Electrostatics – Magnetostatics L1

I Introduction and concepts

1) **Electrostatics devices**
2) **Electric charges**
    a. Ponctual charges
    b. Continuous charges distributions
3) **Electric forces and Electric fields**
    a. The Coulomb law
    b. The electric field
    c. Field lines
4) **Electric potential and energy**
    a. Work of an electric force
    b. Electric potential
    c. Equipotential lines
5) **Electric field created by superposition of charges**
    a. Two electric charges: Shape of the field lines
    b. N electric charges
    c. Continuous charges distribution
    d. Energy of a system of charges
6) **Symmetries of the electric field**

Chapter I Introduction and concepts

1) Electrostatics devices

- electrostatic pendulum
- electrostatic machines
- Leiden jar
- capacitors

$\boxed{\text{See slides I.1)}}$

Objectives of the lecture:

→ How to calculate the electric field produced by charged objects?

→ How to connect them to capacitance? Voltage? Energy?

Ligne de discussion des diapositives (pour moi à l'oral)

- il existe une propriété : l'électricité (des charges) que l'on peut créer, propager, stocker.
- ces propriétés induisent des forces (interactions) et une notion : le champ electrique
- How to calculate electric field produced by generic objects
- How to connect to capacitance? Voltage? Energy?

2) Electric charges

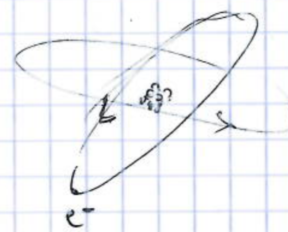
a- Ponctual charges

→ a neutral body get electrisation and acquires an electric charge Q (in coulomb. C)

$Q = \pm Ne$ N : integer

- e = <u>elementary charge</u> = $1,602\ 10^{-19}$ Coulomb
- elementary particle of atoms:

classical picture of Atom



electron : $m_{e^-} = 9,11 \times 10^{-31}$ kg

$q = -e = -1,602\ 10^{-19}$ C

proton : $m_p = 1,67 \cdot 10^{-27}$ kg

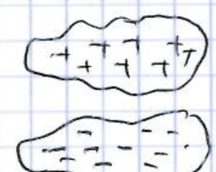
$q = +e$

neutron $m_n = 1,67\ 10^{-27}$ kg

$q = 0$

<u>Electrons responsible of electrization</u> : lighter than protons and protons "have to stay" in the nucleus for nucleus stability. $m\vec{a} = q\vec{E}$; ratio $\frac{q}{m_{e^-}} > \frac{q}{m_p}$

→ Body charged positively : lack of electron



negatively : excess of electrons

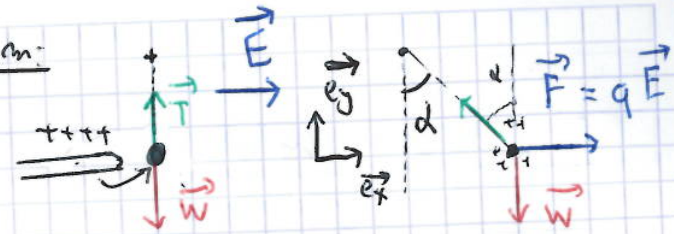
→ Electrisation in friction experiment. The electric charge is usually between milli Coulomb (mC) or micro coulomb (µC)

How many electrons?

$Q = -10^{-3}$ C $\Rightarrow N = \frac{Q}{-e} = 6,25\ 10^{15}$ electrons

if $Q = -10^{-6}$ C $= -1\mu$C $\Rightarrow N \sim 6,25\ 10^{12}$ electrons

- Electrostatic pendulum:



$$m\vec{a} = \sum \vec{F} = \vec{0}$$

$$\vec{W} + \vec{T} + \vec{F} = \vec{0}$$

$$\begin{cases} T\cos\alpha - mg = 0 \\ -T\sin\alpha + F = 0 \end{cases} \Rightarrow \underline{\vec{F} = mg\tan\alpha\,\vec{e_x}}$$

Knowing q or $\|\vec{E}\|$ can give acces to the other.

$m = 2\,g$ $\quad \|\vec{F}\| = 2\times10^{-3} \times 10 \approx 0{,}02\,N = \|q\vec{E}\|$

$\alpha = 45°$ $\quad \|\vec{g}\| \sim 10\ m/s^2$

+ Approximation of ponctual point or charge.

 $\xrightarrow[\text{mechanics}]{\text{usually done in}}$ + Mechanics of material point

q  $\xrightarrow[\text{approximation}]{}$ + q Electrostatics of ponctual charge

or

q  Spherical distribution

b. Continuous charge distribution

infinitesimal volume $d\tau = dx\,dy\,dz = dV$

has electric charge dq

- volumic charge density at point M $\boxed{\rho(M) = \frac{dq}{d\tau}}$ $C.m^{-3}$
- Total electric charge:

$$Q = \iiint_{\text{volume}} d\tau\,\rho(M) \underset{\substack{\uparrow \\ \text{if } \rho \text{ is uniform}}}{=} \rho \iiint_{\text{volume}} d\tau = \rho V$$

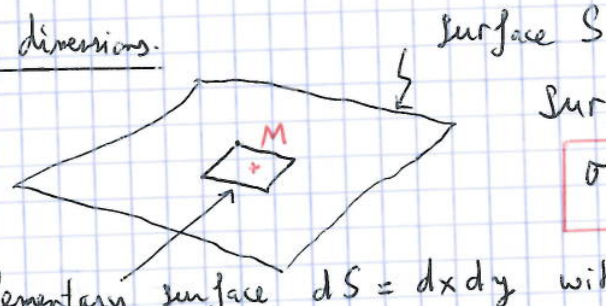
Remark: in cartesian coordinates $d\tau = dx\,dy\,dz$

cylindric $d\tau = r\,d\theta\,dr\,dz$

spherical $d\tau = dr\,r^2 d\theta \sin\theta\,d\varphi$

$\boxed{\text{see slides I.2.b)}}$

2 dimensions.



Surfacic charge density

$$\boxed{\sigma(M) = \frac{dq}{dS}} \quad C.m^{-2}$$

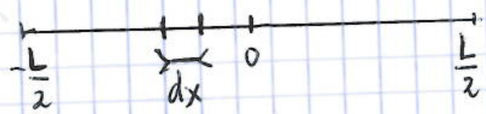
elementary surface $dS = dx\,dy$ with charge dq

$$Q = \iint_{surface} \sigma(M)\, dS \underset{\uparrow}{=} \sigma \iint_{surface} d\tau = \sigma S$$

if σ = constant uniform

cartesian $dS = dx\,dy$
cylindric $dS = r\,d\theta\,dz$
spherical $dS = r \sin\theta\, d\varphi\; r\,d\theta = r^2 \sin\theta\, d\theta\, d\varphi$.

