

# Physics-L2 Electromagnetism

## Approximative program

- 
- 1) Sources –Fields -interactions
  - 2) Fundamentals of magnetism

Chap 1: Electrostatics

Chap 2: Magnetostatics

**Chap 3: Time-dependent regime-Induction phenomena**

Chap 4: Maxwell equations

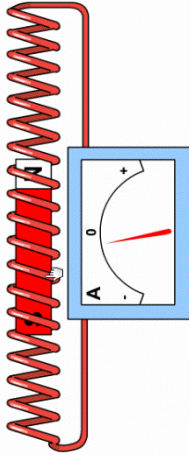
Chap 5: Dielectric media and applications

Chap 6: Conducting media and applications

Chap 7: Magnetic media and applications

week	Magistral lectures
1	Electrostatics
2	Electrostatics
3	Electrostatics
4	Electrostatics
5	Magnetostatics
6	Magnetostatics
7	<b>Induction</b>
8	<b>Induction</b>
9	Maxwell equations
10	Maxwell equations
11	Dielectric media
12	Dielectric / Metallic media
13	Metallic Media
14	Magnetic media

# Electromagnetic Induction -L2



## 1) Experimental approach

### a) i) Faraday experiment.

ii) Notion of electromotive force and field.

### b) Neumann induction. Time-dependent magnetic field and rigid circuit.

i) Examples

ii) Faraday law and electromotive field

iii) Lenz law

### c) Lorentz induction. Constant magnetic field and moving circuit.

i) example with « rail of Laplace ». Deformable circuit

ii) Cut Flux, Faraday law and electromotive field

iii) Generalisation to moving but not deformable circuit.

## 2) Theoretical description

## 3) Inductance and self inductance

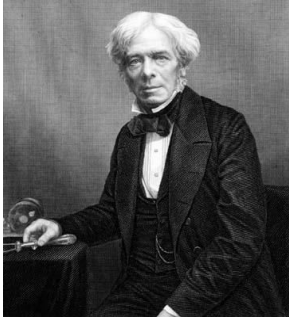
## 4) Applications

i) Electric transformer

iii) Induction heating

ii) Electro-mechanical conversion: DC and AC motors

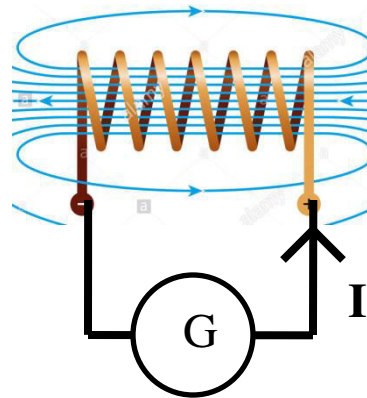
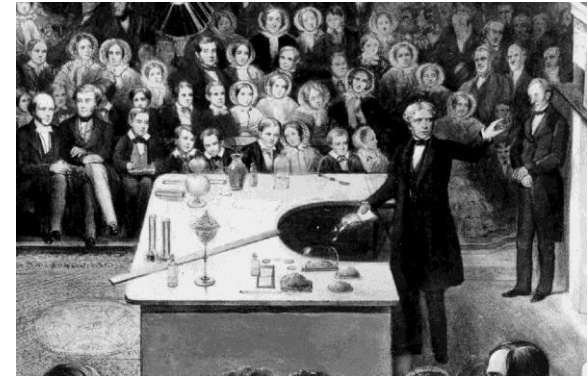
## 1831: *Magnetism can induce electricity*



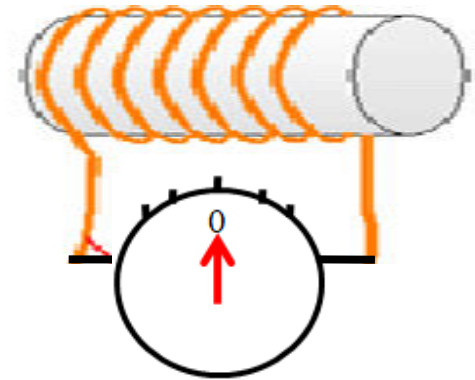
*Michael Faraday*  
1791-1867

### Important contribution

- Chemistry (electrolysis)
- Notion of *vectors fields*
- *Electromagnetic Induction*

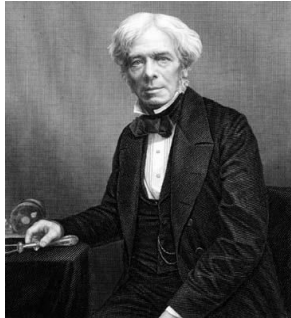


A magnetic coil  
that produces  
a magnetic field



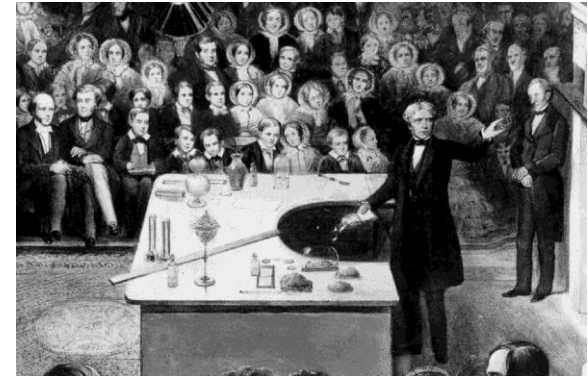
A magnetic coil related to  
a voltmeter.

