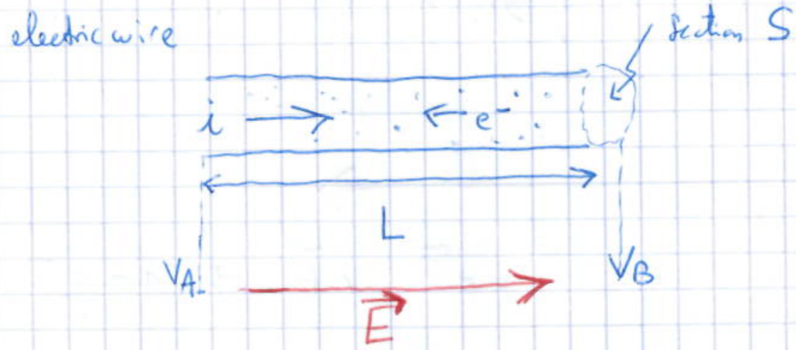
CLASSICAL LIGHT-MATTER INTERACTION IN A METALIC MATERIAL - CONDUCTIVITY

1.) Electric conductivity

I) Conductance - Conductivity - Ohm law



Ohm law : $U = V_A - V_B = Ri$

· $\vec{E} = -\vec{grad}V = -\frac{\Delta V}{L}\vec{u} = -\frac{(V_B - V_A)}{L}\vec{u}$ so:

$= \frac{U}{L}\vec{u}$

· $i = \iint \vec{j}\cdot\vec{dS} = jS$

$U = Ri$ or $i = \frac{U}{R} = GU$

Resistance R in Ω
Conductance G in Siemens

$jS = G\,E.L$

$j = \frac{GL}{S}E$

$\boxed{\vec{j} = \gamma\vec{E}}$ local Ohm law

γ = conductivity : $= \frac{GL}{S}$: $[S.m^{-1}]$

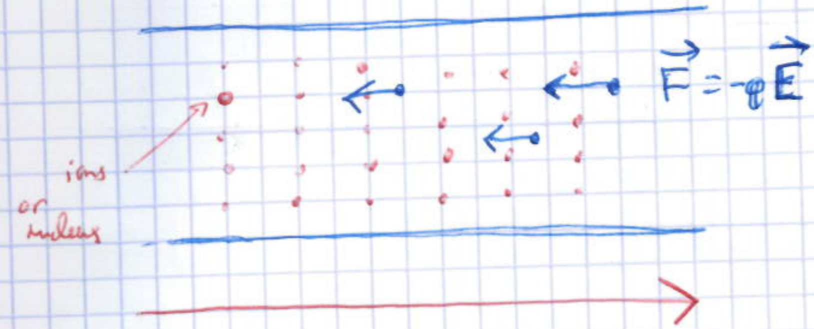
$\vec{j} = -\gamma\,\vec{grad}\,V$: analogy : Fourier law $\vec{j} = -\lambda\,\vec{grad}\,T$

diffusion, Fick law $\vec{j} = -D\,\vec{grad}\,n$

$\gamma_{Cu} = 5{,}8\,10^7\ S.m^{-1}$
$\gamma_{Ag} = 6{,}2\,10^7\ S.m^{-1}$ } metal

$\gamma_{glass} \simeq 10^{-11}\ S.m^{-1}$
(dielectric)

- Microscopic motion of charges driven by electric field
- free electron can move and perform collision with
  - other nucleus
  - other electr.



2nd Newton law: $\begin{cases} - m\vec{a} = q\vec{E} = -e\vec{E} \Rightarrow a = \frac{q}{m}E \\ -\text{velocity} = \frac{qE}{m} t = v \\ -\text{position} = \frac{qE}{m}\frac{t^2}{2} = x \end{cases}$

- But motion is stopped because collisions: and in general, the motion is:

Random "driven" trajectory.

