PW n°1: Study of the signal propagating in a coaxial cable

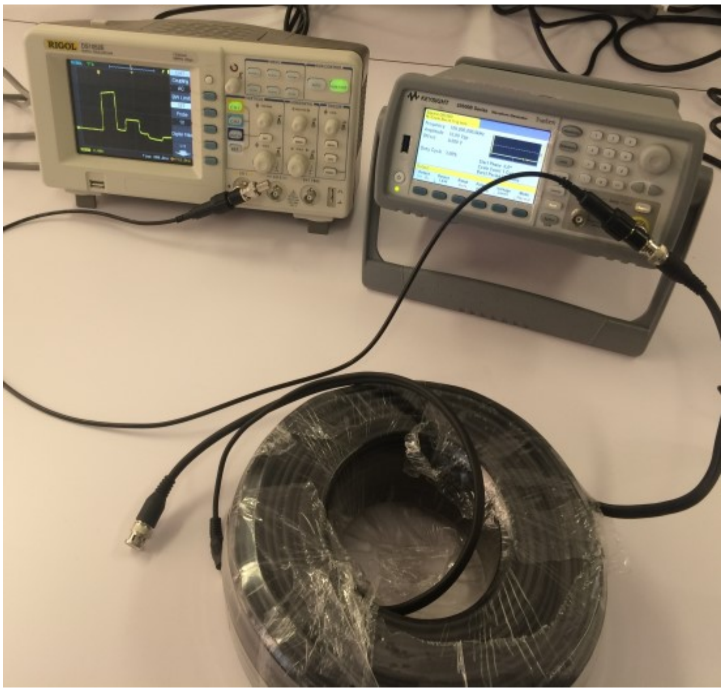
Coaxial cable was invented in the first half of the twentieth century. Its fundamental interest is that the signal passing through it is not influenced by the external electromagnetic environment and that the signal is not responsible for an external emission. The propagation occurs inside the cable, the wave is guided and the the cable can be seen as a waveguide. The coaxial cable is widely used for the wiring of antennas that they are repeaters or receivers such as radio antennas, television. It is also much used for data transmission in the field of sound or image. However, optical fiber is preferred for long distance transmissions when distances are in the order of one kilometer or beyond. The coaxial cable is characterized by a characteristic impedance often between 50 Ω or 75 Ω. Its attenuation is expressed in decibel per unit length, it is of the order of 10 $dB.km^{-1}$. The optical fiber has a much lower attenuation of the order of 0.1 $dB.km^{-1}$. Even if there are great differences between an optical fiber and a coaxial cable, the study of the cable is not without interest because it is similar to the case of an optical fiber.

I. Objectives

In this PW, we will measure the two main characteristics of the cable: the speed of propagation of information (group velocity) and the attenuation coefficient. We will also study the phenomena of wave reflection related to a change in the nature of the propagation medium realized by the connection of a resistance at the end of the cable (change of impedance). This question will lead us to study the question of impedance matching. We will also see that some cable end resistors favor the reflection of the signal sent in the cable. This situation is conducive to the formation of a standing wave system in the cable that we will characterize.

II. Apparatus

We have at our disposal a wave generator (Keysight), an oscilloscope and a coaxial cable of characteristic of length L=50 m with a characteristic impedance to be determined. The wave generator will be used to build fine electrical pulses by playing with the duty-cycle function that permits to select a given percentage of an alternative signal. The propagating signal will be seen with the oscilloscope as well as the reflecting signal. In a second part, the study of the impedance adaptation will be done using a resistance box delivering resistance values between 1 Ω and 10 Ω .



Oscilloscope, wave generator and 50 m coaxial cable

Figure 1.

III. Theoretical aspects and preparing work

We consider a infinitesimal length element dx of the coaxial cable element described by the following electrical model.

