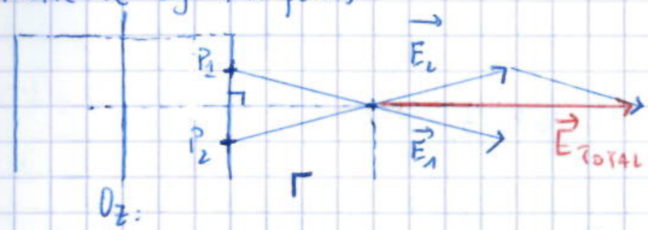
I. Coaxial cable

A. Study of the Electric field in region II

1) $\oiint \vec{E} \cdot \vec{dS} = \dfrac{Q_{int}}{\varepsilon_0}$

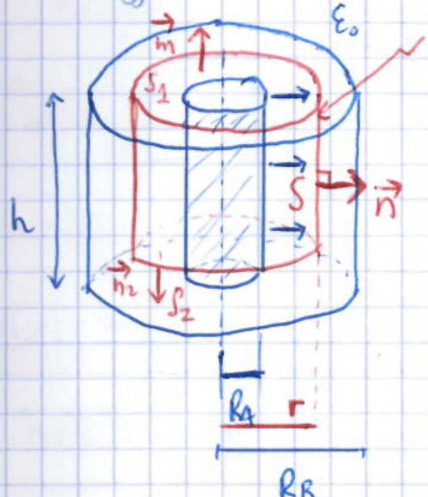
2) Take 2 symmetric points

i)



+ ii) infinite: invariant along Oz

3) $\oiint \vec{E} \cdot \vec{dS} = \dfrac{Q_{int}}{\varepsilon_0} \iff \iint_{S_1} \vec{E} \cdot dS \vec{n_1} + \iint_{S_2} \vec{E} \cdot dS \vec{n_2} + \iint \vec{E} \cdot dS \vec{n} = \dfrac{Q_{int}}{\varepsilon_0}$



With $\vec{E} \perp \vec{n_1}$

$\vec{E} \perp \vec{n_2}$

$\vec{E} \parallel \vec{n}$ and $\vec{E} = E(r)\,\vec{e_r}$ only

$$\iff 0 + 0 \quad \iint \vec{E}(r) \cdot dS \vec{n} = \frac{Q_{int}}{\varepsilon_0}$$

$$\iff E(r) \cdot \underbrace{\iint dS}_{\text{red cylinder}} = \underbrace{\iint \sigma_A \, dS}_{S \text{ small cylinder}}$$

$$\iff E(r)\, 2\pi r\, h = \frac{\sigma_A\, 2\pi R_A h}{\varepsilon_0}$$

$$\iff \boxed{E(r) = \frac{\sigma_A R_A}{\varepsilon_0 r}}$$

4) $$V(r) = -\int \vec{E}(r) \cdot \vec{dr} = -\int \frac{\sigma_A R_A}{\varepsilon_0 r} \vec{e_r} \cdot dr \vec{e_r}$$

$$= -\int \frac{\sigma_A R_A}{\varepsilon_0 r} dr = -\frac{\sigma_A R_A}{\varepsilon_0} \ln r + C$$

and with $V(R_A) = V_A = -\dfrac{\sigma_A R_A}{\varepsilon_0} \ln R_A + C$ we find C leading to

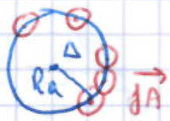
$$\boxed{V(r) = -\frac{\sigma_A R_A}{\varepsilon_0} \ln \frac{r}{R_A} + V_A}$$

5) Total length ℓ so $C_x = \frac{C}{\ell} = \frac{2\pi\varepsilon_0 \ell}{\ell \ln\left(\frac{R_B}{R_A}\right)} = \frac{2\pi\varepsilon_0}{\ln(R_B/R_A)}$

$$C_x = \frac{2\times\pi\times 8{,}85\,10^{-12}}{\ln\left(\frac{0{,}5}{0{,}15}\right)} = 4{,}61\,10^{-11} \text{ F/m}$$

B. Study of magnetic field in region II

6) Propagation only on surface.



so $i_A = 2\pi R_a j_A$.