- We define as $\underline{\tau}$ the time "average" over all electron between two collisions.
- Electron position is $\langle x \rangle = \frac{qE}{m2}\langle t^2 \rangle \equiv \frac{qE}{2m}\tau^2$

velocity $\langle v \rangle = \frac{qE}{m}\tau$

current density: $\vec{j} = n_v q\vec{v}$ will be

$$\boxed{\begin{aligned} j &= \frac{n_v q^2 E \tau}{m} = \gamma E \\ \gamma &= \frac{n_v q^2 \tau}{m} \end{aligned}}$$

depends on:
- τ
- n_v

} material and its internal ordering

$n = \frac{\rho N_A \times 10^6}{M} = \frac{8{,}96 \times 6{,}02\,10^{23} \times 10^6}{63{,}5} = 8.10^{28}.m^{-3}$

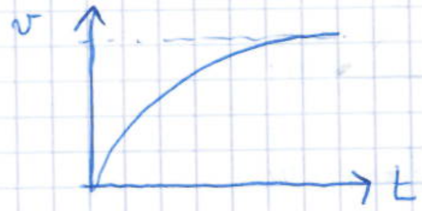
$n_v^{Cu} \approx 8.10^{28}.m^{-3}$

$\gamma_{Cu} \approx 6.10^{7}\,S.m^{-1}$

$\tau \sim \frac{m\gamma}{q^2 n_v} \sim \frac{9.10^{-31}}{(1{,}6\,10^{-19})^2} \cdot \frac{6.10^{7}}{8.10^{28}} \approx \underline{2{,}6\,10^{-14}\,s}$

→ collisions parameter will be modeled by a friction force

$\vec{f} = -\frac{m}{\tau}\vec{v}$ to have $m\frac{d\vec{v}}{dt} + \frac{m}{\tau}\vec{v} = q\vec{E}$ whose solution is



$$\vec{v} = \frac{qE\tau}{m}\left(1 - e^{-t/\tau}\right)$$

Time dependent electric field:

Free electron (no Hooke force)

$$m\vec{a} = q\vec{E}(t) - \frac{m\vec{v}}{\tau}$$

$\cdot\ E(t) = E_0 e^{-i\omega t}$

so $\vec{v} = v_0 e^{-i\omega t}$

and $\vec{a} = -i\omega\vec{v}$

$$i\omega v + \frac{v}{\tau} = \frac{qE}{m}$$

$$v\left(i\frac{\omega\tau}{\tau} + \frac{1}{\tau}\right) = \frac{qE}{m}$$

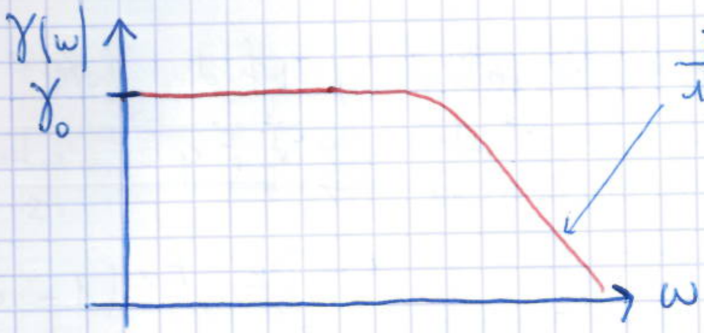
$$v = \frac{qE\tau}{m(1 - i\omega\tau)} \Rightarrow j = n_v q v$$

$$j = \frac{n_v q^2\tau}{m(1 - i\omega\tau)} = \gamma(\omega)E$$

$$\boxed{\gamma(\omega) = \frac{n_v q^2\tau}{m(1 - i\omega\tau)} = \frac{\gamma_0}{1 - i\omega\tau}}$$

$\cdot\ \omega\tau \ll 1$ if $\tau \ll \frac{1}{\omega} \sim T$ small frequency

$\cdot\ \omega\tau \gg 1$ high frequency



$$\frac{\gamma(0)}{1+i\omega\tau} \sim \frac{\gamma_0}{\omega\tau} \xrightarrow[\omega \nearrow]{} 0$$

No conductivity at high frequency...