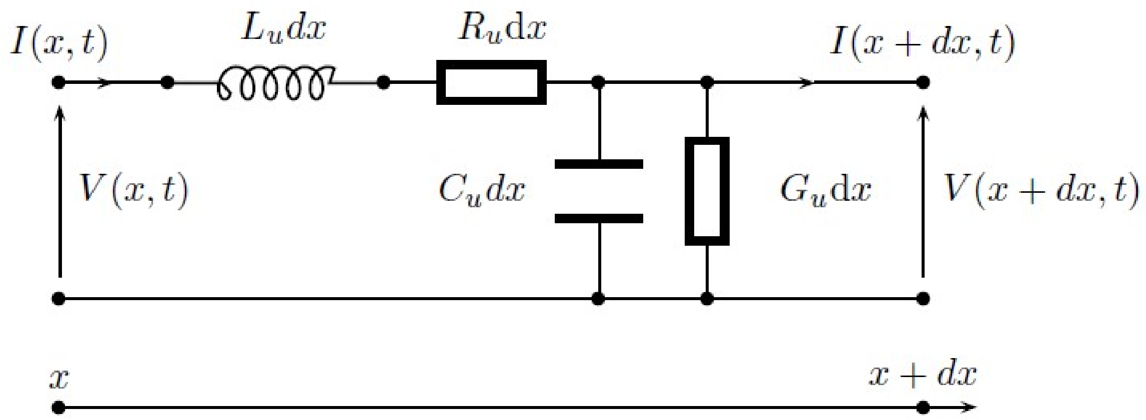


Figure 2. Electric model for the coaxial cable including resistive effects

The coaxial cable is characterized by the following parameters L_u, C_u, R_u and G_u assumed to be constant over the entire length of the cable. R_u is the linear resistance of the cable in $\Omega.m^{-1}$, G_u is the leakage linear conductance in $\Omega^{-1}.m^{-1}$. The insulation between the central conductor and the cylindrical conductor that surrounds it is not the vacuum but polyethylene. The relative permittivity of polyethylene is $\varepsilon_r = 2.3$ when the material is full, it turns out that in cheap cables, it presents alveoli, we then speak of cellular dielectrics. Its relative permittivity is then much weaker, it approaches in a way that of the vacuum ($\varepsilon_r = 1$). It can be in range $1.3 < \varepsilon_r < 1.5$. Under these conditions, it can be shown in Electromagnetism that the linear capacitance C_u in $F.m^{-1}$ and the linear inductance L_u in $H.m^{-1}$ of the cable are given by:

$$C_u = \frac{2\pi\varepsilon_0\varepsilon_r}{ln\frac{b}{a}}$$

$$L_u = \frac{\mu_o}{2\pi} ln\frac{b}{a}$$

The relative permittivity of a dielectric medium (insulator) is related to the refractive index n in this medium by the relation $\varepsilon_r = n^2$. It is recalled that $\varepsilon_o \mu_o c^2 = 1$.

ANSWER to the following questions:

1) Using the Kirchhoff laws write the differential coupled relations between $I(x,t)$ and $V(x,t)$.

2) By eliminating the current $I(x,t)$ show that one can obtain the following equation for $V(x,t)$

$$\frac{\partial^2 V}{\partial x^2} = C_u L_u \frac{\partial^2 V}{\partial t^2} + (L_u G_u + R_u C_u)\frac{\partial V}{\partial t} + R_u G_u V$$

3) Show that without friction elements, the wave equation is transformed into the usual D'Alemebert Wave equation

$$\frac{\partial^2 V}{\partial x^2} = \frac{n^2}{c^2}\frac{\partial^2 V}{\partial t^2}$$

where $v = \frac{c}{n}$ is the phase velocity. Give the expression v as a function of ε_0, μ_0 and ε_r.

In the case of an ideal coaxial cable of characteristic impedance Z_c, the solutions of the propagation equation can be written as:

$$I(x,t) = I_0 e^{i(\omega t - kx)} + I_1 e^{i(\omega t + \quad)}$$
$$V(x,t) = Z_c I_0 e^{i(\omega t - \quad)} + Z_c I_1 e^{i(\omega t + kx)}$$

Where $Z_c = \frac{V(x,t)}{I(x,t)}$ is the cable impedance. We put at the end of the cable at $x = L$ an impedance $= R$. The incident propagating wave $\approx A_i e^{i(\omega t - kx)}$ in medium of impedance Z_c can reflect itself at the interface with the medium of impedance R leading to a reflective propagating wave $\approx A_r e^{i(\omega t + kx)}$. We remember that the ratio of the incident wave amplitude to the incident one defines the Fresnel reflection coefficient in amplitude as

$$r = \frac{A_r}{A_i} = \frac{R - Z_c}{Z_c + R}$$

4) Using the boundaries and the expression of r, show that the expression of the voltage in the cable becomes:

$$V(x,t) = Z_c I_o e^{i\omega t}\left[\exp(-ikx) - \frac{Z_c - R}{Z_c + R}\exp ik(x - 2L)\right]$$

5) Discuss the value of the reflected wave amplitude in the following cases.

- When $0 < R < Z_c$
- When $Z_c = R$
- When $Z_c < R$
- When $R \to \infty$ (open)

IV. Experiments

IV.1 Configuration of the electrical pulse

Connect the signal generator to the oscilloscope using the small coaxial cable. As depicted below on the electric picture, connect the coaxial cable to a T connector and then connect the T to the Channel 1 of the oscilloscope.