7) Ampere theorem: $\oint \vec{B}\cdot d\vec{\ell} = (\Sigma\, i_{encircled})\,\mu_0$

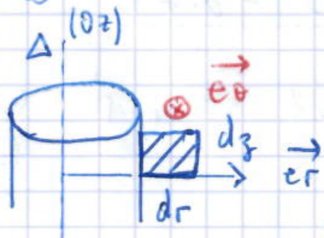
$$B.2\pi r = \mu_0 i_A$$

$$B(r) = \frac{\mu_0 i_A}{2\pi r}$$

and $\vec{B}(r) = B(r)\,\vec{e_\theta}$

8) $\Phi = \iint \vec{B}(r)\,d\vec{S} = \int_{R_A}^{R_B}\int_0^{\ell} \frac{\mu_0 i_A}{2\pi r}\,\vec{e_\theta}\,.\,dr\,dz\,\vec{e_\theta}$

9)



$$= \frac{\mu_0 i_A}{2\pi}\int_{R_A}^{R_B}\frac{dr}{r}\,\underbrace{\vec{e_\theta}.\vec{e_\theta}}_{1}\int_0^{\ell} dz$$

$$\Phi = \frac{\mu_0 i_A}{2\pi}\left[\ln r\right]_{R_A}^{R_B}\left[z\right]_0^{L}$$

$\Phi = \frac{\mu_0 i_A}{2\pi}\ln\frac{R_B}{R_A}\,.\,\ell =$ i (with $i = i_A$)

So $\boxed{L = \frac{\mu_0 \ell}{2\pi}\ln\left(\frac{R_B}{R_A}\right)}$

10) $L_x = \frac{L}{\ell} = \frac{\mu_0}{2\pi}\ln\left(\frac{R_B}{R_A}\right) = \frac{4\pi.10^{-7}}{2\pi}\ln\left(\frac{0{,}5}{0{,}15}\right) = 2{,}4\,10^{-7}$ H/m

11) $v = \frac{1}{\sqrt{C_x L_x}} = \frac{1}{\sqrt{\frac{2\pi\varepsilon_0}{\ln\left(\frac{R_B}{R_A}\right)}\times\frac{\ln\left(\frac{R_B}{R_A}\right)\mu_0}{2\pi}}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \equiv c$!!!

II. Electromagnetic wave propagation. (A) in the vacuum

12) Vacuum: $\rho = 0$ and $\vec{j} = \vec{0}$ so

$$\mathrm{div}\,\vec{E} = 0 \qquad \mathrm{div}\,\vec{B} = 0$$
$$\vec{\mathrm{rot}}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\mathrm{rot}}\,\vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

13) $\vec{\mathrm{rot}}\left(\vec{\mathrm{rot}}\,\vec{E}\right) = -\vec{\mathrm{rot}}\left(\frac{\partial \vec{B}}{\partial t}\right)$ and $\frac{\partial}{\partial t}\vec{\mathrm{rot}}\,\vec{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\frac{\partial \vec{E}}{\partial t}$

$$\vec{\mathrm{grad}}\,\underbrace{\mathrm{div}\,\vec{E}}_{0} - \Delta\vec{E} = -\vec{\mathrm{rot}}\,\frac{\partial \vec{B}}{\partial t} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\Leftrightarrow \quad \Delta\vec{E} = \mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\Leftrightarrow \quad \Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0. \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

14) $\left.\begin{aligned} \Delta\vec{E} &= -k^2\vec{E} \\ \frac{\partial^2 \vec{E}}{\partial t^2} &= -\omega^2 \vec{E} \end{aligned}\right\}$ Wave eq° $\Rightarrow$ $-k^2 - \frac{1}{c^2}(-\omega^2) = 0.$

$\Leftrightarrow \quad k^2 = \omega^2 c^2$

and $v_\varphi = \frac{\omega}{k} = c$

(B) in dielectric material

i) $\mathrm{div}\,\vec{E} = \frac{\rho}{\varepsilon_0} = \frac{\rho_F + \rho_B}{\varepsilon_0} = \frac{0 - \mathrm{div}\,\vec{P}}{\varepsilon_0}$

