Physics-L2 Electromagnetism

Approximative program

1) Sources –Fields -interactions

2) Fundamentals of magnetism

Chap 1: Electrostatics

Chap 2: Magnetostatics

Chap 3: Time-dependent regime-Induction phenomena

Chap 4: Maxwell equations

Chap 5: Dielectric media and applications

Chap 6: Conducting media and applications

Chap 7: Magnetic media and applications

week	**Magistral lectures**
1	Electrostatics
2	Electrostatics
3	Electrostatics
4	Electrostatics
5	**Magnetostatics**
6	**Magnetostatics**
7	Induction
8	Induction
9	Maxwell equations
10	Maxwell equations
11	Dielectric media
12	Dielectric / Metallic media
13	Metallic Media
14	Magnetic media

Physics-L2 Electromagnetism

To learn and understand well the lecture, important to connect it with the Tutorial

<table>
<tr><th>week</th><th>Magistral lectures</th><th>Tutorial</th></tr>
<tr><td>1</td><td>Electrostatics</td><td></td></tr>
<tr><td>2</td><td>Electrostatics</td><td rowspan="4">Electrostatics</td></tr>
<tr><td>3</td><td>Electrostatics</td></tr>
<tr><td>4</td><td>Electrostatics</td></tr>
<tr><td>5</td><td>Magnetostatics</td></tr>
<tr><td>6</td><td>Magnetostatics</td><td rowspan="3">Magnetostatics</td></tr>
<tr><td>7</td><td>Induction</td></tr>
<tr><td>8</td><td>Induction</td></tr>
<tr><td>9</td><td>Maxwell equations</td><td></td></tr>
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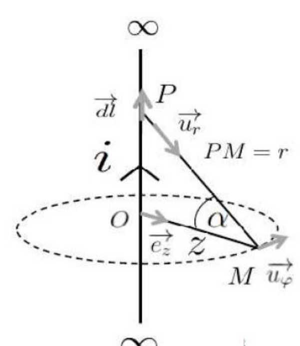
UFAZ-L2-Magnetostatics

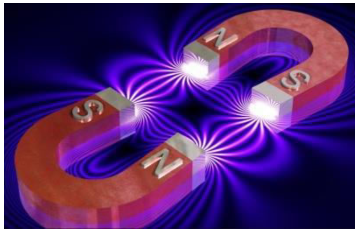
Exercise 1: Magnetic field created by an infinite wire of current

We consider an electric current i propagating in an electric wire. We assume that the wire can be considered as infinite in the region where we study its action on point M located at a distance z from the wire axis. With the Biot and Savart law show that the magnetic field produced by the current at a distance z from the wire is given by

$$\vec{B} = \frac{\mu_0 i}{2\pi z}\vec{u_\varphi}$$



Magnetostatics-L2
A-Sources- Magnetic Fields -interactions



1) Origin(s) and highlightings

a) Natural Magnets –magnetic compass- magnetic poles – Magnetic Field lines

b) Similar effects with an electric current- Biot and Savart Law

2) Magnetic field calculations produced by electric currents

Rect wire of current, loop of current, solenoid, field lines analysis

3) Geometric properties of the magnetic field

Flux conservation and Ampere theorem

4) Magnetic interactions

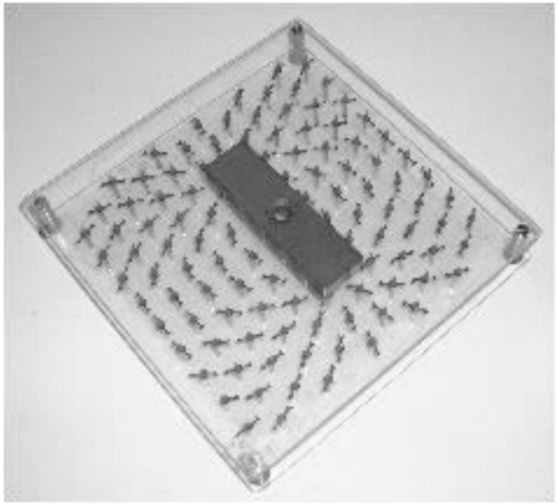
Lorentz forces – Laplace Force

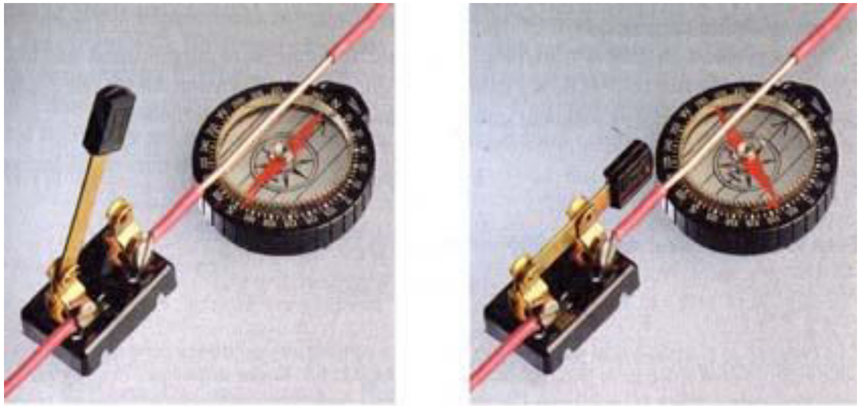
5) Some applications

How can we obtain magnetic field ?

1. With Magnet

2. With electric current



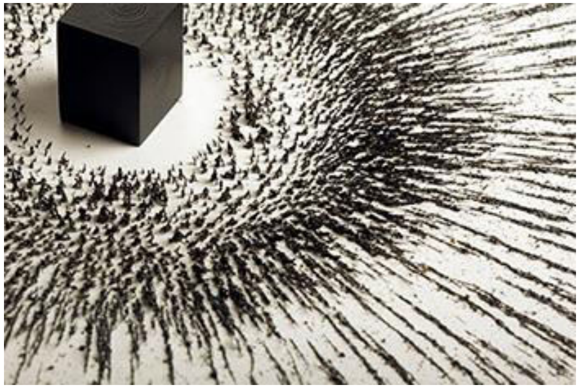


(a) (b)

Magnet

Magnetite (Iron oxide) has attractive or Repulsive properties (known since antiquity)

If one object has bigger mass, the interaction mechanicaly affects the smaller



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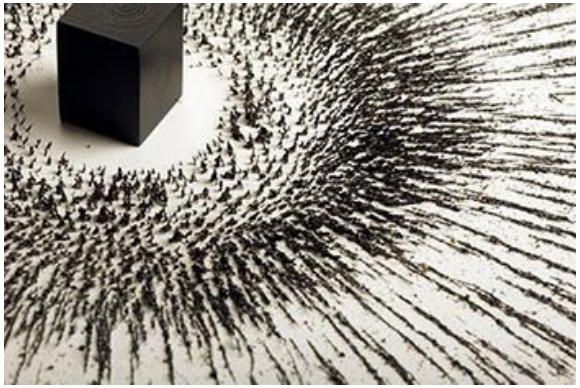
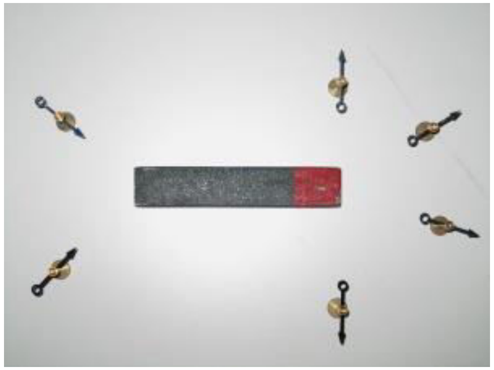
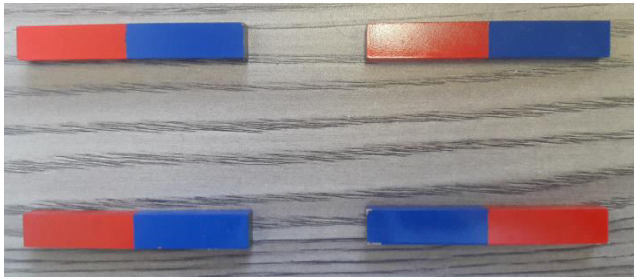


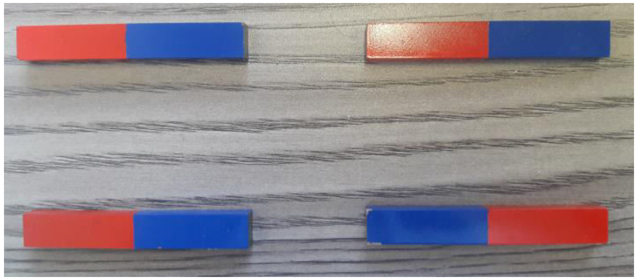
Magnetic object can oriente themselves in a presence of another magnetic object



Relative orientation of two big objects

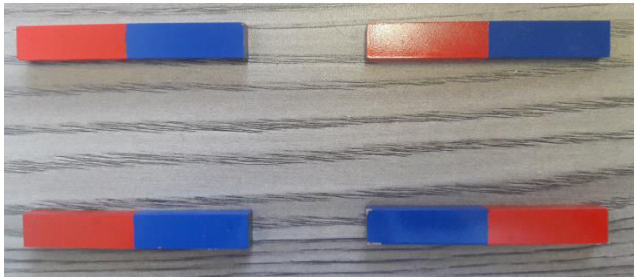


Relative orientation of two big objects



We add a third small object: iron fillings to illustrate the difference

Relative orientation of two big objects



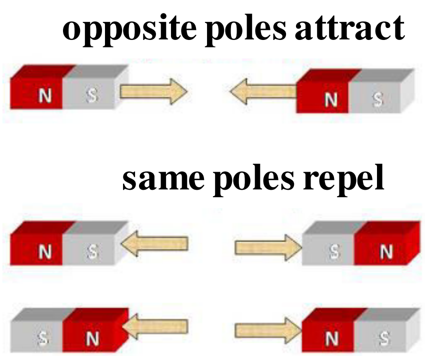
We add a third small object: iron fillings to illustrate the difference

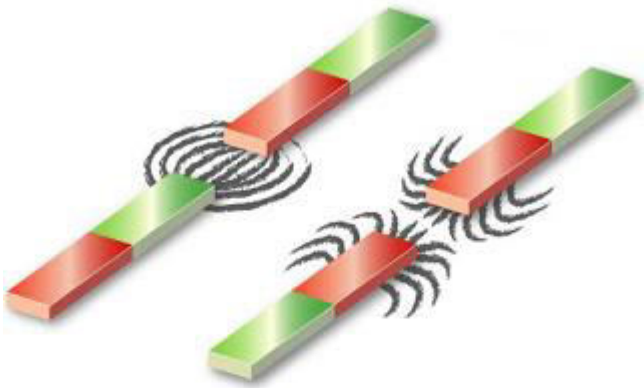
attraction

repulsion

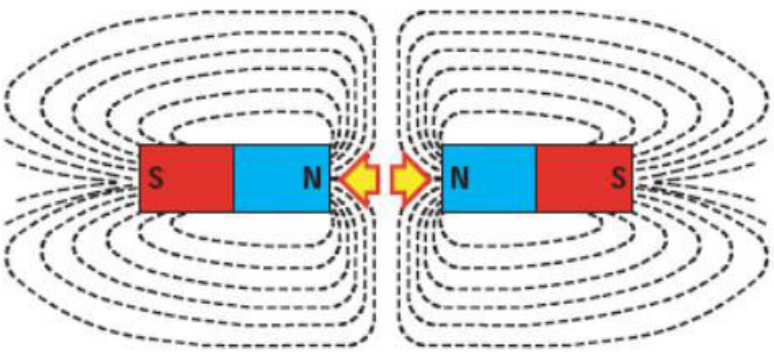
Origin and higlightings

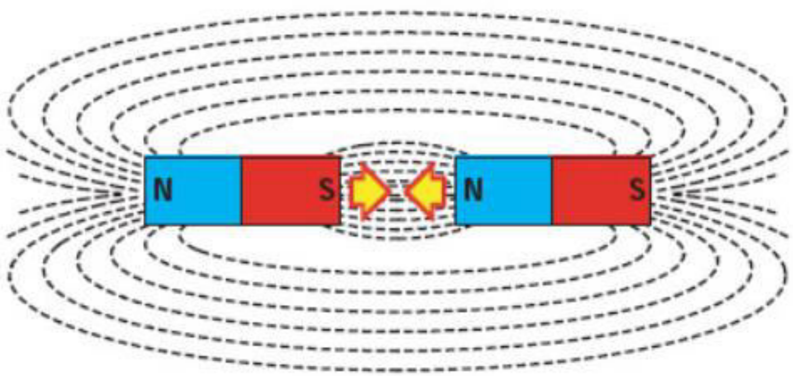
We define the arbitrary notion of poles: Noth and South poles





We use the iron fillings to figure out « some directions » due to the perturbation

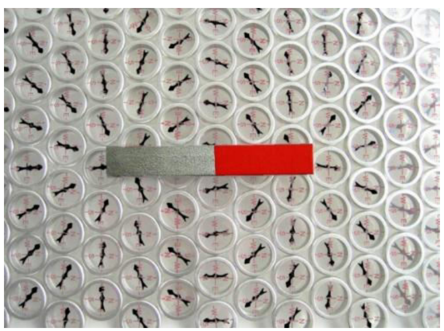


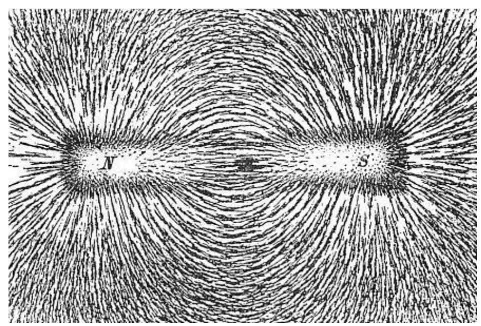


Magnetic field

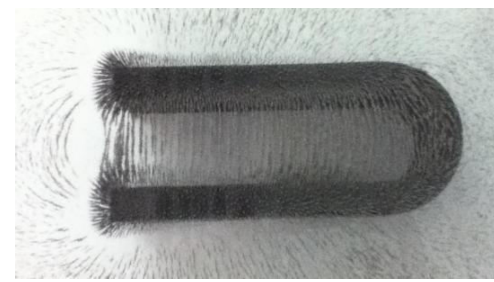
We call magnetic field the property emitted by a magnetic object and that can act on other magnetic object.

If one object has less ability to make moving the other object (big magnet VS iron fillings or magnetic compass), we can distinguish **the field lines of the magnetic field created by the big one**

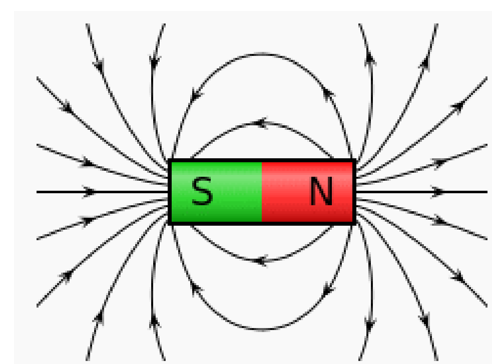




Iron fillings



Poles are defined as:
Magnetic field enters by the south and leaves by the north



Natural Magnetic orientation on Earth

Existence of a natural geographic orientation…. Showing the geographic North

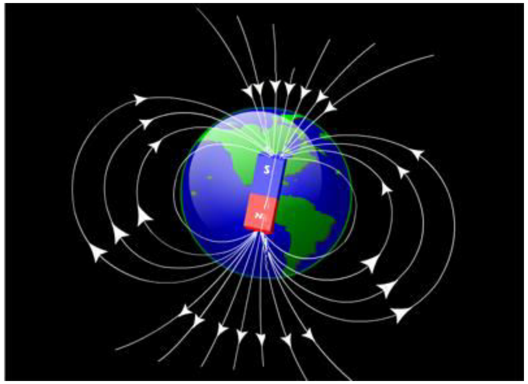
magnetic compass

Earth Magnetism

Geographic North is a magnetic south

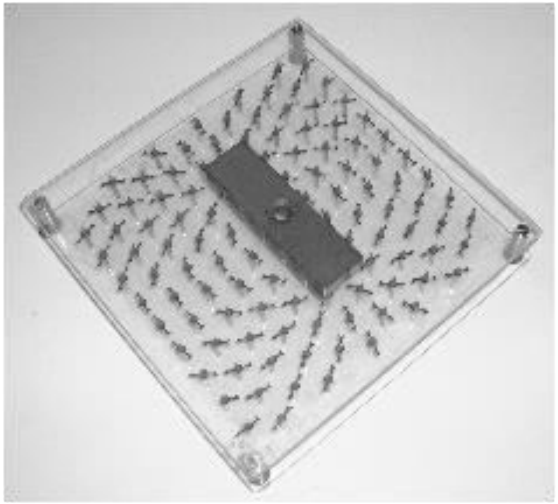
William Gilbert
1544-1603

The magnetic compass should react in the same way than with a magnet

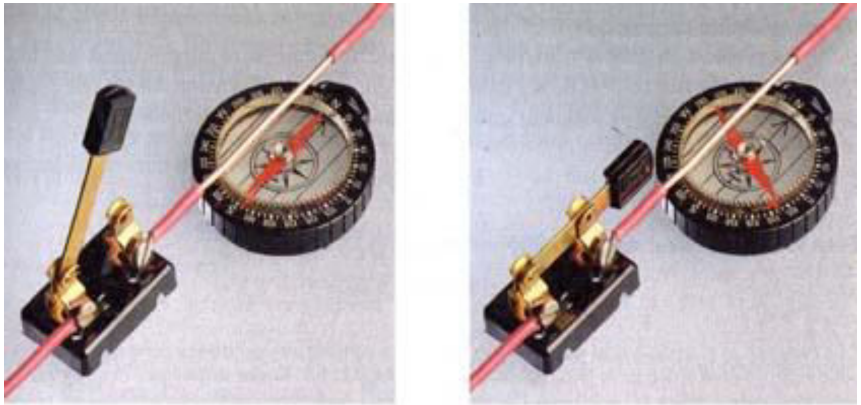


How can we obtain magnetic field ?

1. With Magnet



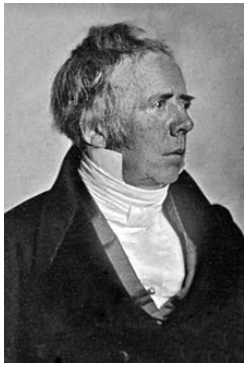
2. With electric current



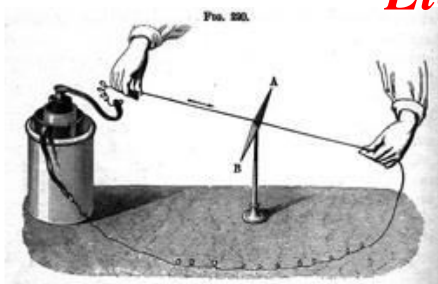
(a) (b)

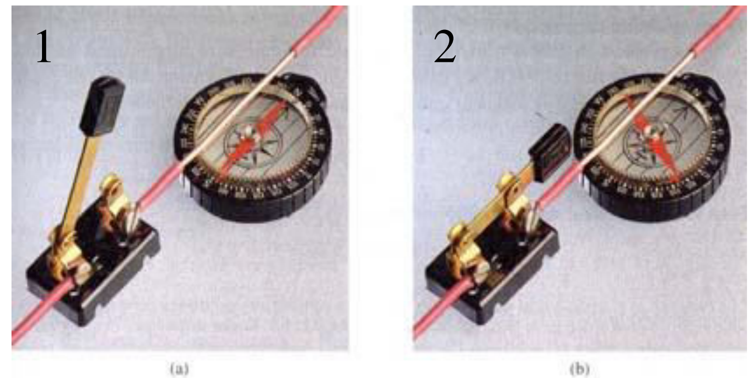
1819: *A Link between electricity and magnetism*

Electricity induces motion of magnetic compass



Hans. C Oersted
1777-1851





1. No conduction: initial orientation of magnetic compass

2. Circulation of electricity modifies the orientation of magnetic compass.

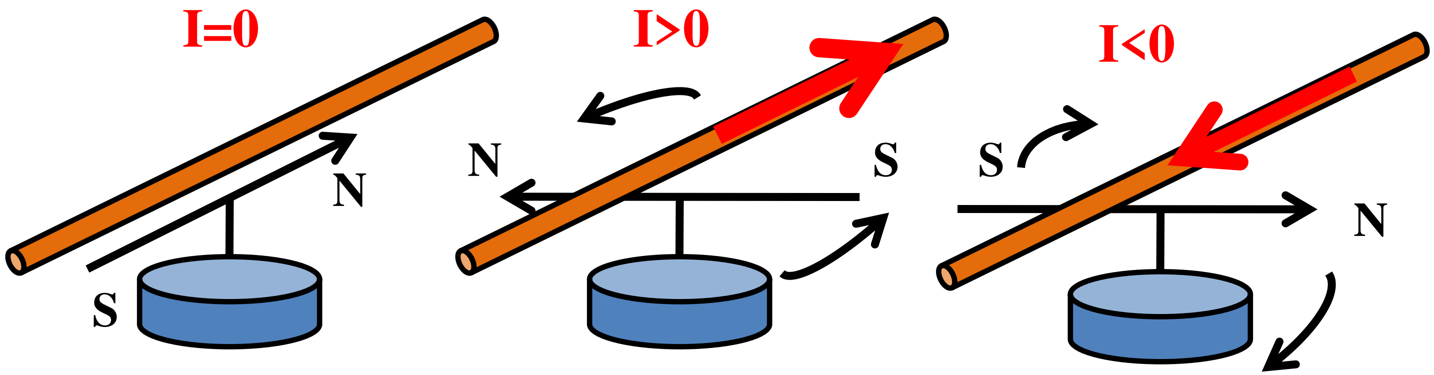
Electricity can have similar action than a natural magnet

Eletrodynamic theory

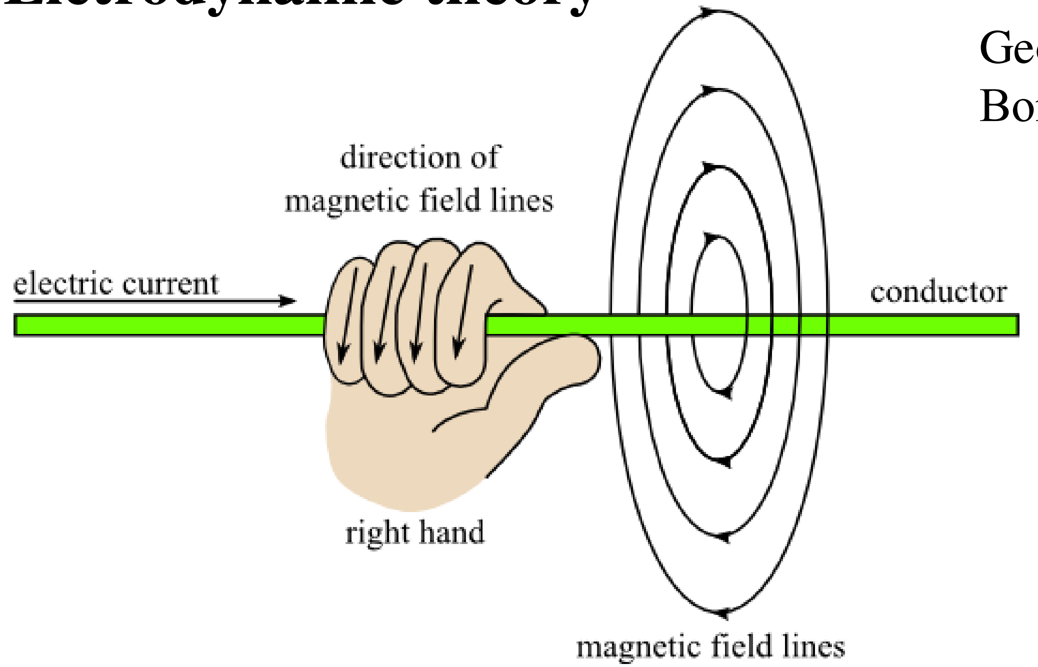
Interpretation of Oersted Experiment
Electric current creates magnetic action

- Notion of **electric current I**: motion of microscopic electric charges
- Direction of the current I influences direction of magnetic compass

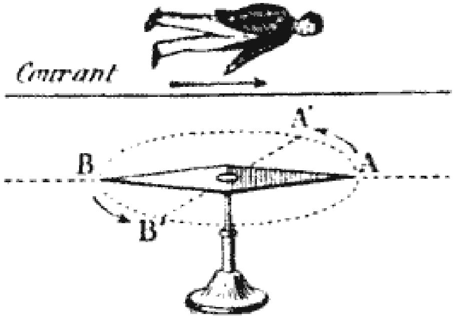
André-M. Ampère
1775-1836



Eletrodynamic theory



Geometric analysis: Right-hand rule
Bonhomme d'Ampère



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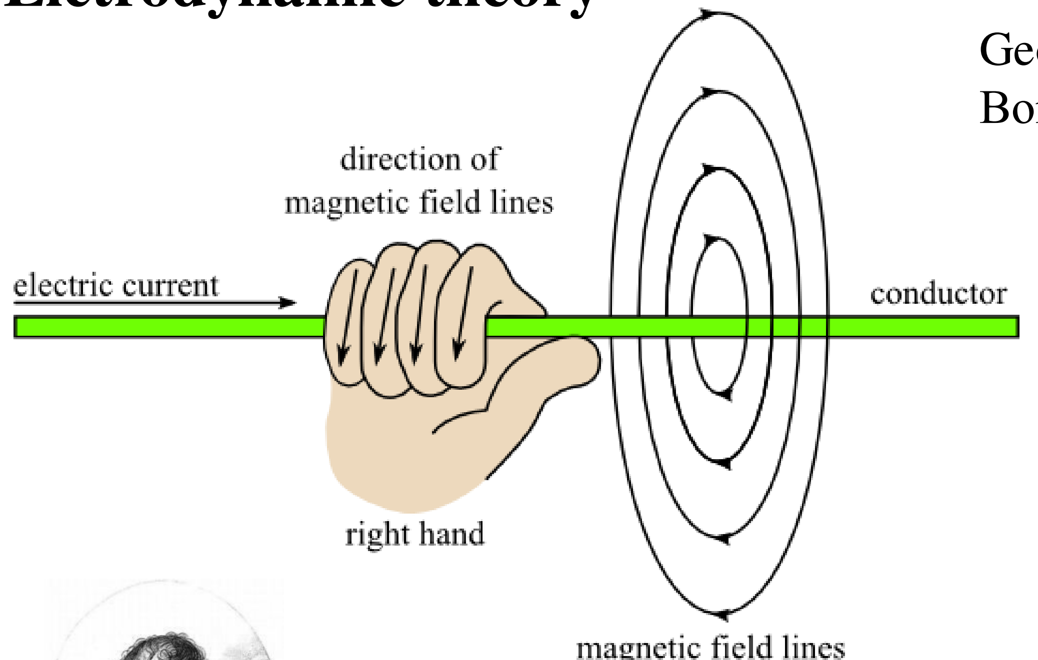

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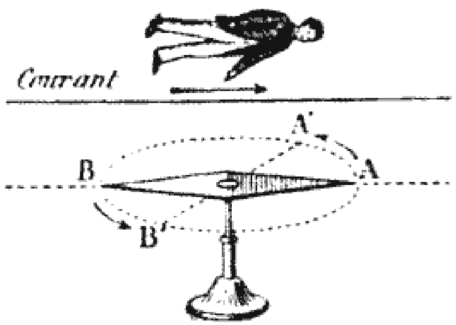
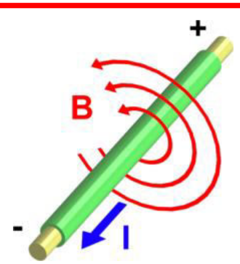


J.B Biot
1774-1862

Biot-Savart Law

$$\overrightarrow{B} = \int \frac{\mu_0 i \, \overrightarrow{dl}}{4\pi r^2} \wedge \overrightarrow{u_r}$$



Unit of B in Tesla (T)
$\mu_0 = 4\pi 10^{-7}$ H/m is the Magnetic permeability

Felix Savart
1791-1841

Magnetic field produced by an electric current

Biot-Savart Law

$$\overrightarrow{B} = \int \frac{\mu_0 i \, \overrightarrow{dl}}{4\pi r^2} \wedge \overrightarrow{u_r}$$

$$d\overrightarrow{B} \approx i\,\overrightarrow{dl} \wedge \overrightarrow{u_r}$$

What is the direction of the magnetic field ?