## 1831: *Magnetism can induce electricity*

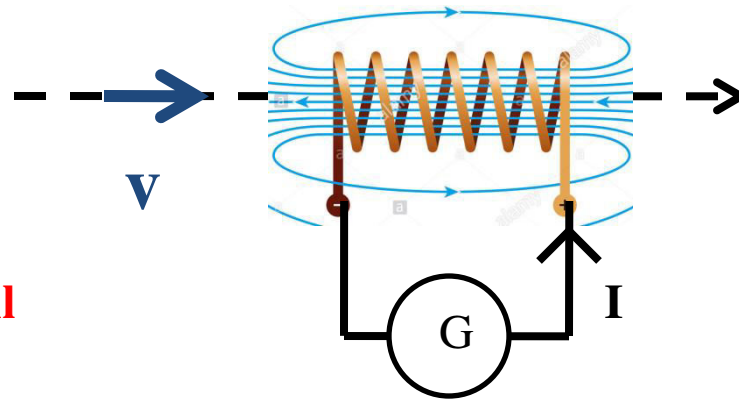


### Important contribution

- Chemistry (electrolysis)
- Notion of *vectors fields*
- *Electromagnetic Induction*



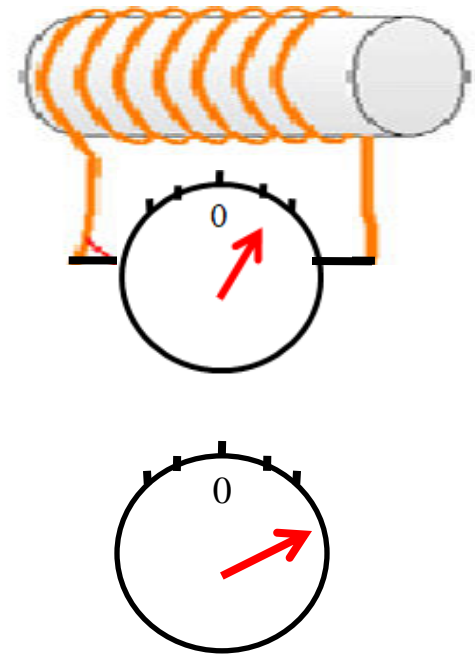
*Michael Faraday*  
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**We move the first coil close to the second and a voltage appears during the motion !**

**We say also: an electromotive force**

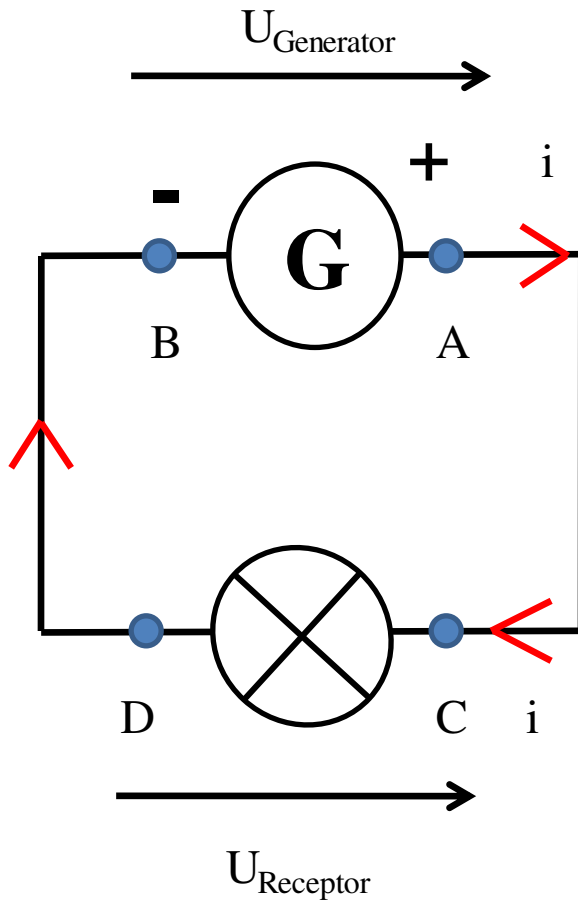
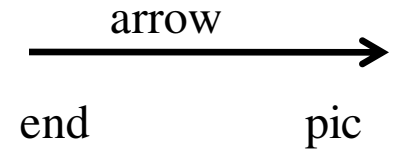
If motion is faster, voltage  
In the second coil is higher !



## Convention about electric circuit

The value of the voltage will be

$$U = V_{\text{pic}} - V_{\text{end}}$$



### For Generator:

Current  $i$  goes from  $+$  to  $-$

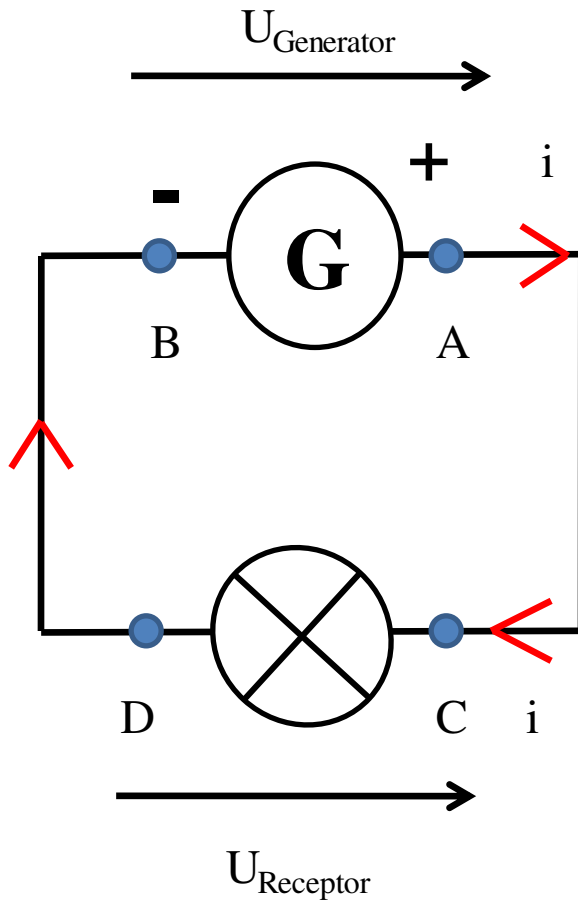
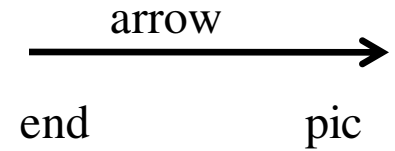
Voltage and current show the same direction.

