary, and orient it (choice of the direction of the $\vec{dS}$)

One shows that $\iint_{\mathcal{S}} \vec{j}\cdot\vec{dS}$ is independent of $\mathcal{S}$, dependent only on $\mathcal{C}$

$\boxed{I_{\mathcal{C}} = \iint_{\mathcal{S}} \vec{j}\cdot\vec{dS}}$ is the intensity passing across $\mathcal{C}$

- Rk: I_0 is >0 if $\vec{j}$ points roughly in the same direction as the choice one made for $\vec{dS}$



⚠ if $I>0$, e^- are moving in the other direction!

- Rk: $[j\,\vec{dS}] = C\cancel{L}^{-3}\cancel{L}T^{-1}\cancel{L^2} = CT^{-1}$ = Ampère.

- Rk: imagine a river full of pebbles (ions +). In the stream one can measure in each point $\rho(\vec{r})\vec{v}$ ← celerity, ↑ volumic mass. Flow rate of the river $= \iint \rho\vec{v}\cdot\vec{dS}$, S across the river

4)

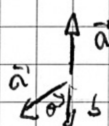
II) ~~Field~~ Magnetic fields generated by current distribution

→ Central question : knowing $\vec{j}(\vec{r})$ (distribution of current)
what is $\vec{B}(\vec{r})$? (consequence)

1) The general formula : Biot & Savart's

$$\boxed{\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint d\tau' \, \frac{\vec{j}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}}$$ → give also $\frac{\mu_0 I}{4\pi} \oint \frac{\vec{dr'} \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$

→ Recall : vectorial product



$\vec{a} \times \vec{b} \perp (\vec{a} \text{ and } \vec{b})$
$|\vec{a} \times \vec{b}| = |a||b| \sin\theta$
screwdriver's law :
screw from $\vec{a}$ to $\vec{b}$ the tool ~~progress~~ moves ~~toward~~ according to $\vec{a} \times \vec{b}$

$$\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}_R \quad \vec{b} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}_R$$

$$\hookrightarrow \vec{a} \times \vec{b} = \begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix}$$

→ $\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$
$\vec{a} \times \vec{a} = \vec{b} \times \vec{b} = \vec{0}$

→ B&S shows clearly that $\vec{B}$ is ~~generated~~ created by the current $\vec{j}$

→ μ_0 = ~~diamagnetic permittivity of vacuum~~ vacuum permeability !
$= 4\pi \, 10^{-7} \ N/A^2$ exact value.

We have an exact relation $\boxed{\varepsilon_0 \mu_0 c^2 = 1}$ c: light velocity

→ We have in electrostatics

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iiint d\tau' \, \frac{\rho(\vec{r}\,') \cdot (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$

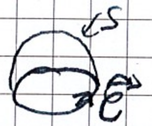
→ similar ~~close~~ to BS
with $\begin{cases} \rho \to \vec{j} \\ \varepsilon_0 \to \frac{1}{\mu_0} \\ \cdot \to \times \end{cases}$

→ similar but ≠ !

2) A useful relation : Ampère theorem

- The BS relation is exact but quite complicated to use
- Historically Ampère found beforehand a very general relation shedding light on the properties of $\vec{B}(\vec{r})$:

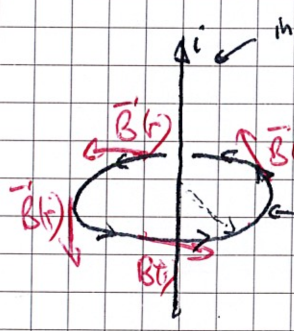
  - Take a closed oriented curve, and put on it an oriented surface
  The two orientations MUST fulfill the screwdriver rule



$$\boxed{\forall \mathcal{C} \quad \oint_{\mathcal{C}} \underbrace{\vec{B}\cdot\vec{dl}}_{\text{along } \vec{\mathcal{C}}} = \mu_0 I_{\mathcal{C}}}$$

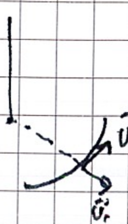
"The circulation of the magnetic field ~~around~~ along $\vec{\mathcal{C}}$ oriented equals the intensity enclosed by $\mathcal{C}$"

Ex : ⚠ very important



$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{\mathcal{C}}$$

Symmetry arguments show that $\vec{B}(\vec{r})$ | is $/\!/ \vec{u}_\theta$ | depends only on r



$$\begin{cases} \vec{B}(\vec{r}) = B(r)\,\vec{u}_\theta \\ \vec{dl} = r\,d\theta\,\vec{u}_\theta \end{cases}$$