1 Dimension



linear charge density $\lambda(M) = \frac{dq}{dx}$. $C.m^{-1}$

$$Q = \int_{-\frac{L}{2}}^{\frac{L}{2}} \lambda(M)\, dx \underset{\uparrow}{=} \lambda \int_{-\frac{L}{2}}^{L/2} dx = \lambda . L$$

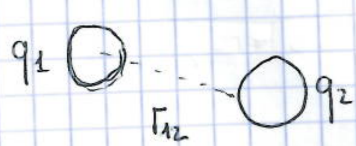
$\lambda(x) = \lambda$ uniform

3) Electric forces and Electric fields - Field Lines

a. The Coulomb law

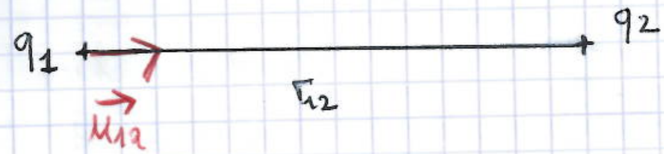
- Charles Augustin Coulomb 1736 - 1806
- See Slides I.3.a

· Force intensity $F \sim \frac{q_1 q_2}{r_{12}^2}$ - 1784



· intuition from Newton law of gravitation $F \sim \frac{m_1 m_2}{r_{12}^2}$

Today: 2 charges q_1 and q_2



$$\vec{F}_{1\to 2} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\vec{u}_{12}}{r_{12}^2} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\vec{r}_{12}}{r_{12}^3} = -\vec{F}_{2\to 1}$$

action from 1 on 2
(felt by 2)

- if $(q_1, q_2) > 0$ or $(q_1, q_2) < 0$: $q_1 \cdot q_2 > 0$ repulsion

- if $(q_1 < 0 ; q_2 > 0)$ or $(q_1 > 0 ; q_2 < 0)$: $q_1 q_2 < 0$ attraction

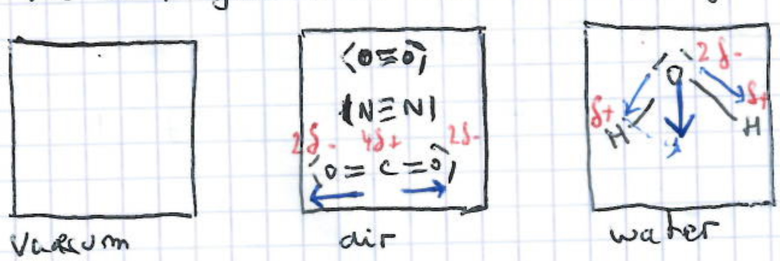
→ $\varepsilon_0 = 8{,}85 \times 10^{-12}\ F.m^{-1}$ (Farad: F)
dielectric permittivity of vacuum.

→ in material medium $\varepsilon_0 \mapsto \varepsilon_0 \varepsilon_r$

$\varepsilon_r(\text{air}) = 1{,}0006$ at 20°C

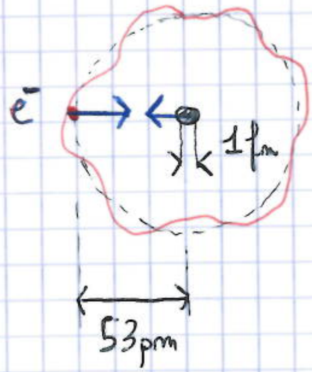
$\varepsilon_r(\text{water}) = 80$ at 20°C

Due to the fact that matter is made of "dipoles":



Exercise 1.

- Atom Hydrogen



$$\|\vec{F}_{elec}\| = \left\| \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}^2} \right\| = \frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{(1{,}6\ 10^{-19})^2}{4 \times \pi \times 8{,}85\ 10^{-12} \times (53 \times 10^{-12})^2}$$

$$= 8{,}19 \times 10^{-8}\ N$$

$$\|\vec{F}_{grav}\| = \left\| G \frac{m_1 m_2}{r_{12}^2} \right\| = \frac{6{,}67 \times 10^{-11} \times 9\ 10^{-31} \times 1{,}67 \cdot 10^{-27}}{(53 \times 10^{-12})^2}$$

$$= 3{,}6 \times 10^{-47}\ N$$

Ratio: $\dfrac{\|\vec{F}_{elec}\|}{\|\vec{F}_{grav}\|} = \dfrac{8{,}19\ 10^{-8}}{3{,}6\ 10^{-47}} \approx 10^{39}$!!!

Z

- Intensity in a nucleus: between two protons:

$r \approx 10^{-15}$ m.

$\|F_{elec}\| = 230$ N !!

$F_{\substack{repulsion \\ protons}} \gg F_{\substack{attraction \\ e^- - proton}}$:

- Stability of nucleus due to strong interaction (not an electromagnetic phenomena)

- Maybe tabular with fundamental interactions (slides)

Exercise 2:

- 

$m = \rho V = n \times M = \frac{N}{N_a} M$

$V = 1\,cm^3$
$M = 63\,g/mol$
$\rho = 8{,}7\,g/cm^3$

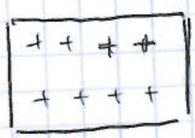
- $N = \frac{\rho V N_a}{M} = \frac{8{,}7 \times 1 \times 6{,}02\,10^{23}}{63} = 8{,}31\,10^{22}$ atoms

- We assume only one electrons per atom can be free.