The electrical pulse will have the following input parameters that you can select with the different menus of the wave generator. Select a rectangular waveform and in *Parameters*, set up the frequency to 100 kHz, the amplitude to 10 Volt peak-to-peak. Set a duty-cycle at 3% and an offset to 0 V.

Then, in the Burst mode, check that the successive parameters in the menu are: On, Ncycle, #cycle, Start Phase 0, Cycle Count 1 cyc and Burst period 10 ms.

When all the parameters are ok, you have also to press Channel and select On in the Output menu.

Then observe the signal on the oscilloscope screen by selecting the adapted horizontal and vertical sensitivity (time and amplitude).

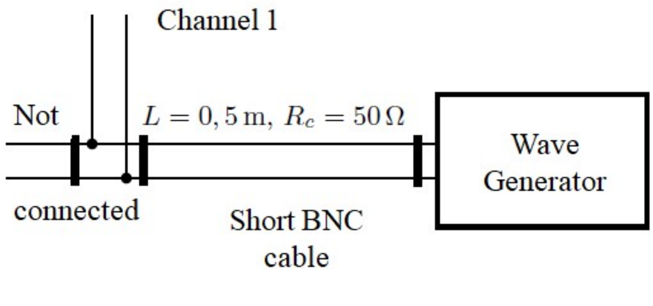


Figure 3. Settings without impedance adaptation

Adding a 50 Ω resistance or a 50 Ω Plug Terminator

It is possible that the signal presents some defects. To clean the pulse it is possible to put on the second part of the T, an element having a 50 Ω impedance.
The input impedance of the oscilloscope being 1MΩ it is clearly different from the characteristic impedance of the black coaxial cable usually closed to 50 Ω. Connect the resistance box to the second part of the T connector and set its value to 50 Ω like depicted in Figure 4. Has the signal a better shape? Modify the value of the resistance in the resistance box and observe what happens. The change of shape of the signal is due to the parasite reflection occurring in the T connector. They amplitude will depend on the value of the resistance ending the circuit.
Once done, remove the resistance box and plug directly the 50 Ω Plug Terminator on the second part of the T connector. We will work in these conditions for the remaining part of the work.

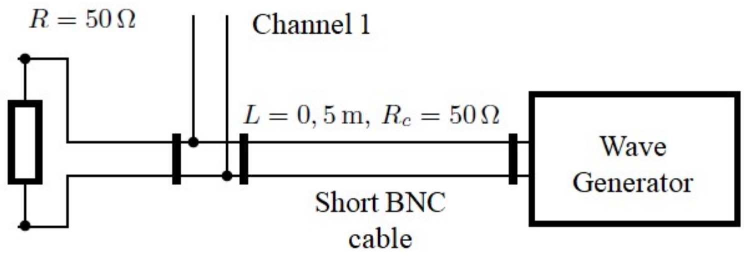


Figure 4a. Settings with impedance adaptation.

Figure 4b. Plug Terminator

We propose to study this aspect of impedance adaptation in the next section IV.2.

IV. 2 Reflections at the end of the cable

Realize the circuit as depicted in Figure 5. Use the Plug Terminator to adapt the impedance of the small cable and plug a resistance box at the end of the second cable depicted by the symbol R_u. Pay attention that 50 m coaxial cable is not send to the oscilloscope.

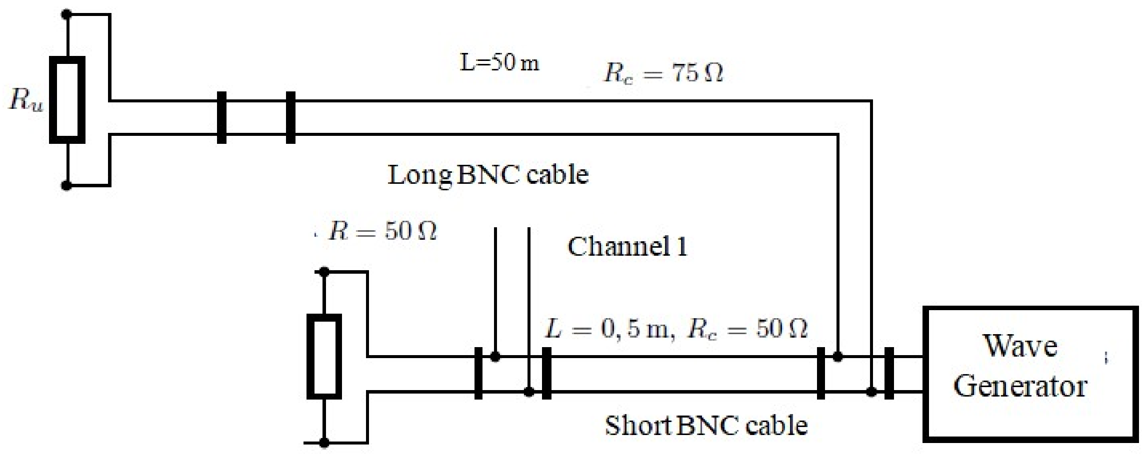


Figure 6.