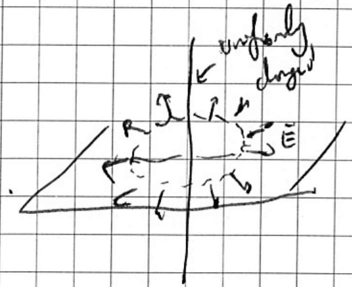
$$\Rightarrow \oint \vec{B}\cdot\vec{dl} = \int_0^{2\pi} B(r)\,r\,d\theta\,\underbrace{\vec{u}_\theta^{\,2}}_{1} = B(r)\,r\int_0^{2\pi} d\theta = 2\pi r B(r)$$

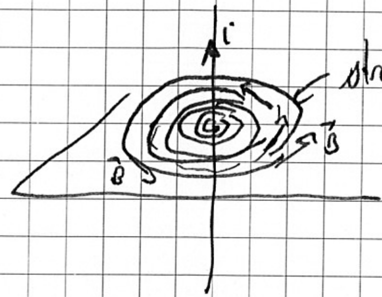
$$\Rightarrow 2\pi r B(r) = \mu_0 I_{\mathcal{C}} = \mu_0 i \Rightarrow \boxed{B(r) = \frac{\mu_0 i}{2\pi r}}$$

- Ampère allows exact calculation
- $B(r)\,\vec{u}_\theta$

Notice the "whirling" shape of the magnetic field!

→ completely ≠ from the electric field!





streamlines = circles centered on the wire

⇒ Each time a current distribution is highly symmetric, the Ampere's law can be used to compute (with minimal efforts) the magnetic field it creates.

⇒ Analogous to the Gauss theorem of electrostatics

3) <u>What about a potential for $\vec{B}$</u> ?

- electrostatics = 2 laws $\mathrm{div}\,\vec{E} = \rho/\varepsilon_0$ ⇔ Gauss th.
  $\vec{\mathrm{rot}}\,\vec{E} = \vec{0}$ ⇔ ∃ pot $\vec{E} = -\vec{\nabla} V$
- magnetostatics = Ampere theorem ($\vec{\mathrm{rot}}\,\vec{B} = \mu_0 \vec{j}$)
  - 2^{nd} law? <u>yes</u>

    $\boxed{\mathrm{div}\,\vec{B} = 0}$

→ it means macroscopically $\oiint_{\partial V} \vec{B}\cdot\vec{dS} = 0$ whatever the vol. V!

[$\vec{B}$ is a zero-flux field] ~~which conserves the flux~~

→ there exists the notion of a potential for $\vec{B}$, but it is a vectorial potential $\vec{A}(\vec{r})$ such that $\vec{B} = \vec{\mathrm{rot}}\,\vec{A}$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint d^3r' \frac{\vec{j}(\vec{r}')}{|\vec{r}-\vec{r}'|} \longleftrightarrow V = \frac{1}{4\pi\varepsilon_0} \iiint d^3r' \frac{\rho(r')}{|r-r'|}$$

never mind...

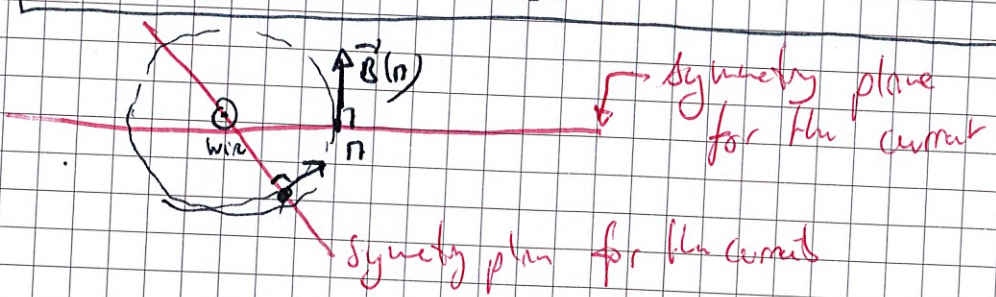
4) Symmetry argument for magnetic fields

a) • Whence comes the fact that $\vec{B}$ is orthoradial for the ~~so~~ straight wire

from the following theorem:

> If P is a symmetry plane for the current, and if $M \in P$
> then $\vec{B}(M)$ is $\perp$ Π, always

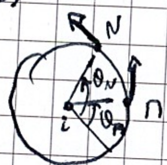
Observer un contre exemple : le P' pas plan de symétrie de courant !!



• Whence comes the fact that $\vec{B} = B(r,\cancel{\theta},\cancel{z})\,\vec{u}_\theta = B(r)\,\vec{u}_\theta$?