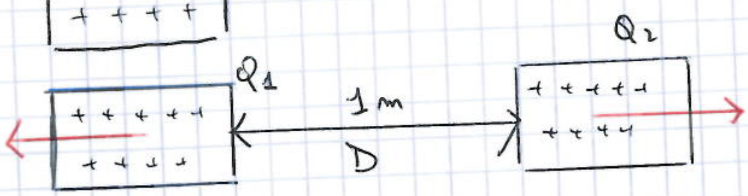
- Removing $1/1000000 = 10^{-6}$ fraction
- So free electron: $8{,}31 \times 10^{22} \times 10^{-6} = 3{,}81 \times 10^{16}$ electrons

and so total charge is

$$q = -ne = -3{,}81\,10^{16} \times 10^{-19} = -3{,}81\,10^{-3}\,C$$



$q = +3{,}81\,10^{3}\,C$



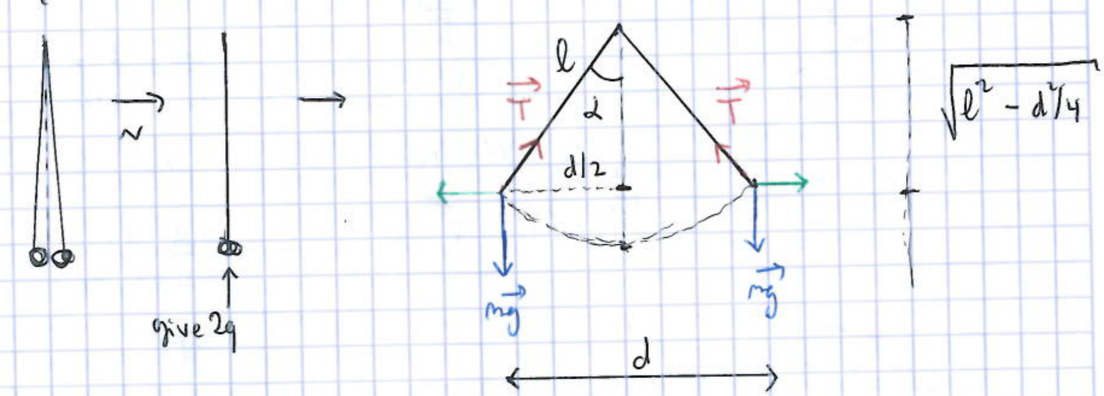
$$\|\vec{F}\| = \frac{Q_1 Q_2}{4\pi\varepsilon_0 D^2} = \frac{(3{,}81 \times 10^{3})^2}{4\pi \times 10^{-12} \times 8{,}85 \times 1^2} = 1{,}30\,10^{17}\,N \gg 1$$

The electric charge is too large !!!!

Exercise 3

$m_1 = m_2 = m = 0{,}2g$ $\quad d = 10\,cm = 0{,}1\,m$

$l_1 = l_2 = 30\,cm$



We know $\|\vec{F}\| = mg \tan\alpha$ (see I-2-a)

and $\tan\alpha = \dfrac{d}{2} \times \dfrac{1}{\sqrt{\ell^2 - d^2/4}}$

$$\|\vec{F}\| = \frac{q^2}{4\pi\varepsilon_0 d^2} = \frac{mg\,d}{2\sqrt{\ell^2 - d^2/4}}$$

$$q^2 = \frac{mg\,4\pi\varepsilon_0\,d^3}{2 \times \sqrt{\ell^2 - d^2/4}}$$

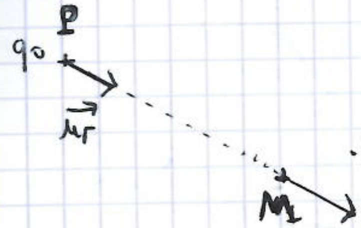
$g \sim 10\,m/s^2$

$$q = \left[\frac{0{,}2 \times 10 \times 4 \times \pi \times 8{,}85 \times 10^{-12} \times (0{,}1)^3}{2 \times \sqrt{(0{,}3)^2 - \frac{0{,}01}{4}}}\right]^{1/2} \approx \sqrt{3{,}76\,10^{-13}}$$

$$= 6{,}13\,10^{-7}\,C = 0{,}61\,\mu C$$

b- The electric field

* → test charge : q_0 at a point P



- We put in M a charge q_1:

$$\vec{F}_{0\to 1} = \vec{F_1} = \frac{q_0 q_1}{4\pi\varepsilon_0 PM^2}\vec{u_r}$$

- We remove q_1 and put q_2 in M

$$\vec{F}_{0\mapsto 2} = \vec{F_2} = \frac{q_0 q_2}{4\pi\varepsilon_0 PM^2}\vec{u_r}$$

- We remove q_2 and put q_3 ..... an so .. on with N charges

The quantity:

$$\frac{\vec{F_1}}{q_1} = \frac{\vec{F_2}}{q_2} = \dots = \frac{\vec{F_N}}{q_N} = \frac{q_0\,\vec{u_r}}{4\pi\varepsilon_0 PM^2}$$

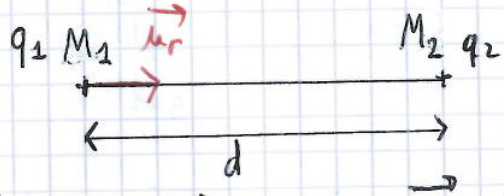
depends only on q_0 and traduce a physical property at point M created by q_0 in P

$$\vec{E}(M) = \frac{q_0}{4\pi\varepsilon_0 PM^2}\vec{u_r}$$ is called the electric field created by q_0.

Unit Volt/meter V/m

* Fields of two interacting charges

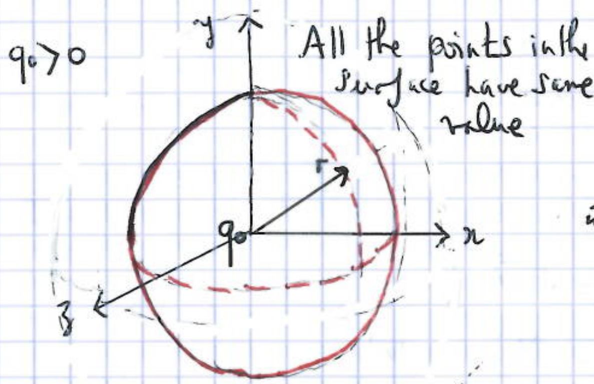
- For a charge q at M, it will feel the force $\vec{F} = q\,\vec{E}(M)$

