Application of electromagnetic induction: electro-mechanical conversion

Exercise 1: Lecture point: electro-mechanical conversion

Write the relation between the electrical power and the mechanical power related to the Laplace force.

Exercise 2: Principle of a DC Motor

We consider the pictures below illustrating the principles of a DC motor (Direct current). A rectangular and rigid electric circuit of length $L = MN = QP$ and width $l = NP = QM$ is set in the air gap of a magnet. The circulation of an electric current i in the presence of a magnetic field $\vec{B} = B\overrightarrow{e_x}$ generates Laplace forces $\vec{F}$ and torque $\vec{\Gamma}$ being able to move the rectangle. The magnet is part of the stator that is fixed while the rectangle that can move is part of the rotor. The rotor is made of many rectangular loops whose width NP crosses the axis of rotation $\overrightarrow{e_y}$. In Fig 1.c two of them are represented.

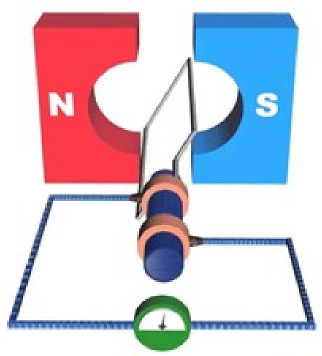

Fig1.a

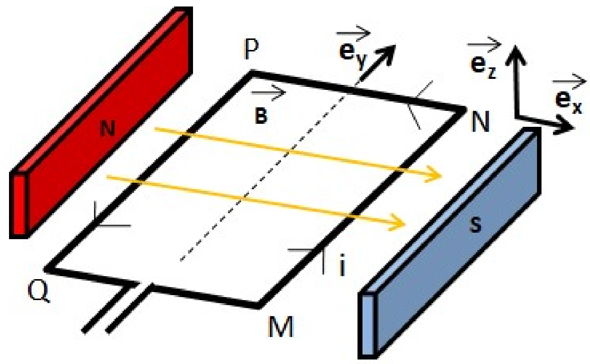

Fig1.b

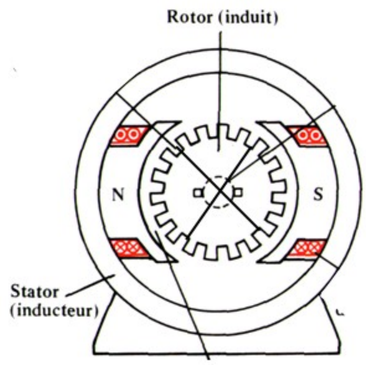

Fig1.c

Working as motor

1) Calculate the resultant of the Laplace forces acting on the rectangle MNPQ.

2) Calculate the torque acting on the square MNOP.

Working as alternator

3) We assume the rectangle MNQP rotates about $\overrightarrow{e_y}$ axis at angular velocity $\vec{\omega}$. Deduce the expression of the magnetic flux crossing the rectangle and the expression of the electromotive force.

4) How can we obtain a DC electric signal taking with the contribution of all rectangular loops in the rotor?

Exercise 3: Rotating magnetic field and synchrone AC motor

We consider three coils like depicted on the fig. 2 having AC current $i_k(t) = i_0 \cos(\omega t + \varphi_i)$ producing an AC magnetic field along their respective axis whose amplitude is respectively $B_k(t) = B\cos(\omega t + \varphi_i)$. We assume $\varphi_1 = 0$, $\varphi_2 = \frac{\pi}{3}$ and $\varphi_3 = \frac{2\pi}{3}$.

1) Draw on the same graph, the three function $B_1(t), B_2(t)$ and $B_3(t)$.

2) Calculate the total magnetic field at the intersection A of the three axis.

3) What happens when putting a "small magnet" at point I?

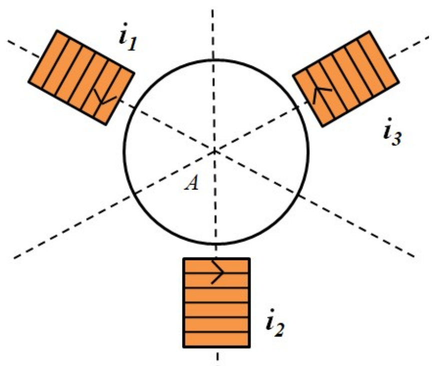

Figure 2.