$$U_{\text{Generator}} = V_A - V_B$$

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### For receptor

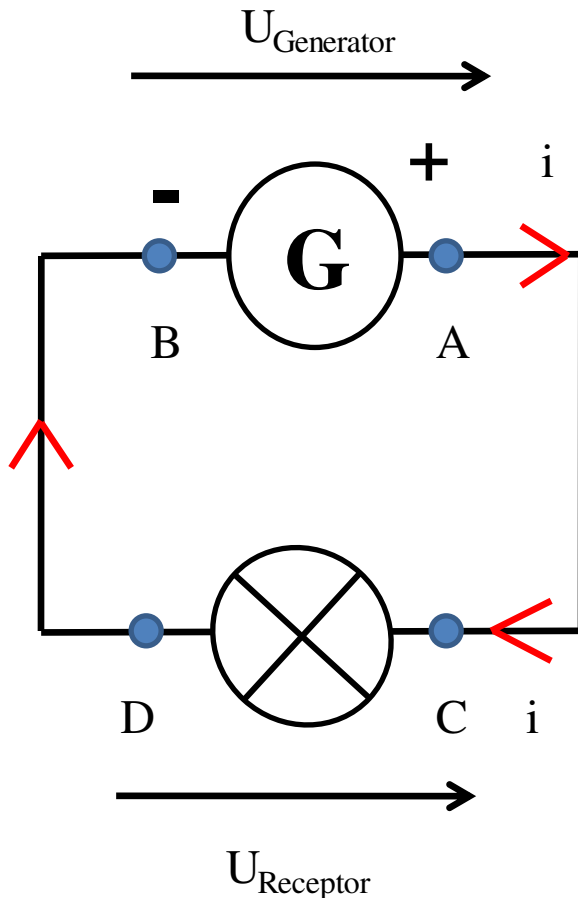
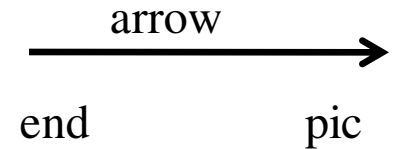
Voltage and current show opposite directions

$$U_{\text{Receptor}} = V_C - V_D$$

## Convention about electric circuit

The value of the voltage will be

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### For Generator:

Current  $i$  goes from  $+$  to  $-$

Voltage and current show the same direction.

$$U_{\text{Generator}} = V_A - V_B$$

### For receptor

Voltage and current show opposite directions

$$U_{\text{Receptor}} = V_C - V_D$$

Also, an electric wire has the same potential in the beginning and in the end  $V_A = V_C$  and  $V_B = V_D$

$$U_{\text{Receptor}} = V_C - V_D = V_A - V_B = U_{\text{Generator}}$$

# Notion of electromotive field or electromotive force

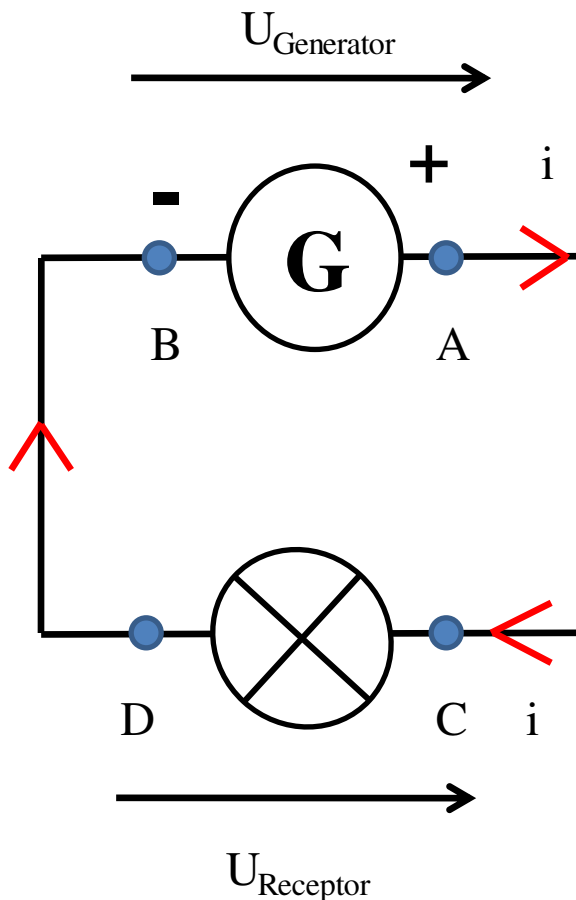
$$U = V_A - V_B = \int_A^B \vec{E}_m \cdot \vec{dr} = e$$

$\vec{E}_m$  is called the electromotive field, related to a Electromotive force  $\vec{f}$ :

$$\vec{E}_m = \frac{\vec{f}}{q}$$

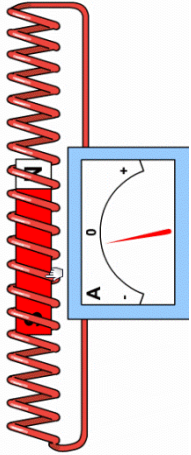
By abuse of language, we can call  $e$  as electromotive force:

$$e = \int_A^B \vec{E}_m \cdot \vec{dr}$$





# Electromagnetic Induction -L2



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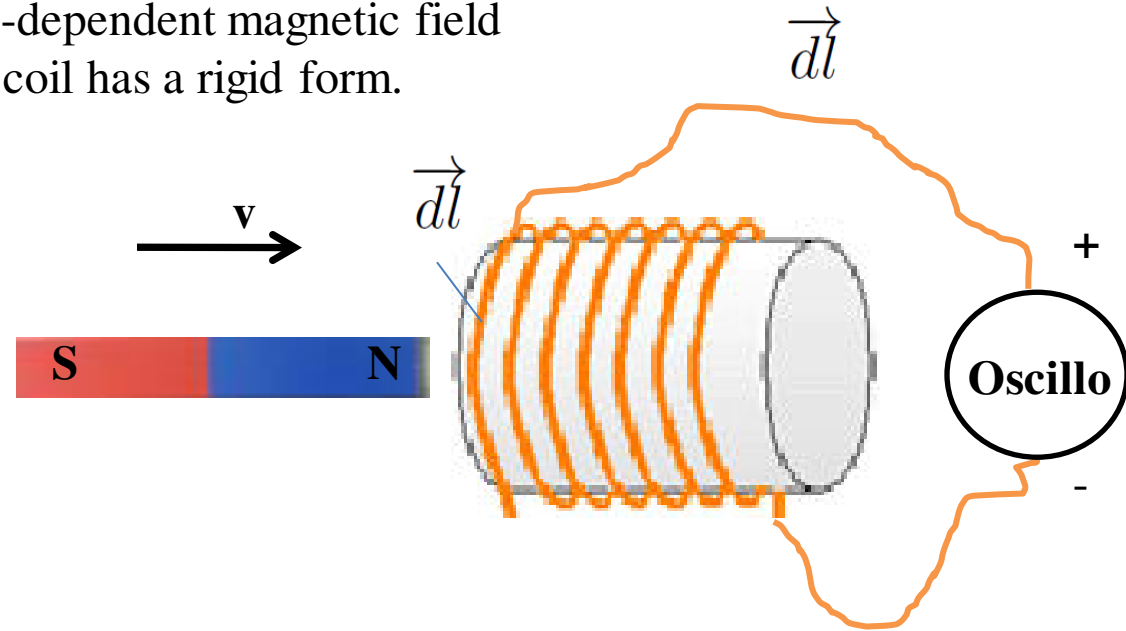
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## Neumann induction.