→ Two points located at the same distance from the wire play identical roles ⇒

rotation invariance of the current



$\Rightarrow B(M) = B(N) \Rightarrow B(r,\theta_M,z) = B(r,\theta_N,z)$

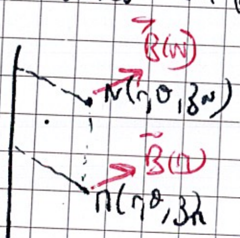
↓

B indep from θ

→ Two points located on top of each other play identical roles (no wire)

translation invariance of the current.

⇒

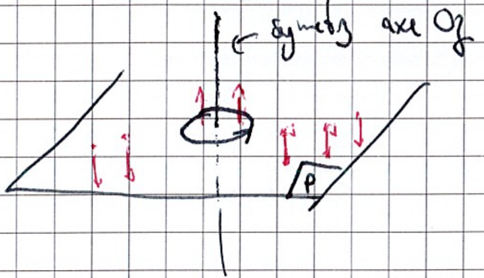


$B(M) = B(N) \Leftrightarrow B(r,z_M) = B(r,z_N)$

↓

$B(r,z) = B(r)$

b) Symmetry argument for the single current loop



top view

* Question : how is the $\vec{B}_{(M)}$, qualitatively, for $M \in P$?

Answer : P = symmetry plane for the current loop

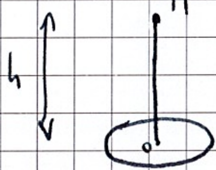
$\Rightarrow M \in P \Rightarrow \vec{B}(M) \perp P$!!

↳ a little bit counter-intuitive

* Question : how is $\vec{B}(M)$, for $M \in O_z$

M is rotationally invariant $\Rightarrow \vec{B}(M) \,//\, O_z$

Let $M = (0,0,h)$



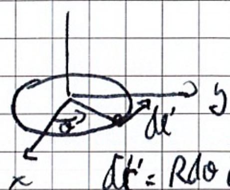
Compute $\vec{B}(M)$

→ one knows already $\vec{B}(M) = B_{(M)}\vec{e}_z = B(h)\vec{e}_z$

→ Ampère's theorem? No, not enough symmetries

→ Compelled to use B&S's law

$$\vec{B}(M) = \frac{\mu_0 i}{4\pi}\oint \frac{\vec{dl'} \wedge (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$



$\vec{dl'} = R\,d\theta\,\vec{u}_\theta$

$\vec{r}' = R\vec{u}_r$

$\vec{r} = h\vec{u}_z$

$$= \frac{\mu_0 i}{4\pi}\int_0^{2\pi} \frac{R\,d\theta\;\vec{u}_\theta \wedge (h\vec{u}_z - R\vec{u}_r)}{|h\vec{u}_z - R\vec{u}_r|^3}$$

$$= \frac{\mu_0 i R}{4\pi (h^2+R^2)^{3/2}} \int_0^{2\pi} d\theta\,(\underbrace{\vec{u}_r h}_{\text{I know it is } \vec{0}} + R\vec{u}_z)$$

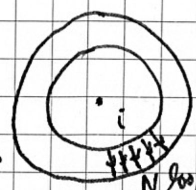
$$= \frac{\mu_0 i R^2}{2(h^2+R^2)^{3/2}}$$

Rq : $B \to 0$ as $\frac{1}{h^3}$ (not $\frac{1}{h^2}$!!)

5) Solénoïds

Imagine a circular cylinder of insulating material on which a wire is coiled making N loops:

top view



↳ the ≠ loops do not touch each other (no short circuit)

N loops 1 single circuit intensity i

Symmetry considerations give

$\rightarrow |\vec{B}|(r,\theta,z) = |\vec{B}|(r,z)$ (invariance by rotation)

$\rightarrow$ vertical symmetry plane $\Rightarrow \boxed{\vec{B} = B(r,z)\,\vec{u}_\theta}$

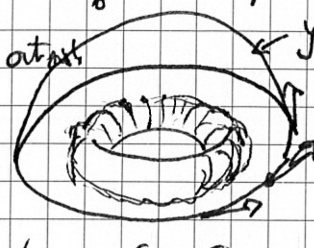
We gonna see that $\vec{B}$ is ~~constant~~ (non) within the torus indep of z

0 outside.

Ampère theorem: $\mathcal{C}$ = circle passing inside

- $\vec{B}(z).2\pi r = \mu_0 N i \qquad B(r,z) = \frac{\mu_0 N i}{2\pi r}$ indep of z!

- outside



$\leftarrow S$ to compute $I_C = \iint \vec{j}.d\vec{S}$

↳ $\oint \vec{B}.\vec{dl} = 2\pi r B(r,z) = \mu_0 I_C = 0 \Rightarrow B = 0$ outside.