2) Propagation of an electromagnetic field in a metalic medium

a) Maxwell equations

$$\begin{cases} \operatorname{div} \vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \vec{\operatorname{rot}}\, \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \end{cases} \qquad \operatorname{div} \vec{B} = 0 \qquad \vec{\operatorname{rot}}\, \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

- Conducting media:

$$\rho = \rho_{Free} + \cancel{\rho_{bound}} = \rho_F$$

$$\vec{j} = \vec{j}_{free} = \gamma \vec{E}$$

- Simplification:

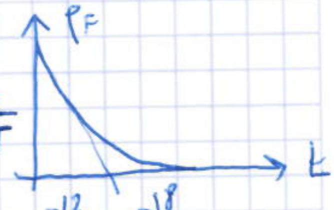
$$\frac{\partial \rho_F}{\partial t} + \operatorname{div} \vec{j_F} = 0$$

$$\frac{\partial \rho_F}{\partial t} + \operatorname{div} \gamma \vec{E} = 0$$

$$\frac{\partial \rho_F}{\partial t} + \frac{\gamma \rho_F}{\varepsilon_0} = 0$$

Solution

$$\rho(t) = \rho_0 e^{-\frac{t}{T}}$$



$$T = \frac{\varepsilon_0}{\gamma_F} \sim \frac{8{,}10^{-12}}{10^{7}} \sim 10^{-18}\,s \ll 1$$

We will set $\operatorname{div} \vec{E} \approx \vec{0}$

b) Wave equation:

- $$\vec{\operatorname{rot}}(\vec{\operatorname{rot}}\, \vec{E}) = -\vec{\operatorname{rot}} \frac{\partial \vec{B}}{\partial t} = \operatorname{grad} \underbrace{\operatorname{div} \vec{E}}_{0} - \Delta \vec{E}$$

- and

$$\frac{\partial}{\partial t}\left(\vec{rot}\,\vec{B}\right) = \frac{\partial}{\partial t}\left[\mu_0 \underbrace{\vec{j}}_{\gamma\vec{E}} + \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right]$$

$$= \mu_0\gamma \frac{\partial \vec{E}}{\partial t} + \mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Finally:

$$\boxed{\Delta\vec{E} = \mu_0\gamma\frac{\partial \vec{E}}{\partial t} + \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}}$$

if $|\vec{j} = \gamma\vec{E}| \gg |\vec{j}_D = \varepsilon_0\frac{\partial\vec{E}}{\partial t}| \sim |\varepsilon_0\omega\vec{E}|$

$\left|\frac{\gamma E}{\omega\varepsilon_0 E}\right| \gg 1 \iff \omega \ll \frac{\gamma}{\varepsilon_0} \sim 10^{+18}$ rad/s

$\rightarrow$ $\Delta\vec{E} = \mu_0\gamma\frac{\partial\vec{E}}{\partial t}$: diffusion equation: at low frequency

Dispersion relation:

with $\vec{E} = \vec{E}\,e^{i(kr-\omega t)}$ plugged in wave equation

$$\Delta\vec{E} = -k^2\vec{E}\ ;\ \frac{\partial\vec{E}}{\partial t} = -i\omega\vec{E}\ ;\ \frac{\partial^2\vec{E}}{\partial t^2} = -\omega^2$$

it gives

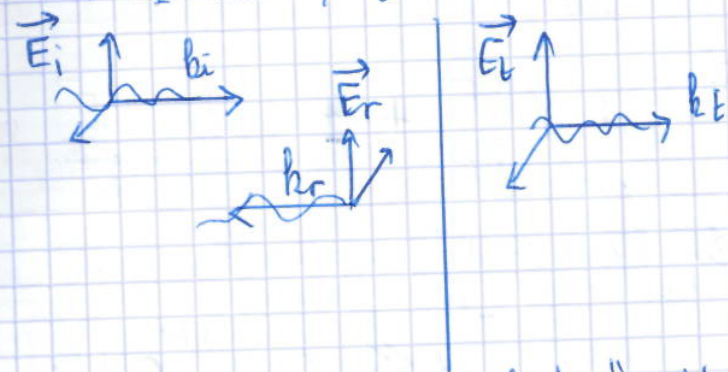
$$-k^2 = -i\omega\mu_0\gamma - \frac{\omega^2}{c^2}$$

