UFAZ-L1-Electrostatics

Part 1: Electric charges – Electric fields –Fields lines

Exercise 1

1) The Hydrogen Atom is composed of a nucleus made of one proton ($m_p = 1.67 \times 10^{-2}$ kg and $q = e = 1.602 \times 10^{-1}$ C) and one electron ($m_{e-} = 9.11 \times 10^{-31}$ kg and $q = -e = -1.602 \times 10^{-19}$C) whose average distance from the nucleus is $d = 53$ pm. Calculate and compare the magnitude of the electric force and the gravitational force between the electron and the proton. We give $\varepsilon_0 = 8.85 \times 10^{-12}$ F.m^{-1} and G=6.67× 10^{-1} $m^3.kg^{-1}.s^{-2}$.

2) The size of an atomic nucleus made of protons and neutrons is in the order of magnitude of the femtometer. Calculate the electrostatic repulsion between two protons. Comments.

Exercise 2

Copper molar mass is $M = 63$ g/mol and its volumic mass is $\rho = 8{,}7$ g/cm^3. Calculate the number of atoms in 1 cm^3 of copper. We will assume that there is one free electron per copper atom. We give Na=6.023×10^{23} mol^{-1}.

1) What would be the electric charge of 1 cm^3 of copper if we remove $1/1000000$ of its free electrons?

2) What would be the intensity of the electric force between two identical pieces of copper like described in the first question and separated from 1 meter?

Exercise 3

Two identical electrostatic pendulums of length $l = 0.3$ m and mass $m = 0.2$ g, attached in the same point are in contact at initial situation. We assume the radius of the spheres are negligible so that the initial angles made by the pendulums with the vertical is taken to be zero. We give them the same electric charge q. What is the value of that charge if the equilibrium distance between the two pendulums is $d = 10$ cm?

Exercise 4

1) Draw the field lines of an electrostatic doublet: (-q ;+q) and then (+q ;+q).

2) We consider an equilateral triangle with electric charges on its summits. We have three configuration (+q;+q;+q) , (+q;+q;+2q) and (+q,+q,-2q). Calculate the electric field in the center of the triangle for each case. Draw the field lines and determine the axis of symmetry.

3) We consider four electric charges in the summits of a square ABCD of size a. Two are positive and two are negative. In the first configuration, the positive charges share the same side of the square (A and B) and in the second configuration, the positive charges occupy the diagonal (A and C). Draw the field lines and determine the axis of symmetry. For the second configuration, calculate the electric force felt by the charge in A and then the electric force in the center of the square when the electric charges are 0.4 µC, -0.8 µC and if the size of the square a is 10 cm.

Part 2: Calculations of electric fields produced by charged geometric objects

Exercise 1: Electric charged wire

We consider a wire of size 2a and a negligible section with a uniform lineic density of charge λ.

1) Show that the electric field at a point M that belongs to the system symmetry axis at a distance z from the wire is written as:

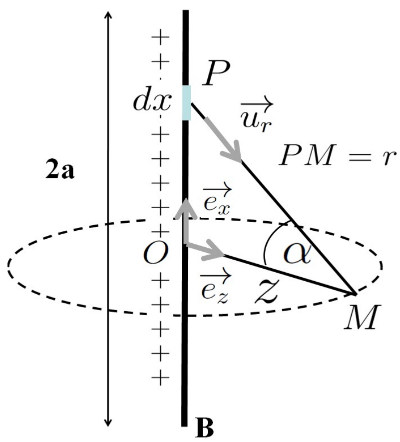
$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0} \frac{a}{z\sqrt{z^2 + a^2}} \overrightarrow{e_z}$$

2) Check that the obtained formula is homogenous to an electric field.

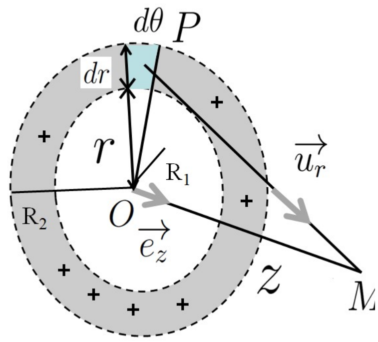
3) What is the expression of the electric potential V(M) ?

4) How is transformed the previous results when the wire is considered as infinite?

5) Discuss the orientation of the field lines.



Exercise 2: electric charged crown, disc and plane



We consider a crown of internal radius R_1 and external radius R_2 and center O, having a uniform surface density of charge σ.

1) Calculate the expression of the electric field vector produced at a point M in the symmetry axis of the system at a distance z from O. Deduce the expression of the electric potential V(z).

2) How are modified the results if the crown is transformed into a disc of radius R?
3) Deduce that the electric field vector created by a charged infinite plane of electric density σ at a distance z is:

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\overrightarrow{e_z}$$

4) Give the expression of the electric potential V(z).

Exercise 3: Plane Capacitor

We consider a capacitor made of two rectangular metallic plates of section S separated from a distance d. The first plate P_1 located at $z = -d/2$ has a uniform surface density $\sigma^+ > 0$ and the second plate P_2 located at $z = d/2$ has a uniform negative surface density $\sigma^- < 0$. We assume that the electric fields can be taken as the ones created by infinite planes.

1) Draw the graphical representation of the electric field $E_1(z)$ and $E_2(z)$ produced respectively by P_1 and P_2. Deduce the graphical representation of the total electric field produced by the two plates $E(z) = E_1(z)+ E_2(z)$. Give the expression of the electric field vector inside and outside the two plates. How are modified the results if we invert the sign of σ^+ and σ^- ?

2) Calculate the expression of the electric potential $V(z)$ and draw its graphical representation. We give $V(z = -d/2) = V_1$ and $V(z = d/2) = V_2$.

3) The voltage difference between the two plates is given by $\Delta V = V_1 - V_2$ and we assume that the electric charges on the plates are given by Q_1 and Q_2 with $|Q_1| = |Q_2| = Q$. Write the voltage difference ΔV as a function of electric charge Q.

4) The capacitance C of the capacitor is defined as the ratio between the electric charge and the voltage $C = Q/\Delta V$. For a plane capacitor show that C is given by:

$$C = \frac{\varepsilon_0 S}{d}.$$

5) What is the unit of C? Discuss the influence of the plate's surface and the distance d.
6) Considering that the plates have a charge Q_1 and a charge $Q_2 = -Q_1$ express the potential energy E_p of this two-charge system. The voltage difference is written $\Delta V = U$. Show that the capacitor energy reads

$$E_p = \frac{1}{2}CU^2.$$

7) A capacitor of capacity C=0.6 pF (pico farad) is submitted to a voltage difference of 4 V. Calculate the number of electrons collected at the negative plate. We assume the plates to be a square of size d. Deduce also the distance between the two plates and the electric surface density of charge.