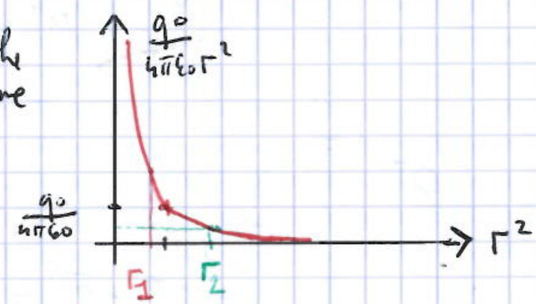


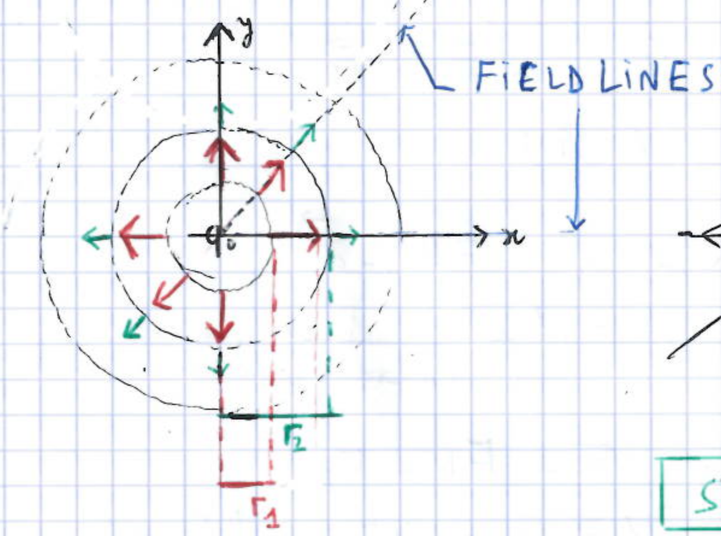
$$\vec{F}_{2\mapsto 1} = q_1\,\vec{E_2}(M_1) \text{ and } \vec{F}_{1\to 2} = q_2\,\vec{E_1}(M_2)$$

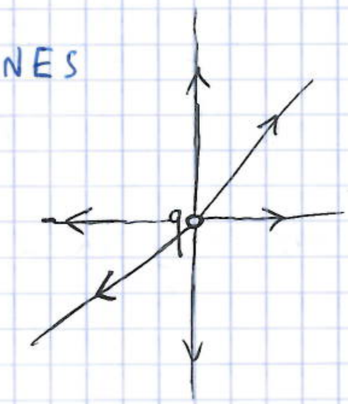
c- Field lines

- Charge q_0 creates Electric field in the space
- At a distance r from q_0: $\vec{E} = \dfrac{q_0}{4\pi\varepsilon_0 r^2}\vec{u_r}$
- intensity depends on $\dfrac{1}{r^2}$
- direction depends on sign of q_0:



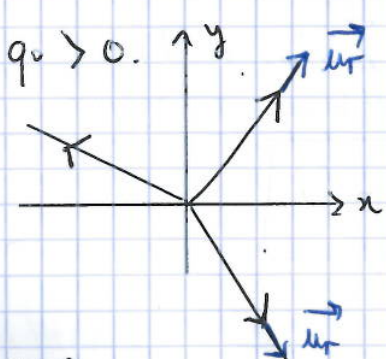


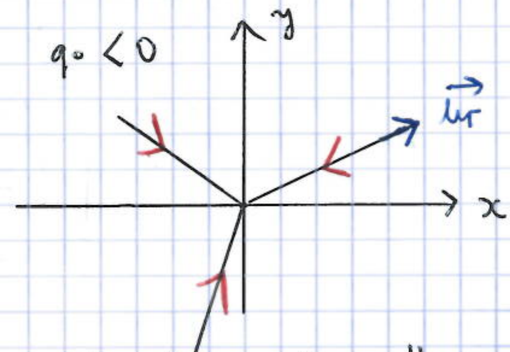




$\boxed{\text{See Slides I.2.c}}$

Field lines are tangent to the electric field vector





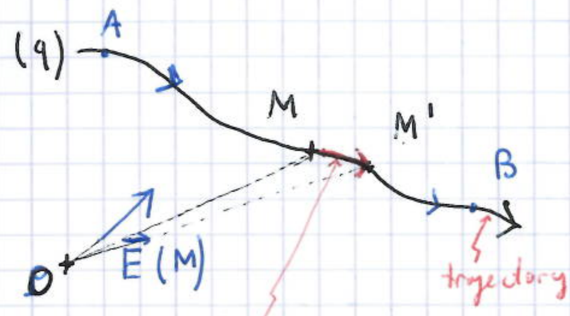
Field lines go out

Field line go "inside"

4) Electric potential and energy

a- Work of an electric force

- We move an electric charge q in a region where lives an electric field $\vec{E}(M)$



$\vec{dl} = \vec{MM'}$
$= \vec{OM'} - \vec{OM} = d\vec{OM}$

(i) . Elementary work between M and M'

$$\delta W = \vec{F} \cdot \vec{dl} = q\,\vec{E}(M) \cdot \vec{dl}$$

- Total Work between A and B

$$W_{AB} = \int_A^B \delta W = q \underbrace{\int_A^B \vec{E}(M) \cdot \vec{dl}}_{\text{cirulation of electric field}}$$

(ii) E total energy $\quad E = E_k + E_p$

if E = constante $\quad \Delta E = 0 = \Delta E_k \quad \Delta E_p = 0$

So $\quad \Delta E_k = -\Delta E_p$

But we know: $\quad \Delta E_k = \sum W_{ork} = W$

Theorem of kinetic energy

So $\quad \boxed{\Delta E_p = -W}$

We can write : $\Delta E_p = -W$

$$= -q \int_A^B \vec{E}(M) \cdot \vec{dl}$$

We define a function V called <u>electric potential</u> to be

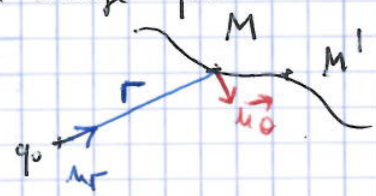
$$\Delta E_p = -q \left[-V \right]_A^B = -q(-V_B + V_A) = q(V_B - V_A)$$

and connected to potential energy.

b- Electric potential

$$\int_A^B \vec{E}(M) \cdot \vec{dl}$$

Let's consider $\vec{E}(M)$ created by a charge q_0:



$$\vec{E}(M) = \frac{q_0}{4\pi\varepsilon_0 r^2} \vec{u_r}$$

$$\vec{dl} = d\vec{OM} = d(r\vec{u_r})$$

$$= dr\,\vec{u_r} + r\,d\theta\,\vec{u_\theta}$$

$$\int_A^B \vec{E}(M) \cdot \vec{dl} = \int_A^B \frac{q_0\,dr}{4\pi\varepsilon_0 r^2} \underbrace{(\vec{u_r} \cdot \vec{u_r})}_{1} + \frac{q_0}{4\pi\varepsilon_0 r^2} r\,d\theta \underbrace{\vec{u_r} \cdot \vec{u_\theta}}_{0}$$

$$= \int_A^B \frac{q_0}{4\pi\varepsilon_0} \frac{dr}{r^2} = \left[-\frac{q_0}{4\pi\varepsilon_0 r} + cte \right]_A^B$$

$$\equiv \left[-V(r) \right]_A^B$$

$$\boxed{V(r) = \frac{q_0}{4\pi\varepsilon_0 r} + cte}$$

. usually we take cte as potential value for $r = \infty$ that is 0

. cte = 0

$$\lim_{r \to \infty} V(r) = cte \underset{\text{choice}}{\equiv} 0$$

Unit: Volt.