Observe the two pulses in the oscilloscope. The second one is delayed. It corresponds to the reflected signal having done the round trip in the long cable. Its amplitude depends on the value of the resistance set in the resistance box that simulates the change of medium.
The goal is to measure the coefficient of reflection in amplitude r given by

$$r = \frac{A_r}{A_i} = \frac{R - Z_c}{Z_c + R}$$

and to look its influence with the resistance given by the box. To this end, one needs to determine the amplitude of the incident and the reflected signal in the oscilloscope. This can be done accurately by using the cursors Mode of the oscilloscope and by measuring the amplitude (reading ΔY on the screen). However, close to the value of Z_c the top of the square pulse will start to show a slope between the beginning and the end of the signal. In that case, set the Y cursor in the middle.

The value for which $r = 0$ will determine the value of the characteristic impedance of the coaxial cableZ_c . Fill a tabular like below for values of R taken from the different decades.

R	A_i	A_r	r
...			

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Take 5 values between 0 and 10 Ω. Then take a measurement point every 10 Ω from R= 10 Ω to R=60 Ω . In the interval [60,90] Ω reduce the interval to 5 Ω. Then from 90 Ω to 150 Ω repeat an interval of 10 Ω. Then, take 5 values in the interval [200, 1000] Ω and again 5 values in [1000, 10000] Ω. **You can enter also directly your experimental values in the data sheet of *Origin* software and calculate *r* with the computer.**
Observe the change of sign in the amplitude of the reflected signal and determine the value of Z_c.
Draw the graphic $r = f(R)$ with the software *Origin* and set a log10 basis for the abscise interval starting at 1 Ω. Determine the value of Z_c obtained graphically.

IV.3 Determination of the velocity of propagation in the cable

In that section, we use the 50 m-coaxial cable whose characteristic impedance is closed to 75 Ω . Realize the circuit like depicted in Figure 5. Plug a T connector at the output of the wave generator and connect the small and long coaxial cable. At the end of each of them, plug a T connector that you connect respectively at the terminals CH1 and CH2 of the oscilloscope. Adapt the impedance of the first connection with the 50 Plug Terminator Ω and the impedance of the second with the resistance box set at 75 Ω

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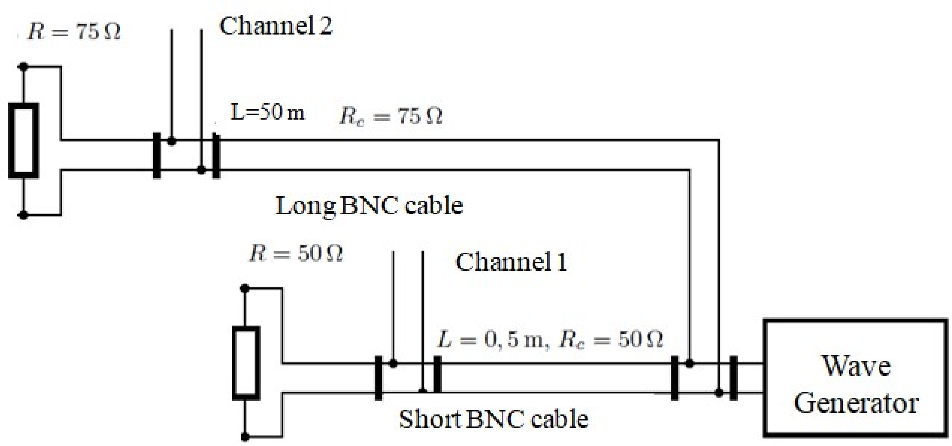

Figure 5.

Observe the pulses detected on both channels. They are delayed in time due to the higher distance covered by the pulse propagating in the long cable. Propose **a method to determine the velocity of propagation in the cable.** With the obtained result, propose a value for the optical index of the material and for the dielectric constant ε_r. It will be useful to know the value of the speed of the light ☺ .