Exercise 4: Electrodynamic Loud speaker

An electrodynamic loudspeaker is made of:
-a magnet of axis $x'x$ creating a radial magnetic field $\vec{B}$ of constant magnitude B in the air gap

-a rigid solenoid of axis $x'x$ made of N circular loops of radius a set in the region of the air gap of the magnet.
-a membrane, perpendicular to axis $x'x$ attached to the solenoid able to perform small motions about its equilibrium position with the help of an elastic system modeled by a spring of stiffness k.
The sytem { solenoid+membrane} of mass m having position $x(t)$ during its motion is submitted to following forces
- the weight (vertical) and the reaction of support (opposite to the weight)
- the spring force

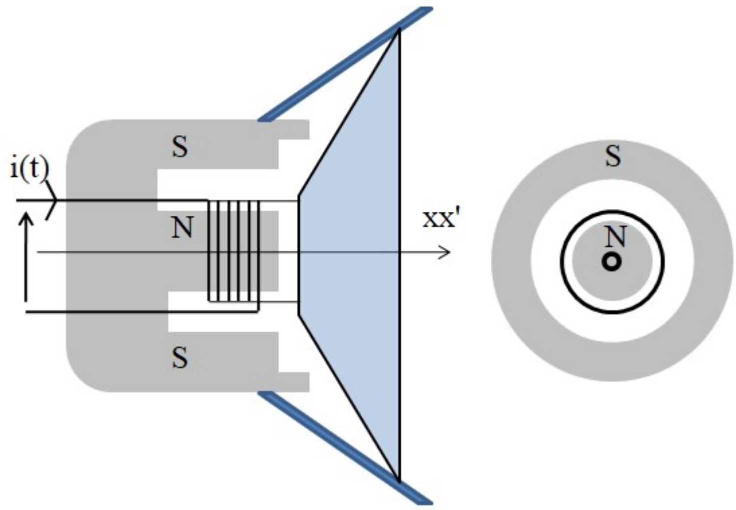

Fig 3.

- sum of the Laplace forces exerted by the magnet on the solenoid when submitted to current $i(t)$
- a friction force proportional to the velocity with friction coefficient α

1) Make a picture of the by showing the circuit orientation for an elementary force $\overrightarrow{df}$ acting on a current element $\overrightarrow{dl}$ assumed to be positive. Express this elementary force and deduce the all resultant $\vec{f}$ acting on the solenoid (we will assume that total length of electric wire is $l = N2\pi a$.
2) Apply the Newton law to the solenoid and project it onto $x'x$ axis to obtain the mechanic equation of the motion:

$$\frac{d^2x}{dt^2} - \frac{\alpha}{m}\frac{dx}{dt} - \frac{k}{m}x = -\frac{Bl}{m}i$$

3) The solenoid moves in the air gap to velocity $\vec{v} = \frac{dx}{dt}\overrightarrow{e_x}$, determine the electromotive force (emf) $e(t)$ induced by the motion as a function of B, l and v. We assume a positive emf if the induced current is positive. Make the electrical scheme of the device: the coil as a resitance R, a inductance L connected to a power supply of voltage $u(t)$. Show that the differential equation for the current $i(t)$ is given by

$$L\frac{di}{dt} + Ri - Blv = u$$

4) We propose to solve the equations system in the sinusoidal regime where $u(t) = U\cos\omega t$ and to focus on the driven regime where system is excited at angular frequency ω. We associate to $u(t)$ its complex form $\underline{u(t)} = Ue^{j\omega t}$ where $j^2 = -1$. Consequently the current $i(t)$ and the velocity $v(t)$ will be written under their complex form $\underline{i(t)}$ and $\underline{v(t)}$. In the complex form, the Ohm law is written $\underline{U} = \underline{Z}\underline{i}$ where $\underline{Z}$ is called the impedance.
5) What is the coil impedance $\underline{Z_L}$? Show that we can write the total quantity $\underline{Z} = \underline{Z_L} + \underline{Z_m}$ where $\underline{Z_m} = R_m(\omega) + jX_m(\omega)$.