Magnetic field produced by an electric current

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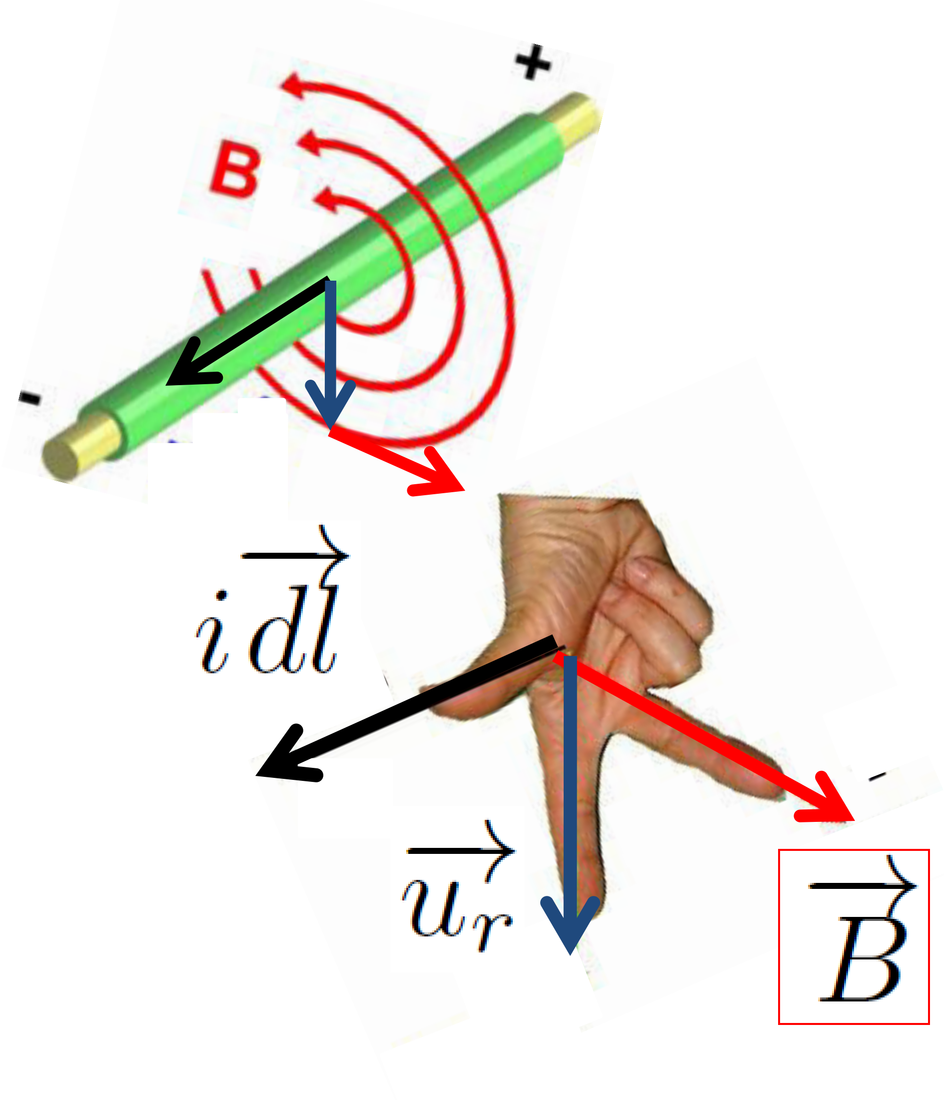

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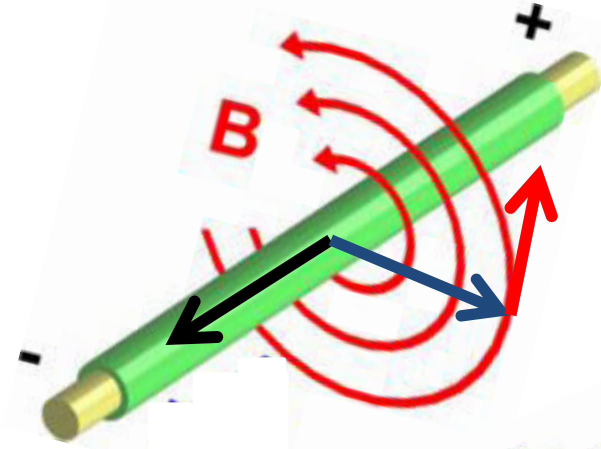
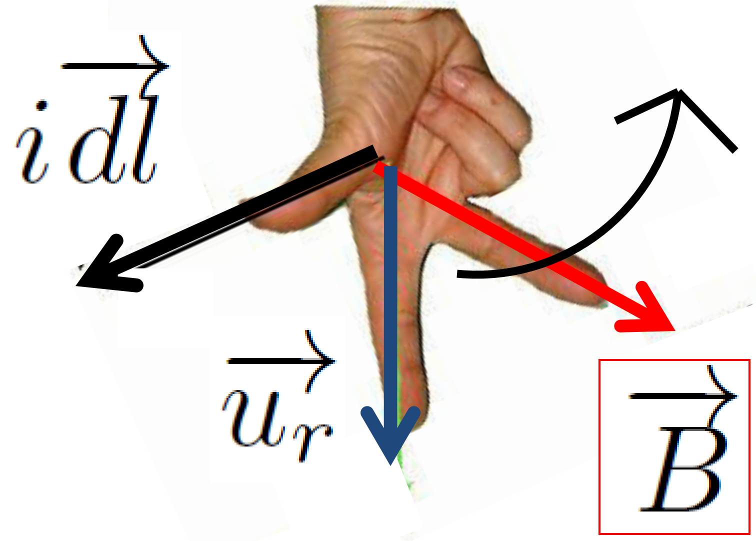
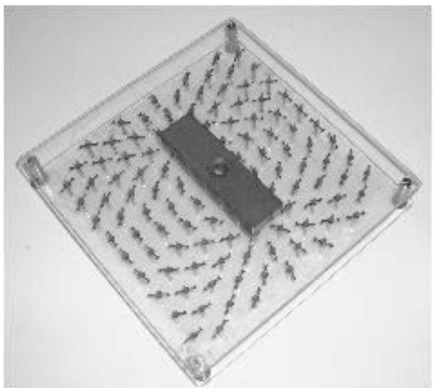


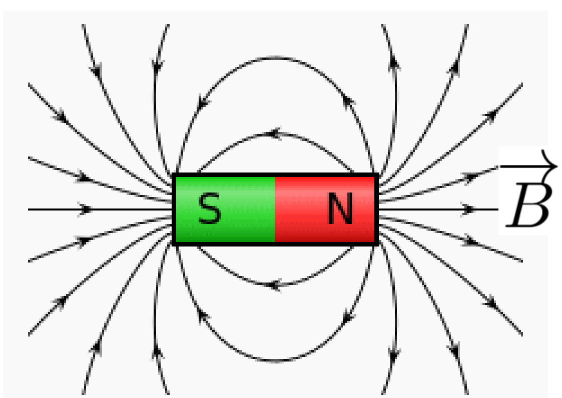


What is the direction of the magnetic field ?

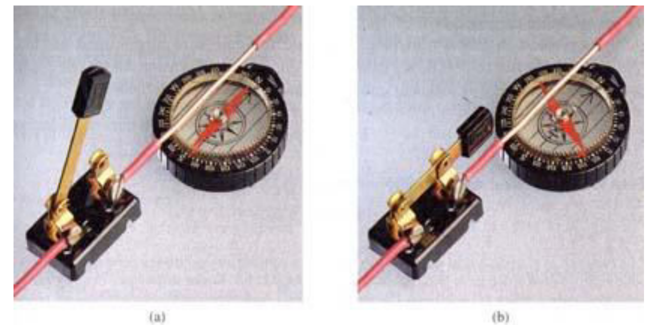
How can we obtain magnetic field ?

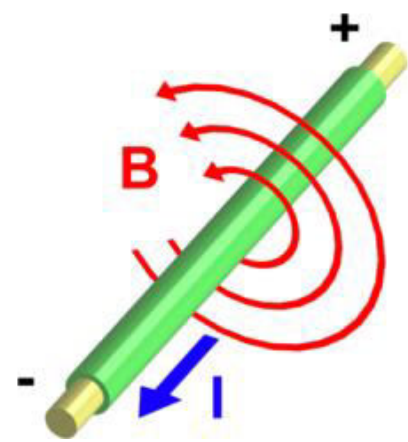
1. With Magnet





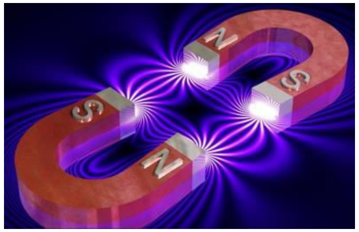
2. With electric current





Magnetostatics-L2

A-Sources- Magnetic Fields -interactions



1) Origin(s) and highlightings

a) Natural Magnets –magnetic compass- magnetic poles – Magnetic Field lines

b) Similar effects with an electric current- Biot and Savart Law

2) Magnetic field calculations produced by electric currents

Rect wire of current, loop of current, solenoid, field lines analysis

3) Geometric properties of the magnetic field

Flux conservation and Ampere theorem

4) Magnetic interactions

Lorentz forces – Laplace Force

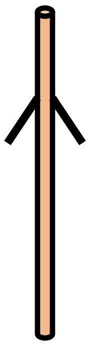
5) Some applications

2) Magnetic field calculations produced by electric currents

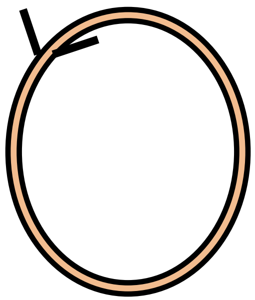
This part corresponds to the three first exercises of the tutorial

UFAZ-L2-Magnetostatics

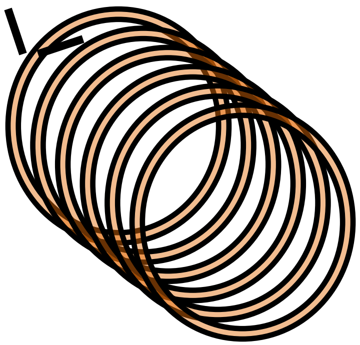
Exercise 1: Magnetic field created by an infinite wire of current



Exercise 2: Magnetic field created by a loop of current



Exercise 3: Magnetic field created by a finite and infinite solenoid



In the Lecture we show the main results but the calculations and details are done in the exercises

HOW is The Magnetic field ?

Exercise 1: Magnetic field created by an infinite wire of current

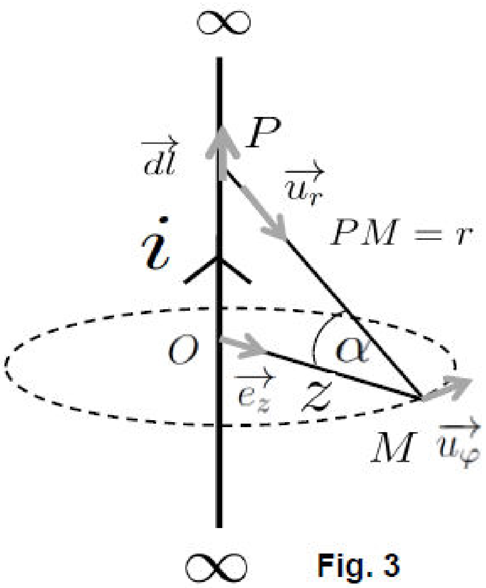
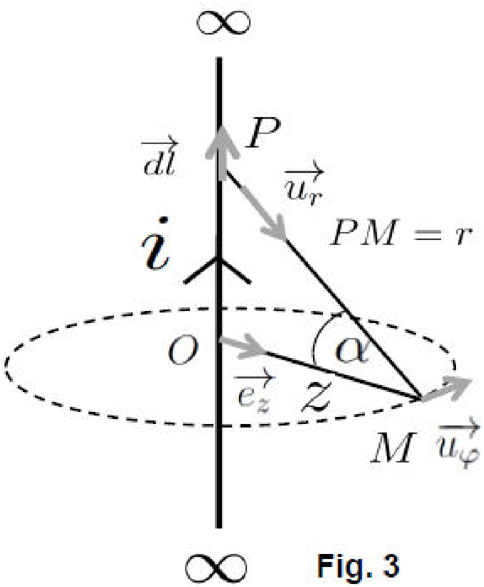


Fig. 3

- We apply Biot Savart Law for an elementary current element

$$d\overrightarrow{B} = \frac{\mu_0 i \overrightarrow{dl}}{4\pi r^2} \wedge \overrightarrow{u_r}$$

Exercise 1: Magnetic field created by an infinite wire of current



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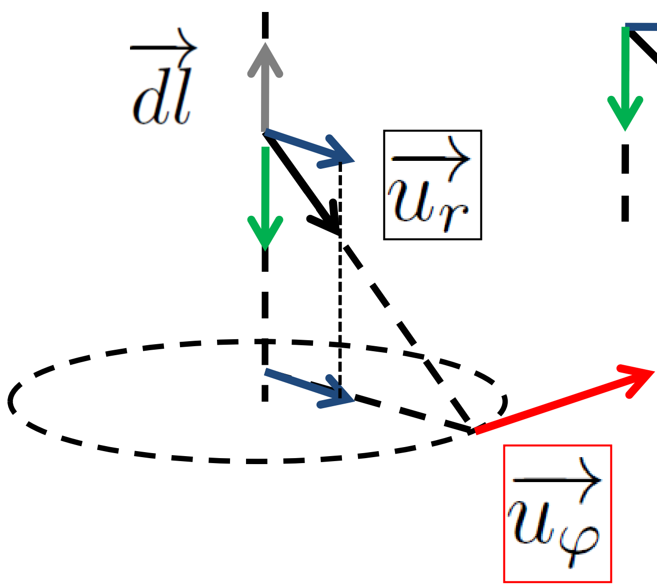
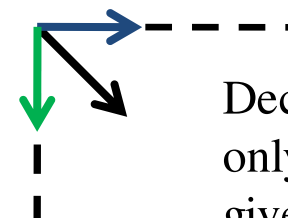
- We apply Biot Savart Law for an elementary current element

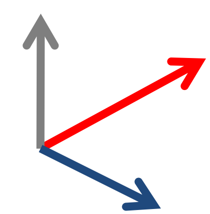
$$d\vec{B} = \frac{\mu_0 i \, \vec{dl}}{4\pi r^2} \wedge \vec{u_r}$$

- The important point is the calculation of the vectorial product





Decomposition of $\mathbf{u_r}$
only its blue component can give a non-zero contribution with vector **dl**
leading to vector: dl cos α **uφ**



Direct Basis

Exercise 1: Magnetic field created by an infinite wire of current

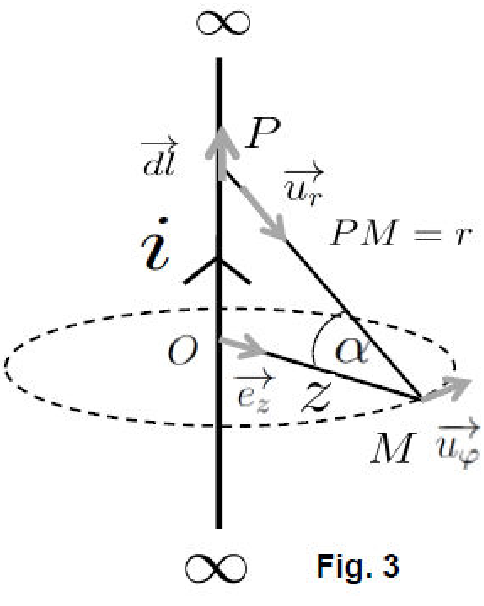


Fig. 3

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$$d\overrightarrow{B} = \frac{\mu_0 i \overrightarrow{dl}}{4\pi r^2} \wedge \overrightarrow{u_r}$$

- The important point is the calculation of the vectorial product:
  dl X **ur =** : dl cos α **uφ**

- By performing the appropriate integration

$$\overrightarrow{B} = \int d\overrightarrow{B} = \int \frac{\mu_0 i dl \cos\alpha \overrightarrow{u_\varphi}}{4\pi r^2}$$

Exercise 1: Magnetic field created by an infinite wire of current

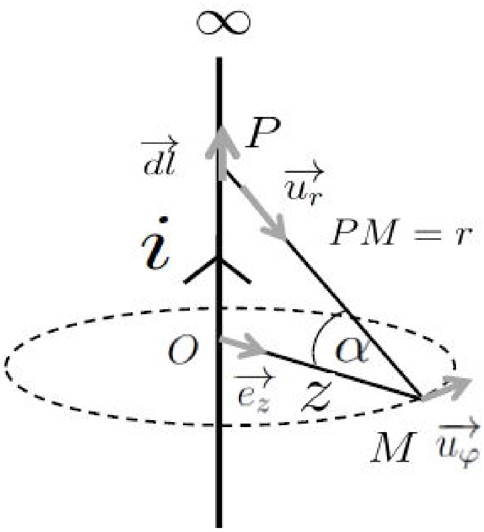
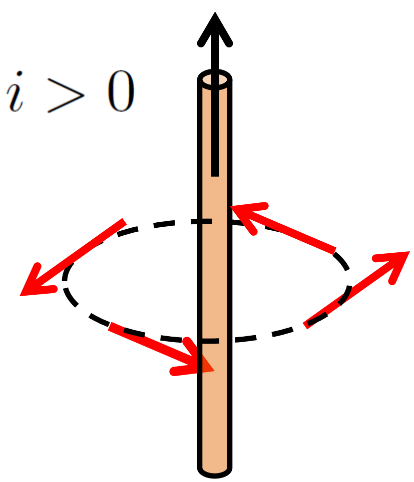


Fig. 3



• We apply Biot Savart Law for an elementary current element

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• The important point is the calculation of the vectorial product:
dl X **ur =** : dl cos α **uφ**

• By performing the appropriate integration

$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0 i dl \cos \alpha \vec{u_\varphi}}{4\pi r^2}$$

• We obtain:

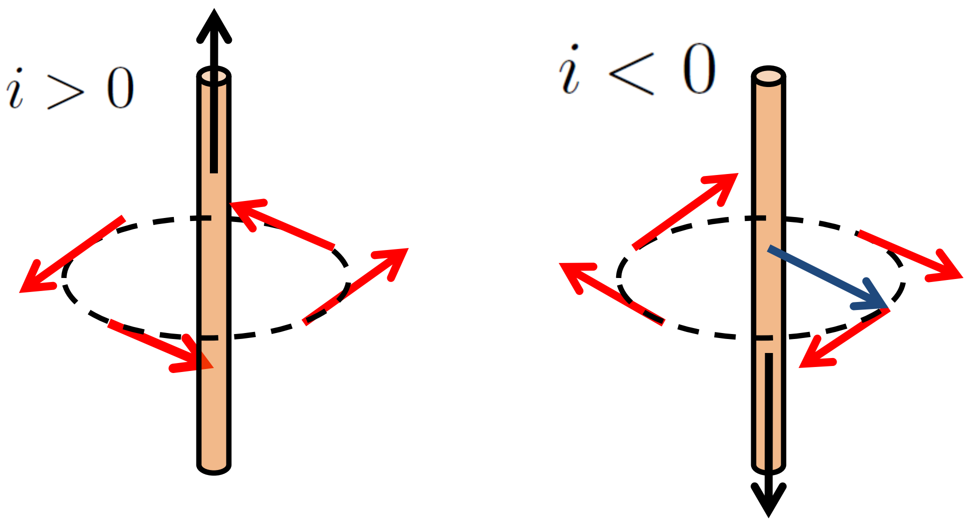
$$\boxed{\vec{B} = \frac{\mu_0 i}{2\pi z} \vec{u_\varphi}}$$

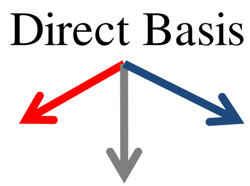
Magnetic field orthoradial vector perpendicular to wire axis

Exercise 1: Magnetic field created by an infinite wire of current

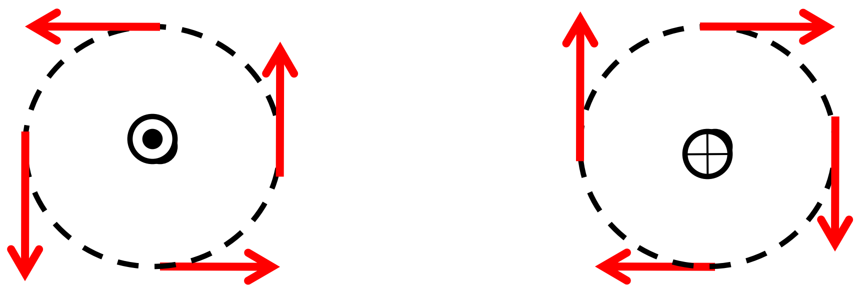
Influence of electric current sign.

$$\overrightarrow{B} = \frac{\mu_0 i}{2\pi z}\overrightarrow{u_\varphi}$$





Magnetic field vector in the opposite direction when i changes



Exercise 1: Magnetic field created by an infinite wire of current

$$\boxed{\overrightarrow{B} = \frac{\mu_0 i}{2\pi z}\overrightarrow{u_\varphi}}$$

Magnetic field vector is tangent to the fields lines

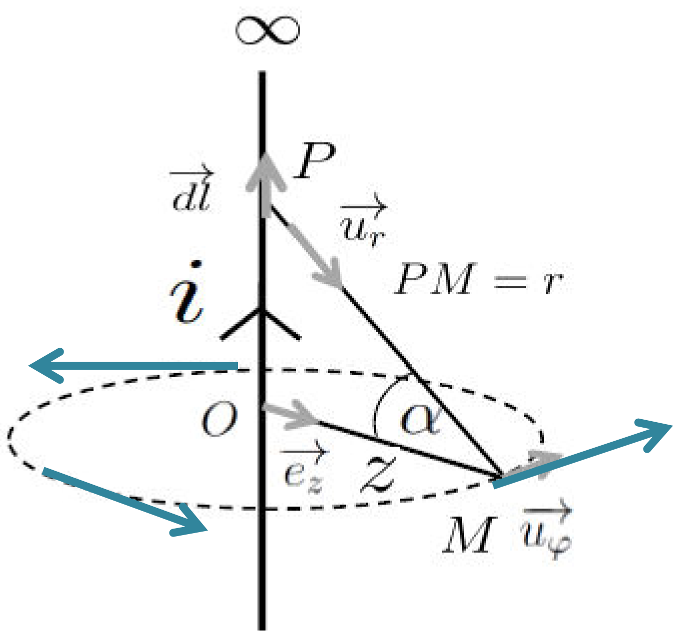
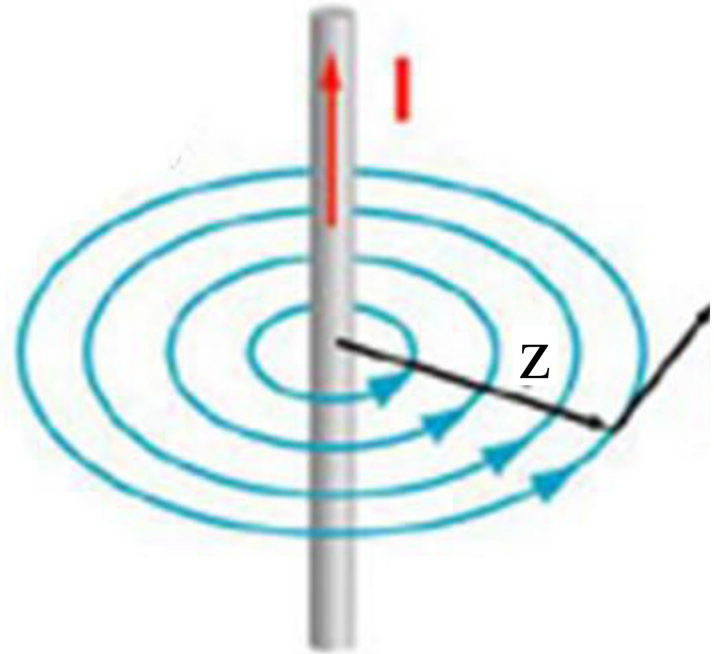


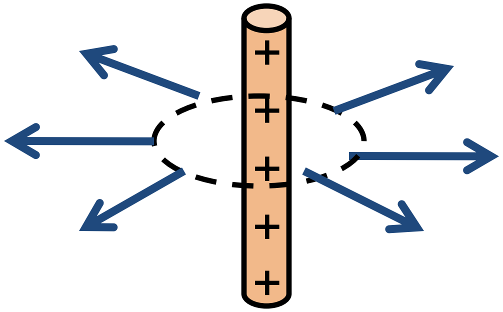
Fig. 3

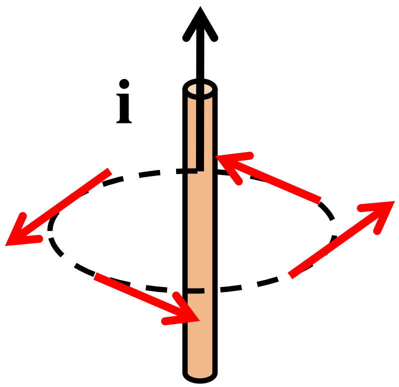


Right hand

***Difference** between electric field and magnetic field*

$\boxed{\overrightarrow{E}}$



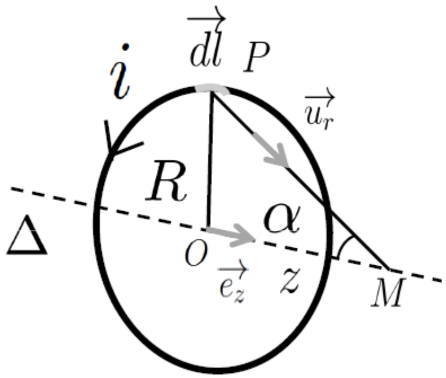


$\boxed{\overrightarrow{B}}$

Electric field vector is radial

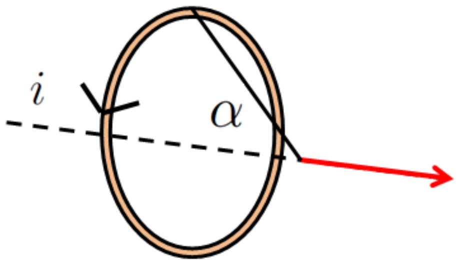
Magnetic field vector is orthoradial

Exercise 2: Magnetic field created by a loop of current



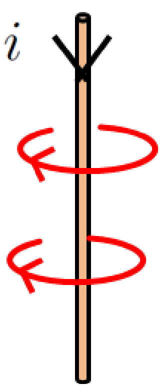
- Applying Biot and Savart Law to the current element **dl** and then, performing an integration will give for a point M that belongs to the axis, a contribution along the axis

$$\boxed{\vec{B} = \frac{\mu_0 i}{2R} (\sin\alpha)^3 \vec{e_z}}$$

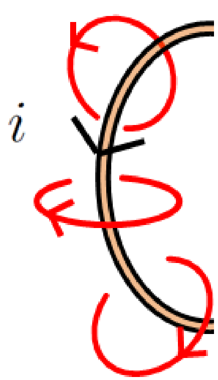


- Interpretation with field lines and rect wire

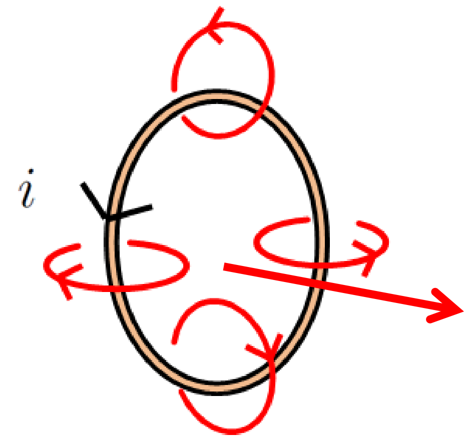
Rect wire



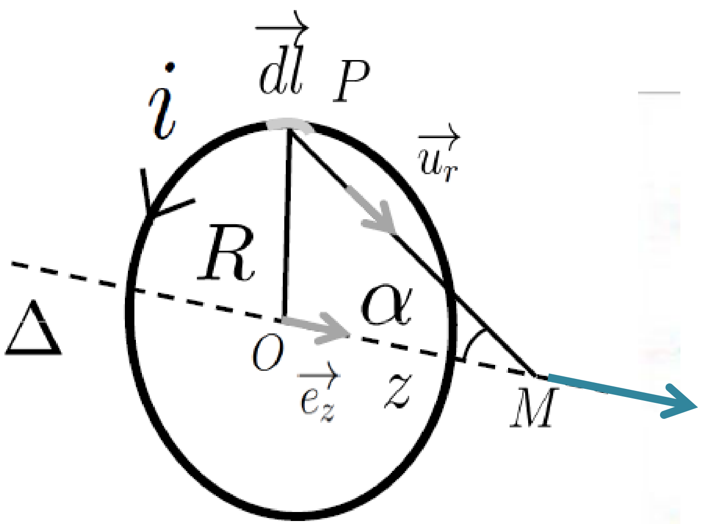
Bended wire



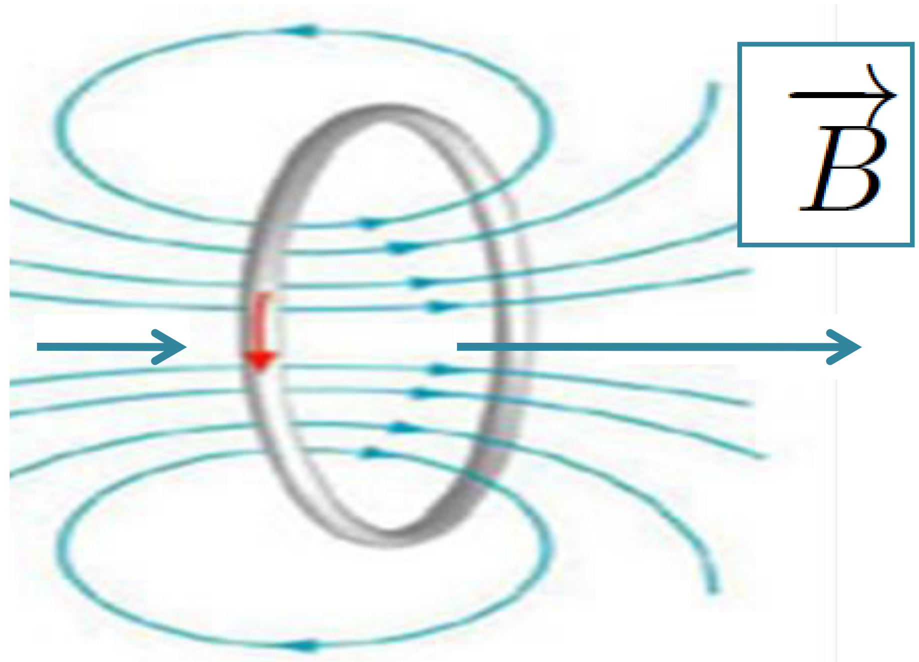
Closed wire: loop



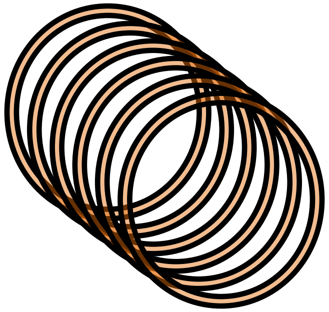
Exercise 2: Magnetic field created by a loop of current