Solénoïde infini

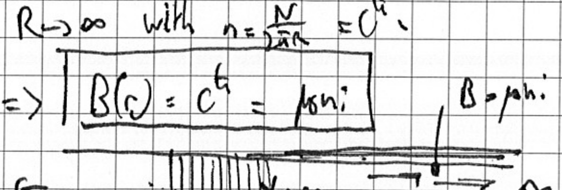
regardons $B(r)$ pour $r = R + \delta$ R = ~~rayon du~~ torus big radius, δ = small

$$B(r) = \frac{\mu_0 i N}{2\pi R\left(1+\frac{\delta}{R}\right)} = \mu_0 i \underbrace{\frac{N}{2\pi R}}_{n} \frac{1}{1+\delta/R}$$

n = nb of loops per unit length

$R \to \infty$ with $n = \frac{N}{2\pi R} = C^{te}$

$\Rightarrow \boxed{B(r) = C^{te} = \mu_0 n i}$

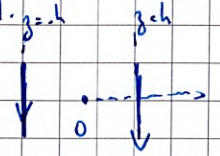


Application: very important role in electronics

6) Helmholtz coils

- Solenoids are the way to produce constant and tuned magnetic fields
- The pb is that the interior of a sol. is not easily accessible.

→ Helmholtz invented a device to produce cheaply a zone with an almost constant field.



we recycle the result for the single loop $B = \frac{\mu_0 i R^2}{2}\left[\frac{1}{[(z-h)^2+R^2]^{3/2}} + \frac{1}{[(z+h)^2+R^2]^{3/2}}\right]$

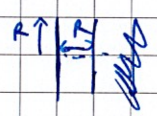
how to choose h to get B "as constant as possible" on the axis Oz around O ($z=0$) ?

$$B(z \text{ small}) \simeq \frac{\mu_0 i R^2}{2}\left[\frac{1}{(R^2+h^2)^{3/2}\left[1+\frac{z^2-2zh}{h^2+R^2}\right]^{3/2}} + \circlearrowleft_{z\to -z}\right]$$

$$= \frac{\mu_0 i R^2}{2[R^2+h^2]^{3/2}}\left[1 - \frac{3}{2}\frac{z^2-2zh}{h^2+R^2} + \left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\frac{1}{2}\frac{4z^2h^2}{(h^2+R^2)^2} + \circlearrowleft_{z\to -z}\right]$$

$$= \frac{\mu_0 i R^2}{[R^2+h^2]^{3/2}}\left[1 + \frac{z^2}{[h^2+R^2]^2}\underbrace{\left[-\frac{3}{2}(h^2+R^2) + \frac{15}{2}h^2\right]}_{0 \text{ if } 12h^2 = 3R^2} + O(z^4)\right]$$

$$\boxed{2h = R}$$



~~III) Magnetic fields generated by~~

III.1) Magnetic dipoles

. You have probably remarked a major difference between electrostatics and magnetostatics : whereas there is electric charge there is no magnetic charge (called often the magnetic monopole).

. Thus, can we define an elementary distribution of current, playing the role of charges in electrostatics
→ definitely not

. The best one can do is the equivalent of dipole

Def: a magnetic dipole is a closed circuit $\mathcal{C}$ with intensity I and surface vector $\vec{S} = \iint_{\mathcal{C}} d\vec{S}$, which is observed at distances $r \gg$ size of the circuit $\sqrt{|\vec{S}|}$
For this dipole, one defines the magnetic moment $\vec{m} = I\vec{S}$

Rk: if one has a current distribution $\vec{j}$, one assumes it can be decomposed into closed loops : $\vec{m} = \text{"}\sum\text{"}\ i\oint_{\Sigma} \vec{r}\wedge d\vec{r}$ $= \frac{1}{2}\text{"}\sum\text{"}\oint d\vec{S}\ \vec{r}\wedge\vec{j}\,dl = \frac{1}{2}\iiint \vec{r}\wedge\vec{j}\,d\tau$
↳ petit vect surface

Rk': Consider a sphere with constant charge density ρ and mass density μ
The total kinetic moment is $\vec{\sigma} = \iiint_{\text{sphere}} \vec{r}\wedge\mu\vec{v}$ if the sphere is for instance rotating
whereas the total magnetic moment induced by the rotation
$$\vec{m} = \frac{1}{2}\iiint \vec{r}\wedge\rho\vec{v}$$
If $\rho(\vec{r})$ is proportional to $\mu(\vec{r})$ for instance $\rho = c^{te}$ and $\mu = c^{te}$ $\Rightarrow \frac{\rho}{\mu} = \frac{q}{m}$
$\Rightarrow \vec{\sigma} = \frac{2m}{q}\vec{m}$ $\quad \vec{m} = \frac{q}{2m}\vec{\sigma}$ $\quad \frac{q}{2m}$ = gyromagnetic ratio γ

Remind : e^- have intrinsic $\vec{m}$!