$$\boxed{k^2 = i\omega\mu_0\gamma + \frac{\omega^2}{c^2}}$$

3) Reflection and Transmission at the interface of a metallic medium

a) Fresnel Coefficient

→ normal incidence
→ n_1 = air ; n_2 = metal = n



We have seen in "Wave lecture" that.

$$r = \frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2} \quad \text{and} \quad R = r^2 =$$

$$t = \frac{2n_1}{n_1 + n_2} \qquad T = \frac{n_2}{n_1} t^2$$

amplitude coefficient

energy coefficient
with $R + T = 1$

n_1 = air
$n_2 = n$ metal

$$r = \frac{1 - n}{1 + n} \qquad t = \frac{2}{1 + n}$$

b) Influence of frequency - Limit cases

b 1) Low frequency and Skin effect

free current $\vec{j}_{free} = \gamma \vec{E}$ is larger than displacement current $\vec{j}_D = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ when $\omega \ll 10^{18}$ rad/s

Wave equation is $\Delta \vec{E} = \mu_0 \underline{\gamma}(\omega) \dfrac{\partial \vec{E}}{\partial t}$ and the dispersion relation gives with $\vec{E} = \vec{E_0}\, e^{i(kr - \omega t)}$

$$-k^2 = -i\omega\, \mu_0 \gamma(\omega)$$

and $\gamma(\omega) = \dfrac{\gamma_0}{1 - i\omega\tau} \approx \gamma_0$ at Low frequency

$\omega\tau \ll 1$

$T > 10^{-4}$ s.

$\omega < 10^{14}$ rad/s

$$k^2 = i\, \omega \mu_0 \gamma_0$$

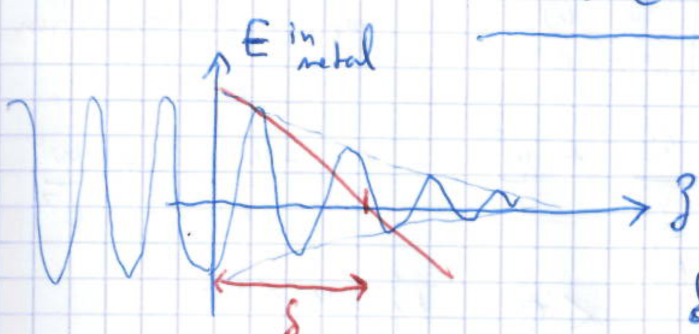
$$i = \frac{(1+i)^2}{2} = (1+i)^2 \frac{\mu_0 \omega \gamma}{2}$$

So $k = \sqrt{\dfrac{\mu_0 \omega \gamma}{2}}\,(1+i) = \dfrac{(1+i)}{\delta} = k' + ik''$

$\delta = \sqrt{\dfrac{2}{\mu_0 \omega \gamma}}$ depht of attenuation

$\vec{E}$ in metal (1D) $= \vec{E} = \vec{E_0}\, e^{i\left(\left(\frac{1+i}{\delta}\right)x - \omega t\right)}$

$$= \vec{E_0}\, e^{-\frac{x}{\delta}}\, e^{i\left(\frac{x}{\delta} - \omega t\right)}$$