Magnetic field vector is tangent to the fields lines

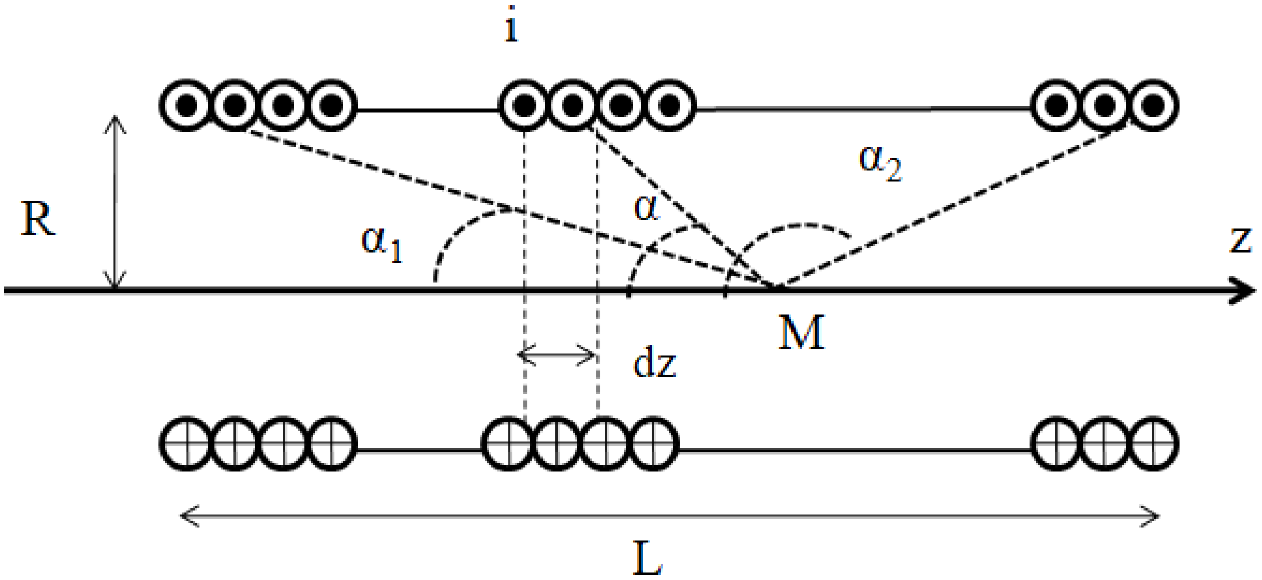


Exercise 3: Magnetic field created by a finite and infinite solenoid

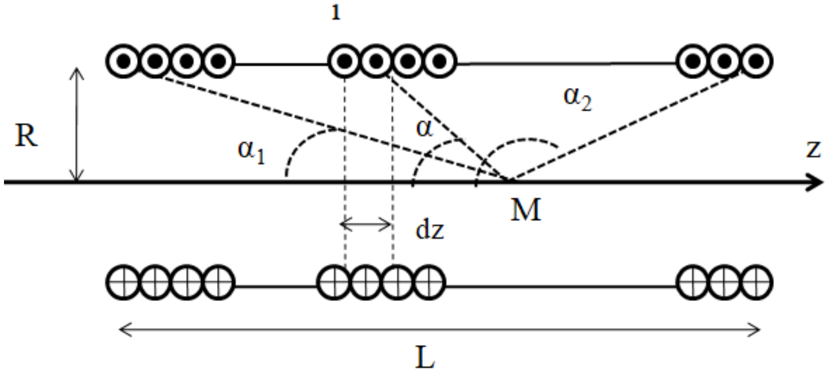


A solenoide is a many-loop device. If N is the number of loop and L the length of the solenoide, we can define the loop density:

$$n=N/L$$



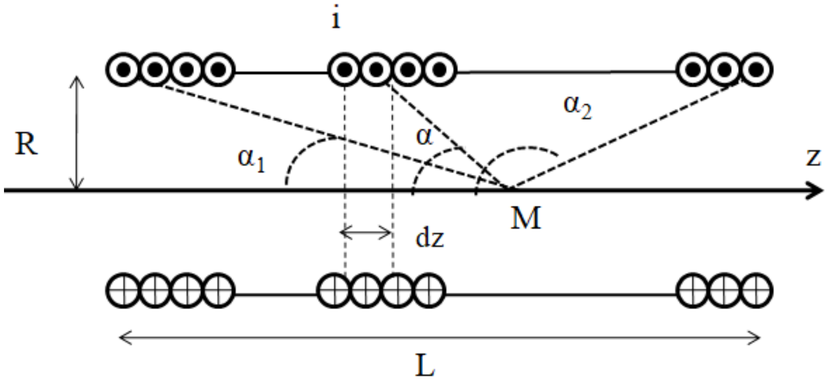
Exercise 3: Magnetic field created by a finite and infinite solenoid



Using the result obtained for one loop, we write the elementary field created at point M by a length dz having ndz loop is

$$d\overrightarrow{B} = \frac{\mu_0 indz}{2R} \sin^3 \alpha \overrightarrow{e_z}$$

Exercise 3: Magnetic field created by a finite and infinite solenoid



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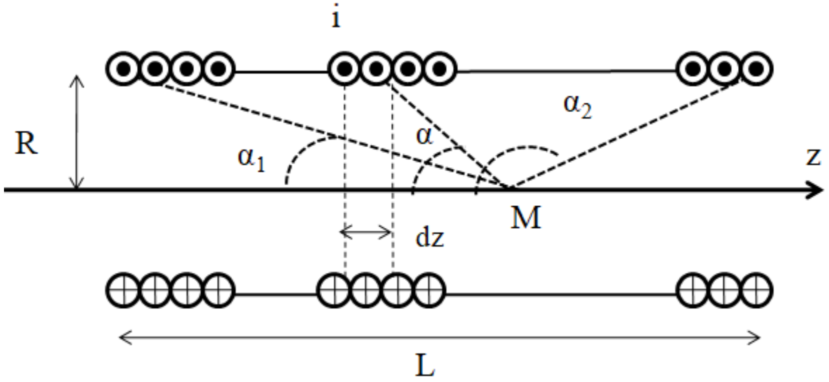
$$d\overrightarrow{B} = \frac{\mu_0 indz}{2R}\sin^3\alpha\overrightarrow{e_z}$$

By integrating over all the loops, we obtain for a finite solenoide

$$\overrightarrow{B} = \int d\overrightarrow{B} = \frac{\mu_0 ni\alpha}{2}\left(\cos\alpha_1 - \cos\alpha_2\right)\overrightarrow{e_z}$$

Exercise 3: Magnetic field created by a finite and infinite solenoid



Using the result obtained for one loop, we write the elementary field created at point M by a length dz having ndz loop is

$$d\overrightarrow{B} = \frac{\mu_0 i n dz}{2R} \sin^3 \alpha \overrightarrow{e_z}$$

By integrating over all the loops, we obtain for a finite solenoide

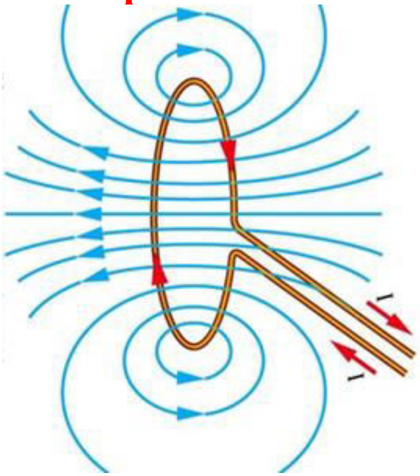
$$\overrightarrow{B} = \int d\overrightarrow{B} = \frac{\mu_0 n i \alpha}{2} \left(\cos \alpha_1 - \cos \alpha_2\right) \overrightarrow{e_z}$$

That becomes for an infinite solenoide where $\alpha 1 \rightarrow 0$ and $\alpha 2 \rightarrow \pi$

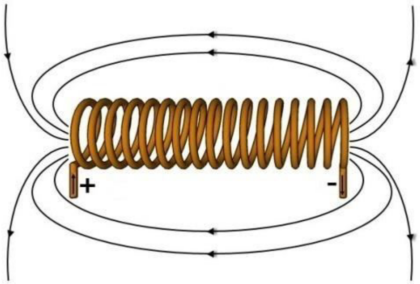
$$\boxed{\overrightarrow{B} = \frac{\mu_0 N i}{L} \overrightarrow{e_z} = \mu_0 n i \overrightarrow{e_z}}$$

Exercise 3: Magnetic field created by a finite and infinite solenoid

1 loop of current



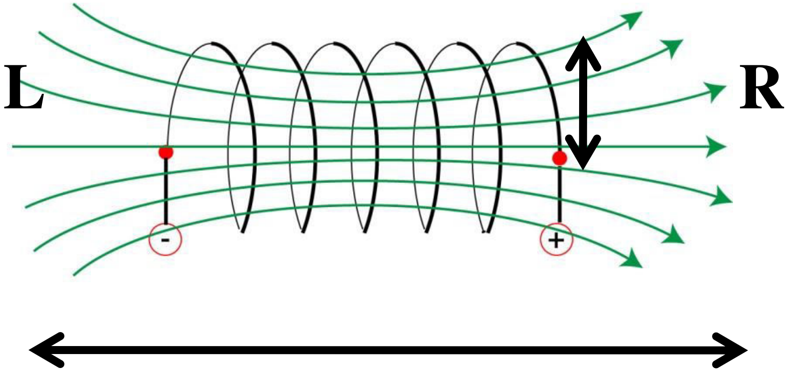
Many loops of current



Creation of magnetic field inside the solenoide

Magnetic field vector is tangent to the fields lines

Infinite solenoid: L >> R:



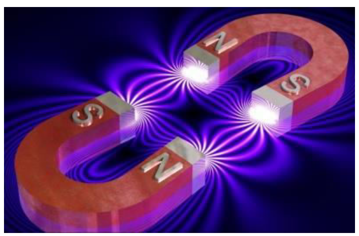
$$\boxed{\overrightarrow{B} = \frac{\mu_0 N i}{L}\overrightarrow{e_z} = \mu_0 n i \overrightarrow{e_z}}$$

Source	Approximate Magnetic Field Strength
Neuron depolarization (imaged by MEG)	0.5 pT (5×10^{-13} T)
Earth's magnetic field	0.5 G (50 μT)
Refrigerator magnet	50 G (5 mT)
Junkyard electromagnet	1 T
Clinical MRI scanners	0.5 - 3.0 T (typical)
Research MRI scanners (human)	7.0 T – 11.7 T
Laboratory NMR spectrometers	6 - 23 T
Largest pulsed field created in lab nondestructively	97 T
Largest pulsed field created in lab (destroying equipment but not the lab)	730 T

Don't trust two last values
Surface of application is not specified

Magnetostatics-L2

A-Sources- Magnetic Fields -interactions



1) Origin(s) and highlightings

a) Natural Magnets –magnetic compass- magnetic poles – Magnetic Field lines

b) Similar effects with an electric current- Biot and Savart Law

2) Magnetic field calculations produced by electric currents

Rect wire of current, loop of current, solenoid, field lines analysis

3) Geometric properties of the magnetic field

Flux conservation and Ampere theorem

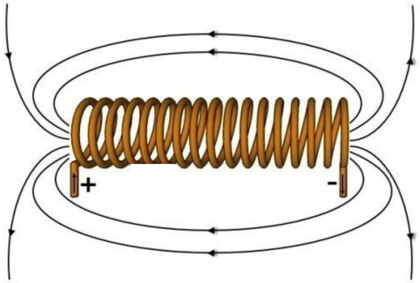
4) Magnetic interactions

Lorentz forces – Laplace Force

5) Some applications

Similar properties between the magnetic field lines creadted by a solenoid and the ones created by a natural magnet

Many loops of current



Creation of magnetic field inside the solenoide

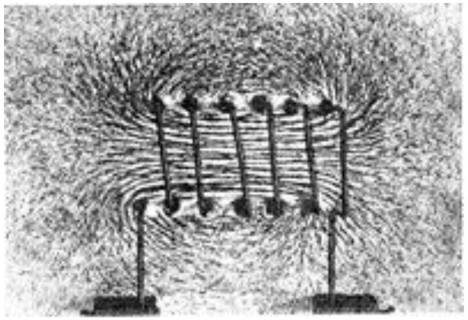
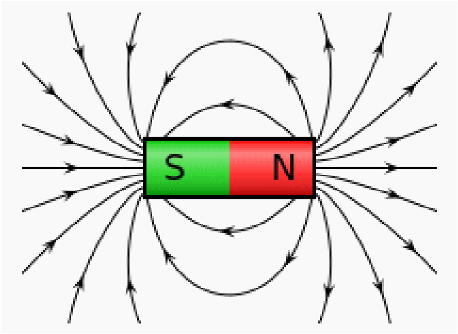
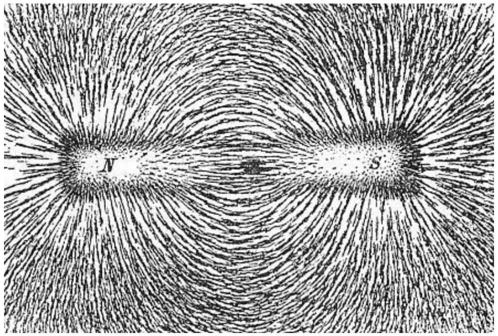


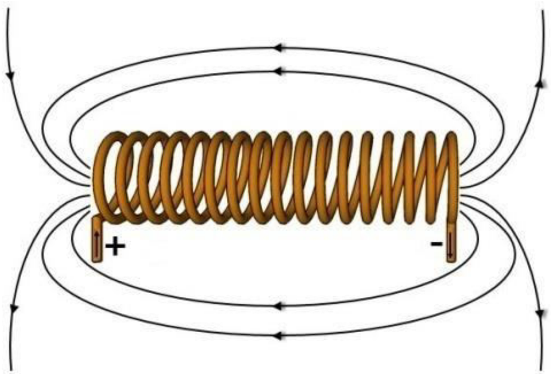
Fig. 3

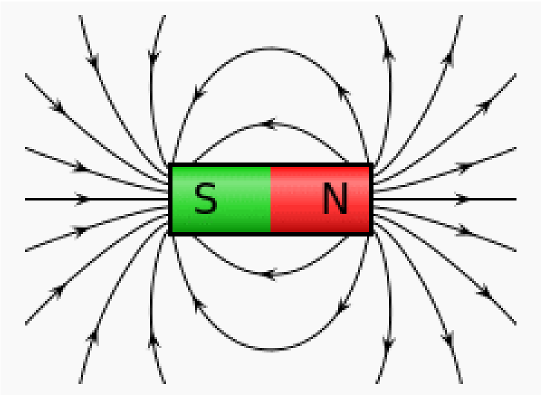


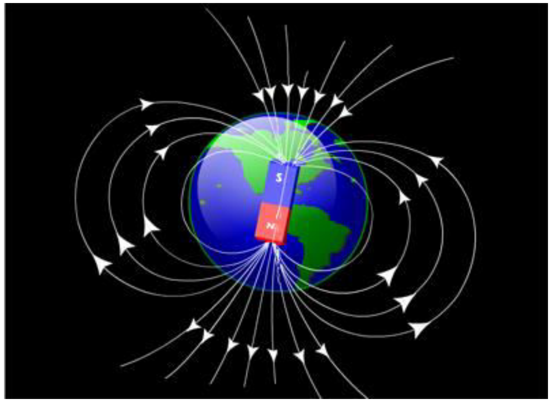


Conservation of Magnetic flux

The flux of magnetic field through a closed surface is zero.
All the field lines that leaves the closed surface (volume of magnet) will enter back again

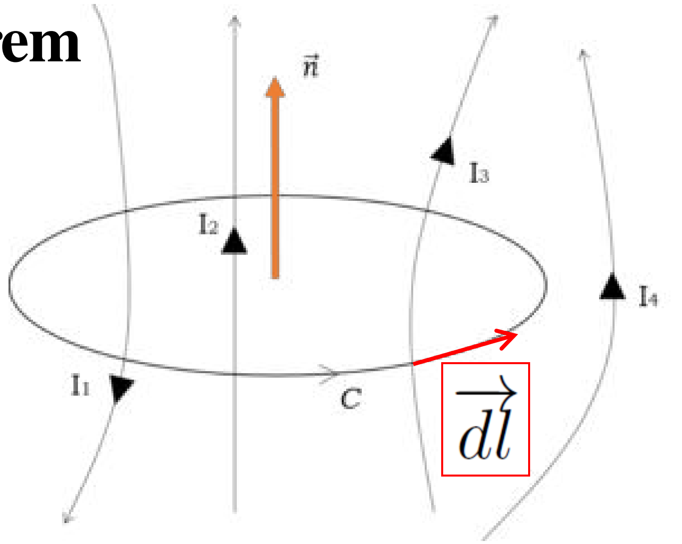






$$\oiint \overrightarrow{B} \cdot \overrightarrow{dS} = 0$$

Magnetic field circulation – Ampere theorem



Lets consider
i) an ensemble of current
ii) an oriented surface S defined by a closed curve C

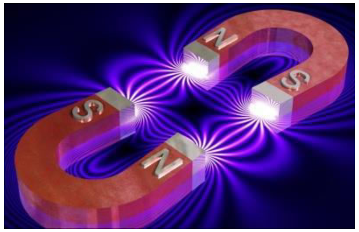
The currents generates magnetic field.

Ampere theorem states that the circulation (integral line) of the magnetic field along the closed curve is equal to the sum of current encircled by the the curve C.

$$\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \mu_0 \sum_k i_{\mathbf{k} \quad \mathbf{encir}}$$

Magnetostatics-L2

A-Sources- Magnetic Fields -interactions



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Flux conservation and Ampere theorem

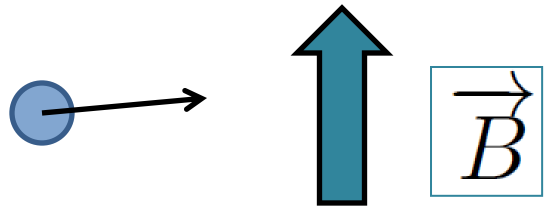
4) Magnetic interactions

Lorentz forces – Laplace Force

5) Some applications

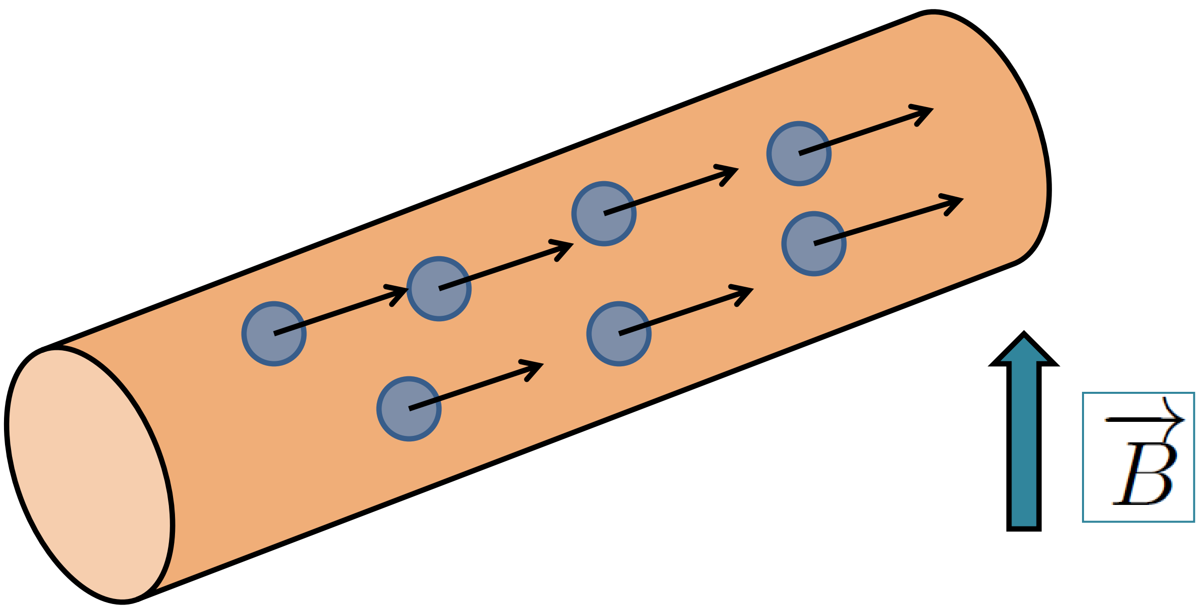
Effects of magnetic field on moving charges

i) One moving charge in a magnetic field



Lorentz force

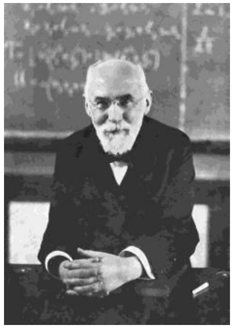
ii) Many moving charges in a rigid solid in a magnetic field……



Laplace force

Lorentz force: Electric and Magnetic forces

$$\frac{d\vec{p}}{dt} = \sum \vec{F} = \left(q\vec{E} + q\vec{v} \wedge \vec{B} \right)$$

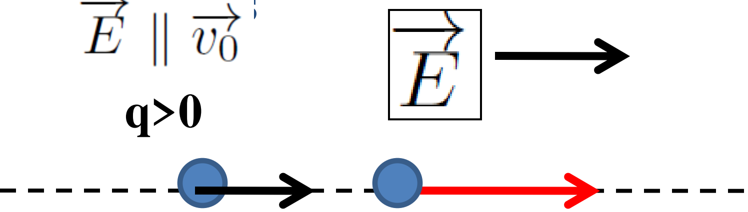


Hendrick. Lorentz
1853-1928

Accelerates (and deviates) charge

$\vec{E} \parallel \vec{v_0}$

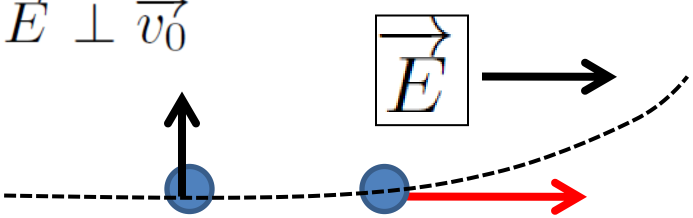
q>0



$$x = \frac{qEt^2}{2m} + v_0 t$$

Accelerated rect motion (1D)

$\vec{E} \perp \vec{v_0}$



$$x = \frac{qEt^2}{2m}$$

$$y = v_0 t$$

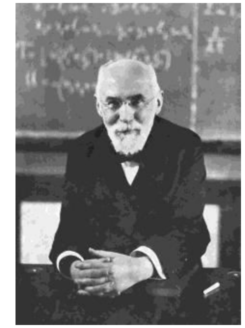
q>0

Accelerated parabolic motion (2D)

Magnetic interaction

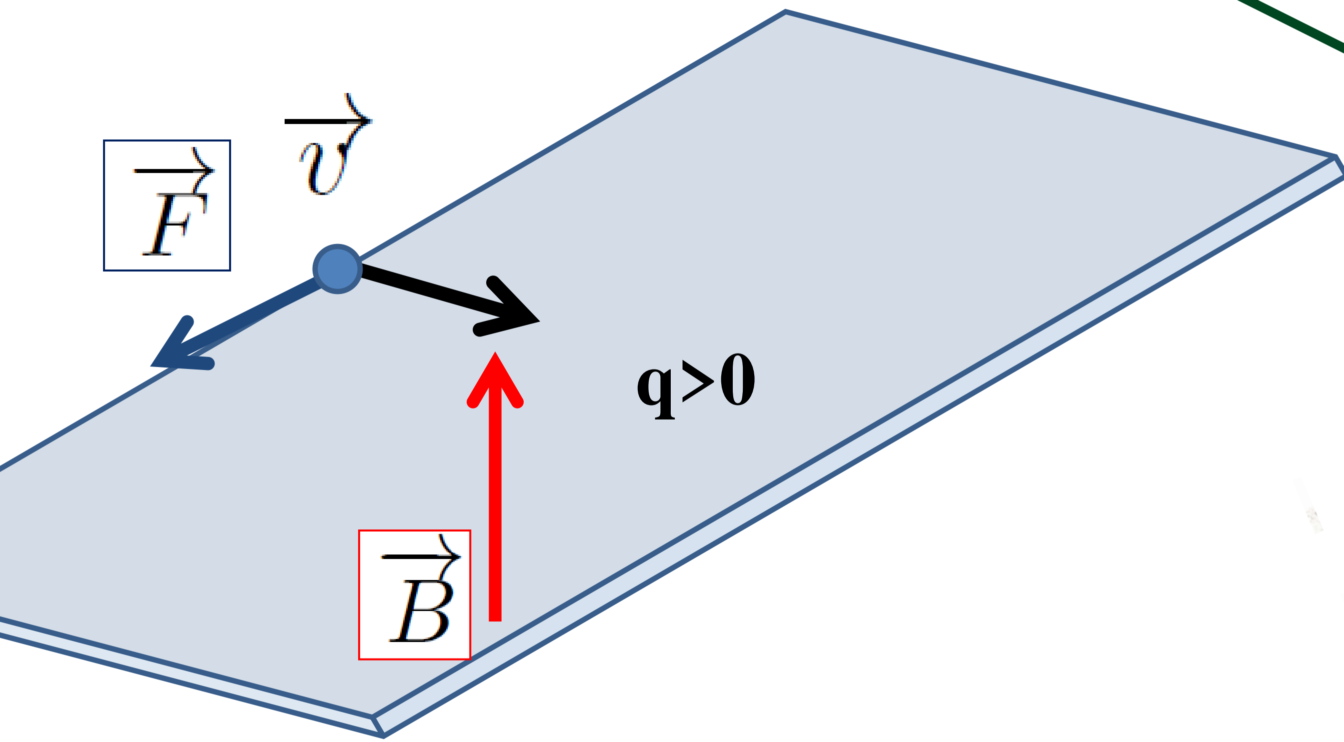
Lorentz force: Electric and Magnetic forces

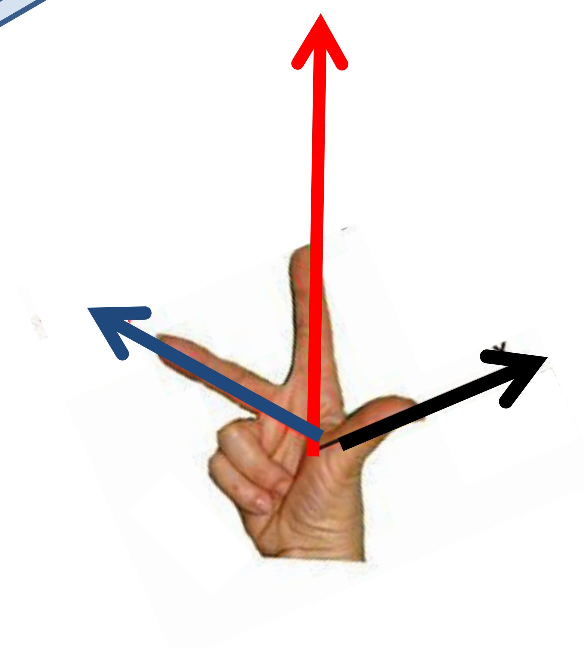
$$\frac{d\overrightarrow{p}}{dt} = \sum \overrightarrow{F} = \left(q\overrightarrow{E} + \underbrace{q\overrightarrow{v} \wedge \overrightarrow{B}}\right)$$



Hendrick. Lorentz
1853-1928

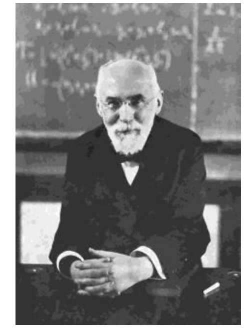
Deviates charge





Lorentz force: Electric and Magnetic forces

$$\frac{d\vec{p}}{dt} = \sum \vec{F} = \left(q\vec{E} + q\vec{v} \wedge \vec{B}\right)$$