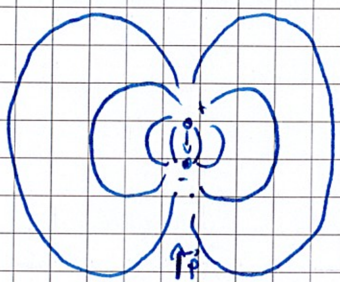
This is a classical model. For an electron an intrinsic kinetic moment exists (classical picture = the e^- spins around itself), as well as an intrinsic magnetic moment
Quantum effects yields however $\gamma = \frac{q}{m}$ and not $\frac{q}{2m}$

Rk: The name "magnetic dipole" is uncorrect: there is no such thing as two "poles" of opposite charges here. Why then do we use such an improper denomination?

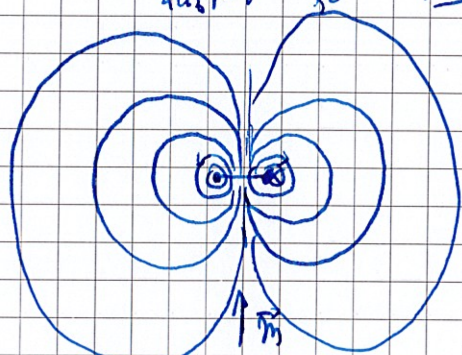
→ if one computes the "far-field" of a current loop of magnetic moment $\vec{m}$, one gets [COMPLICATED CALCULATION!]

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi r^3}\left[\frac{3(\vec{m}\cdot\vec{r})\,\vec{r}}{r^2} - \vec{m}\right]$$

electric field of an electric dipole: $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0 r^3}\left[\frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^2} - \vec{p}\right]$



electric dipole



magnetic dipole

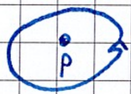
IV ~~Other sources~~ Static sources of magnetic field

- Up to now we have seen that current loops are producing $\vec{B}$
- But one knows materials that produce $\vec{B}$ without any current applied to the magnets. So what?

materials for magnets : lodestone (magnetite, ?? magnetit)

→ Actually, there are potential sources of magnetic fields in every single atom

take an atom H



The e^- rotates around proton
→ tiny current loop

with ~~$i = \frac{+e}{2\pi a}$~~ = ~~$\frac{1{,}6 \cdot 10^{-19}}{2\pi \cdot 0{,}5} A \approx 10^{-8} A$~~

$$i = \frac{e}{T} \quad \leftarrow \text{period}$$

$$= \frac{ev}{2\pi a} = \frac{e\sqrt{a}}{2\pi a}\sqrt{\frac{v^2}{a}}$$

$$= \frac{e\sqrt{a}}{2\pi a}\sqrt{\frac{e^2}{4\pi\varepsilon_0 a^2 m}}$$

$$= \frac{e^2}{2\pi\sqrt{4\pi\varepsilon_0}\, a^{3/2}\sqrt{m}}$$

$\to \vec{m} = \frac{e}{2\pi a}\pi a^2 \vec{n}_z = \frac{ea}{2}$

~~$a = \frac{\hbar}{m_e}$~~

$$m = i\pi a^2 = \frac{e^2\sqrt{a}}{4\sqrt{\pi\varepsilon_0 m}}$$

The kinetic moment is quantified $\sigma_z \in \hbar\mathbb{N}$
$\sigma \in \sqrt{\sigma(\sigma+1)}\,\hbar$

↳ but very rapidly the direction of $\vec{m}$ explores all directions → $\langle \vec{m} \rangle = \vec{0}$

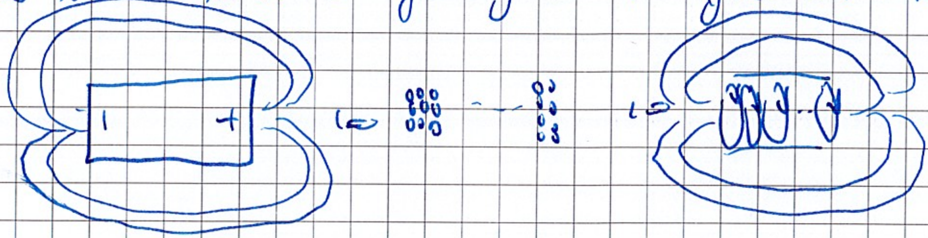
→ in Quant Mech it is reflected by a fundamental state with full SO symmetry

→ There are also intrinsic magnetic moment
→ spins of e^-
→ spins of nucleus. (10^3 times weaker) negligible (p, neut, et e^- = spin $\frac{1}{2}$) ← fermions

→ Usually, a macroscopic piece of body has a zero magnetic moment because every atom has a total magnetic moment with a random direction

→ For some materials (including iron) a macroscopic piece of body can display a macroscopic magnetic moment, resulting from a global alignment of microscopic $\vec{m}$ in the same direction

This situation is called "ferromagnetism" and gives birth to magnets



The complete theory of magnetic materials is pretty complicated!