$$\int \vec{E}\cdot\vec{dr} = [-V]$$
$$= -\int dV$$

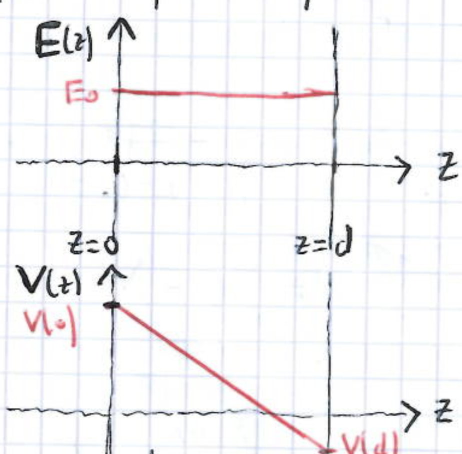
So $$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$
$$= \vec{grad}\,V \cdot \vec{dr} = \vec{\nabla} V \cdot \vec{dr}$$

with $\vec{grad} = \vec{\nabla} = \begin{vmatrix} \vec{e_x}\,\frac{\partial}{\partial x} \\ \vec{e_y}\,\frac{\partial}{\partial y} \\ \vec{e_z}\,\frac{\partial}{\partial z} \end{vmatrix} = \begin{vmatrix} \vec{e_r}\,\frac{\partial}{\partial r} \\ \frac{\vec{e_\theta}}{r}\,\frac{\partial}{\partial \theta} \\ \vec{e_z}\,\frac{\partial}{\partial z} \end{vmatrix}$

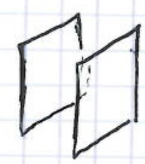
cartesian — cylindric

Finally: $$\boxed{\vec{E} = -\vec{grad}\,V}$$
$$\boxed{\int \vec{E}\cdot\vec{dr} = -[V]}$$

<u>Example</u>: in plane capacitor where $\vec{E}$ is constant



$\vec{E} = E_0\,\vec{e_z}$



$\vec{E} = -\vec{grad}\,V$

$E_0\,\vec{e_z} = -\left[\frac{\partial V}{\partial x}\vec{e_x} + \frac{\partial V}{\partial y}\vec{e_y} + \frac{\partial V}{\partial z}\vec{e_z}\right]$

$E_0\,\vec{e_z} = -\frac{\partial V}{\partial z}\,\vec{e_z}$

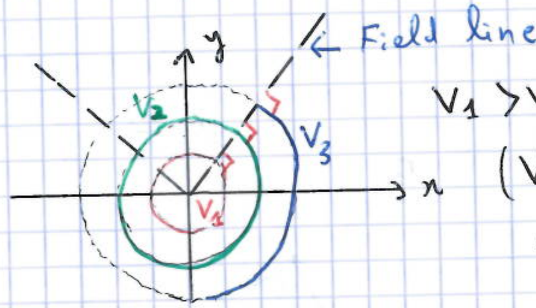
$$\int_{z=0}^{z=d} dV = -\int_{z=0}^{z=d} E_0\,dz = -\left[E_0 z + cte\right]_{z=0}^{z=d} = V(d) - V(0)$$

$$E_0 = \frac{V(0) - V(d)}{d}$$

c- Equipotential lines

- Lines where $V(r)$ as same value is called equipotential

<u>Ex:</u>



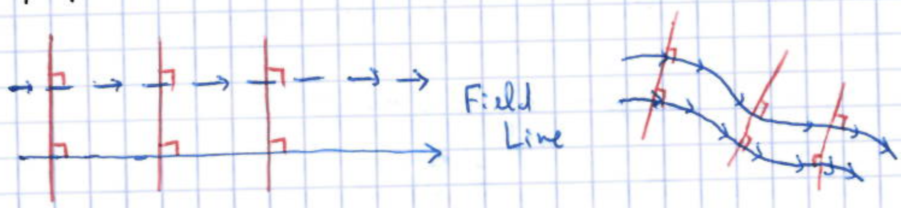
$V_1 > V_2\ V_3$

(V_1, V_2, V_3) are 3 ≠ equipotential line

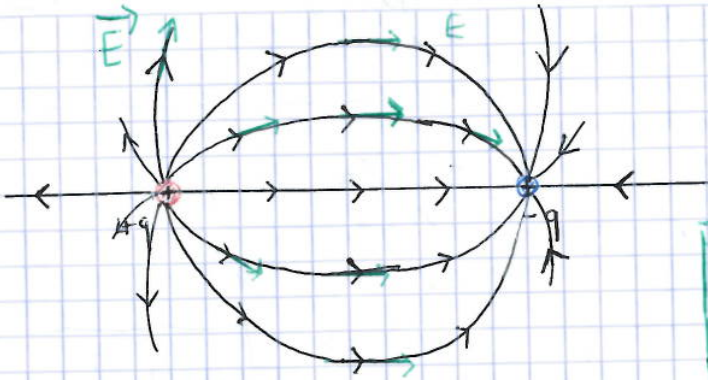
$$V = \frac{q_0}{4\pi\varepsilon_0 r} \quad ; \quad \vec{E} = \frac{q_0\, \vec{u_r}}{4\pi\varepsilon_0 r^2}$$