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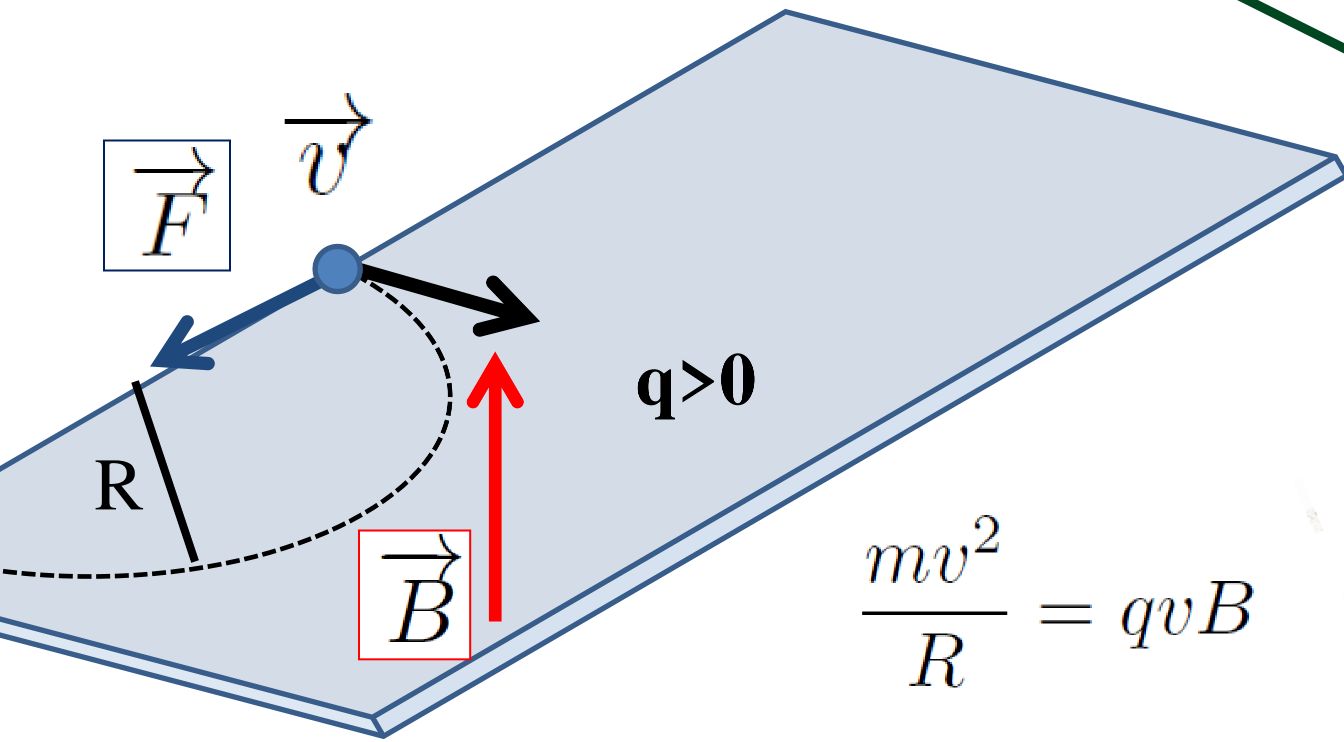
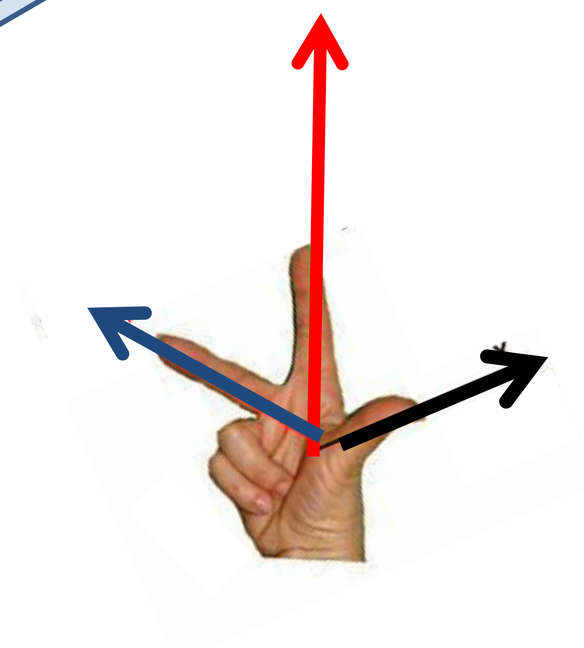


$$\frac{mv^2}{R} = qvB$$



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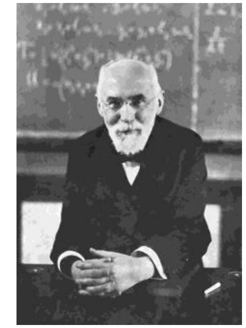

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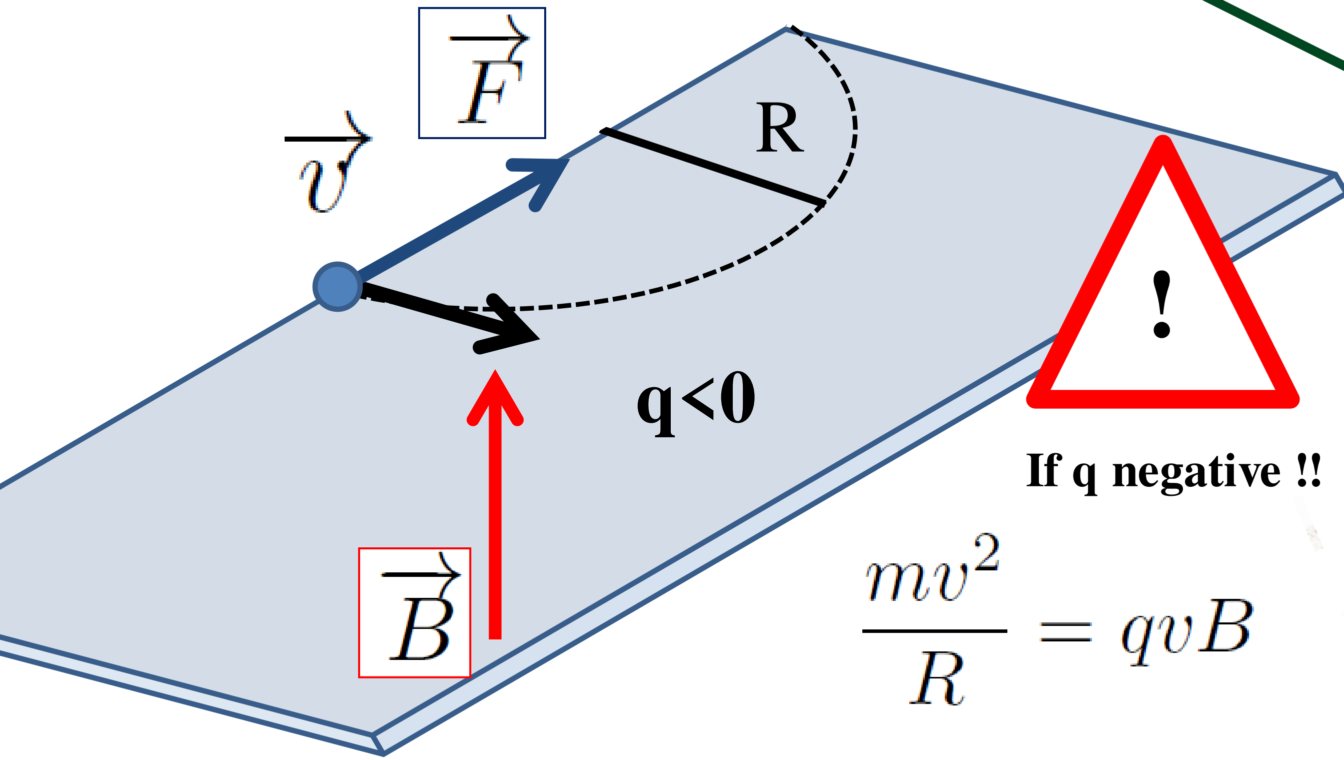
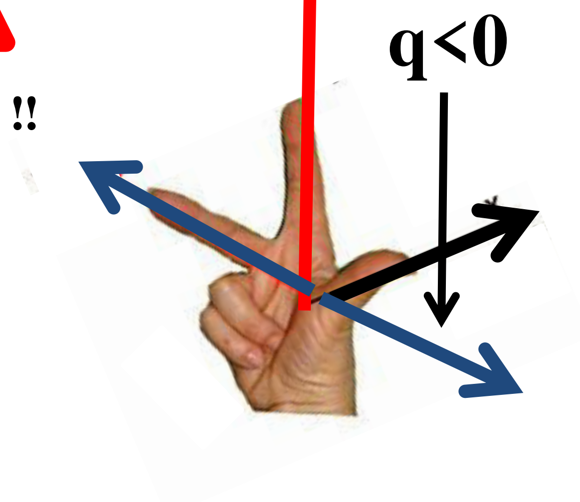


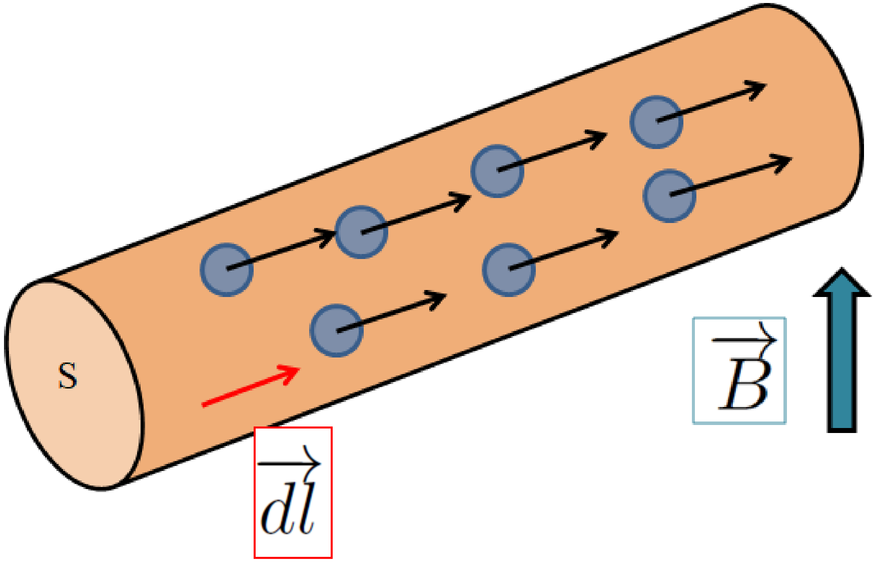
If q negative !!

$$\frac{mv^2}{R} = qvB$$



ii) Many moving charges in a rigid solid in a magnetic field……

$$dq = \rho dV$$



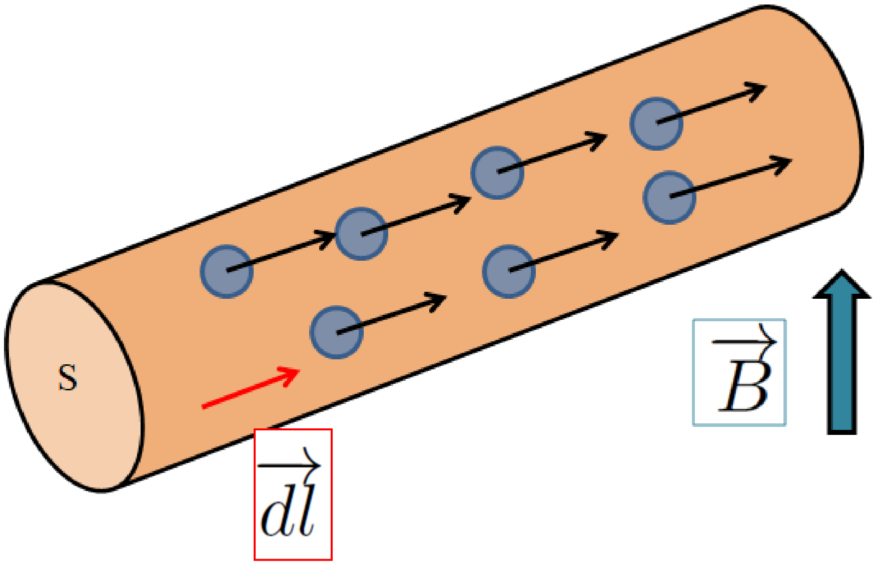
Force felt by a volume dV of charge dq

$$\begin{aligned}\overrightarrow{d^3F} &= dq\,\overrightarrow{v} \wedge \overrightarrow{B} \\ &= \rho dV\,\overrightarrow{v} \wedge \overrightarrow{B} \\ &= (\rho\overrightarrow{v})(dSdl) \wedge \overrightarrow{B} \\ &= (\rho v)(dS\,\overrightarrow{dl}) \wedge \overrightarrow{B}\end{aligned}$$

$$\overrightarrow{v} \parallel \overrightarrow{dl}$$

ii) Many moving charges in a rigid solid in a magnetic field……

$$dq = \rho dV$$



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&= \rho dV\,\overrightarrow{v} \wedge \overrightarrow{B} \\
&= (\rho\overrightarrow{v})(dSdl) \wedge \overrightarrow{B} \\
&= (\rho v)(dS\overrightarrow{dl}) \wedge \overrightarrow{B}
\end{aligned}$$

$$\overrightarrow{v} \parallel \overrightarrow{dl}$$

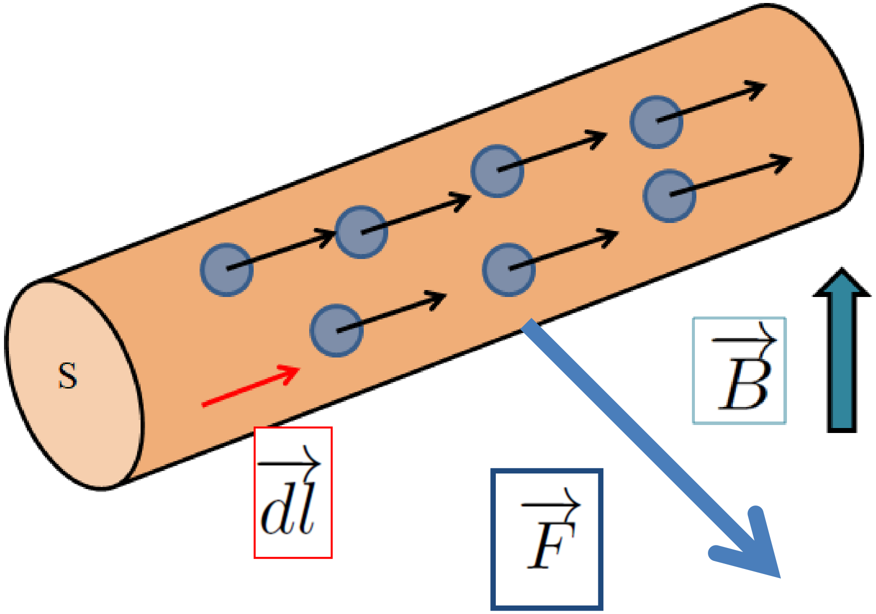
We integrate over the surface S

$j = \rho v$ Volumic curent

$$\begin{aligned}
\overrightarrow{dF} &= (jS)\overrightarrow{dl} \wedge \overrightarrow{B} \\
&= i\,\overrightarrow{dl} \wedge \overrightarrow{B}
\end{aligned}$$

ii) Many moving charges in a rigid solid in a magnetic field……

$$dq = \rho dV$$



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$j = \rho v$ Volumic curent

$$\begin{aligned}\overrightarrow{dF} &= (jS)\overrightarrow{dl} \wedge \overrightarrow{B} \\ &= i\,\overrightarrow{dl} \wedge \overrightarrow{B}\end{aligned}$$

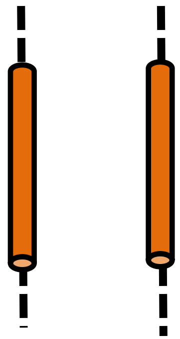
Force over all the solid is the Laplace force

$$\boxed{\overrightarrow{F} = \int i\,\overrightarrow{dl} \wedge \overrightarrow{B}}$$

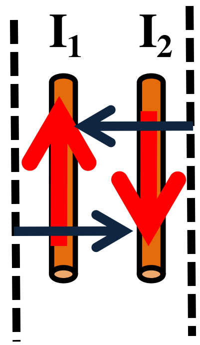
Particular effects due to Laplace force

Interaction between two electric currents

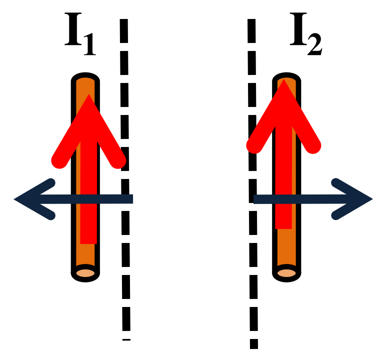
$I_1 = I_2 = 0$



Initial position



$I_1 > 0$ and $I_2 < 0$
Or $I_1 < 0$ and $I_2 > 0$
ATTRACTION

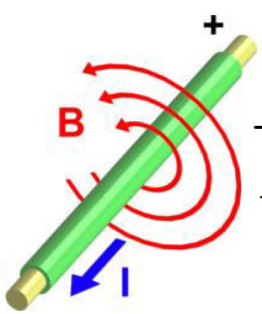


$I_1 > 0$ and $I_2 > 0$
Or $I_1 < 0$ and $I_2 < 0$
REPULSION

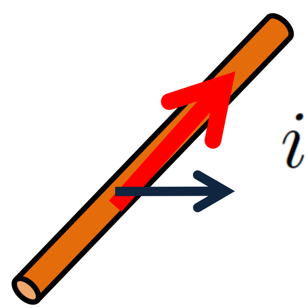
Definition of Ampere:
Unit of electric current:

1 A= current needed to have a lineic force of 2*10^7 N/m between two infinite wires separated from one meter.

Magnetic field acts also on other electric current



$\overrightarrow{B}$



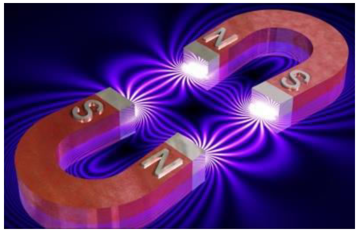
$i\,\overrightarrow{dl}$

Laplace Force

$$d\overrightarrow{F} = i\,\overrightarrow{dl} \wedge \overrightarrow{B}$$

Magnetostatics-L2

A-Sources- Magnetic Fields -interactions



1) Origin(s) and highlightings

a) Natural Magnets –magnetic compass- magnetic poles – Magnetic Field lines

b) Similar effects with an electric current- Biot and Savart Law

2) Magnetic field calculations produced by electric currents

Rect wire of current, loop of current, solenoid, field lines analysis

3) Geometric properties of the magnetic field

Flux conservation and Ampere theorem

4) Magnetic interactions

Lorentz forces – Laplace Force

5) Some applications

They are various applications (by applications we mean things we can do)

- **Helmholtz coils:** device used to produce a constant magnetic field

- **Hall Probe:** (modern) device used to measure the amplitude of a magnetic field

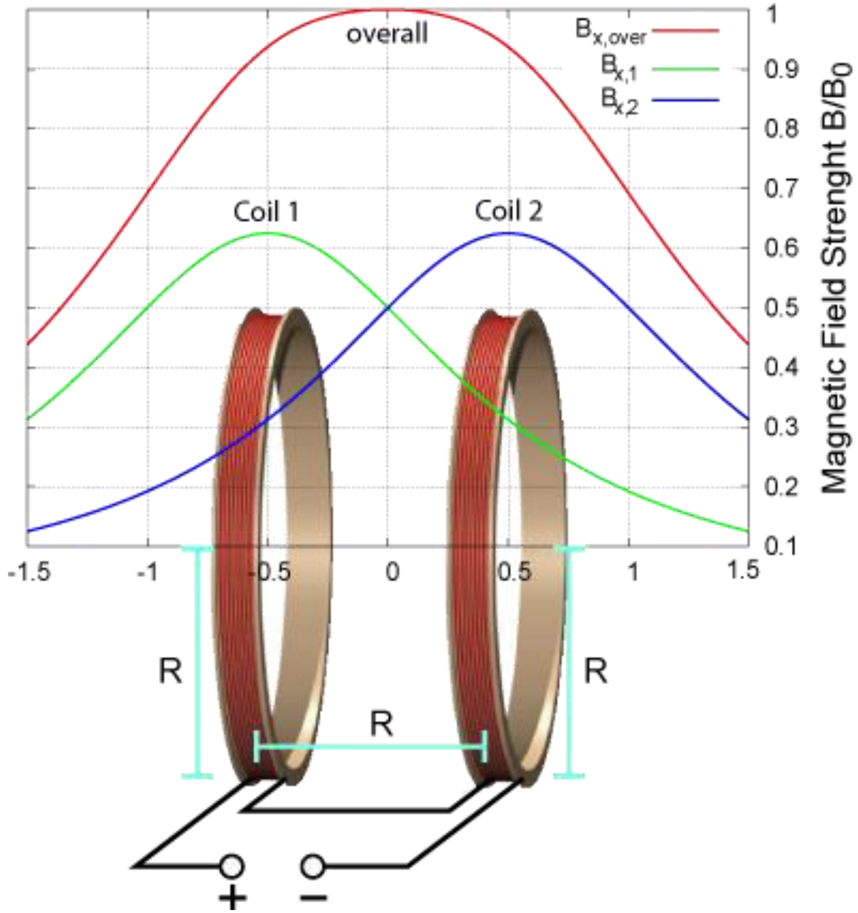
- Using **Laplace Forces** in electric motors (mainly in induction lecture: next chapter)

- Using magnetic field to **deviate charged particles**…..and to study fundamental particles and interactions

- Using magnetic field in medecine: **IMR (imaging by magnegtic resonance)** **next year in L3**

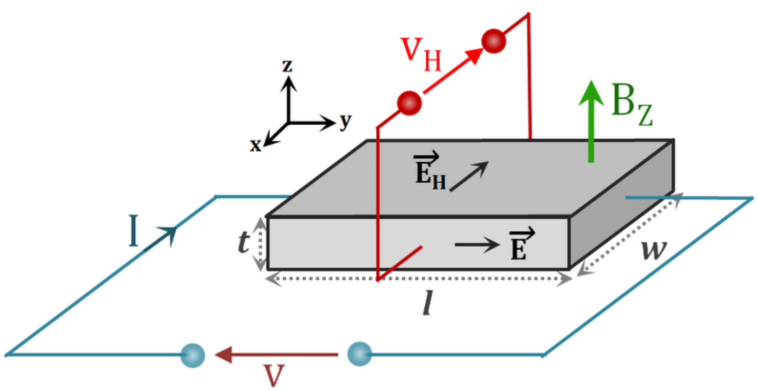
Helmholtz coils

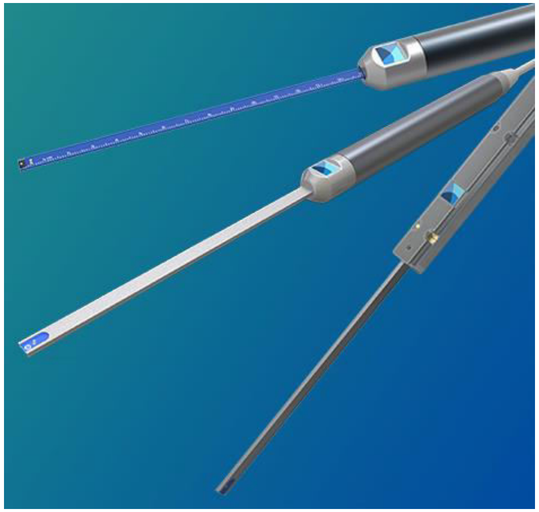
Exercise 4 : Uniform magnetic field obtained with Helmholtz coils



Hall Probe: (modern) deviced used to measure the amplitude of a magnetic field

Exercise 6: Hall effect: how to measure a magnetic field ?



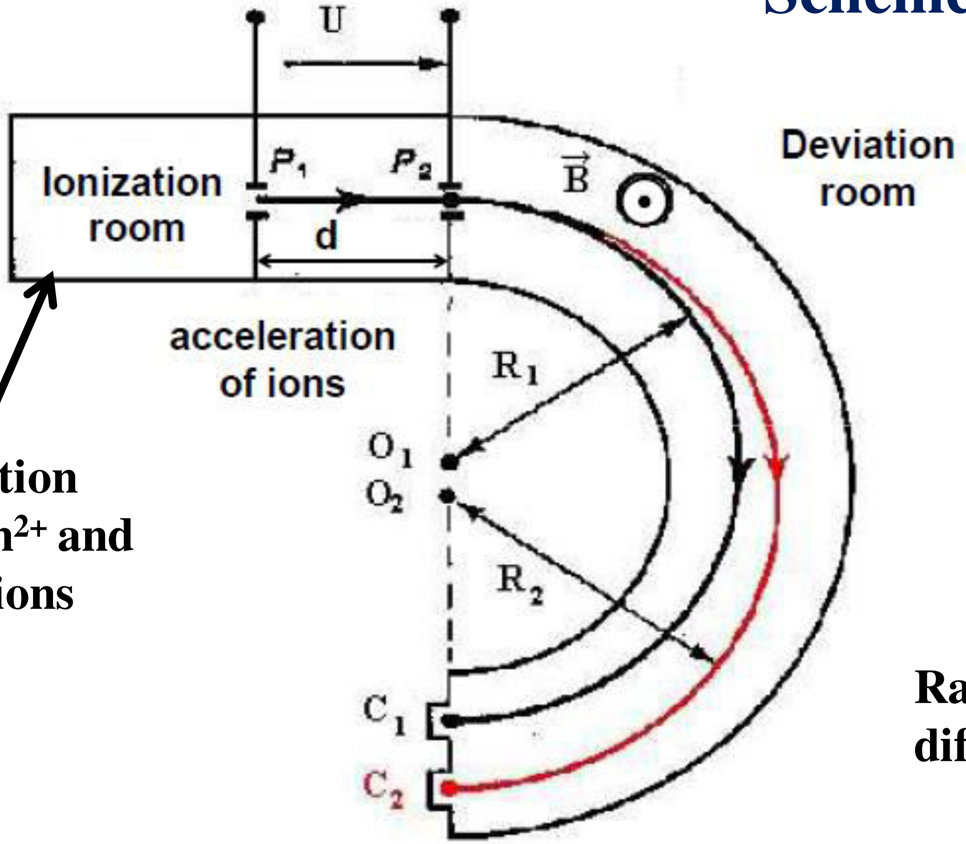


Deviation of a charged particle by a magnetic field

See: Tutorial of electromagnetism. Part 3.