$\Leftrightarrow \quad \mathrm{div}\left(\varepsilon_0\vec{E} + \vec{P}\right) = 0$ with $\varepsilon_0\vec{E} + \vec{P} = \varepsilon_0\vec{E} + \varepsilon_0\chi\vec{E}$
$= \varepsilon_0(1+\chi)\vec{E}$
$= \varepsilon_0\varepsilon_r\vec{E}$

$\Leftrightarrow \quad \mathrm{div}\left(\varepsilon_0\varepsilon_r\vec{E}\right) = 0$

$\varepsilon_0\varepsilon_r\,\mathrm{div}\,\vec{E} = 0.$

ii) $$\vec{\mathrm{rot}}\,\vec{B} = \mu_0\vec{J_F} + \mu_0\vec{J_B} + \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}$$
$$= 0 + \mu_0\frac{\partial \vec{P}}{\partial t} + \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t} = \mu_0\frac{\partial}{\partial t}\Big[\underbrace{\vec{P} + \varepsilon_0\vec{E}}_{\varepsilon_0\varepsilon_r\vec{E}}\Big]$$
$$= \mu_0\varepsilon_0\varepsilon_r\frac{\partial \vec{E}}{\partial t}$$

When doing $\vec{\mathrm{rot}}(\vec{\mathrm{rot}}\,\vec{E})$ and $\frac{\partial}{\partial t}(\vec{\mathrm{rot}}\,\vec{B})$ we obtain finaly

$$\Delta\vec{E} - \mu_0\varepsilon_0\varepsilon_r\frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{so} \quad v = \frac{1}{\sqrt{\varepsilon_r\mu_0\varepsilon_0}}$$

16) dispersion relation gives $\omega^2 = k^2\left(\frac{1}{\mu_0 \varepsilon_0 \varepsilon_r}\right)$

So $\omega = \frac{k}{\sqrt{\mu_0 \varepsilon_0}\sqrt{\varepsilon_r}} = \frac{k c}{\sqrt{\varepsilon_r}}$

and $v_\varphi = \frac{\omega}{k} = \frac{c}{\sqrt{\varepsilon_r}} \equiv \frac{c}{n}$ and $n = \sqrt{\varepsilon_r}$ optical index

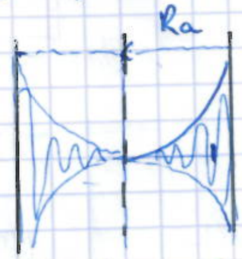
C) Wave propagation in metallic material

17) $\vec{j} = -\gamma \vec{\nabla} \vec{\ } = -\gamma\, \vec{grad}\ \vec{\ } = \gamma \vec{E}$

18) $\delta = \frac{2}{\sqrt{\mu_0 \omega \gamma}} = \frac{2}{\sqrt{4\pi \cdot 10^{-7} \times 2\pi \times 100 \cdot 10^{3} \times 6 \cdot 10^{7}}} = \underline{2{,}9\, 10^{-4}\, m}$

$\omega = 2\pi f$

19) $R_a = 0{,}15$ cm $= 1{,}5$ mm



$\delta = 0{,}3$ mm

$5\delta = 1{,}5$ mm

The wave penetration is important : not fully on surface only.

III Laplace forces and induction

20) $\vec{F} = \int_0^L i\, \vec{dl} \wedge \vec{B} = \int_0^L i\, dl\, \vec{e_y} \wedge (-B\vec{e_z}) = -iLB\vec{e_x}$

21) $e = \int_0^L \vec{E_m} \cdot \vec{dl} = \int_0^L (\vec{V} \times \vec{B}) \cdot \vec{dl}$

$= \int_0^L VB\, dl\, \left(-\vec{e_x} \wedge (-\vec{e_z})\right) \cdot \vec{e_y}$

$= \int_0^L VB\, dl\, \underbrace{\vec{e_y} \cdot \vec{e_y}}_{1} = VBL$

22) $e = -\frac{d\phi}{dt}$; $d\phi < 0$ because surface ↘ with motion along $-\vec{e_x}$

So $\boxed{e > 0}$