V) Effect of magnetic fields on charges

1) Lorentz law

- Up to now we have seen how moving charges in electric circuits can ~~form~~ create magnetic fields, in exactly the same way ele. charges create $\vec{E}$
- We know in electrostatics the converse effect: a charge q in $\vec{E}$ experiences a force $\vec{F} = q\vec{E}$ (and accelerates)
- In magnetism we have the same effect

$$\boxed{\begin{array}{l}\text{Lorentz force: a moving charge } q \text{ with velocity } \vec{v} \text{ at } \vec{r} \\ \text{experiences the force } \vec{F}_{Lo} = q\vec{v} \times \vec{B}(\vec{r})\end{array}}$$

Rk1: if no motion ($v=0$) no force!

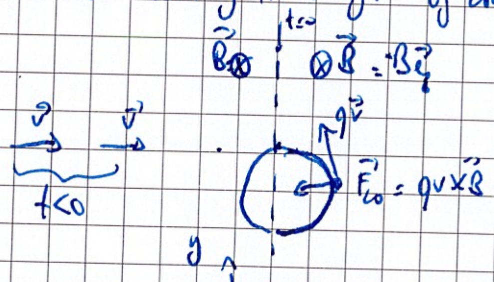
Rk2: the Lorentz force has a very special property: it does not exert any work on the particle $\Rightarrow$ no energy transfer

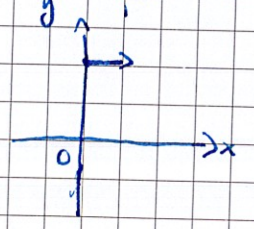
$$\delta W_{Lo} = \vec{F}_{Lo} \cdot d\vec{r} = \vec{F}_{Lo}\, \vec{v}\, dt = q\, \underbrace{(\vec{v} \times \vec{B})}_{\perp \vec{v}} \cdot \vec{v}\, dt = 0$$

Rk3: The general electromagnetic force on a charged particle is

$$\vec{F} = q\,[\vec{E} + \vec{v} \times \vec{B}] \quad \text{if both } (\vec{E}, \vec{B}) \text{ are present}$$

- Example: e^- coming in a region of constant $\vec{B}$ ($t>0$) ($\vec{B}(t<0) = \vec{0}$)





$$m\dot{v}_x = q v_y B_z - 0$$

$$m\dot{v}_y = q[0 - v_x B_z]$$

$$m\dot{v}_z = 0$$

$$\Rightarrow \quad v_z = 0 \text{ and } \dot{v}_x = -\frac{q}{m} B v_y, \quad \dot{v}_y = \frac{q}{m} B v_x \quad \Big\} \quad \ddot{v}_x + \left(\frac{qB}{m}\right)^2 v_x = 0$$

$\omega = \frac{qB}{m}$ = cyclotron pulsation

$$\begin{cases} v_x = v_x^0 \cos \omega t \\ v_y = \alpha \sin \omega t \end{cases} \quad \text{but } \dot{v_y} = \frac{qB}{m} v_x$$

$$\alpha \omega = \frac{qB}{m} v_0 \Rightarrow \alpha = v_x^0$$

$$\begin{aligned} v_x &= v_x^0 \cos \omega t \\ v_y &= v_x^0 \sin \omega t \end{aligned} \Rightarrow \begin{aligned} x &= +\frac{v_x^0}{\omega} \sin \omega t + 0 \\ y &= -\frac{v_x^0}{\omega} \cos \omega t + \cancel{g(t)}\ c \end{aligned}$$

for sake of symmetry: $y\left(\frac{\pi}{2\omega}\right)$: $y\left(\frac{\pi}{\omega}\right) = -y(0)$

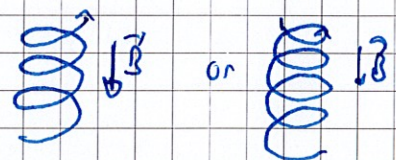
$$\frac{v_x^0}{\omega} + \overset{c}{\cancel{g(t)}} = \frac{v_x^0}{\omega} - c \Rightarrow c = 0$$