Scheme of a mass sprectometer

Separation of isotops



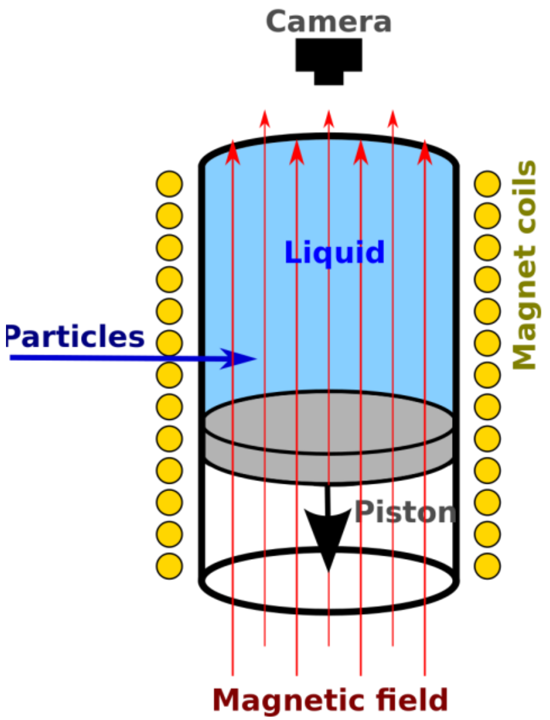
Production Of $^{68}Zn^{2+}$ and $^{70}Zn^{2+}$ ions

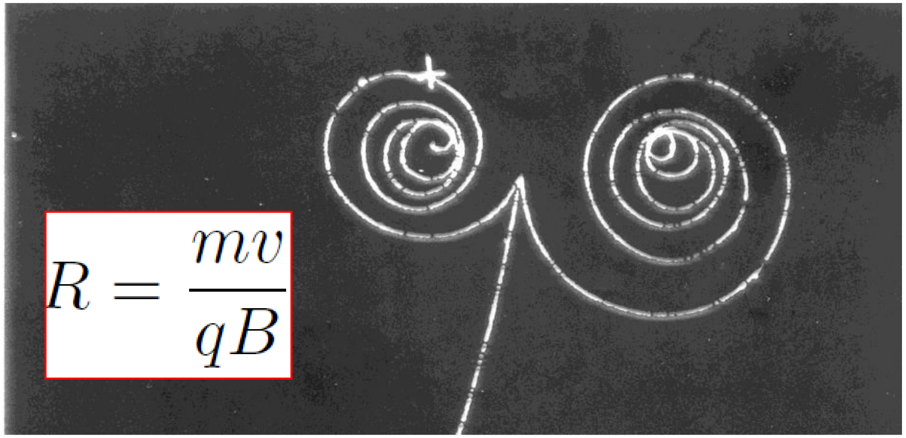
$$\boxed{R = \frac{mv}{qB}}$$

Radius depends on mass and will be different for $^{68}Zn^{2+}$ and $^{70}Zn^{2+}$ ions

Study of elementary particles:

Cloud chamber or bubbles chambers:
They record <u>charged particle trajectories</u>





$$R = \frac{mv}{qB}$$

By measuring radius R we can have acces to mass or charge

Study of elementary particles:

Cloud chamber or bubbles chambers:
They record <u>charged particle trajectories</u>

Discovery of anti-matter:

$$e^{+}$$

A new particle, **the positron (anti-electron):** particle which has *<u>same mass than the electron but an opposite positive charge: +e</u>*

Theoretical Prediction (1928)

Paul A.M Dirac
1902-1984

Experimental Discovery(1932)

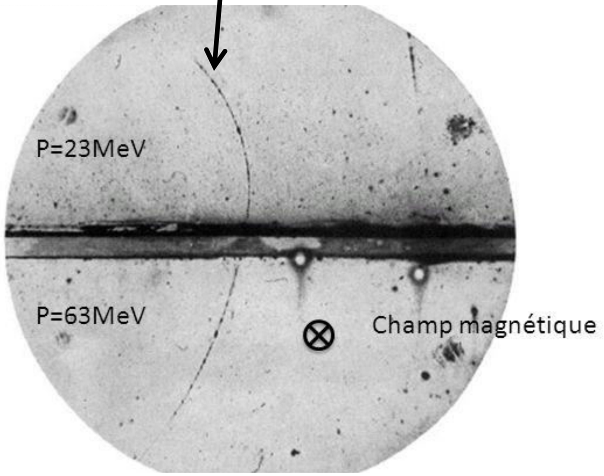
Carl Anderson
1905-1991

Study of elementary particles:

Cloud chamber or bubbles chambers:
They record <u>charged particle trajectories</u>

This radius was too small to be the one of a proton $m_p \approx 2000 m_{e-}$

$$\boxed{R = \frac{mv}{qB}}$$



Discovery of anti-matter:

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Discovery of anti-matter:

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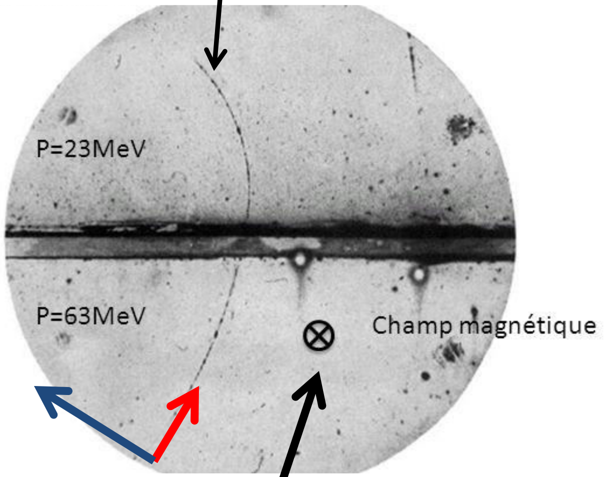


Direction of a positive charge !!

$$\vec{F} = q\vec{v} \wedge \vec{B}$$

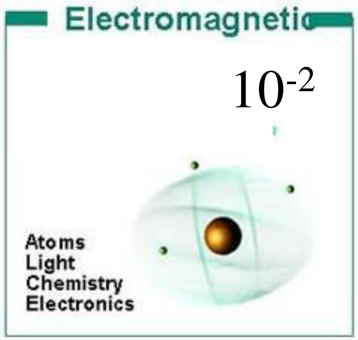
Theoretical Prediction (1928)

Paul A.M Dirac 1902-1984

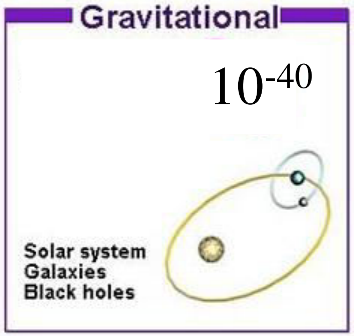
Experimental Discovery(1932)

Carl Anderson 1905-1991

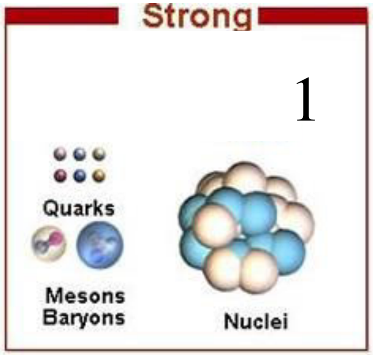
Today we distinguish FOUR fundamental interactions

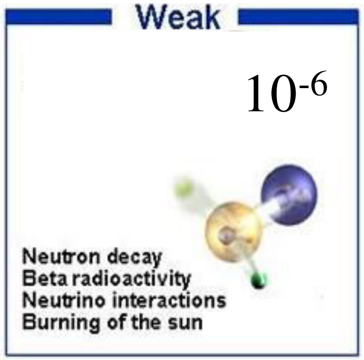


$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\vec{u_r}$$



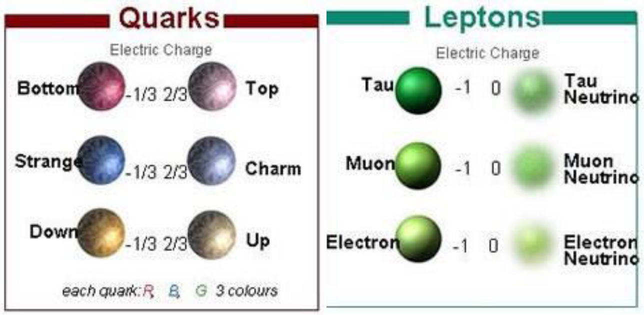
$$\vec{F} = \frac{G m_1 m_2}{r^2}\vec{u_r}$$





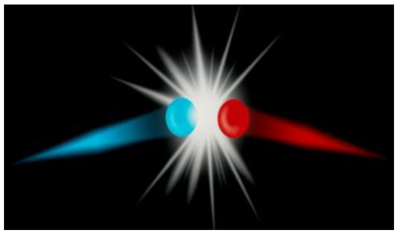
Too complicated …..☺ ….yet

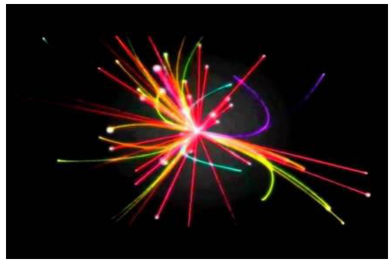
- The particles that feel these interactions are called **fermions** and represent the elementary pieces of the matter.



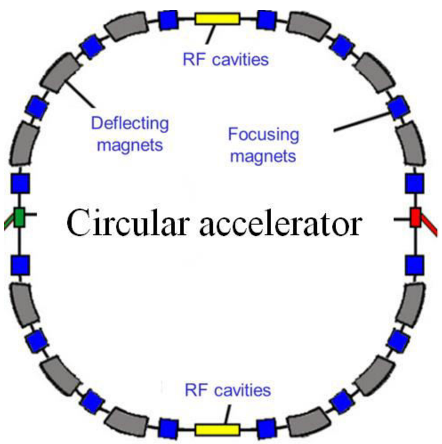
- The interaction is mediated with the help of particles called **bosons**.

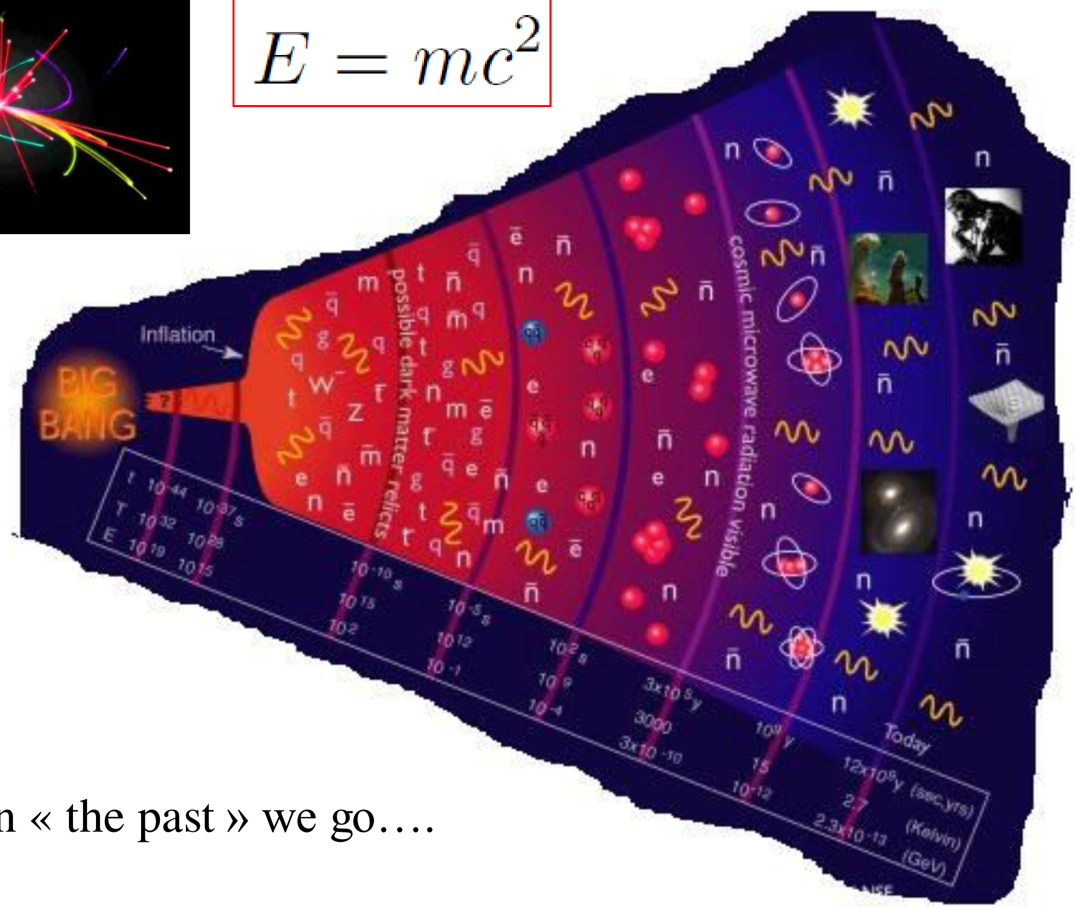
To study these interactions and particles, physicians make colliding particles at high kinetic energy. In a small region of space we have a high energy density where conditions are similar than the univers in the past….just after the BIG BANG





$$E = mc^2$$

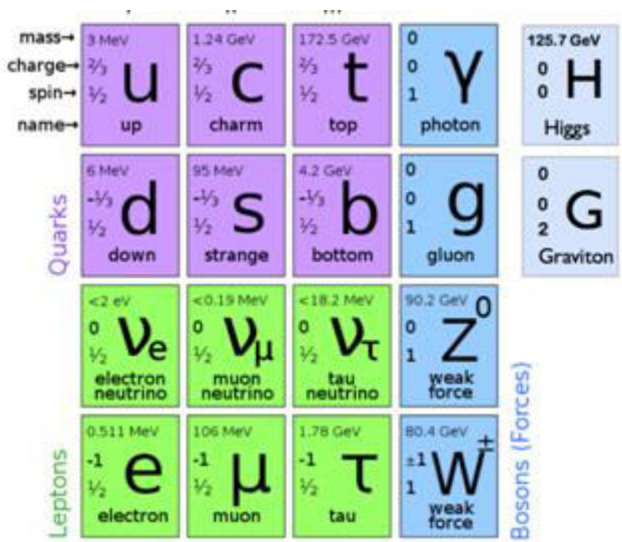


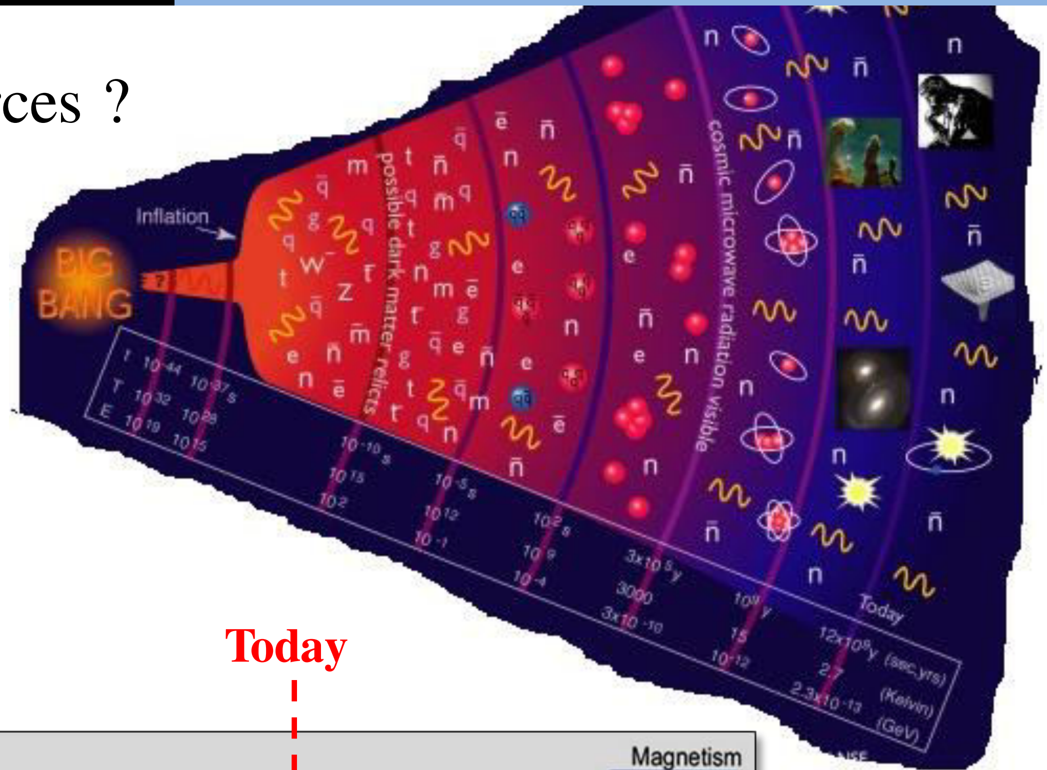


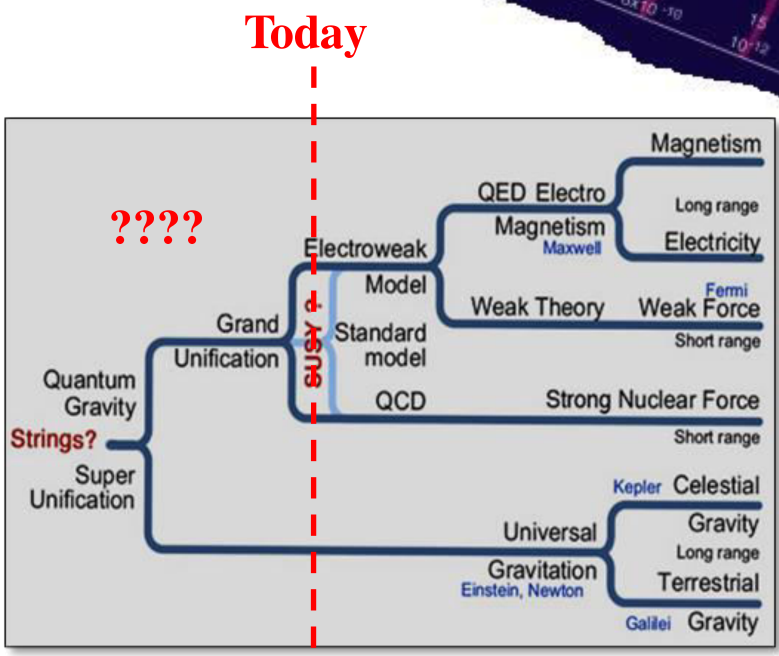
More energy we have, more in « the past » we go….

Toward an unification of forces ?

Current state of achievment
Standard model: with Higgs Boson that would be responsible of mass…

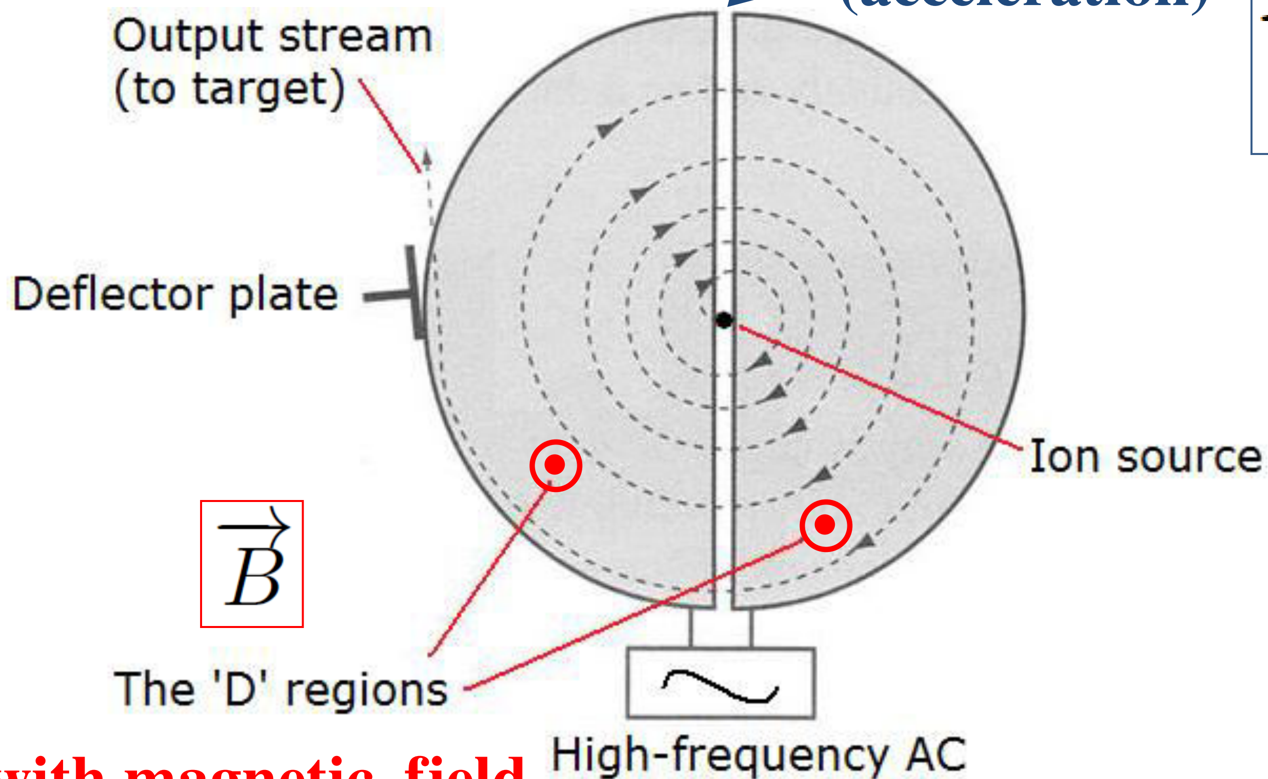






Cyclotron (1930):

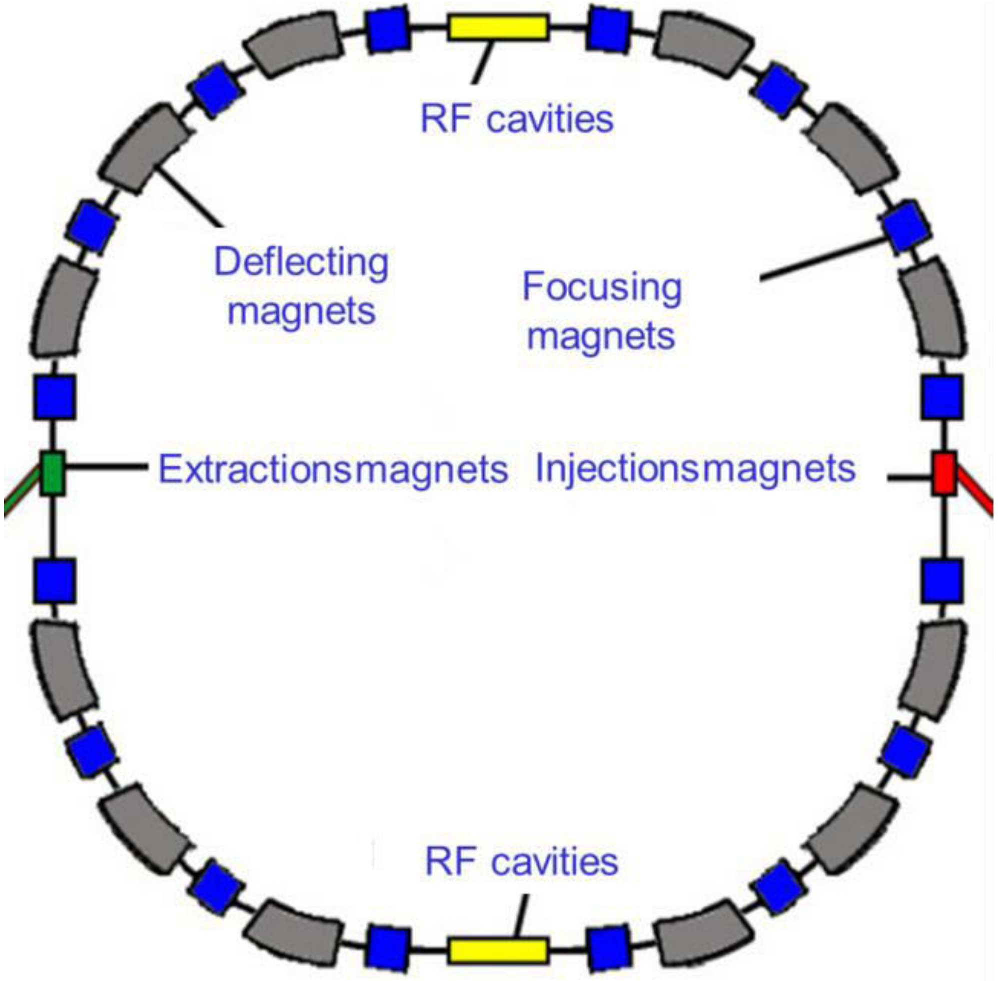
Zone with electric field (acceleration) $\overrightarrow{E}$



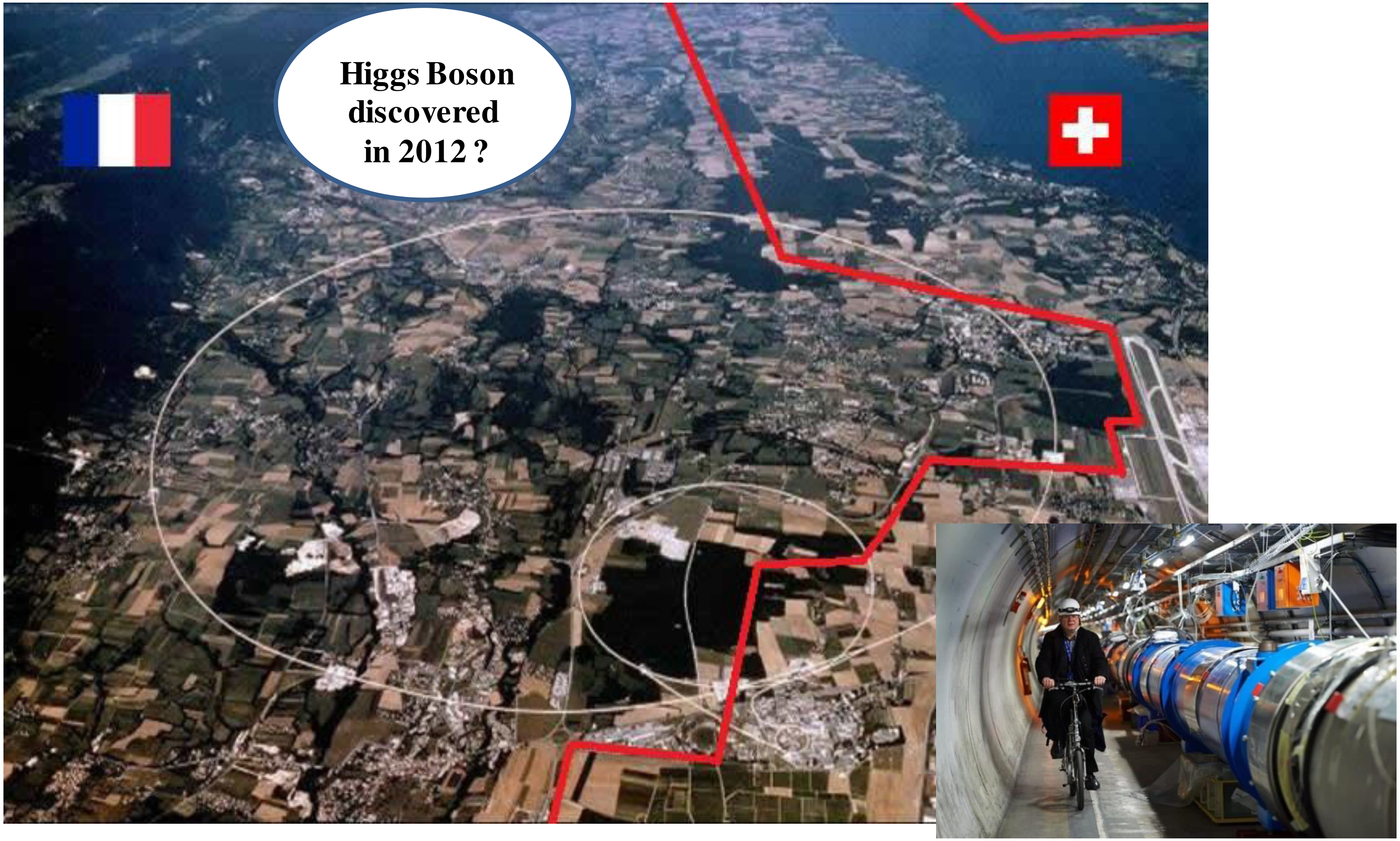
$\overrightarrow{B}$

Zone with magnetic field (deviation)

Circular accelerator :



Large Hadron Collider: CERN: 27 km of circumference



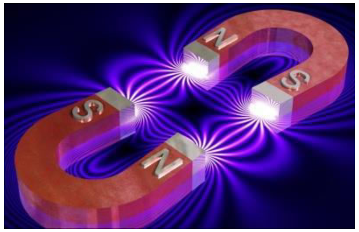
Physics-L2 Electromagnetism
Approximative program

1) Sources –Fields -interactions
2) Fundamentals of magnetism

Chap 1: Electrostatics
Chap 2: Magnetostatics
Chap 3: Time-dependent regime-Induction phenomena
Chap 4: Maxwell equations
Chap 5: Dielectric media and applications
Chap 6: Conducting media and applications
Chap 7: Magnetic media and applications

week	**Magistral lectures**
1	Electrostatics
2	Electrostatics
3	Electrostatics
4	Electrostatics
5	**Magnetostatics**
6	**Magnetostatics**
7	Induction
8	Induction
9	Maxwell equations
10	Maxwell equations
11	Dielectric media
12	Dielectric / Metallic media
13	Metallic Media
14	Magnetic media

Magnetostatics-L2
B-Fundamentals of Magnetism



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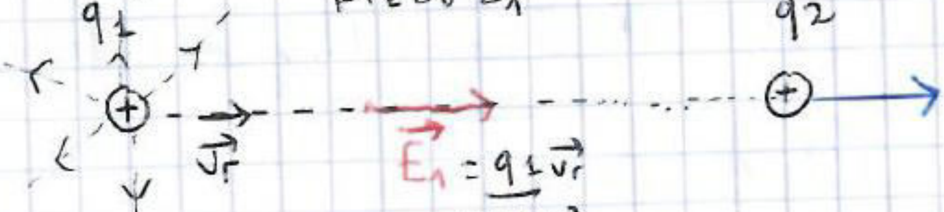
7) Interaction between magnetic dipole
8) Magnetic materials

ML Magnetostatics : Part 2 : Fundamentals

1) Some problematics

a) in Electrostatics

charge 1 — FIELD $\vec{E}_1$ — charge 2 feels a force



$$\vec{E}_1 = \frac{q_1 \vec{u}_r}{4\pi\varepsilon_0 r^2}$$

Coulomb

$$\vec{F} = q_2 \vec{E}_1 = \frac{q_2 q_1}{4\pi\varepsilon_0 r^2} \vec{u}_r$$

b) Magnetostatics

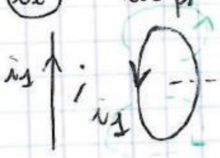
(i) Magnet 1   Magnet 2

concept of field magnetic

or magnetic needle

attraction but what about $\vec{F}_{orce}$?

(ii) loop / wire



$\vec{B}_1$

$$\vec{B}_1 = \int \frac{\mu_0 i \, \vec{dl} \times \vec{u_r}}{4\pi r^2}$$

Biot-Savart

act on the needle

what about $\vec{Force}$?

(iii) wire 1 loop 1 (current 1)   wire 2 loop 2 (current 2)

$\vec{B}_1, \vec{B}_2$ ... Biot-Savart.

Laplace force

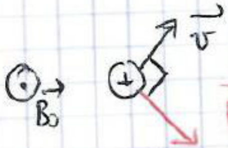
$$\vec{F}_{1\mapsto 2} = \int i_2 \vec{dl}_2 \wedge \vec{B}_1$$

$$\vec{F}_{2\mapsto 1} = \int i_1 \vec{dl}_1 \wedge \vec{B}_2$$

$$= \int i_1 \vec{dl}_1 \wedge \int \frac{\mu_0 i_2}{4\pi}$$

(iv) Any origin

$\left\{ \uparrow i \text{ or } \boxed{N|S} \right\}$ is a B_0



moving charge (current)

Lorentz force

$$\vec{F} = q\vec{v} \wedge \vec{B}$$

How to unify it?

→ current create magnetic field that can act on other current.

$\vec{j}_1 \longrightarrow \vec{B}_1$ that can act on $\vec{j}_2$ (Laplace or Lorentz force)

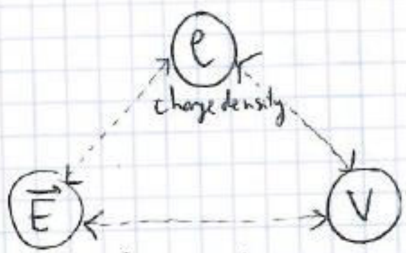
like $q_1 \longrightarrow \vec{E}_1$ act on q_2.

→ 2 problems: What about magnet $\boxed{N|S}$ that creates $\boxed{\vec{B}\ ?}$

and what about the $\boxed{\text{force?}}$ acting on $\boxed{N|S}$ or N|S

c) "Magnetic potential"?