We move the magnet inside the coil that simulate the time-dependent magnetic field  
The coil has a rigid form.



Franz Ernst Neumann  
1798-1895

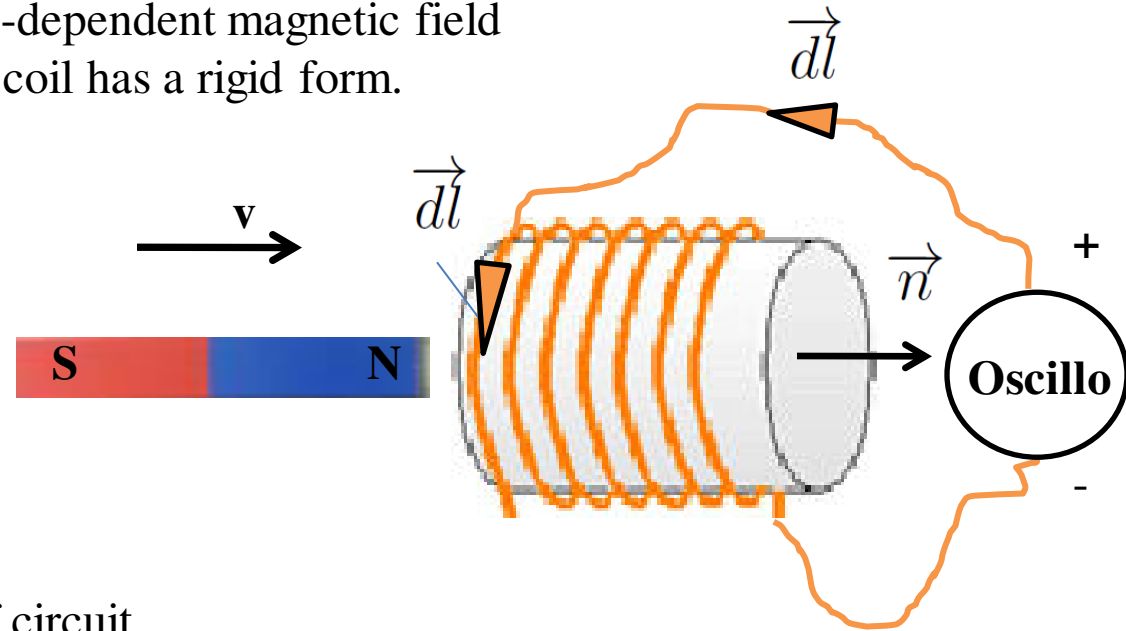


## Neumann induction.



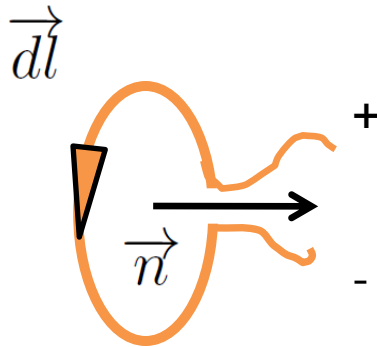
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Before, we need to