Rk: $v_x^2 + v_y^2 = v_{x0}^2 \Rightarrow$ no energy input/output

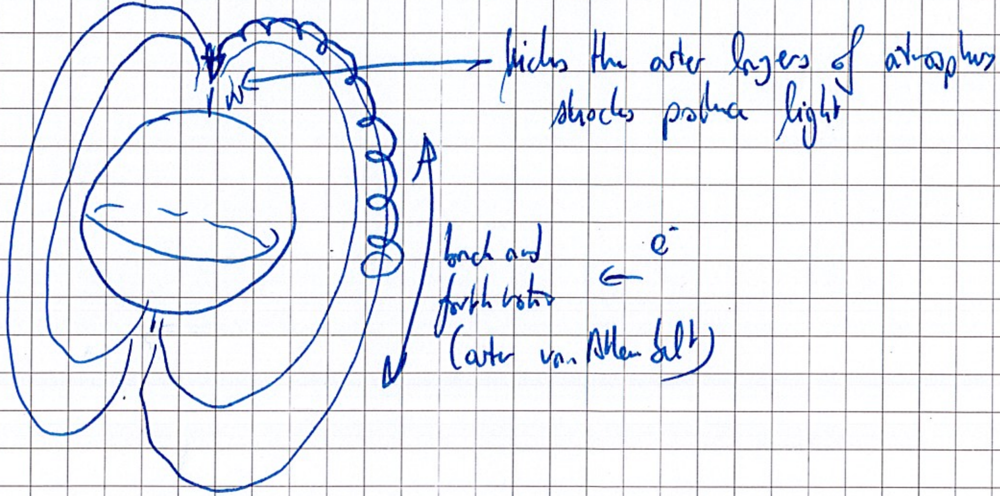
$\Rightarrow$ effect of B = curves the trajectory

converts linear motion to circular motion

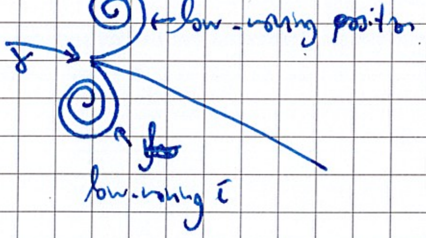
Rk: if $v_y^0 \neq 0 \Rightarrow$ helical motion



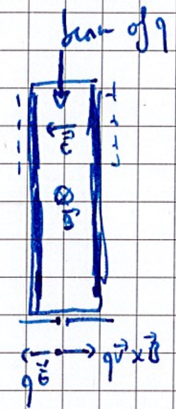
- It is what happens in $\begin{cases}\text{aurora borealis} \\ \text{northern lights}\end{cases}$



• Application : Detection of charge in bubble chambers

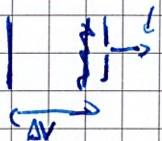


• Application : velocity selector



$\Rightarrow v = E/B$

• JJ Thomson e/m Exp. (1897)

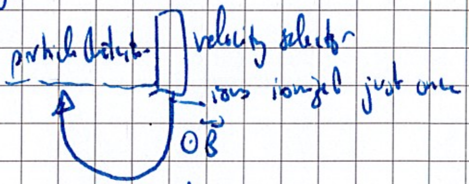


$\frac{1}{2}mv^2 = e\Delta V \quad v = \sqrt{\frac{2e\Delta V}{m}}$

with a velocity selector : $v = \frac{E}{B}$

$\frac{E}{B} = \sqrt{\frac{2e}{m}\Delta V} \quad E, \Delta V, B$ known $\Rightarrow \frac{e}{m}$ known

• Mass spectrometer



↳ radius $= \frac{mv}{qB} \Rightarrow$ access to m.

} discovery of isotopes of Neon Aston (1919)

2) Laplace's law

Consider a portion of a current loop :

the moving [charged] particles in it are described by $\vec{j} = \rho\vec{v}$

→ the charge of such particles $\rho\,\delta\tau$ and $\vec{j}\,\delta\tau = \rho\vec{v}\,\delta\tau$
$\sim \rho\vec{v}\,S\,\vec{dl}$
$\sim I\vec{dl}$

if δl is small

the Lorentz force on such a current element is

$$\delta\vec{F} = \left(\sum dq\right)\vec{v}\wedge\vec{B}$$
$$= \delta\tau\,\vec{j}\wedge\vec{B}$$