Electrostatics

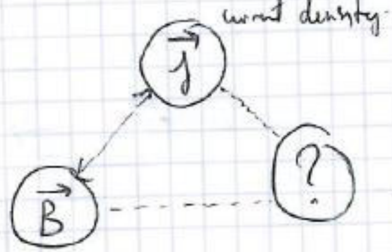


$$\vec{E} = -\vec{\text{grad}}\, V$$

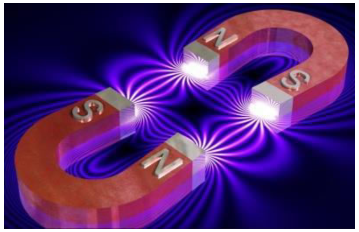
$$V = -\int \vec{E} \cdot \vec{dr}$$

Difference of potential V creates the electric field.

Magnetostatics



Magnetostatics-L2
B-Fundamentals of Magnetism



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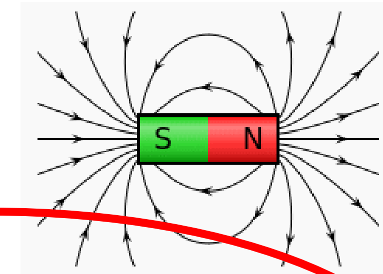
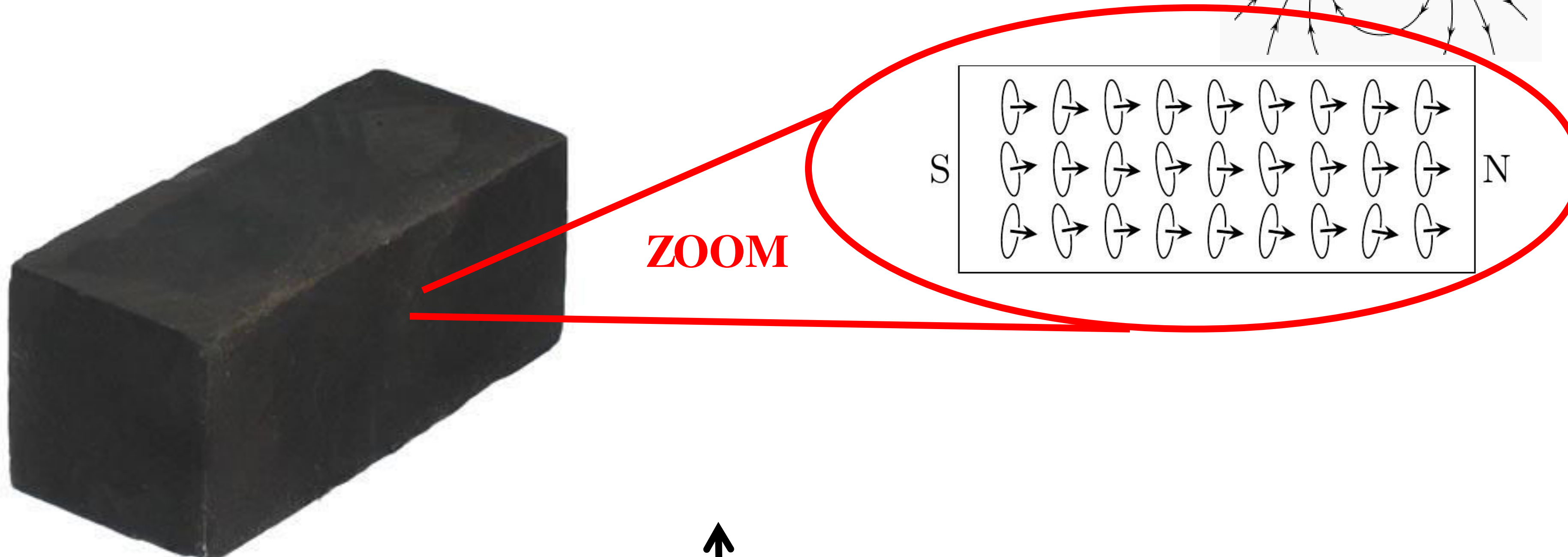
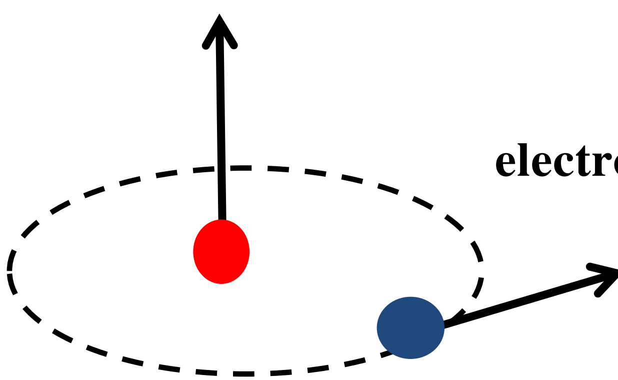
Inside the Magnet: *electronic currents*





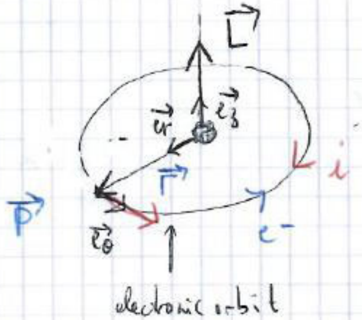
ATOMS have magnetic properties



electronic current

2- Magnetic moment - magnetic dipoles

(a) → Slides: ZOOM on matter made of ATOMS



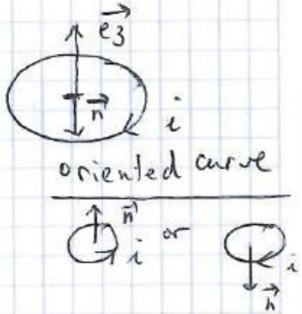
→ e^- has orbital momentum $\vec{L}$

$$\vec{L} = \vec{r} \wedge \vec{p} = \vec{r} \wedge m\vec{v}$$
$$= r\vec{e_r} \wedge m r \dot{\theta}\, \vec{e_\theta} = m r^2 \dot{\theta}\, \vec{e_z}$$
$$= \underline{m r^2 \omega\, \vec{e_z}}$$

→ electric current i

$$i = \frac{dq}{dt} = \frac{e}{T} = \frac{e\omega}{2\pi}.$$

$$\vec{L} = m r^2 \omega\, \vec{e_z} = m r^2 \frac{2\pi}{e} \cdot \frac{e}{2\pi} \omega\, \vec{e_z}$$
$$= 2\frac{m}{e} \cdot (\pi r^2)\, \vec{e_z} \cdot \frac{e\omega}{2\pi}$$
$$= \frac{2m}{e}\, S\, \vec{e_z}\, i$$
$$\vec{L} = -\frac{2m}{e} \underbrace{S \vec{n}\, i}_{\vec{m}}$$

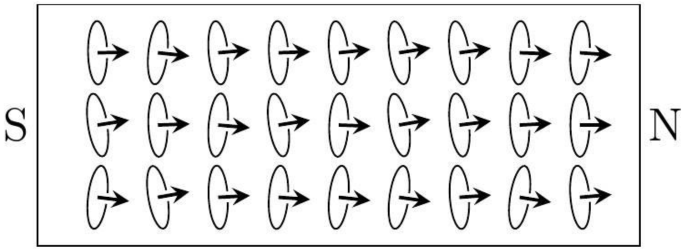


$$\boxed{\vec{m} = -\frac{e}{2m}\vec{L} \equiv i S \vec{n}}$$

magnetic {moment. / dipole. : loop of current is a magnetic dipole

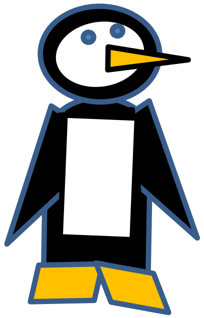
What is the difference between

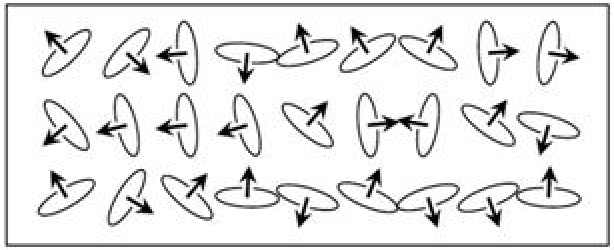
Magnetic materials



All magnetic moments are in the same direction

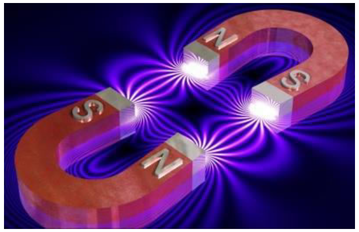
Non Magnetic materials ?





Magnetic moments are zero or in different directions so the total is zero

Magnetostatics-L2
B-Fundamentals of Magnetism



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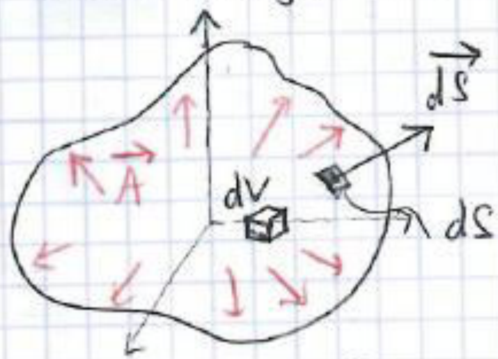
6) **Interaction between a magnetic dipole and a magnetic field**
7) **Interaction between magnetic dipole**
8) **Magnetic materials**

3) Need of geometry:

Remember from L1:

(i) Green-Ostrogradski

a closed surface = 1 volume



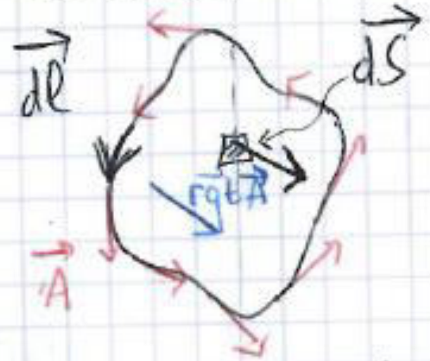
$$\oiint_S \vec{A} \cdot \vec{dS} = \iiint \operatorname{div} \vec{A} \, dV$$

$$\operatorname{div} \vec{A} = \vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

divergence in the internal volume

(ii) Stocks.

a closed curve = a surface



$$\oint_{\mathcal{C}} \vec{A}\cdot\vec{dl} = \iint_S \vec{rot}\,\vec{A}\cdot\vec{dS}$$

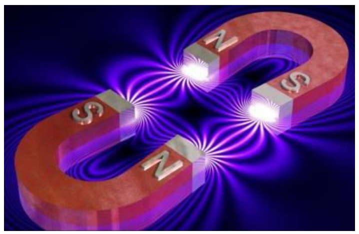
circulation of $\vec{A}$ along the closed curve $\Longleftrightarrow$ flux of a quantity $(\vec{\nabla}\times\vec{A})$ through the surface defined by the closed curve

$$\vec{rot}\,\vec{A} = \vec{\nabla}\times\vec{A} = \vec{curl}\,\vec{A}$$

$$= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{e_x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{e_y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{e_z}.$$

$\left(\vec{rot}\,\vec{A} \perp \vec{A}\right)$

Magnetostatics-L2
B-Fundamentals of Magnetism



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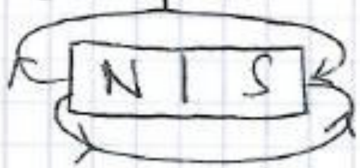
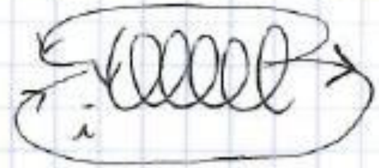
8) Magnetic materials

4) Vector potential.

* Magnetic properties: magnetic field has conservative flux:



or



$$\oiint \vec{B} \cdot \vec{dS} = 0$$

green

$$\Leftrightarrow \iiint \mathrm{div}\, \vec{B} \, d\tau = 0$$

$$\Leftrightarrow \boxed{\mathrm{div}\, \vec{B} = 0} \qquad (\vec{\nabla} \cdot \vec{B} = 0)$$

- NO divergence of the magnetic field contrary to electric field in electrostatics
- NO magnetic $\begin{cases} \text{charge;} \\ \text{monopole} \end{cases}$

But it exist a mathematical property: $\operatorname{div}(\vec{rot}\,\vec{A}) = 0 \quad \forall \vec{A}$

Be $\vec{\nabla} \times \vec{A}$ a vector $\Longrightarrow \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$

$$\frac{\partial}{\partial x}(\vec{\nabla} \times \vec{A})_x + \frac{\partial}{\partial y}(\vec{\nabla} \times \vec{A})_y + \frac{\partial}{\partial z}(\vec{\nabla} \wedge \vec{A})_z$$

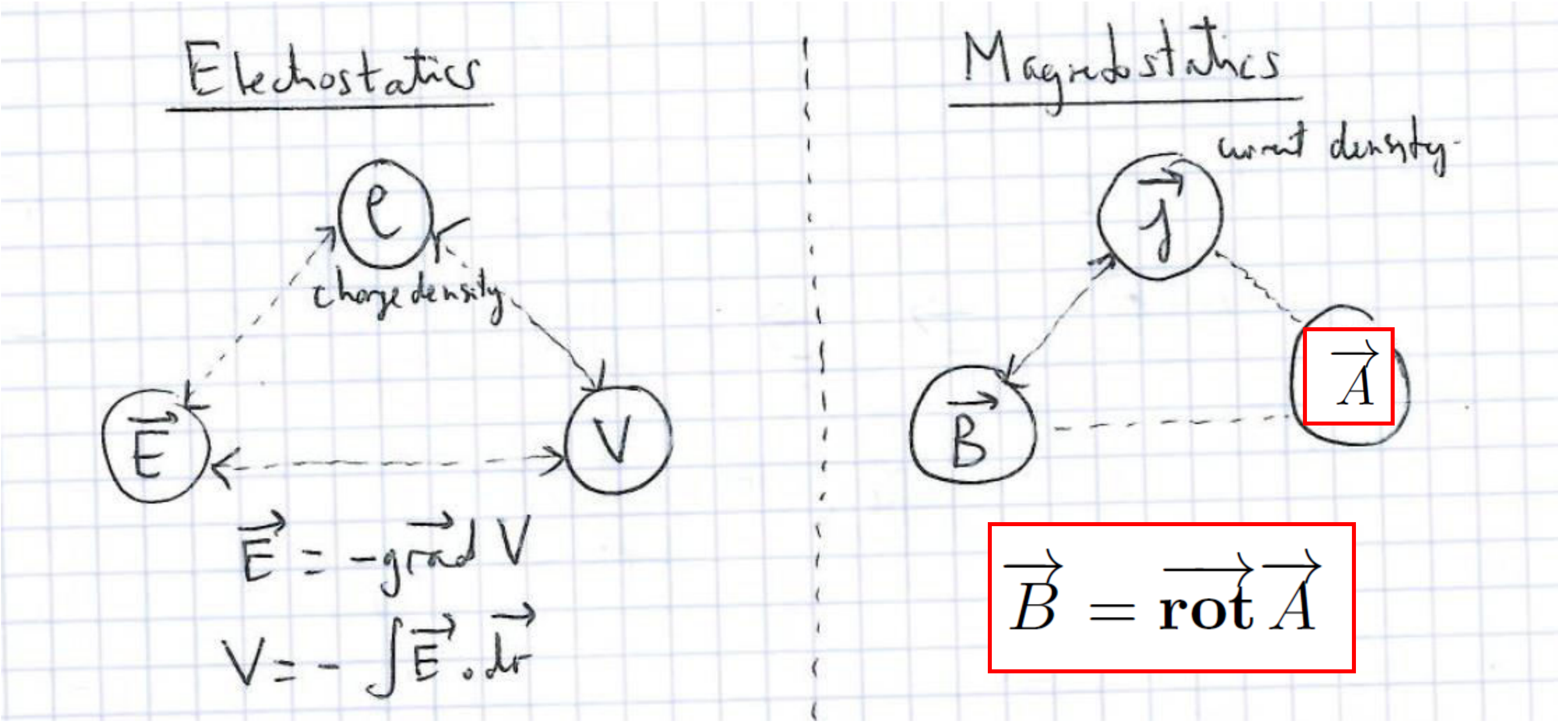
$$\frac{\partial}{\partial x}\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right] + \frac{\partial}{\partial y}\left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right] + \frac{\partial}{\partial z}\left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right] =$$

$$\cancel{\frac{\partial^2 A_z}{\partial x \partial y}} - \cancel{\dots X} + \cancel{\dots} - \cancel{\frac{\partial^2 A_z}{\partial y \partial x}} + \cancel{\dots X} - \cancel{\dots} = 0$$

if $\operatorname{div} \vec{B} = 0 \iff \exists$ 1 vector $\vec{A}$ such

$$\boxed{\vec{B} = \vec{rot}\,\vec{A}}$$

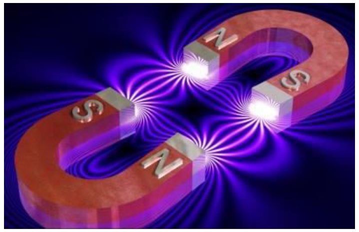
hello vector potential !!



$$V = \iiint \frac{\rho d\tau}{4\pi\epsilon_0 r}$$

$$\vec{A} = \iiint \frac{\mu_0}{4\pi} \frac{\vec{j}\, d\tau}{r}$$

Magnetostatics-L2
B-Fundamentals of Magnetism



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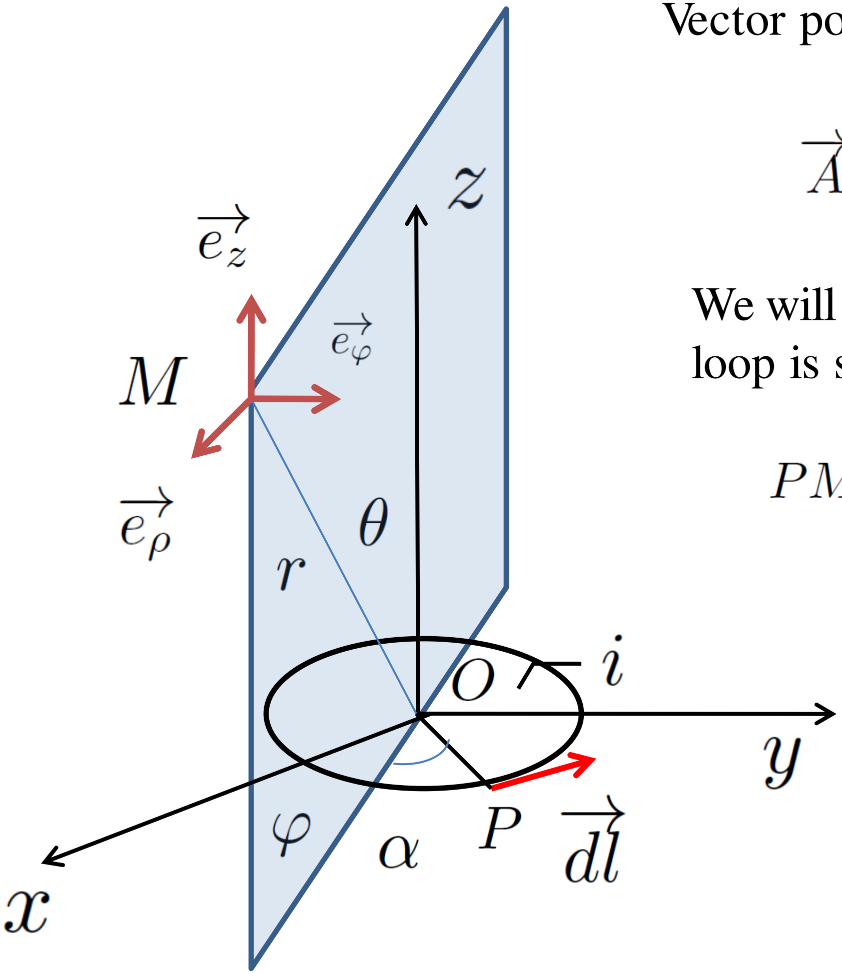


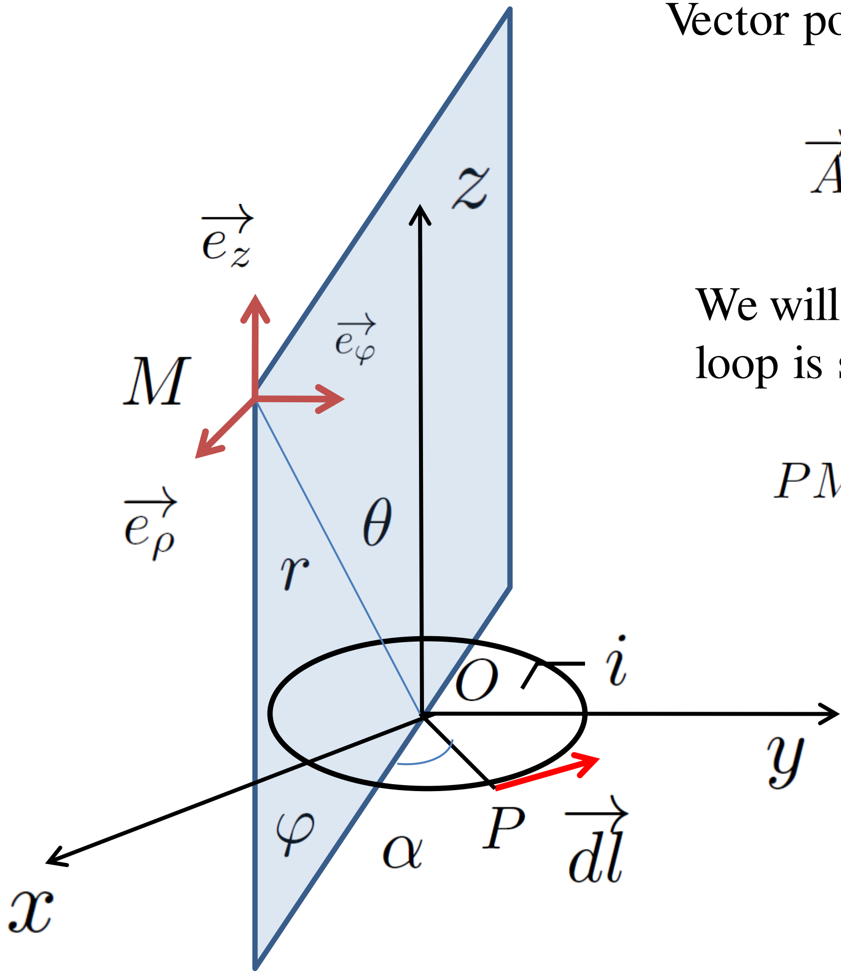
Vector potential created by the lineic loop at point M (r=OM)

$$\overrightarrow{A} = \frac{\mu_0 i}{4\pi} \oint_C \frac{\overrightarrow{dl}}{PM}$$

We will work in the dipolar approximation where R radius of loop is small compared distance r=OM; R << r.

$$PM = \left\| \overrightarrow{OM} - \overrightarrow{OP} \right\| = (r^2 + R^2 - 2\overrightarrow{OM} \cdot \overrightarrow{OP})^{1/2}$$

Vector potential and magnetic field in the dipolar approximation



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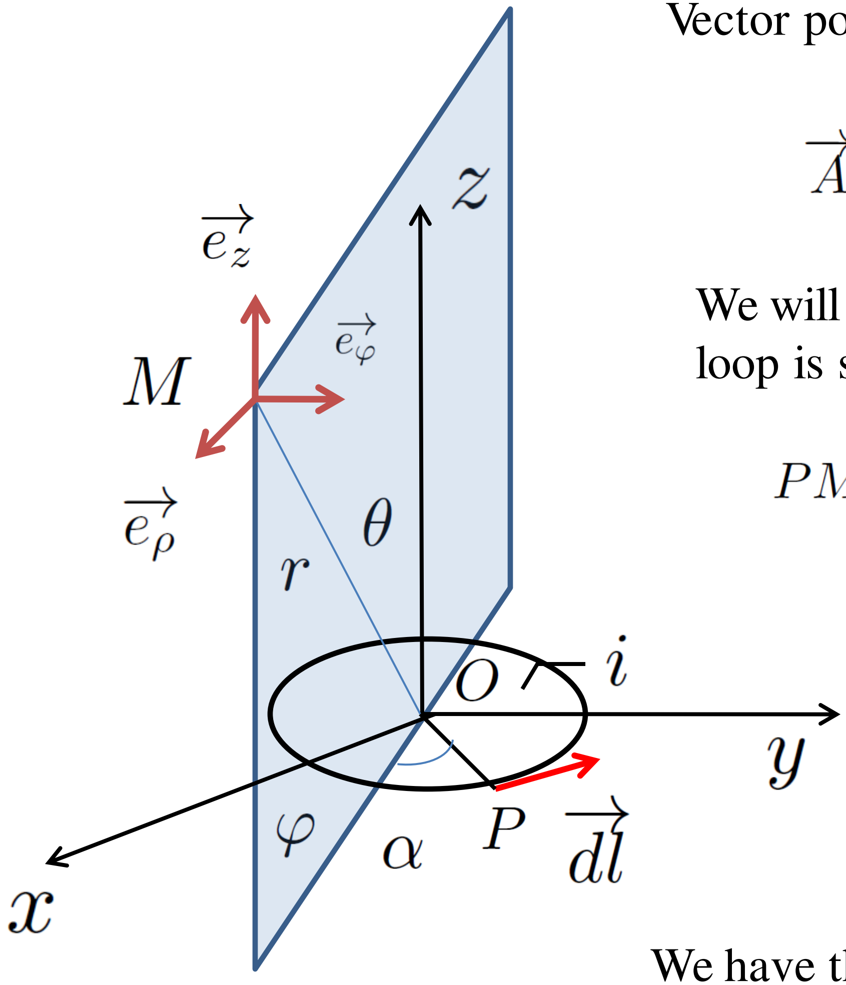
And PM is approximated with Taylor Dev to:

$$\frac{1}{PM} = \frac{1}{r}\left(1 + \frac{R^2}{r^2} - \frac{2\overrightarrow{OM} \cdot \overrightarrow{OP}}{r^2}\right)^{-1/2}$$

$$\approx \frac{1}{r} + \frac{\overrightarrow{e_r} \cdot \overrightarrow{OP}}{r^2}$$

Vector potential and magnetic field in the dipolar approximation



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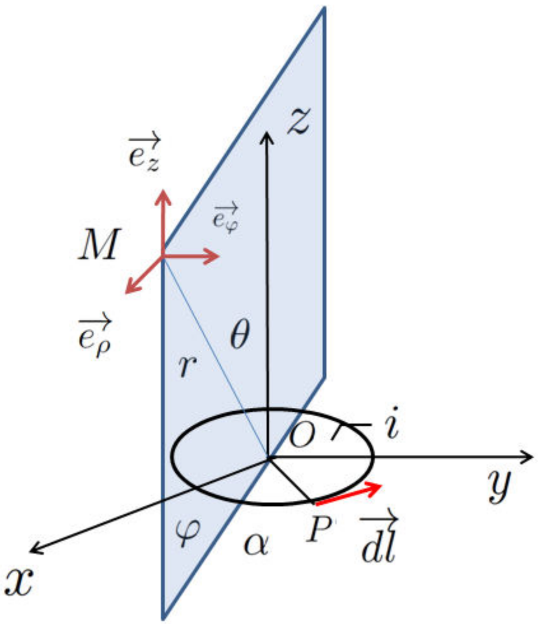
$$\approx \frac{1}{r} + \frac{\overrightarrow{e_r} \cdot \overrightarrow{OP}}{r^2}$$

We have the sum of two terms

$$\overrightarrow{A} = \frac{\mu_0 i}{4\pi}\left[\frac{1}{r}\oint_C \overrightarrow{dl} + \frac{1}{r^2}\oint_C \left(\overrightarrow{e_r} \cdot \overrightarrow{OP}\right)\overrightarrow{dl}\right]$$

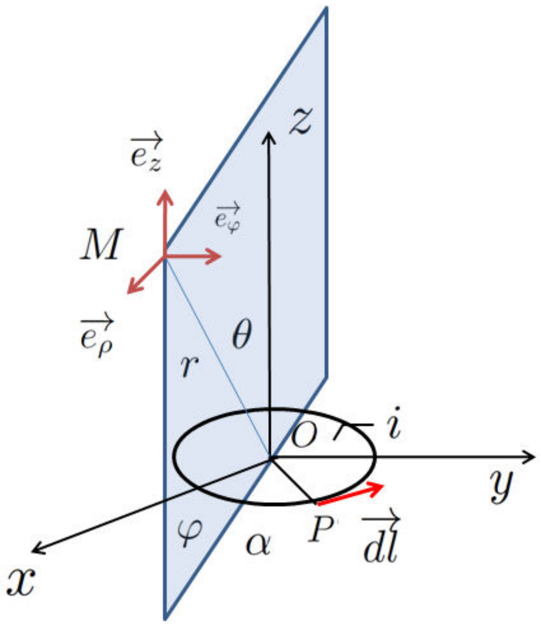


How to continue calculations

$$\overrightarrow{A} = \frac{\mu_0 i}{4\pi}\left[\frac{1}{r}\oint_C \overrightarrow{dl} + \frac{1}{r^2}\oint_C \left(\overrightarrow{e_r}\cdot\overrightarrow{OP}\right)\overrightarrow{dl}\right]$$

First integral is zero

We calculate terms of the second integral in the moving basis $(\overrightarrow{e_\rho}, \overrightarrow{e_\varphi}, \overrightarrow{e_z})$