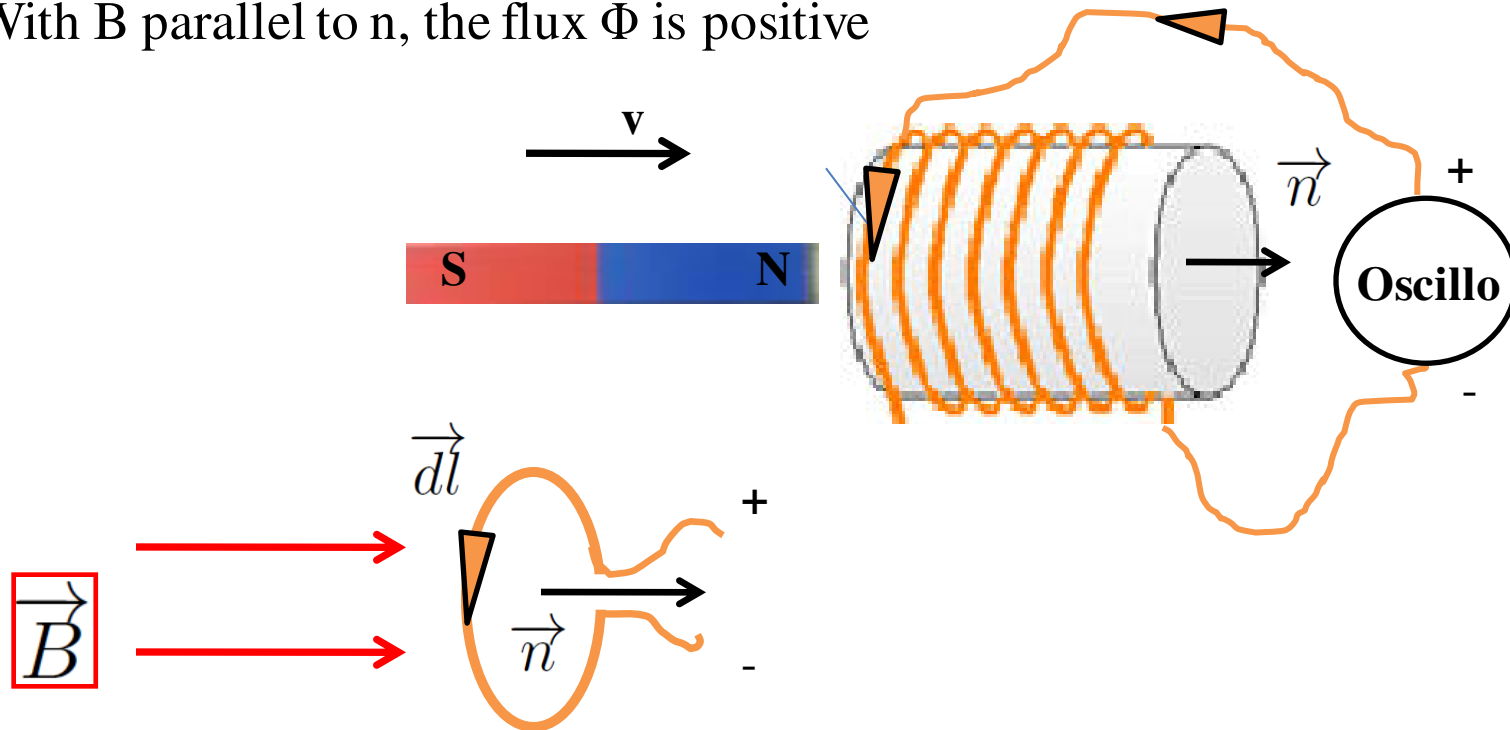
i) Make the orientation of circuit



ii) Discuss the direction of the magnetic field

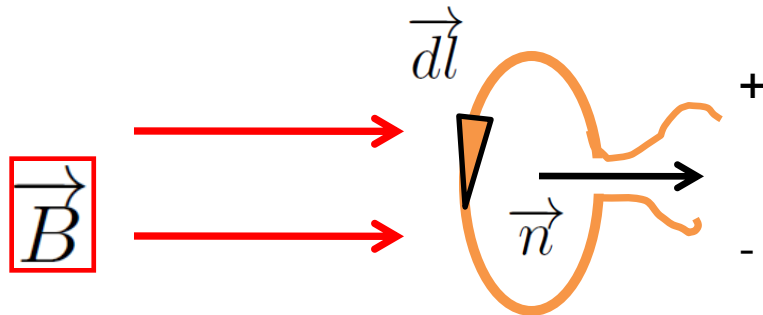
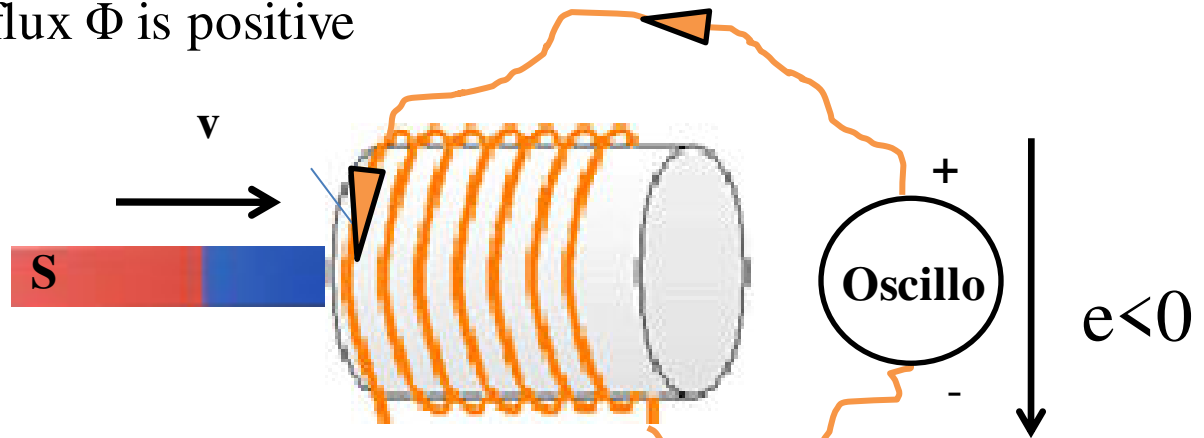


With  $\vec{B}$  parallel to  $\vec{n}$ , the flux  $\Phi$  is positive



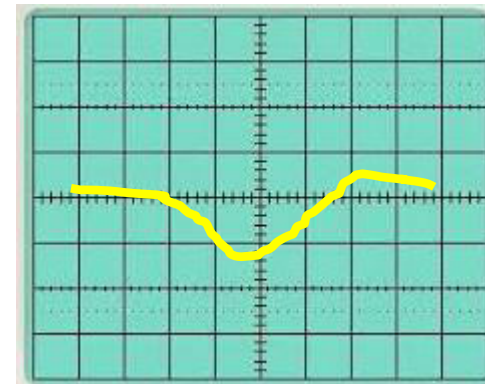
$$\Phi = \iint \vec{B} \cdot dS \vec{n} > 0$$

