# PW n°5: Interferences using Sound waves

## I Elements of theory about waves and interferences

The production and propagation of sounds are linked to the existence of a molecular vibrational motion. Sound waves propagate only on material media: solid, liquid and gaz. At the source level the sound is produced by the deformation of a physical compound. This deformation is then transmitted to the surrounding molecules and brings them to an out of equilibrium position and therefore pushes the neighbor as well. This phenomenon occurs without material transport, the molecules are just brought to an out of equilibrium position and this motion is transmitted to the next neighbors without transport. One often speaks about a compression wave. In a fluid we can show that this surpression due to the sound wave follows the equation:

$$\frac{\partial^2 \Delta P}{\partial t^2} = c^2 \frac{\partial^2 \Delta P}{\partial x^2}$$

with $c = \sqrt{\frac{1}{\rho\chi}}$ is the propagation velocity or wave celerity where $\rho$ is the volume mass of the fluid and $\chi$ its compressibility . Sound celerity in the air depends on the temperature, at ordinary temperature the celerity is around 340m/s. In other materials then air the sound propagates with different velocities. As an example in the water the velocity reaches 1500m/s.

### I .1) Wave propagation

**Observation of a wave at a given point.** A wave (Light, mechanical, or sound…) propagating in one direction and can be represented locally, on a point M on the propagation axis by following expression:

$$s(x_M, t) = s_0 \cos\,(2\pi f(t - \phi) \qquad (1)$$

where $s(x_M, t)$ is the amplitude of the wave at the point M: it is the electrical field in the case of light, the transverse deformation of the wire, the gap from the average pressure in the case of a sound wave, $x_m$ is the abscissa $x$ of the point M, $t$ is the time, $s_0$ the amplitude of the wave, f the frequency of the vibration with $f = \frac{1}{T}$ where T is the temporal period, $\phi$ is the temporal phase shift and represents the delay at the point M compared to the origin O. At the point O ($x_0$=0) the wave equation can be written as follows: $s(x_0, t) = s_0 \cos(2\pi f t)$. The phase shift $\phi$ is therefore dependant on the distance $x_M$ and the wave velocity in the direction x: $\phi = \frac{x_M}{v}$. $s(x_M, t)$ is solution of the wave equation.

Wave shape in space at a given time t. From previous equations waves amplitude $s(x,t)$ can be written:

$$s(x, t) = s_0 \cos\left(2\pi \frac{1}{T}\left(f - \frac{x}{v}\right)\right) \qquad (2)$$

Thus introducing the wavelength $\lambda$ =vT one can write

$$s(x, t) = s_0 \cos\left(2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right),$$

where $\lambda$ is the distance between two points M and M' on the propagation axis for which at a fixed moment $t$ they are in the same state (same elongation and same variation).

$$s(x_{M'}, t) = s_0 \cos\left(2\pi\left(\frac{t}{T} - \frac{x_{M'}}{\lambda}\right)\right) = s_0 \cos\left(2\pi\left(\frac{t}{T} - \frac{x_M + \lambda}{\lambda}\right)\right) = s(x_M, t) \qquad (3)$$

M and M' signals are thus in phase. Let's try to represent the wave along the propagation axis i.e. the elongation $s$ as a function of $x$ for a given time $t$=$t_1$. Expression (2) can thus be rewritten as follows:

$$s(x,t_1) = s_0 \cos\left(2\pi\left(\frac{t_1}{T} - \frac{x}{\lambda}\right)\right)$$

Let's choose $t_1$ such as $t_1=(n+1/2)T$, $n$ is a natural number. Expression (3) can thus be rewritten as:

$$s(x,t_1) = s_0 \cos\left(\pi\left(1 - \frac{2x}{\lambda}\right)\right) = -s_0 \cos\left(\left(\frac{2\pi x}{\lambda}\right)\right)$$

This equation represents the deformation along the propagation axis ($t$ is fixed =a snapshot) and should not be mixed with $s(x_0,t)$ which correspond to the deformation as a function of time (like a boat on a fixed point that move with the waves).