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First integral is zero

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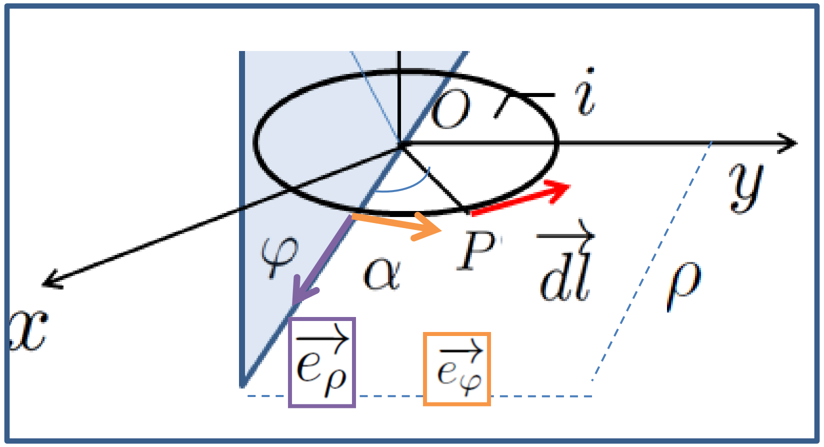
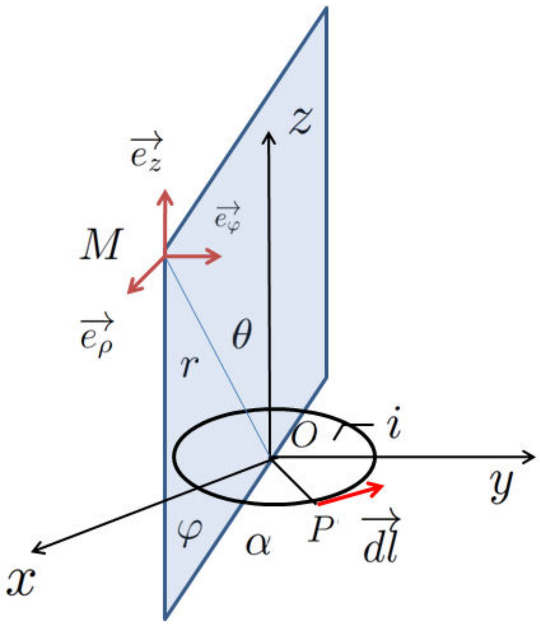
We define $\rho = r\sin\theta$ the orthogonal distance of M to Oz axis.
$\mathbf{e_r}$ is the unit vector along OM. Vector coordinates are:

ZOOM



$$\overrightarrow{e_r} = \left| \begin{matrix} \rho/r \\ 0 \\ z/r \end{matrix} \right. \qquad \overrightarrow{OP} = \left| \begin{matrix} R\cos\alpha \\ R\sin\alpha \\ 0 \end{matrix} \right. \qquad \overrightarrow{dl} = \left| \begin{matrix} -R\sin\alpha d\alpha \\ R\cos\alpha d\alpha \\ 0 \end{matrix} \right.$$

Vector potential and magnetic field in the dipolar approximation



How to continue calculations

$$\vec{A} = \frac{\mu_0 i}{4\pi}\left[\frac{1}{r}\oint_C \vec{dl} + \frac{1}{r^2}\oint_C \left(\vec{e_r}\cdot\vec{OP}\right)\vec{dl}\right]$$

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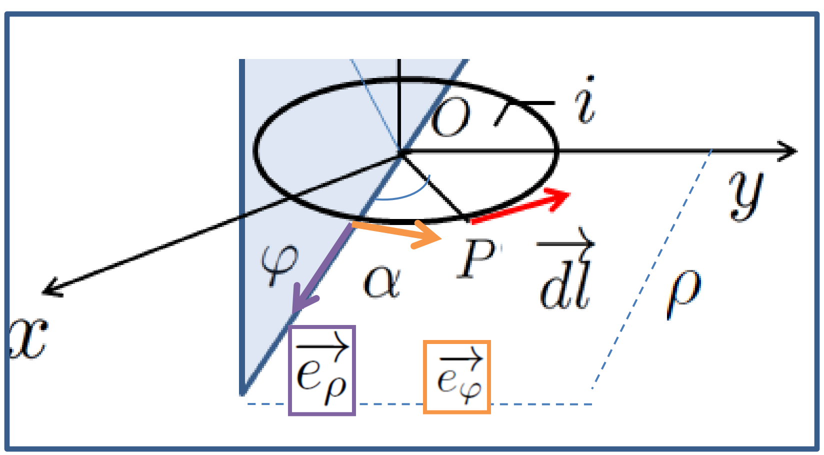


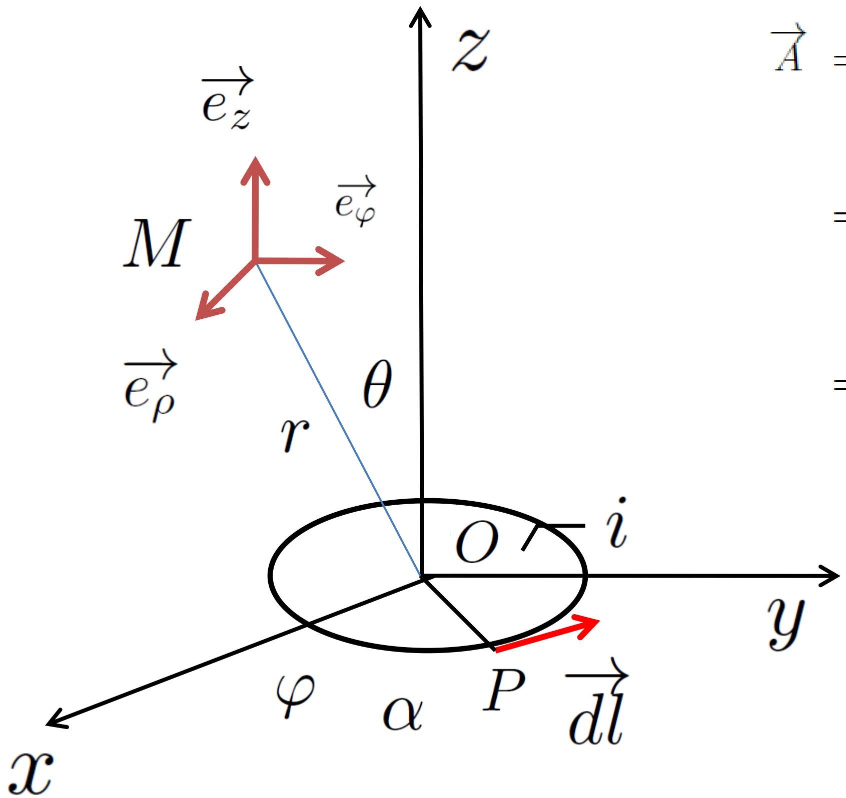
$$\vec{e_r} = \left|\begin{matrix}\rho/r\\ 0\\ z/r\end{matrix}\right. \qquad \vec{OP} = \left|\begin{matrix}R\cos\alpha\\ R\sin\alpha\\ 0\end{matrix}\right. \qquad \vec{dl} = \left|\begin{matrix}-R\sin\alpha d\alpha\\ R\cos\alpha d\alpha\\ 0\end{matrix}\right.$$

Finaly the scalar product :

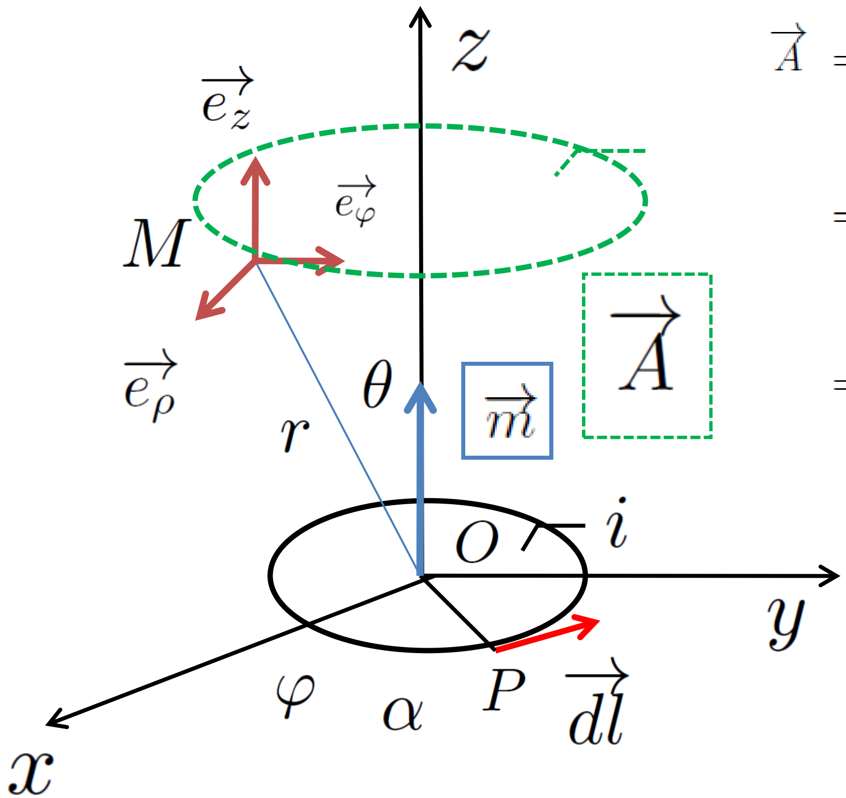
$$\vec{e_r}\cdot\vec{OP} = \frac{\rho}{r}R\cos\alpha$$

The vector potential reads

with $\overrightarrow{dl} = \left| \begin{array}{c} -R\sin\alpha d\alpha \\ R\cos\alpha d\alpha \\ 0 \end{array} \right.$



$$\begin{aligned}
\overrightarrow{A} &= \frac{\mu_0}{4\pi}\frac{iR^2\rho}{r^2}\oint_C \cos\alpha\,\overrightarrow{dl} \\
&= \frac{\mu_0}{4\pi}\frac{iR^2\rho}{r^3}\left(-\overrightarrow{e_\rho}\underbrace{\int_0^{2\pi}\cos\alpha\sin\alpha d\alpha}_{0} + \overrightarrow{e_\varphi}\underbrace{\int_0^{2\pi}\cos^2\alpha d\alpha}_{\pi}\right) \\
&= \frac{\mu_0}{4\pi}\frac{iR^2\rho}{r^3}\pi\overrightarrow{e_\varphi} = \frac{\mu_0}{4\pi}\frac{i\pi R^2\sin\theta}{r^2}
\end{aligned}$$



The vector potential reads

with $\overrightarrow{dl} = \begin{vmatrix} -R\sin\alpha d\alpha \\ R\cos\alpha d\alpha \\ 0 \end{vmatrix}$

$$\overrightarrow{A} = \frac{\mu_0}{4\pi}\frac{iR^2\rho}{r^2}\oint_C \cos\alpha \overrightarrow{dl}$$

$$= \frac{\mu_0}{4\pi}\frac{iR^2\rho}{r^3}\left(-\overrightarrow{e_\rho}\underbrace{\int_0^{2\pi}\cos\alpha\sin\alpha d\alpha}_{0} + \overrightarrow{e_\varphi}\underbrace{\int_0^{2\pi}\cos^2\alpha d\alpha}_{\pi}\right)$$

$$= \frac{\mu_0}{4\pi}\frac{iR^2\rho}{r^3}\pi\overrightarrow{e_\varphi} = \frac{\mu_0}{4\pi}\frac{i\pi R^2\sin\theta}{r^2}$$

We can recognize $\overrightarrow{e_z} \wedge \overrightarrow{e_r} = \sin\theta\overrightarrow{e_\varphi}$

And the **magnetic dipole** $\overrightarrow{m} = i\pi R^2\overrightarrow{e_z} = iS\overrightarrow{e_z}$

$$\boxed{\overrightarrow{A} = \frac{\mu_0}{4\pi}\frac{\overrightarrow{m} \wedge \overrightarrow{e_r}}{r^2}}$$

Vector potential and magnetic field in the dipolar approximation

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{m}\wedge\vec{e_r}}{r^2}$$

Calculation of magnetic field is done with $\vec{B} = \overrightarrow{\mathbf{rot}}\,\vec{A}$

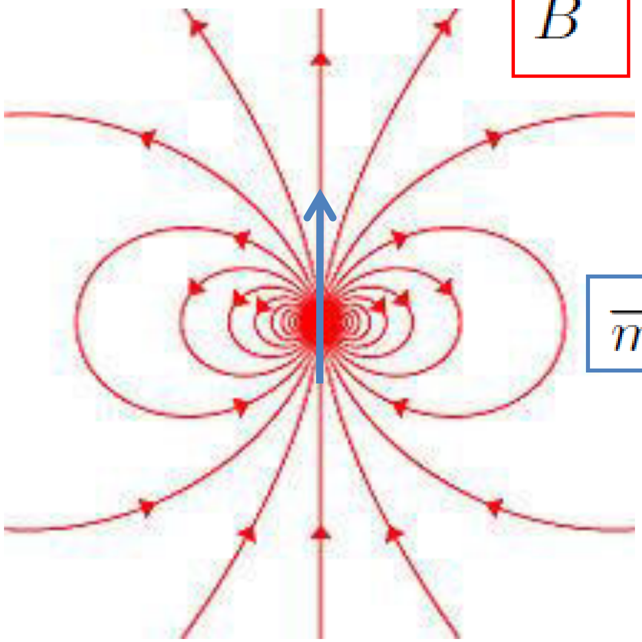
$$\vec{B} = \frac{\mu_0}{4\pi}\overrightarrow{\mathbf{rot}}\left(\frac{\vec{m}\wedge\vec{e_r}}{r^2}\right)$$

Any vector A (not potentiel vector in the below formula)

$$\mathrm{rot}\left(f\vec{A}\right) = f\,\mathrm{rot}\,\vec{A} + \overrightarrow{\mathrm{grad}}\,f \times \vec{A}$$

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{3(\vec{m}\cdot\vec{e_r})\vec{e_r} - \vec{m}}{r^3}\right)}$$

$\boxed{\vec{B}}$



$\boxed{\vec{m}}$

Magnetic field lines created by a magnetic dipole in the dipolar approximation R<<r

Analogy between electrostatic dipole and magnetic dipole in the dipolar approximation

$$V = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{e_r}}{r^2}$$

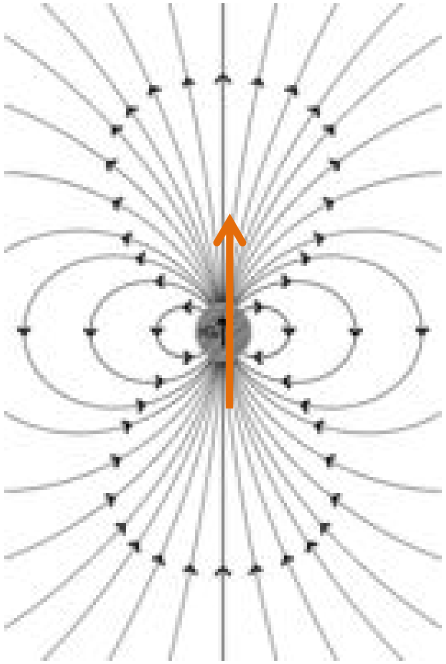
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left(\frac{3(\vec{p}\cdot\vec{e_r})\vec{e_r}-\vec{p}}{r^3}\right)$$

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{m}\wedge\vec{e_r}}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{3(\vec{m}\cdot\vec{e_r})\vec{e_r}-\vec{m}}{r^3}\right)$$

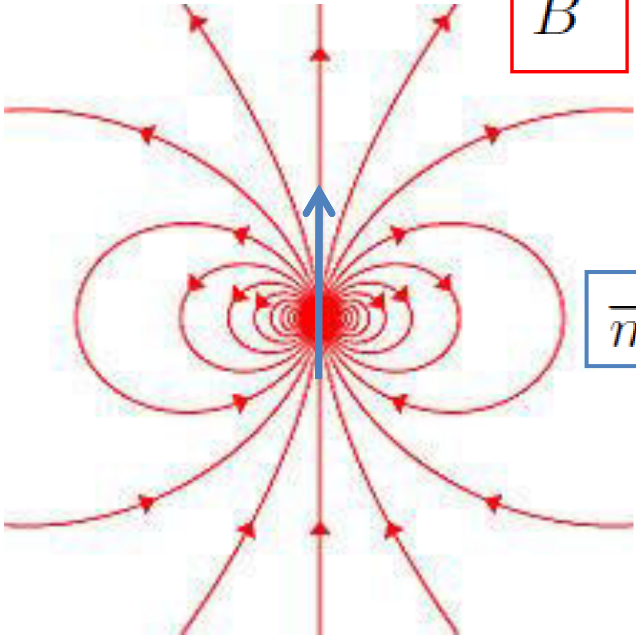
$\boxed{\vec{E}}$

$\boxed{\vec{p}}$

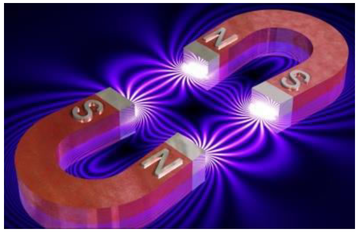


$\boxed{\vec{B}}$

$\boxed{\vec{m}}$



Magnetostatics-L2
B-Fundamentals of Magnetism



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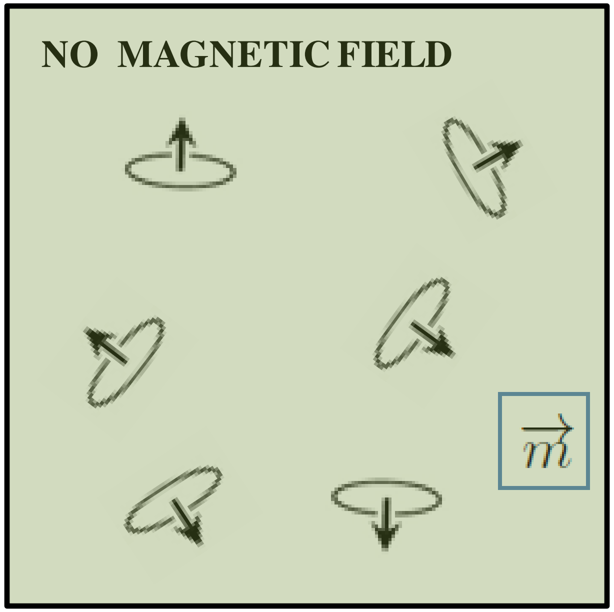
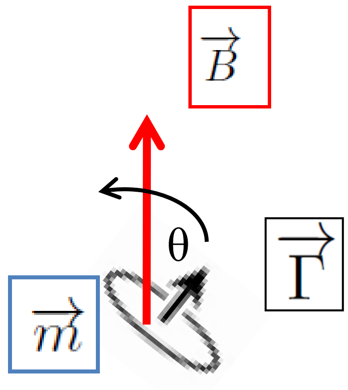
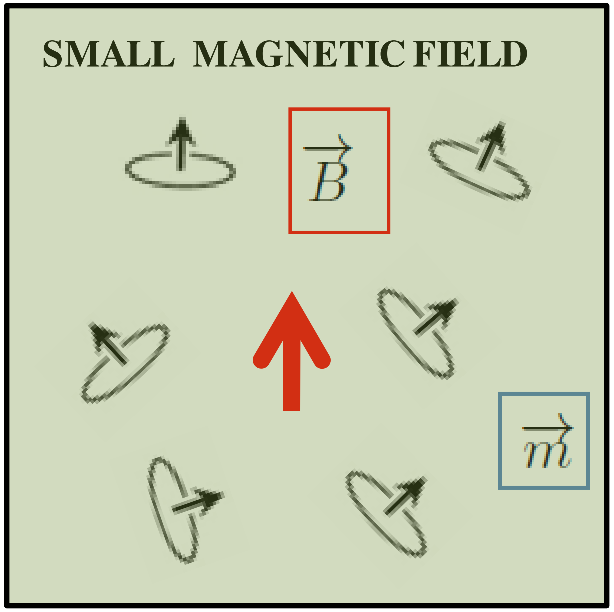
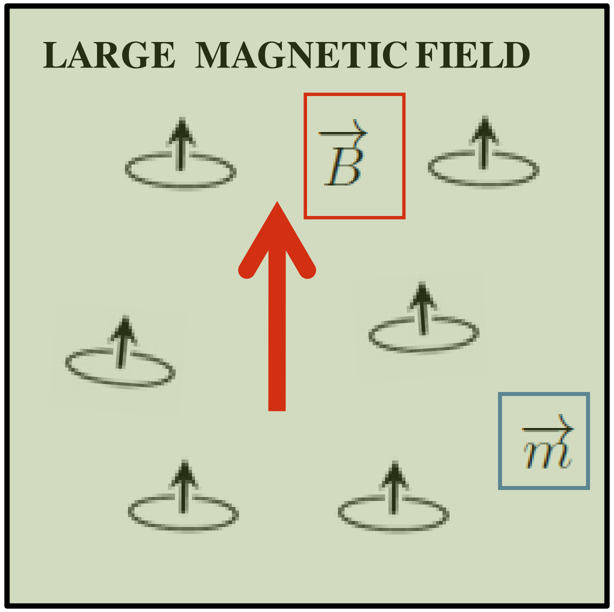
Interaction between a magnetic dipole and a magnetic field





U = -μ B cos θ





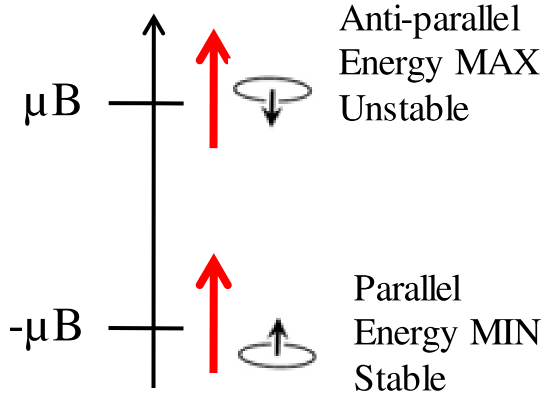
Dipole rotate in the direction of the field

Torque of a force

$$\vec{\Gamma} = \vec{m} \wedge \vec{B}$$

$$U = -\vec{m} \cdot \vec{B}$$

Energy:



Fast communication about Larmor precession:

- Be a loop of current associated to a magnetic dipole $\vec{m}$
- Orbital momentum $\vec{L} = \vec{r} \times \vec{p}$ and $\vec{m} = i S \vec{n}$
  with $\vec{m} = -\frac{e}{2m} \vec{L}$ (see Part B. 2).
- Magnetic field $\vec{B} = B \vec{e_3}$,
- Theorem of orbital momentum for $\vec{L}$:
  - $\frac{d\vec{L}}{dt} = \vec{\Gamma} = \overrightarrow{\text{Torque}} = \vec{m} \times \vec{B}$
  - With $\vec{m} = -\frac{e}{2m} \vec{L} = \gamma \vec{L}$

    γ giro magnetic ratio:

$$(1) \quad \boxed{\frac{d\vec{L}}{dt} = \gamma \vec{L} \times \vec{B}} \boxed{= \gamma B \vec{L} \times \vec{e_3}}$$

$\vec{B}$

motion equation for $\vec{L}$

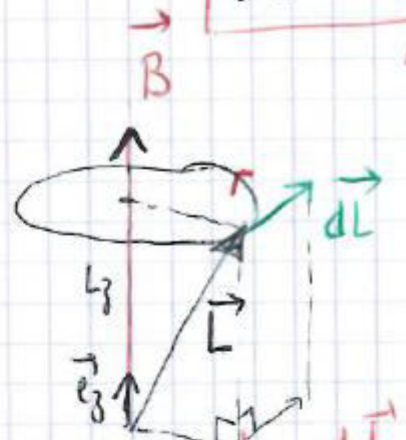
γB: Larmor pulsation

$\boxed{\omega_L = \gamma B}$

$$(1)\quad \boxed{\frac{d\vec{L}}{dt} = \gamma \vec{L} \times \vec{B} = \gamma B\, \vec{L} \times \vec{e_3}}$$

motion equation for $\vec{L}$

γB: Larmor pulsation $\boxed{\omega_L = \gamma B}$



precession motion of $\vec{L}$ about $\vec{B}$

$-\vec{L} \times \vec{e_3}$ gives: direction $\vec{dL}$

From (1): i) $\vec{L} \cdot \frac{d\vec{L}}{dt} = \frac{1}{2}\frac{d\vec{L}^2}{dt} = (\gamma \vec{L} \times \vec{B}) \cdot \vec{L} = \vec{0}$

$\vec{L} = L\vec{e_3} + \vec{L}_\perp$

$L^2 = \text{constant}$

ii) $\vec{e_3} \cdot \frac{d\vec{L}}{dt} = \dot{L_3} = (\gamma B \vec{L} \times \vec{e_3}) \cdot \vec{e_3} = \vec{0}$

$L_3 = \text{constante}$

$\|\vec{L}\| = \text{constant}$ and $L_3 = \text{cte}.$

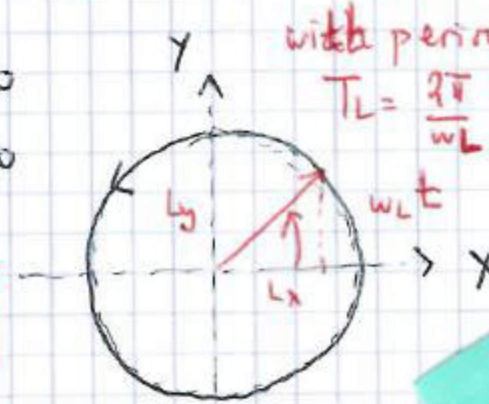
Equation (1) gives also:

$$\begin{cases} \dot{L_x} = \gamma B L_y \\ \dot{L_y} = -\gamma B L_x \\ \dot{L_3} = 0 \rightarrow L_3 = \text{cte} \end{cases} \rightarrow \begin{cases} \ddot{L_x} + (\gamma B)^2 L_x = 0 \\ \ddot{L_y} + (\gamma B)^2 L_y = 0 \end{cases}$$

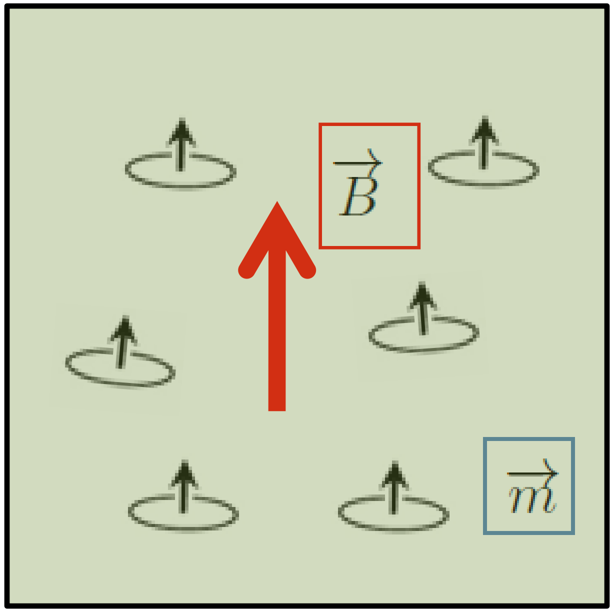
$\vec{L}_\perp = L_x \vec{e_x} + L_y \vec{e_y}$

With . IC

$$\boxed{\begin{aligned} L_x &= A\cos(\omega_L t) \\ L_y &= A\sin(\omega_L t) \end{aligned}}$$



Rotation of $\vec{L_\perp}$ with period $T_L = \frac{2\pi}{\omega_L}$



MAGNETIZATION VECTOR is volumic contribution of all magnetic dipoles:

$$\overrightarrow{M} = \frac{1}{V}\sum_{i=1}^{N}\overrightarrow{m_i}$$

Its amplitude is proportional to the applied magnetic field

$$\overrightarrow{M} = \mu_0 \chi_m \overrightarrow{B}$$

Magnetic succeptibility: χ_m depends on material

Important topic in Physics:

The Entropy /Potential Competition

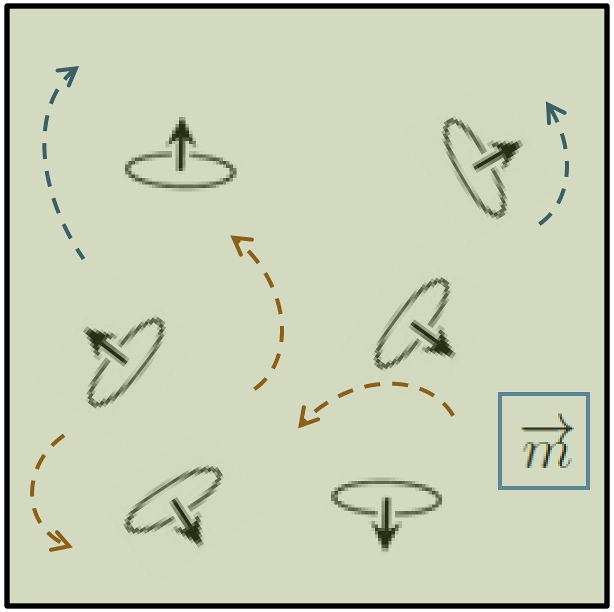
REMEMBER !!!

Some physical effects are an illustration of the competition between two type of phenomena

The ones originating from a **potential energy** that will induces forces and an **ordered behaver** (eletric, magnetic or gravitational ordering)

The ones originating from **the entropy** traducing a **random and chaotic situation** leading to a **disorderd behaver** (like thermal agitation or diffusion phenomena.