$\gamma_{Cu} = 5.10^{7}\ S.m^{-1}$

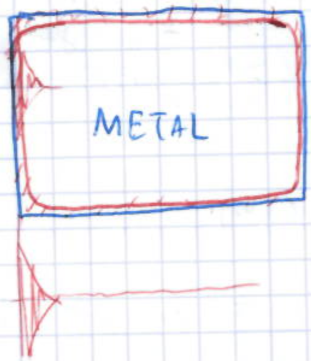
$\mu_0 = 4\pi\, 10^{-7}\ H.m^{-1}$

Value of δ for Cu

$$\delta = \sqrt{\frac{2}{\mu_0 \omega \gamma}} = \frac{0{,}16}{\omega^{1/2}}$$

(s^{-1}) ω	Radio $< 10^{11}$	IR	Visible	UV
δ (m)	$10^{-4} - 10^{-7}$	$10^{-8} - 10^{-9}$	10^{-9}	$10^{-9} - 10^{-10}$

→ E-M waves do not propagate in metal only on the Surface



→ <u>Skin effect</u>

- We see if $\omega \nearrow$ increase $\delta \longmapsto 0$
- $k = \left(\frac{1+i}{\delta}\right) \longmapsto n_2 = n(1+i)$

] TOTAL REFLEXION

b) High frequency - Plasma domain.

→ $\gamma(\omega) = \frac{\gamma_0}{1 - i\omega\tau} \approx \frac{-\gamma_0}{i\omega\tau}$

Relation of dispersion:

$$k^2 = i\omega\mu_0\gamma(\omega) + \frac{\omega^2}{c^2}$$

$$\approx \cancel{i}\cancel{\omega}\mu_0\left[\frac{-\gamma_0}{\cancel{i}\cancel{\omega}\tau}\right] + \frac{\omega^2}{c^2}$$

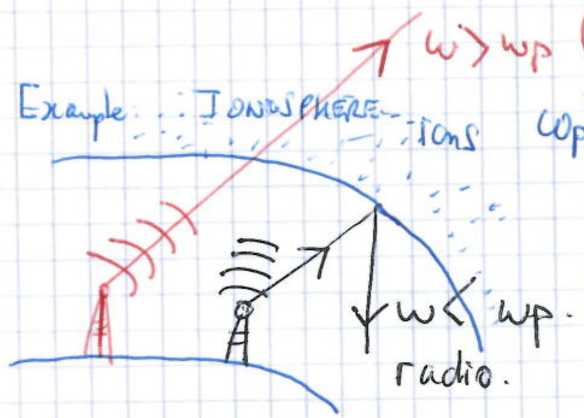
ω_p: plasma pulsation

$$\approx -\mu_0 \frac{n_v q^2 \cancel{\tau}}{m\,\cancel{\tau}}\frac{\varepsilon_0}{\varepsilon_0} + \frac{\omega^2}{c^2} \approx -\frac{\omega_p^2}{c^2} + \frac{\omega^2}{c^2}$$

$$\begin{cases} k^2 = \frac{\omega^2}{c^2}\left[1 - \frac{\omega_p^2}{\omega^2}\right] & < 0 \text{ if } \omega < \omega_p \\ & > 0 \text{ if } \omega > \omega_p \\ k = \frac{\omega}{c}\left[1 - \frac{\omega_p^2}{\omega^2}\right]^{1/2} & \in \mathbb{C} \text{ imaginary } \omega < \omega_p \\ & \in \mathbb{R} \text{ reel } \omega > \omega_p \end{cases}$$

$\omega < \omega_p$ = No transmission $k \in \mathbb{C}$

$\omega > \omega_p$ = total transmission $k \in \mathbb{R}$



$$\omega_p^2 = \frac{N v q^2}{m_{ion}\, \varepsilon_0} \approx 300\ \text{MHz}.$$

c) General overview:

FILTER

$0 \le \omega \ll \frac{1}{\tau}$	$\frac{1}{\tau} \ll \omega \le \omega_p$	$\omega \gg \omega_p$
- reflexion - absorption - attenuation - skin effect	TOTAL REFLEXION NO Transmission	TOTAL TRANSMISSION $\lambda \ll$ object size.