$V = cte$ for $r = cte$

- Equipotential lines are perpendicular to field lines

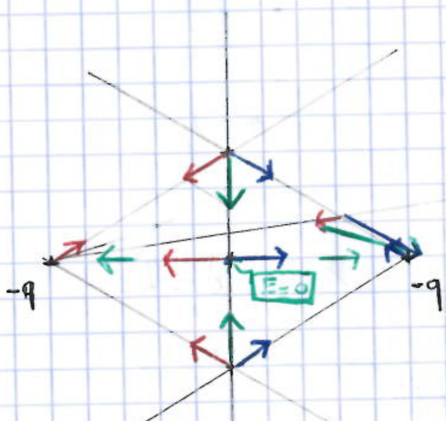


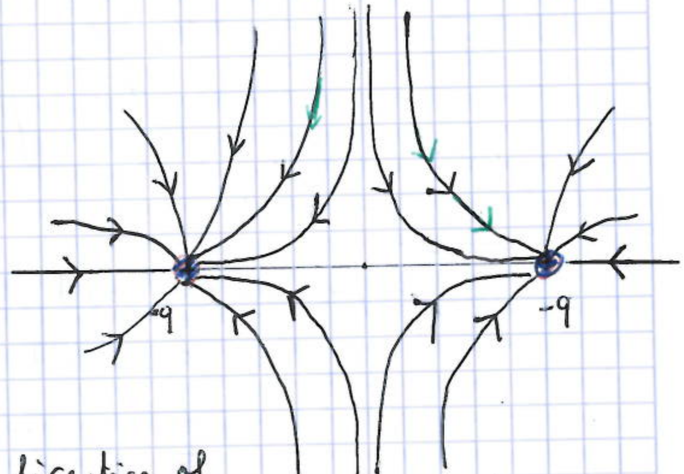
See slids I.4.c
+ topography



See slides I.5.a)
Equipotential b)

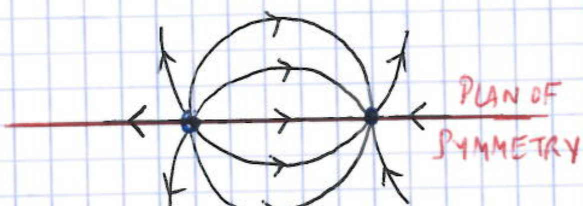
Field lines for $q_1 = -q < 0$ and $q_2 = -q < 0$.

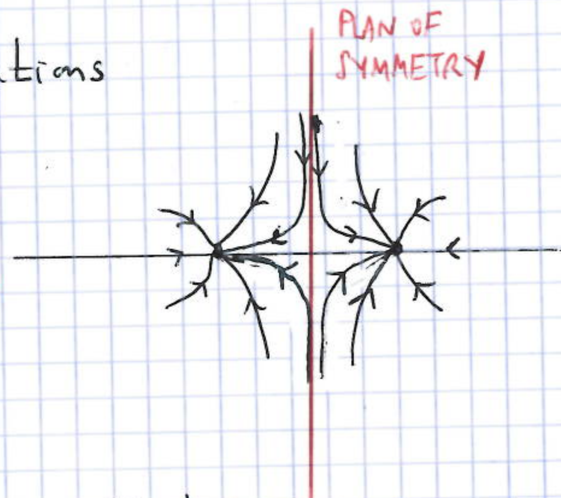




- if $q_1 = q_2 = +q > 0$ direction of field is opposite.

- Symetry of the configurations

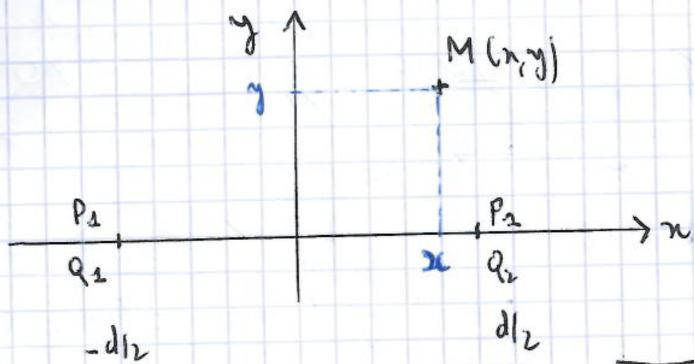




- Equipotential

$$V = cte = V_1(M) + V_2(M)$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{P_1M} + \frac{q_2}{P_2M}\right] = cte$$



$$P_1M = \sqrt{(x_M - x_{P_1})^2 + (y_M - y_{P_1})^2} = \sqrt{(x + d/2)^2 + y^2}$$

$$P_2M = \sqrt{(x - d/2)^2 + y^2}$$

"Complicated" equation: $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{\sqrt{(x+d/2)^2+y^2}} + \frac{q_2}{\sqrt{(x-d/2)^2+y^2}}\right] = cte$

Exercise

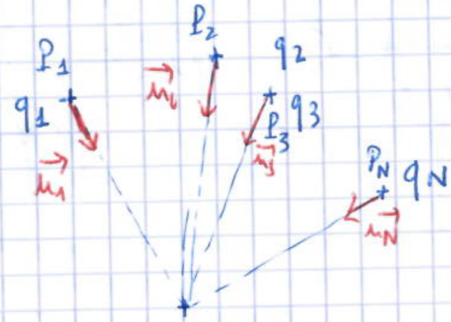
We give $q_1 = -q$ and $q_2 = +2q$.

Show that equipotential $V = 0$ is given by.

$$\left(x + \frac{5d}{6}\right)^2 + y^2 = \frac{4d^2}{9}$$

Equation of a circle of center of radius $R = \frac{2d}{3}$ and center located at point $\left(x = -\frac{5d}{6} \,;\, 0\right)$

b- N electric charges.



$P_iM = r_i$

$\vec{u_i} = \dfrac{\vec{P_iM}}{\|\vec{P_iM}\|}$

$$\vec{E}(M) = \vec{E_1}(M) + \vec{E_2}(M) + \vec{E_3}(M) + \dots \quad \vec{E_N}(M)$$

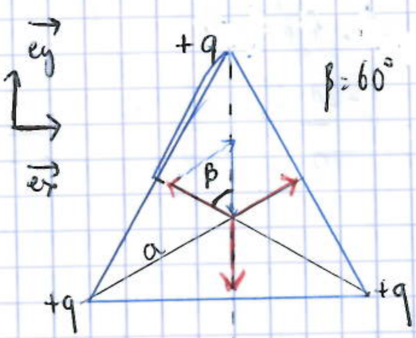
$$= \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_0} \frac{\vec{u_i}}{r_i^2}$$

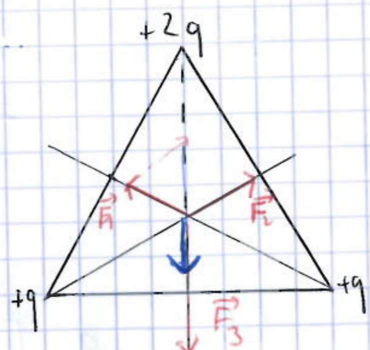
$$V(M) = V_1(M) + V_2(M) + \dots + V_N(M)$$

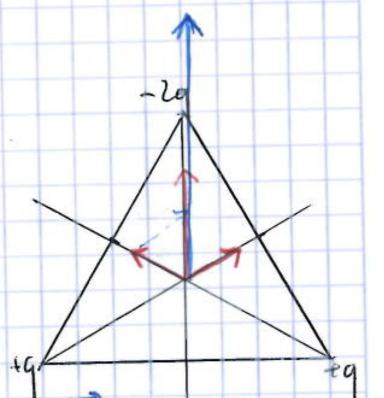
$$= \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_0 r_i}$$

Exercise 4: Part 1:

(1) 3 charges...