With  $\vec{B}$  parallel to  $\vec{n}$ , the flux  $\Phi$  is positive

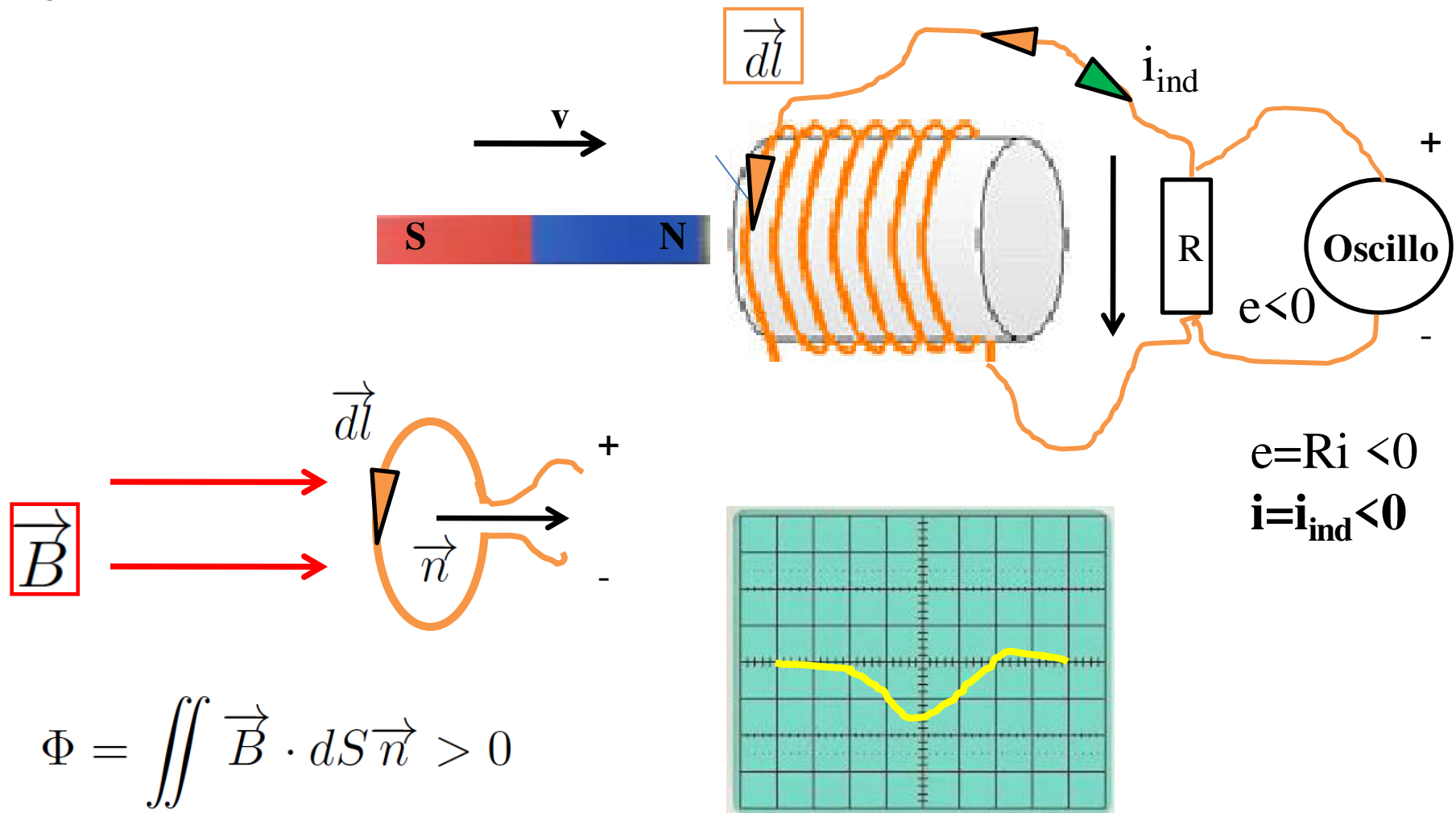


When moving magnet inside the coil  
We observe negative voltage, a negative  
electromotive « force » appears

$$\Phi = \iint \vec{B} \cdot dS \vec{n} > 0$$



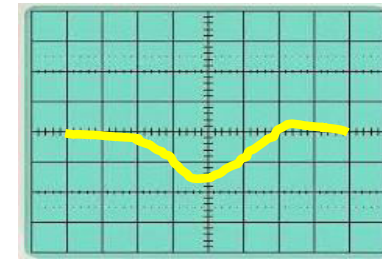
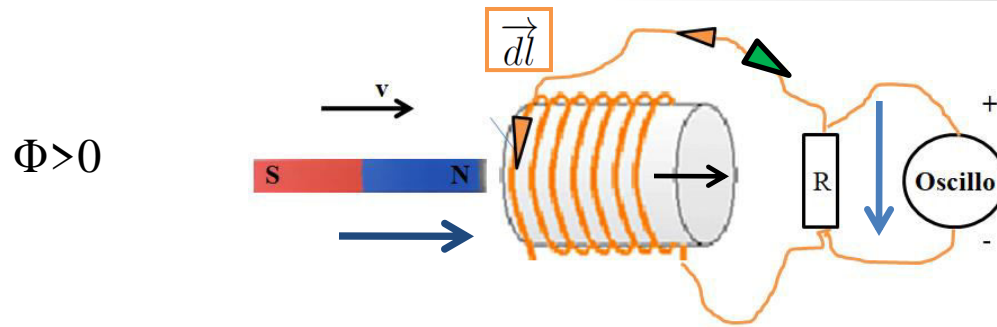
Same experiment by plugging a resistance, whose voltage  $e= Ri$  will be negative.  
The electromotive force is proportional to the current  $i$  so we have apparition of a negative current



Playing with parameters:

Input (the flux)

Output (the electromotive force)



$$e = Ri < 0$$

$$i = i_{\text{ind}} < 0$$

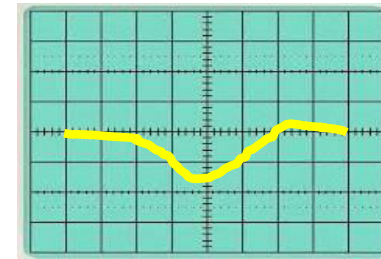
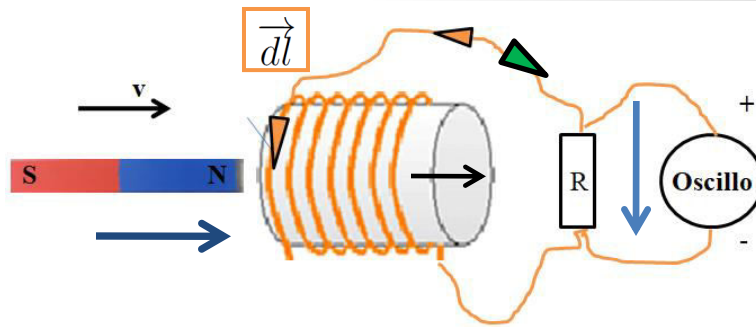


Playing with parameters:

Input (the flux)

Output (the electromotive force)

$$\Phi > 0$$



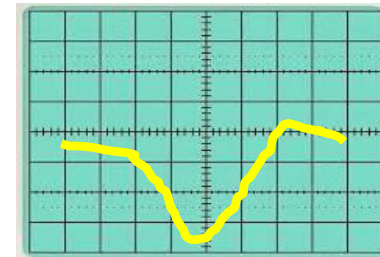
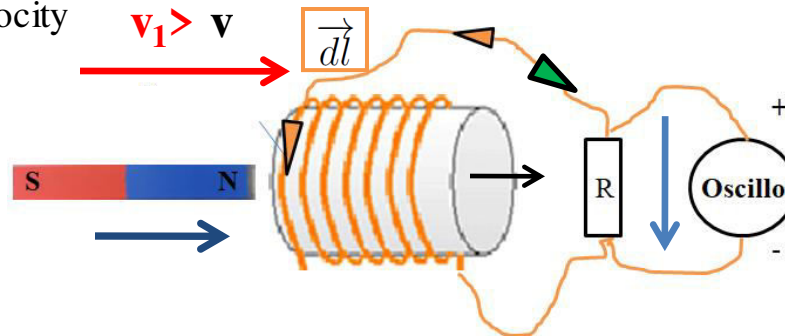
$$e = Ri < 0$$