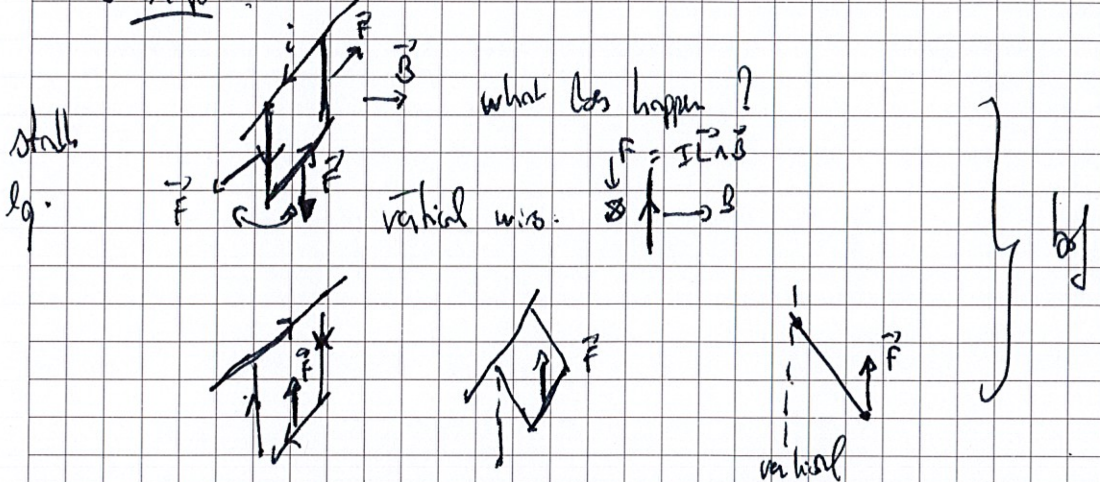
$$\boxed{\delta\vec{F} = I\vec{dl}\wedge\vec{B}}$$

Laplace's law

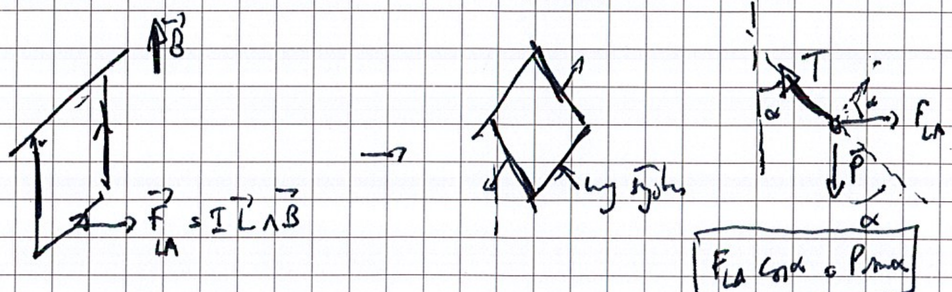
→ this is nothing but the eq. of Lorentz force for a conducting wire.

→ $\vec{dl}$ is oriented along the direction chosen for counting positive I

• Example:



Theory :



bare of 6 cm $\lambda = 0{,}15\ g/cm$ $I = 8A$ $\alpha = \frac{\pi}{4} \Rightarrow F_{LA} = ILB = P$

$ILB = L\lambda g \Rightarrow B = \frac{0{,}15\cdot10^{-3}\times 9{,}8}{8} \simeq 10^{-4}\,T$

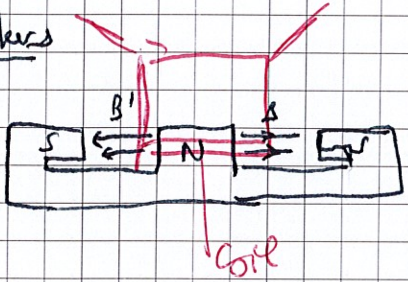
- **Total force on a wire in a homogeneous $\vec{B}$**

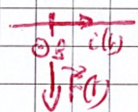
$$\vec{F} = \oint I\, \vec{dl'} \wedge \vec{B} = I \left(\oint \vec{dl'}\right) \wedge \vec{B}$$

⚠ $\boxed{\vec{F}_{tot} = \vec{0} \text{ if } \vec{B} = \vec{cst}}$

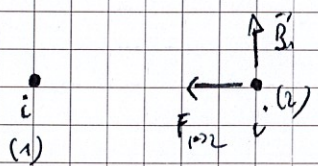
$\oint \vec{dl'} = \vec{0}$

- **Loudspeakers**





- **Two straight wires**



$$|\vec{F}_{1\to 2}| = ILB = \frac{I^2 L \mu_0}{2\pi R}$$

"L'ampère est l'intensité d'un courant constant qui maintenu dans 2 cond. rect. // de long. ∞, de section circ. négligeable, à 1 m l'un de l'autre dans le $\emptyset$ produit une force de 2.10^{-7} N / mètre de fil"

$\Rightarrow$ $\mu_0 = 4\pi.10^{-7}$ SI because of the def of Ampère