3 Forces has same amplitude:

$\|\vec{F_1}\| = \|\vec{F_2}\| = \|\vec{F_3}\| = \dfrac{q}{4\pi\varepsilon_0}\dfrac{1}{a^2} = F$

$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} = 0$

$$\begin{cases} -F\sin\beta + F\sin\beta + 0 = 0 \\ F\cos\beta + F\cos\beta - F = 0. \end{cases}$$

$\dfrac{q}{4\pi\varepsilon_0} \equiv k$

$\|\vec{F_1}\| = \|\vec{F_2}\| = \dfrac{k}{a^2}$

$\|\vec{F_3}\| = \dfrac{2k}{a^2}$

$\boxed{\vec{F} = -F\vec{e_x}}$

$$\begin{cases} F\cos\beta + F\cos\beta - 2F = -F \end{cases}$$

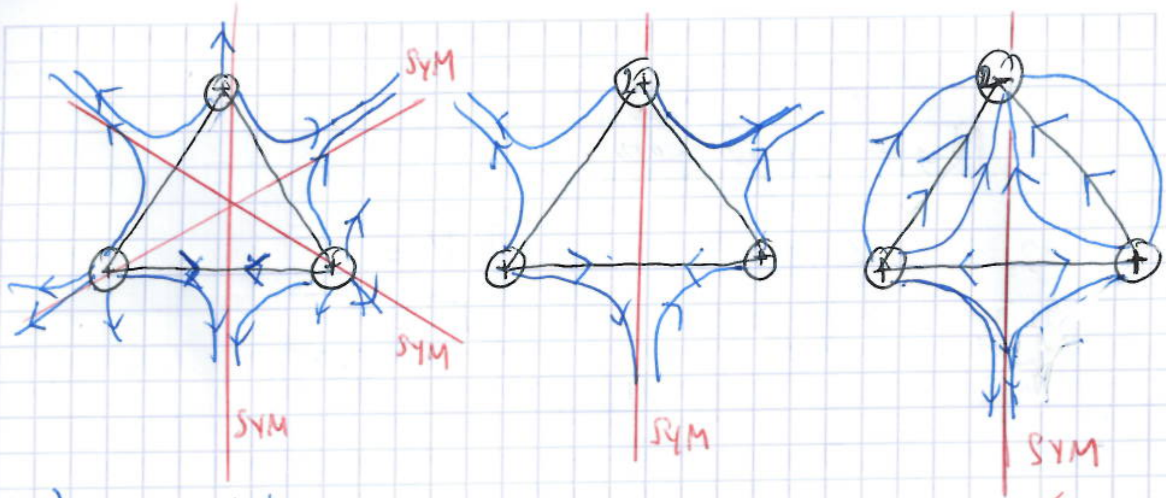
$\vec{F_1} = -F\sin\beta\,\vec{e_x} + F\cos\beta\,\vec{e_y}$

$\vec{F_2} = F\sin\beta\,\vec{e_x} + F\cos\beta\,\vec{e_y}$

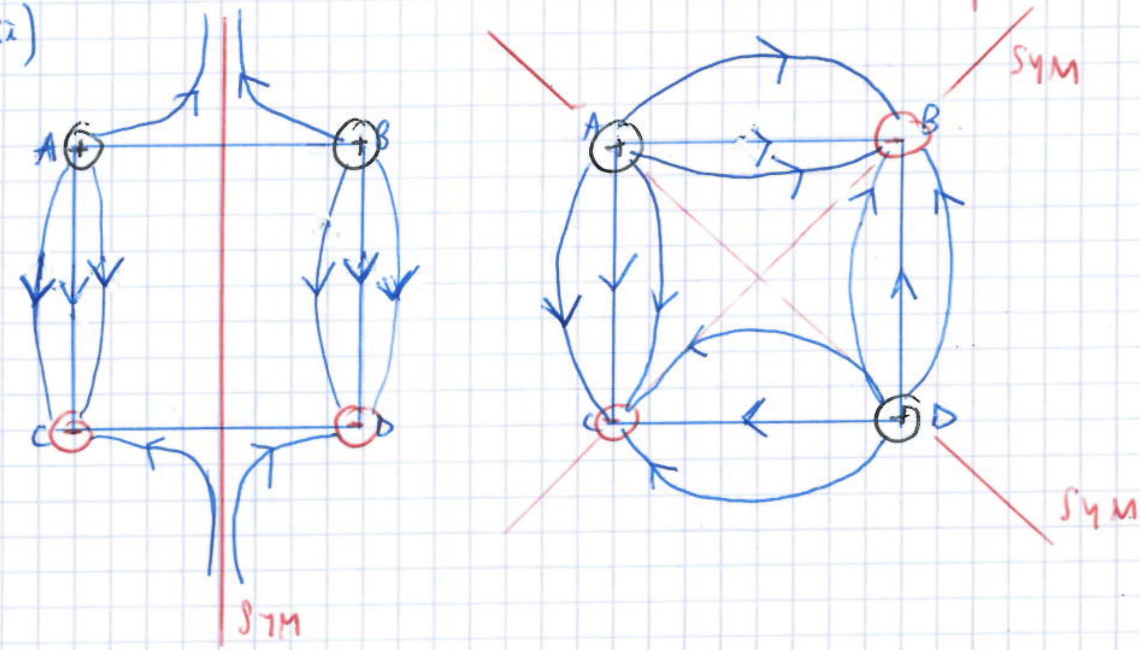
$\vec{F_3} = 2F\vec{e_y}$

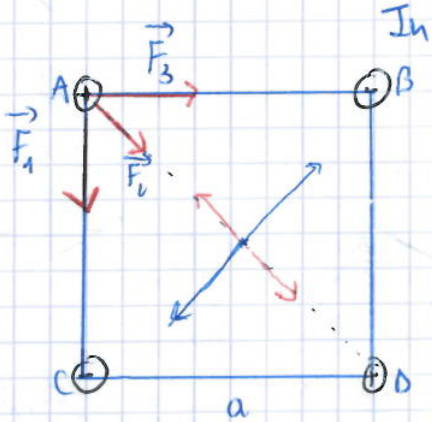
$\vec{F} = 2F(1+\cos\beta)\,\vec{e_y}$

$= 3F\,\vec{e_y}$



(ii)