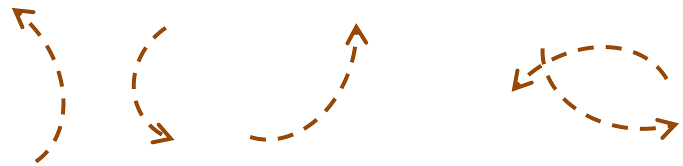
Without considering Phase transition, one can have a competition between:

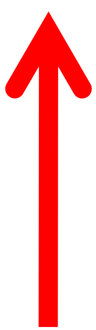


Ordering imposed by the magnetic field:
It wants to impose a direction to the dipoles

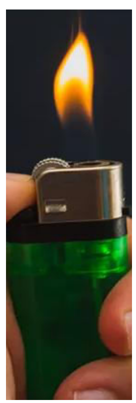
$$U = -\overrightarrow{m} \cdot \overrightarrow{B}$$

Disordering due to thermal agitation: it gives a random orientation to the dipole.





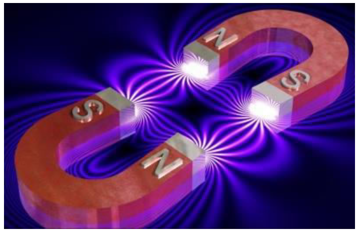
VS



In statistical physics we measure the proability for the dipole to have the energy U with the Boltzmann factor

$$P \approx e^{-\frac{U}{kT}}$$

Magnetostatics-L2
B-Fundamentals of Magnetism



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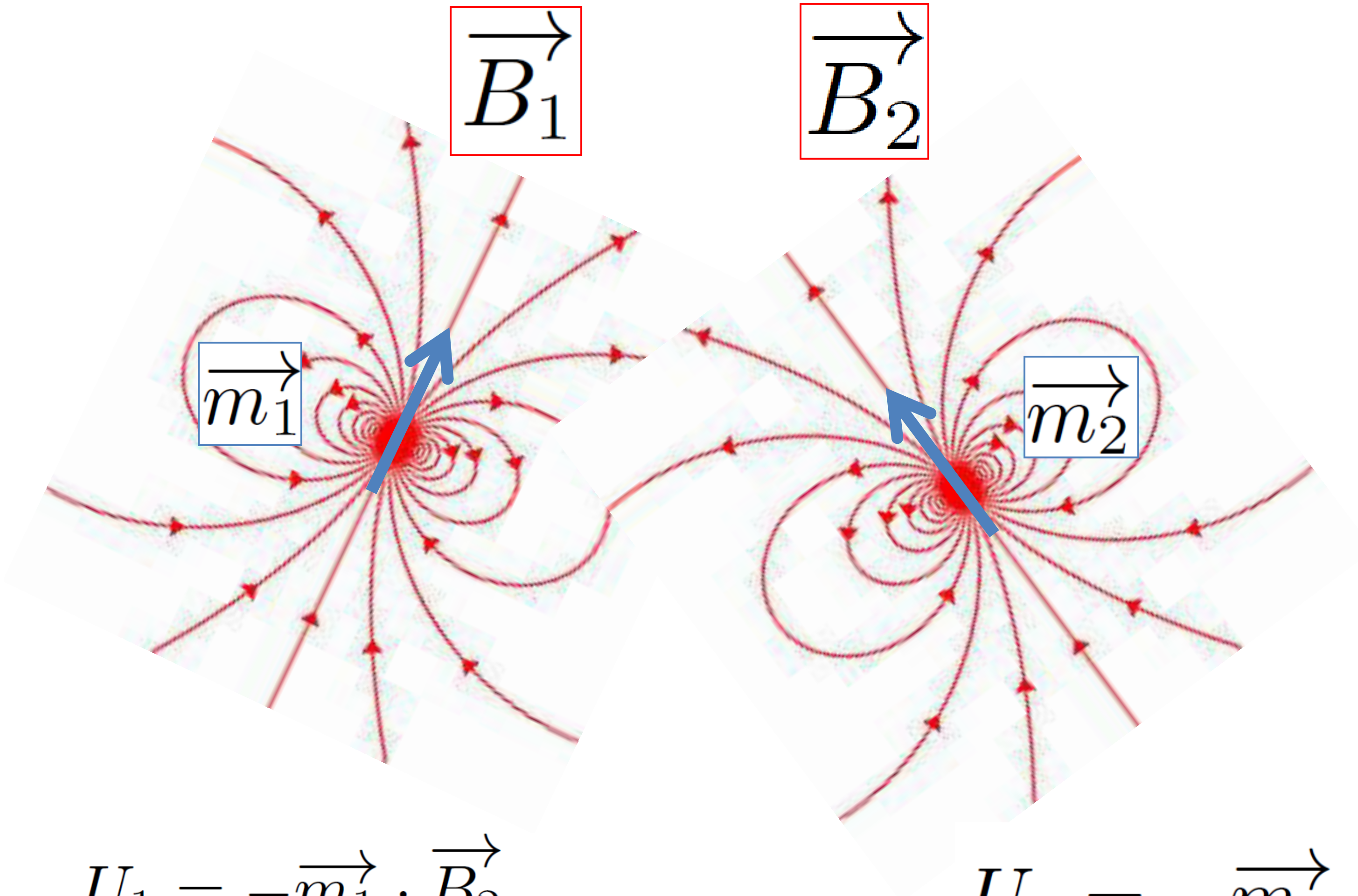
8) Magnetic materials

Interaction between dipoles



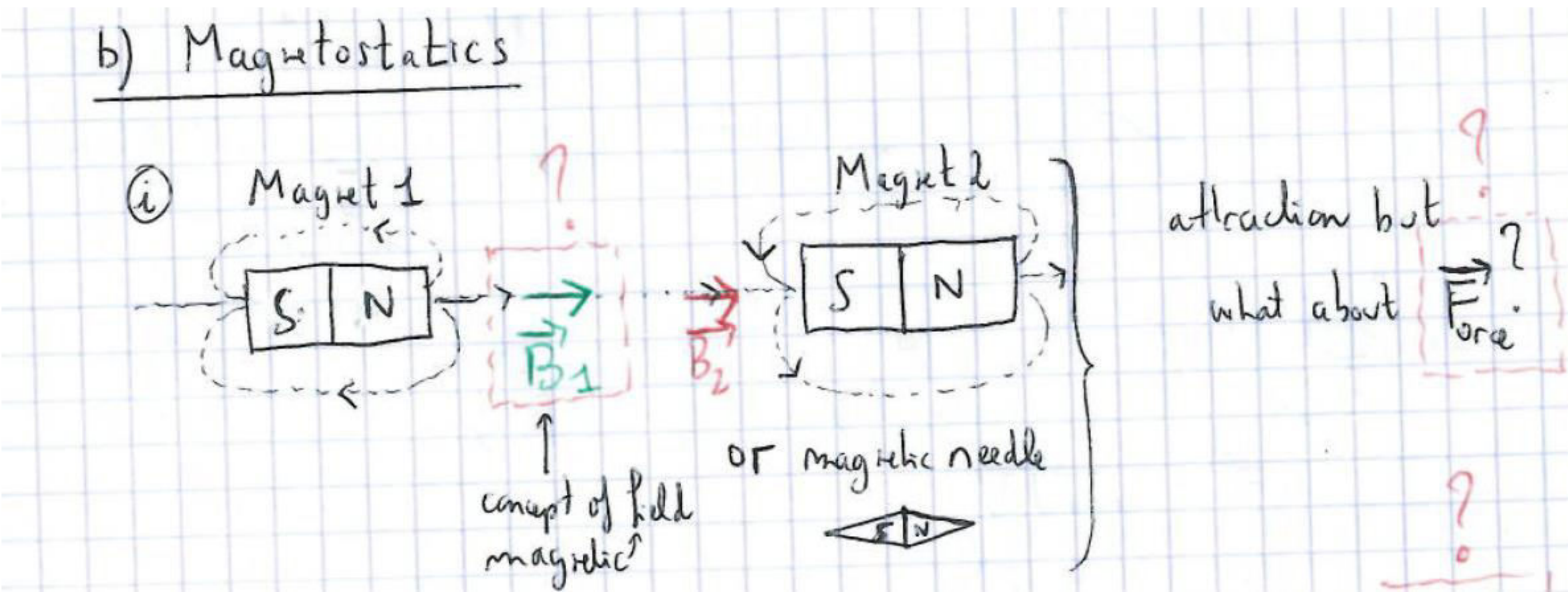
$$U_1 = -\overrightarrow{m_1} \cdot \overrightarrow{B_2}$$

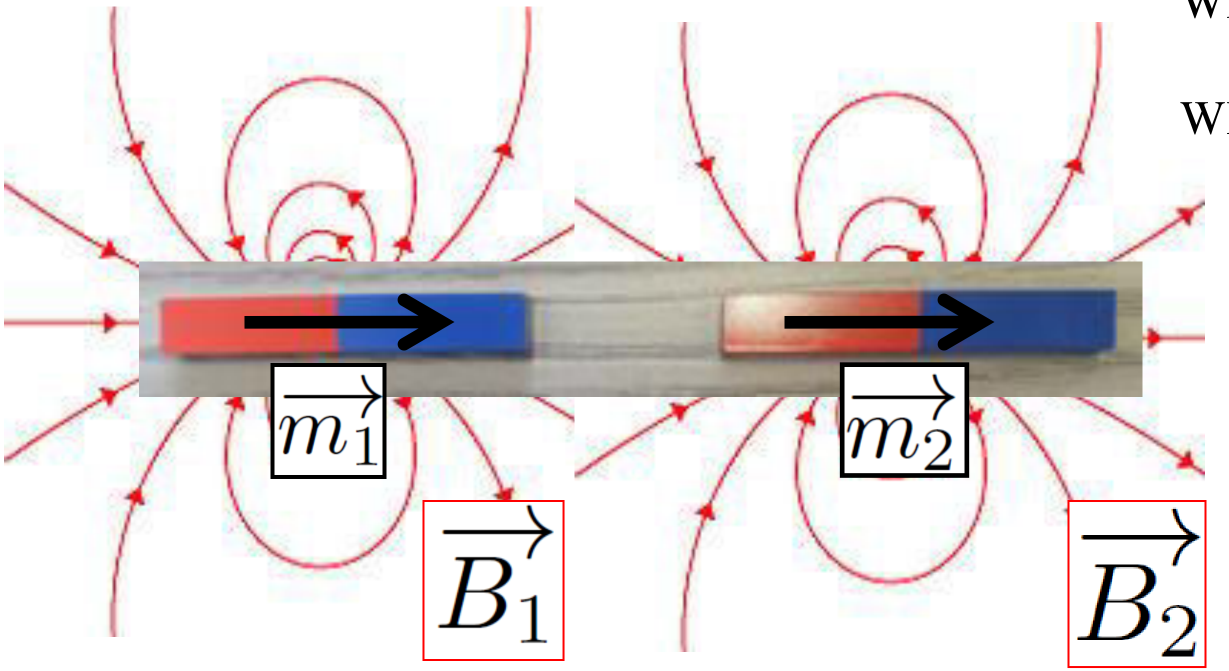
$$U_2 = -\overrightarrow{m_2} \cdot \overrightarrow{B_1}$$

1) Some problematics

WHAT IS THE FIELD ?

WHAT IS THE FORCE ?





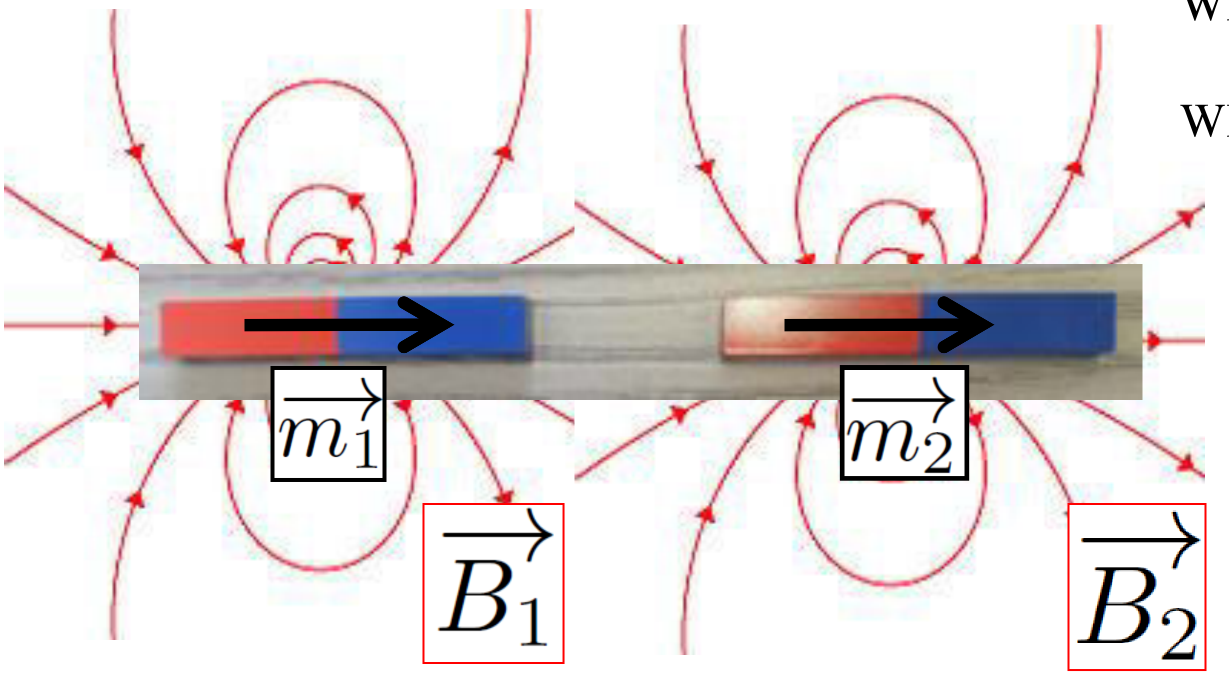
WHAT IS THE FIELD ?

WHAT IS THE FORCE ?

$$\boxed{\overrightarrow{B} = \frac{\mu_0}{4\pi}\left(\frac{3(\overrightarrow{m}\cdot\overrightarrow{e_r})\overrightarrow{e_r} - \overrightarrow{m}}{r^3}\right)}$$

$$U_2 = -\overrightarrow{m_2}\cdot\overrightarrow{B_1}$$

$$U_1 = -\overrightarrow{m_1}\cdot\overrightarrow{B_2}$$



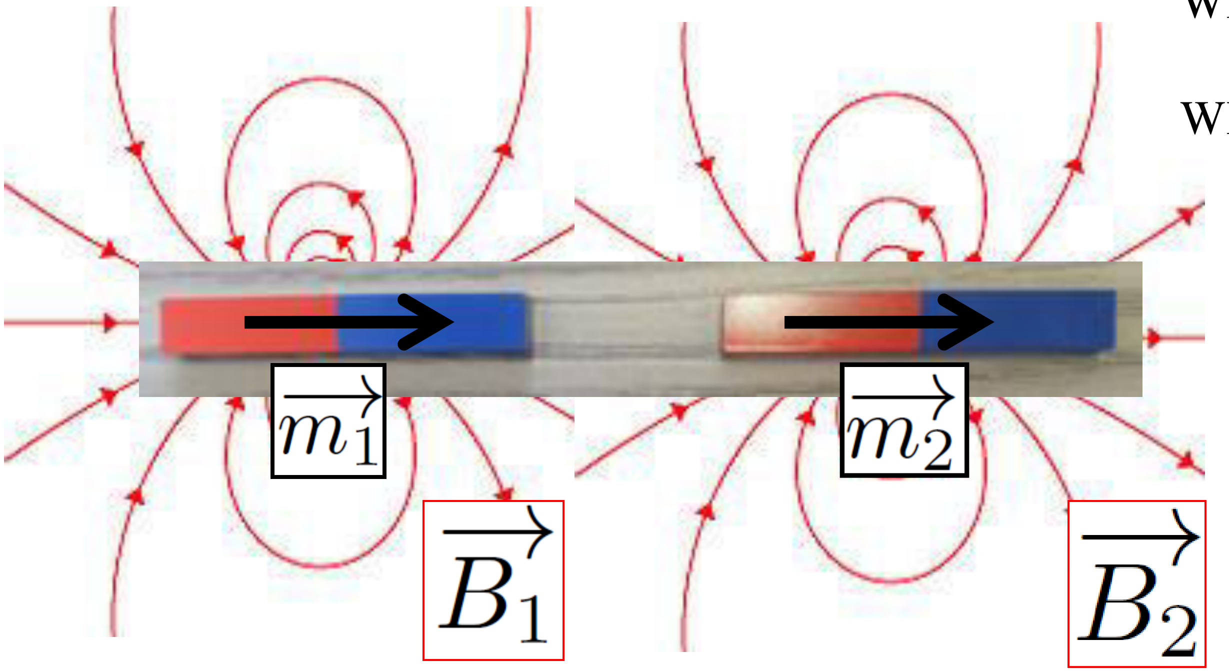
~~WHAT IS THE FIELD ?~~

WHAT IS THE FORCE ?

What about force ??

$$U_2 = -\overrightarrow{m_2} \cdot \overrightarrow{B_1}$$

$$U_1 = -\overrightarrow{m_1} \cdot \overrightarrow{B_2}$$



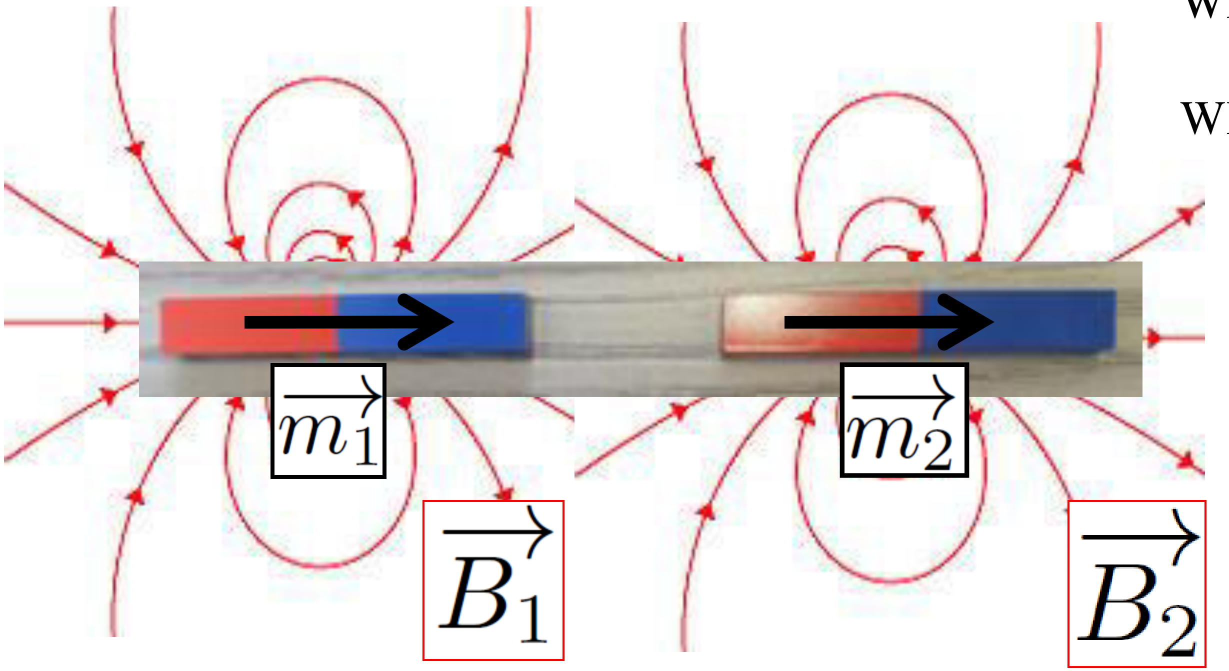
~~WHAT IS THE FIELD ?~~

WHAT IS THE FORCE ?

$$\begin{aligned}\overrightarrow{F} &= -\overrightarrow{\mathrm{grad}}U \\ &= -\overrightarrow{\mathrm{grad}}(-\overrightarrow{m} \cdot \overrightarrow{B})\end{aligned}$$

$$U_2 = -\overrightarrow{m_2} \cdot \overrightarrow{B_1}$$

$$U_1 = -\overrightarrow{m_1} \cdot \overrightarrow{B_2}$$



~~WHAT IS THE FIELD ?~~

WHAT IS THE FORCE ?

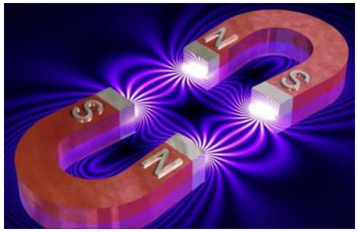
$$\overrightarrow{F}_{1\to 2} = -\overrightarrow{\mathrm{grad}}(-\overrightarrow{m_2}\cdot\overrightarrow{B_1})$$

$$\overrightarrow{F}_{2\to 1} = -\overrightarrow{\mathrm{grad}}(-\overrightarrow{m_1}\cdot\overrightarrow{B_2})$$

$$U_2 = -\overrightarrow{m_2}\cdot\overrightarrow{B_1}$$

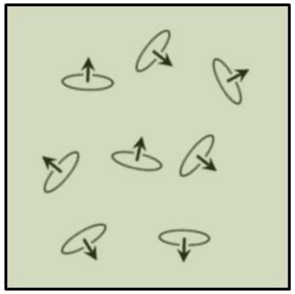
$$U_1 = -\overrightarrow{m_1}\cdot\overrightarrow{B_2}$$

Magnetostatics-L2
B-Fundamentals of Magnetism



1) **Some problematics**
   a) Source –field – interactions in electrostatics
   b) Source -field –interaction in magnetostatics: current and magnets- analogy and difference
   c) where is the magnetic potential ?
2) **Magnetic moment (magnetic dipole)**
   a) atomic current – magnetic moment
   b) magnetic material VS non magnetic material (I)
3) Need of geometry
4) The vector potential
5) Vector potential and magnetic field in the dipolar approximation
6) Interaction between a magnetic dipole and a magnetic field
7) Interaction between magnetic dipoles
8) **Magnetic materials**

NO MAGNETIC FIELD

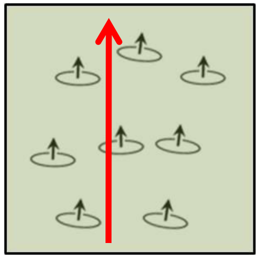


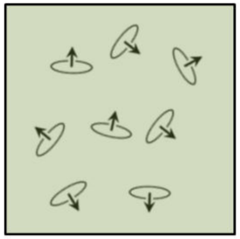
Initially no magnetization

WE APPLY A MAGNETIC FIELD $\vec{M} = \mu_0 \chi_m \vec{B}$

WE CUT THE MAGNETIC FIELD

Paramagnetism

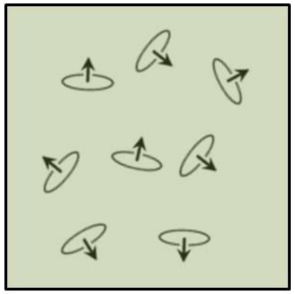




B=0 ; M=0

M parallel to B; Xm ≈ 10^{-3}

NO MAGNETIC FIELD



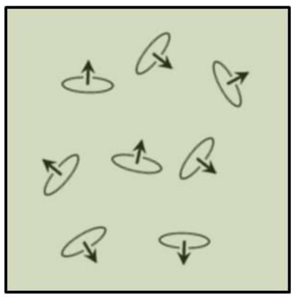
Initially no magnetization

Magnetic material

WE APPLY A MAGNETIC FIELD $\overrightarrow{M} = \mu_0 \chi_m \overrightarrow{B}$

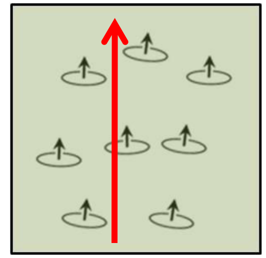
WE CUT THE MAGNETIC FIELD

NO MAGNETIC FIELD

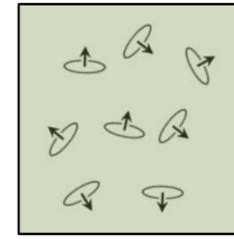


Initially no magnetization

Paramagnetism

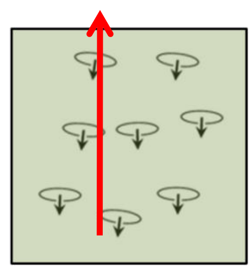


M parallel to B; Xm ≈ 10^{-3}

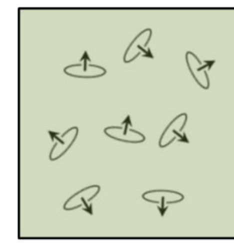


B=0 ; M=0

Diamagnetism



M anti parallel to B; Xm ≈ -10^{-5}



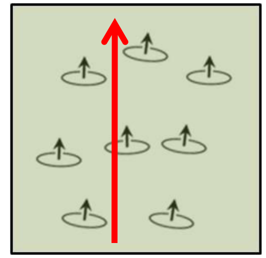
B=0 ; M=0

Magnetic material

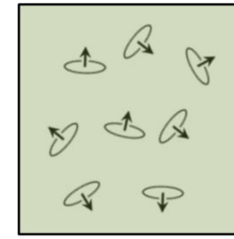
WE APPLY A MAGNETIC FIELD $\overrightarrow{M} = \mu_0 \chi_m \overrightarrow{B}$

WE CUT THE MAGNETIC FIELD

Paramagnetism

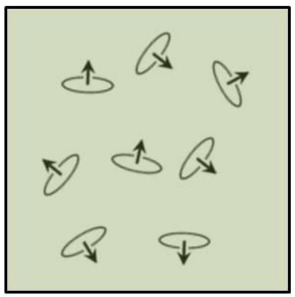


M parallel to B; Xm ≈ 10^{-3}



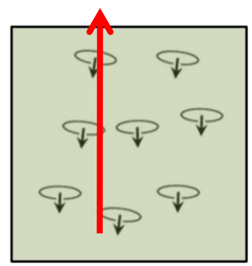
B=0 ; M=0

NO MAGNETIC FIELD

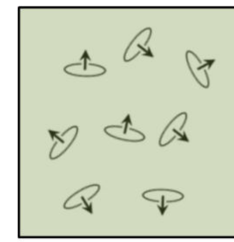


Initially no magnetization

Diamagnetism

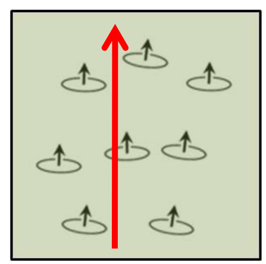


M anti parallel to B; Xm ≈ -10^{-5}

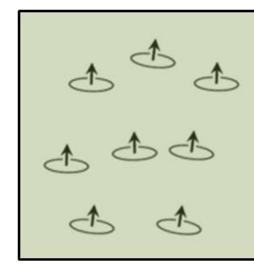


B=0 ; M=0

Ferromagnetism

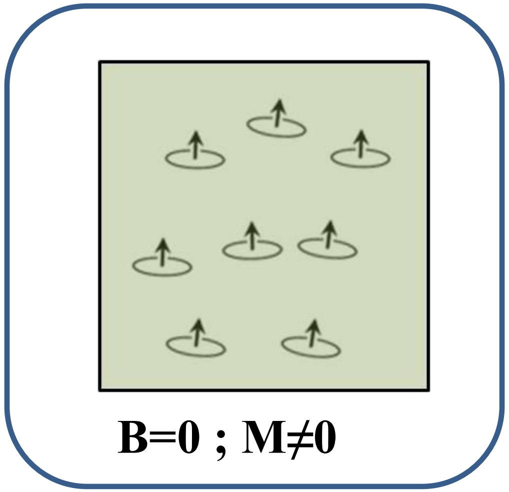


M parallel to B; Xm ≈ 10^{5}



B=0 ; M≠0

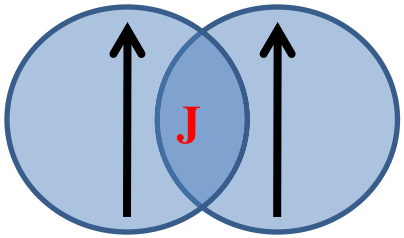
Ferromagnetism



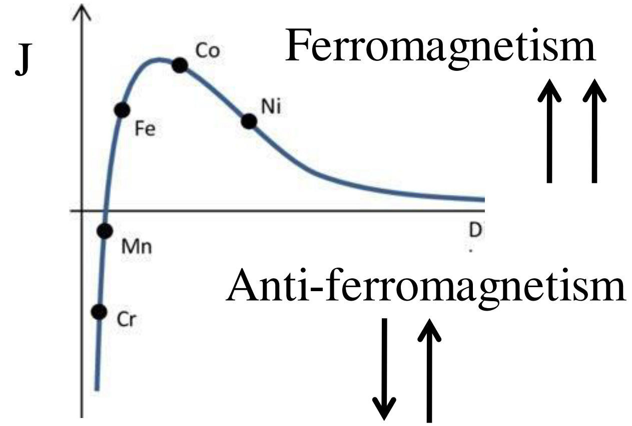
Existence of a <u>spontaneous magnetisation **M**</u>.

i) Dipolar energy CAN NOT explain it because it is smaller than thermal energy kT at room temperature.

ii) It appears to be a <u>quantum effect</u> called *exchange interaction* coming from **Pauli's exclusion principle**. Interaction between SPINS.



$$U = -J\overrightarrow{S_1}\cdot\overrightarrow{S_2}$$



ABOUT MAGNETISM…..