## I.2) Interferences between 2 sound waves

Let's consider 2 emitters $S_1$ and $S_2$ distant from each other by a distance $2a$ and emitting with the same frequency $f$, with the same amplitude and the same propagation direction $x$. To have some interference the waves must be coherent meaning that the phase shift should be constant over time. To address this issue, we will use the same generator to produce the waves and therefore these waves are in phase. In these conditions, these waves can be written with the same expression:

$$s_1 = s_2 = s_0 \cos(2\pi f t)$$

The waves propagating in the same direction $x$ will combine in their common zone of propagation. If we look through a plan perpendicular to the propagation axis at a distance $D$ from the sources, at the point M the waves will have propagate through the distances $x_1$ for the first and $x_2$ for the second. Thus the expression for the 2 waves at the point M:

$$s_1 = s_0 \cos\left(2\pi f(t - \frac{x_1}{v}\right)$$

$$s_2 = s_0 \cos\left(2\pi f(t - \frac{x_2}{v}\right)$$

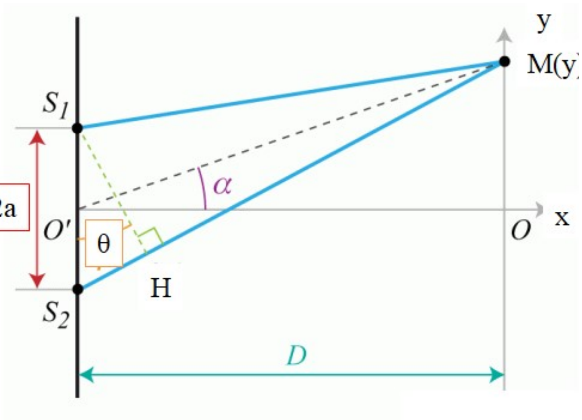


**Figure 1.**

The sum wave S is obtained by summing up the 2 other waves $s_1$ and $s_2$:

$$S = s_1 + s_2 = 2s_0 \cos\left(2\pi f\left(\frac{x_1 - x_2}{2v}\right)\right) cos\left(2\pi f\left(t - \frac{x_1 + x_2}{v}\right)\right)$$

This expression can also be written

$$S = 2s_0 \cos\left(2\pi\left(\frac{x_2 - x_1}{2\lambda}\right)\right) cos\big(2\pi f(t - \phi_m)\big) \qquad (4)$$

Or directly under the form $S = A\cos\big(2\pi f(t - \phi_m)\big)$ . The amplitude A is function of the path difference δ between the waves $s_1$ and $s_2$: at the point M: $\delta = x_2 - x_1 = MS_2 - MS_1$ and depends on the position of the point M. Let's calculate δ as a function of y, in the case $D >>> 2a$.

$$\delta = MS_2 - MS_1 \approx S_2H$$

$S_2H$ is perpendicular to $S_1H$, therefore $\widehat{S_1S_2H} = \theta$ and $S_2H = S_1S_2 \sin\theta$ . With $D>>>2a$ , we assume $\alpha \approx \theta \approx \tan\theta \approx \frac{y}{D}$ and so: $S_2H \approx S_1S_2\left(\frac{y}{D}\right)$. The amplitude A of the resulting wave is therefore twice the amplitude $s_0$ modulated in the y direction by a cosine function depending on the ordinate y:

$$A(y) = 2s_0 \cos\left(2\pi\frac{\delta}{2\lambda}\right) = 2s_0 \cos\left(\pi\frac{2ay}{D\lambda}\right) \quad (5)$$

In fact, we focus on the intensity of the wave at the point M which is proportional to the square of the amplitude:

$$P_{/m^2} \propto A^2 = 4s_0^2 \cos^2\left(\pi \frac{2ay}{D\lambda}\right) \quad (6)$$

The intensity is maximal for $\pi\frac{2ay}{D\lambda} = k\pi$ with k a natural number, thus at the position $y = k\frac{D\lambda}{2a}$. The intensity is minimal for $\frac{2ay}{D\lambda} = (2k = 1)\frac{\pi}{2}$ , thus: $y = \left(k + \frac{1}{2}\right)\frac{D\lambda}{2a}$. From previous equations we can follow the variation between two successive dark fringes or bright once. We described this variation as the interfringe distance $i$ characterized by:

$$i = \frac{\lambda D}{2a} \quad (7)$$

## II. Experimental part

**1)** Explain how experimentally you would access to the time period and the wavelength of a sound wave propagating from an emitter towards a receptor? To this end you may remember (or have a look) to the wavelength determination in PW3: Sound Wave propagation.