$$i = i_{\text{ind}} < 0$$



Enters with faster velocity  
Influence of TIME  
VARIATION

$$\frac{d\Phi}{dt} > 0$$



$$e_1 = Ri < 0$$
  

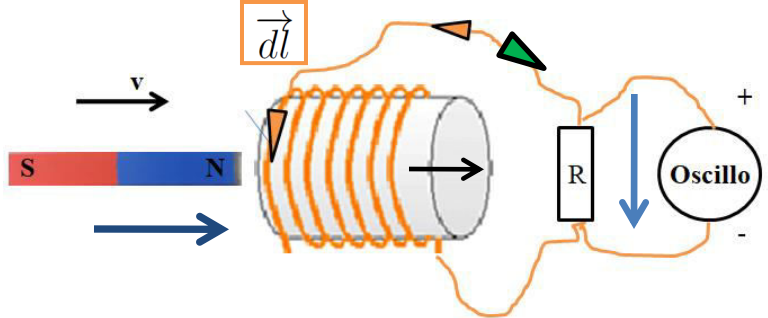
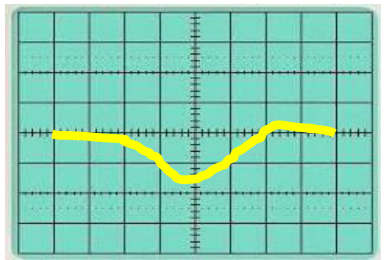

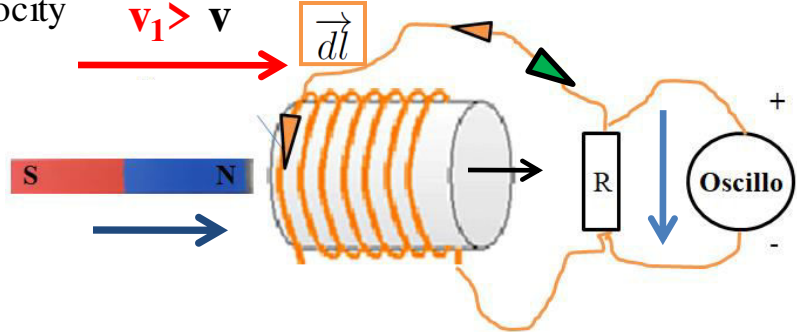
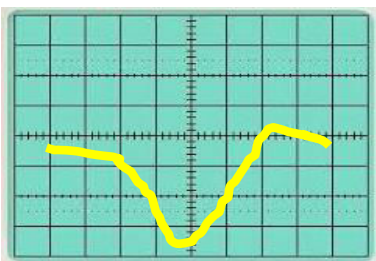
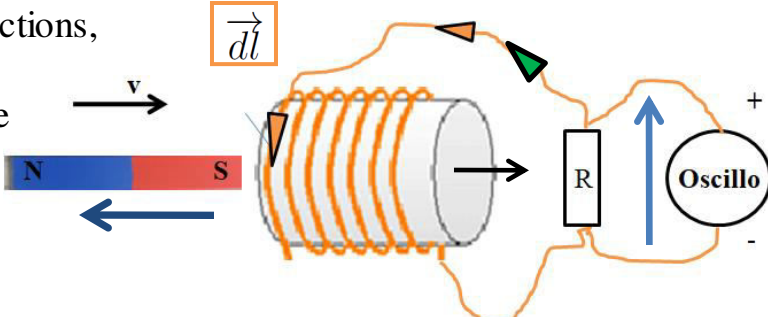
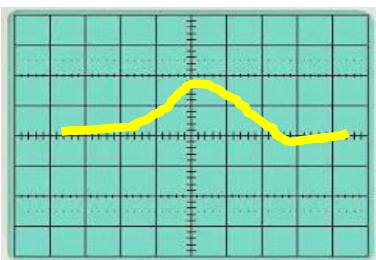
$$\text{and}$$
  

$$|e_1| > |e|$$

$$i = i_{\text{ind}} < 0$$



# Playing with parameters:

	Input (the flux)	Output (the electromotive force)
$\Phi > 0$		 $e = Ri < 0$ $i = i_{ind} < 0$ 
Enters with faster velocity Influence of TIME VARIATION $\frac{d\Phi}{dt} > 0$		 $e_1 = Ri < 0$ and $ e_1  >  e $ $i = i_{ind} < 0$
We switch the pole directions, <b>B</b> opposite to <b>n</b> and $\Phi$ becomes negative $\Phi < 0$ $\frac{d\Phi}{dt} < 0$		 $e = Ri > 0$ $i = i_{ind} > 0$

## Results

Lets look first the units:  $[e]=[V]=\text{Volt}$

$$\left[\frac{d\Phi}{dt}\right] = \frac{[B][S]}{[t]} = \frac{[B][m]}{[t]}[m] = [E][m] = [V] = \text{Volt}$$

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When the flux variation is positive  $\frac{d\Phi}{dt} > 0$  the electromotive force is negative  $e < 0$

When the flux variation is negative  $\frac{d\Phi}{dt} < 0$  the electromotive force is positive  $e > 0$

## Results

Lets look first the units:  $[e]=[V]=\text{Volt}$

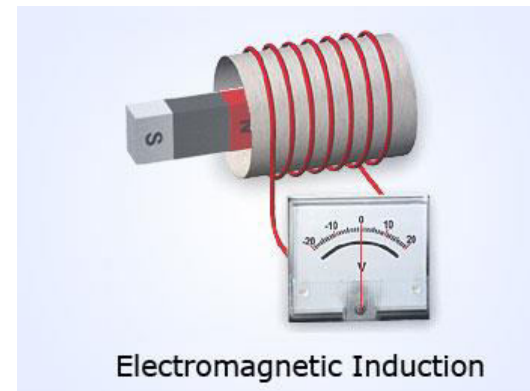
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When the flux variation is negative  $\frac{d\Phi}{dt} < 0$  the electromotive force is positive  $e > 0$

$$\frac{d\Phi}{dt} = -e$$

Faraday law



## Connection with electromotive force

$$\begin{aligned} e &= -\frac{d\Phi}{dt} = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{S} \\ &= -\frac{d}{dt} \iint \operatorname{rot} \vec{A} \cdot d\vec{S} \\ &= -\frac{d}{dt} \oint \vec{A} \cdot d\vec{l} \\ &= \oint -\frac{\partial \vec{A}}{\partial t} \cdot d\vec{l} \\ &= \oint \vec{E}_m \cdot d\vec{l} \end{aligned}$$