In $\vec{F}_{in\,A} = \vec{F}_{C\to A} + \vec{F}_{B\to A} + \vec{F}_{D\to A}$

$$= \frac{1}{4\pi\epsilon_0}\left(-\frac{2q}{a^2}\vec{e_y} + \frac{2q}{a^2}\vec{e_x} + \frac{q}{2a^2}\cos\frac{\pi}{4}\vec{e_x} - \frac{q}{2a^2}\sin\frac{\pi}{4}\vec{e_y}\right)$$

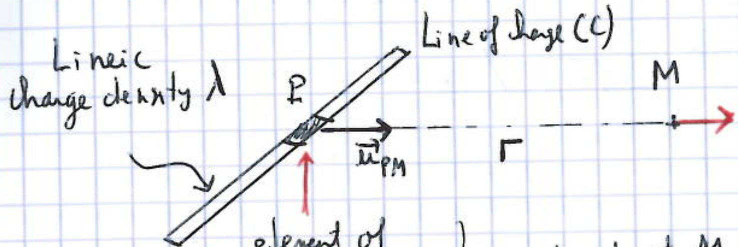
$$= \frac{q}{4\pi\epsilon_0 a^2}\left[\vec{e_y}\left[2+\frac{\sqrt{2}}{4}\right] + \vec{e_y}\left[-2-\frac{\sqrt{2}}{2}\right]\right]$$

$\cos 45 = \frac{\sqrt{2}}{2}$

In center: $\vec{0}$

C- Continuous charge distribution

* idea is similar to a discrete charge distribution



$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0}\frac{\vec{u}_{PM}}{PM^2} =$$

element of length dl and charge dq } produced at M a field $d\vec{E}$

$dq = \lambda dl$

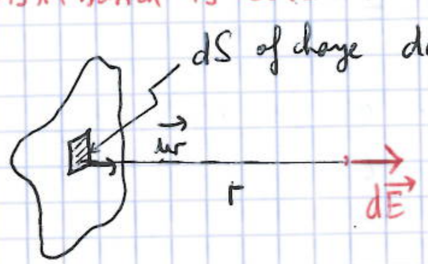
With $PM = r$ and $\vec{u}_{PM} = \vec{u}_r = \frac{\vec{PM}}{\|\vec{PM}\|}$.

• Total field is the contribution of all small elements.

$$\vec{E}(M) = \int_{line} d\vec{E} = \int_{(C)} \frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{r^2}\vec{u}_r$$; $\vec{u}_r$ and r will depend on position

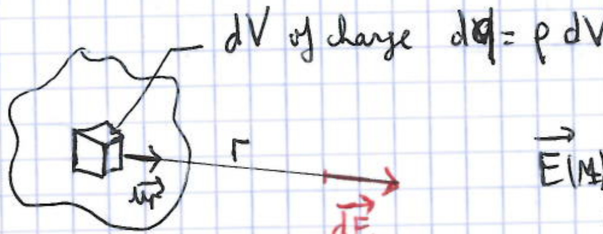
• Distribution is continue so $\sum$ is replaced by $\int$:

2D:



$$\vec{E}(M) = \iint_{Surface} \frac{\sigma dS}{4\pi\varepsilon_0 r^2}\vec{u}_r$$

3D



$$\vec{E}(M) = \iiint_{volume} \frac{\rho dV \vec{u}_r}{4\pi\varepsilon_0 r^2}$$

* For electric potential $V(M)$, the idea is similar

See slides I.5.c

1D: $V(M) = \int \frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{r}$

2D: $V(M) = \iint \frac{\sigma dS}{4\pi\varepsilon_0 r}$

3D: $V(M) = \iiint \frac{\rho dV}{4\pi\varepsilon_0 r}$

$$\begin{tikzcd} \boxed{\text{Electric field } \vec{E}} \arrow[r, bend left, "\int \vec{E}\cdot d\vec{r} = [-V]"] & \boxed{\text{Electric potential } V} \arrow[l, bend left, "\vec{E} = -\vec{grad}\, V"] \end{tikzcd}$$

d- Energy of a system of charges

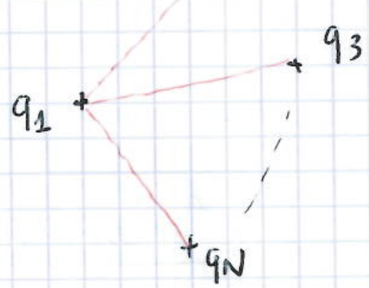
- Let's consider two charges q_1 and q_2

- We have seen that $E_{potential} = qV$
- Potential energy of the system:

$$E_p = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} = q_1 \cdot \frac{q_2}{4\pi\varepsilon_0 r_{12}} = q_1 V_2 = \frac{q_1}{4\pi\varepsilon_0 r_{12}} q_2 = V_1 q_2$$

So we write $\boxed{E_p = \frac{1}{2}(q_1 V_2 + q_2 V_1)}$

- Let's consider N charges:



let's calculate the quantity :

$$q_1 (0 + V_2 + V_3 + \dots + V_N)$$ "energy of 1"

$$+ q_2 (V_1 + 0 + V_3 + \dots + V_N)$$

$$+ q_3 (V_1 + V_2 + 0 + \dots + V_N)$$

$$\vdots$$

$$+ q_N (V_1 + V_2 + \dots + V_{N-1} + 0)$$

SUM

$$\sum_{i=1}^{N} q_i \left(\sum_{i \neq j}^{N} V_j \right) \underset{\uparrow}{=} 2 E_p$$

and $q_i V_j = q_j V_i$ is counted twice for each pair

$$\boxed{E_p = \frac{1}{2} \sum_{i=1}^{N} q_i \left(\sum_{i \neq j}^{N} \frac{q_j}{4\pi\varepsilon_0 r_{ij}} \right)}$$

6- Symmetry of the electric field

See slides I.6)