### II.1 Directivity of the emitter

The emission of the ultra-sound occurs in a preferential direction. It is thus important in the case of studying interferences to determine the propagation area. This area is obtained by moving in a circle of radius *r* from the emitter with the receptor facing the emitter.

**2)** For each *y*-position in a given circle of radius r, measure the amplitude given by the receptor. Repeat the experiment with different radii. Then, draw graph amplitude as a function of r for different radii from the emitter to the receptor.

Is this emission isotropic, give the angular opening?

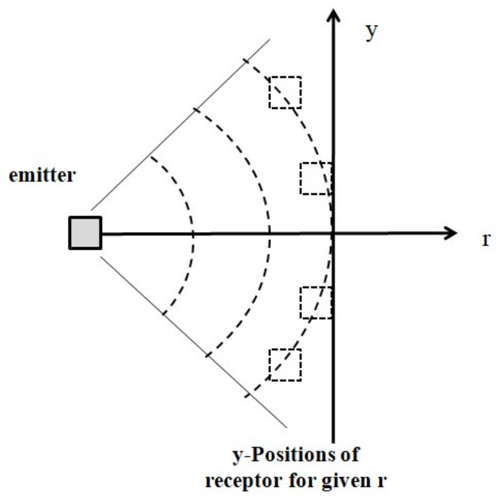


**Figure 2.**

### II.2 Propagation and Attenuation of the wave

One can observe that the amplitude $s_0$ becomes weaker as the wave is propagating. The reason why $s_0$ decreases with x can be shown by studying the dispersion. At the point O where the wave is emitted, the wave is characterized by a power $P_{wave}$. This power is proportional to the square of the amplitude (eq. 12). $P_{/m}{}^2$ evolves as $1/x^2$ and is proportional to $s_0^2$. If the wave propagates in all direction of the space in an isotropic way, the emission power will be distributed over spheres with increasing areas. Because of energy conservation, the total power of the wave remains constant over transport over x direction. This power is only distributed over areas $4\pi x^2$ that are increasing: therefore one can write

$$P_{/m^2} = \frac{dP}{dS} = \frac{P_{total}}{S_{total}}$$

Therefore the wave power $P_{wave}$ is linked to the power of the wave per unit area by $P_{wave} = 4\pi x^2 P_{/m^2} = cte$, or $P_{/m^2} = \frac{P_{wave}}{4\pi x^2}$. We thus expect the amplitude $s_0$ of the wave to decrease as a function of *1/x*.

**3)** Put emitter and receptor in front one from the other increase the distance x between them and measure the amplitude of the signal. With software Origin plot the graph of the amplitude *S=f(1/x)* and show the linear behavior with the inverse of the distance to the source.

## II. 3) Interferences

Use 2 emitters $E_1$ and $E_2$ placed symmetrically from the propagation axis on position $S_1$ and $S_2$. The distance between $S_1$ and $S_2$ is called *2a*. The distance of the receptor that will be moved on y axis is called *D*.

**Measuring interfringe *i* for *D* and *2a* fixed.**

Put them as closed as possible and measure the distance between them $2a_{min}$. Put the receptor at 20 cm from the point in the middle of the line between the two emitters.

**4)** What is the frequency on the receptor? Describe the variation of amplitude when moving the receptor for different positions along y direction (see figure 1 for instance). Compare these variations to the variation observed when determining the propagation area.

**5)** The interference field is limited in space therefore you should observe them in an area between 5cm and -5cm from the propagation axis. Determine the receptor positions for which the amplitude are maximal and minimal. Deduce the value of the interfringe i from your experiments. How to measure the interfringe accurately?

**Measuring interfringe *i* for *D* fixed and *2a* as a parameter.**

**6)** Which graph would you plot to verify the theoretical behavior of the interfringe *i* as a function of parameter *a*? For *D* fixed at 20 cm, make a table of your measurements for $2a=2a_{min}$, 10 cm, 12 cm and 14 cm.

**7)** Measure the interfringe *i* for the different values of *a* and draw the graph $i=f(1/a)$ with Origin software.

**8)** Determine the value of the slope and compare with the one given by the theoretical formula.

**Measuring interfringe *i* as a function of *D* with 2a fixed**

We work now with $2a=2a_{min}$ and we study the influence of distance *D*. By performing a similar study than previously with *D*=25, 20, 15 and 10 cm, check the dependence of the interfringe with the distance *D*.