Stockes

$$\vec{E}_m = -\frac{\partial \vec{A}}{\partial t}$$

**Time-dependent magnetism generates electricity !!!**

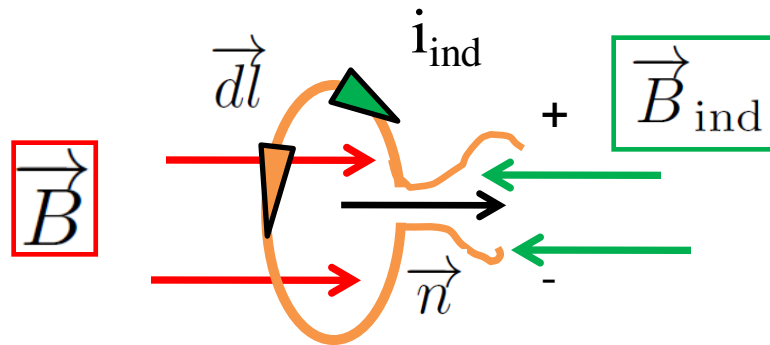
# Lenz Law

## Fact:

a positive variation of flux (an increase) creates an electromotive force that generates a negative current. This negative current will create a magnetic field whose flux is opposite to the initial flux.



Friedrich Emil Lenz  
1804-1865

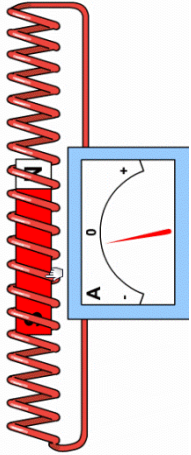


Traduction of the usual french statement about Lenz law

*L'induction génère des effets qui s'opposent aux causes qui leur donnent naissance*

Induction generates effects that oppose themselves to what gave them birth

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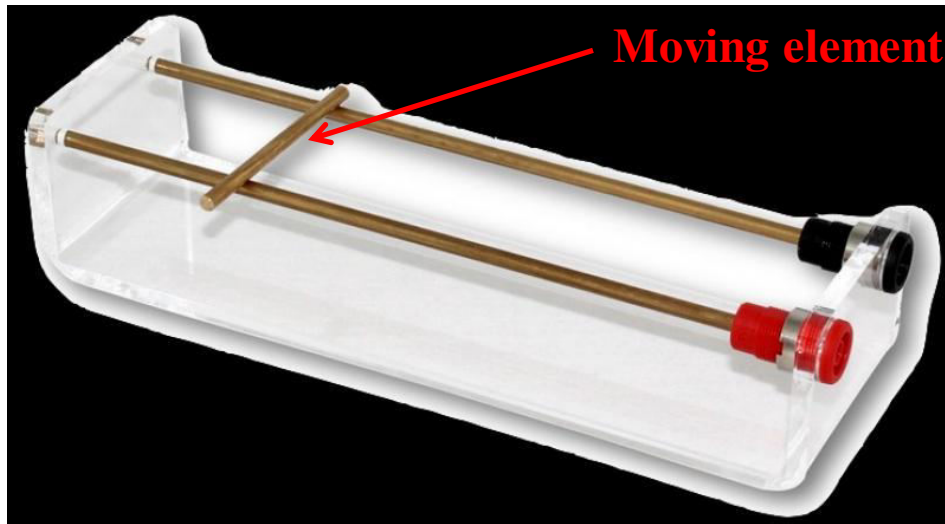
We consider now the opposite case: the magnetic field is constant but the circuit can move

In a first approach, we will consider a deformable circuit : its form can change.

The device is called « **rail de Laplace** » :

It is made of a two parallel conductors in contact with a third one than can move just because it is not fixed to the two first.

Consequently, the circuit can change its form....



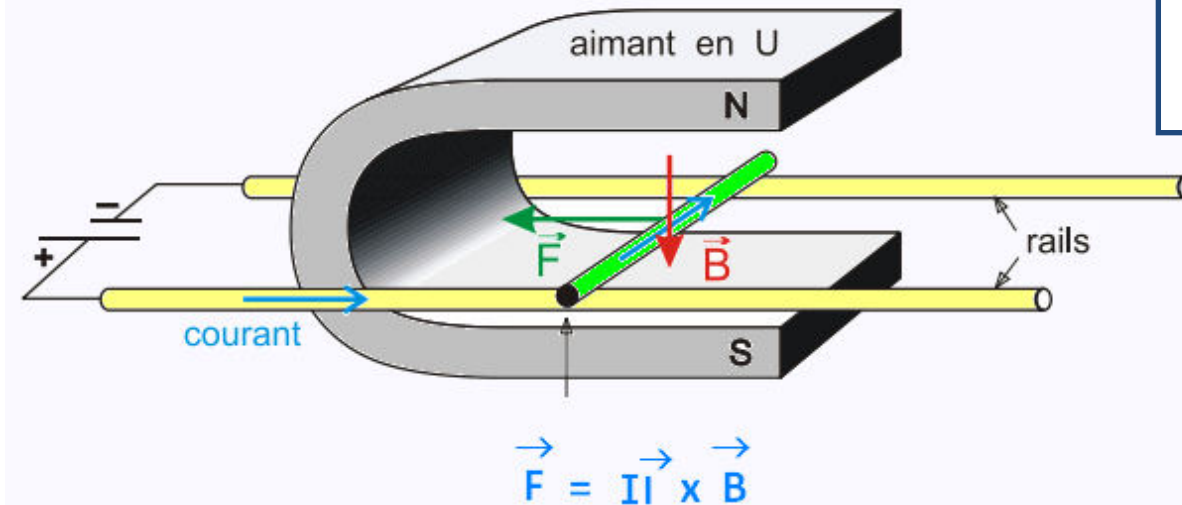
*Rail in french*





## The principle

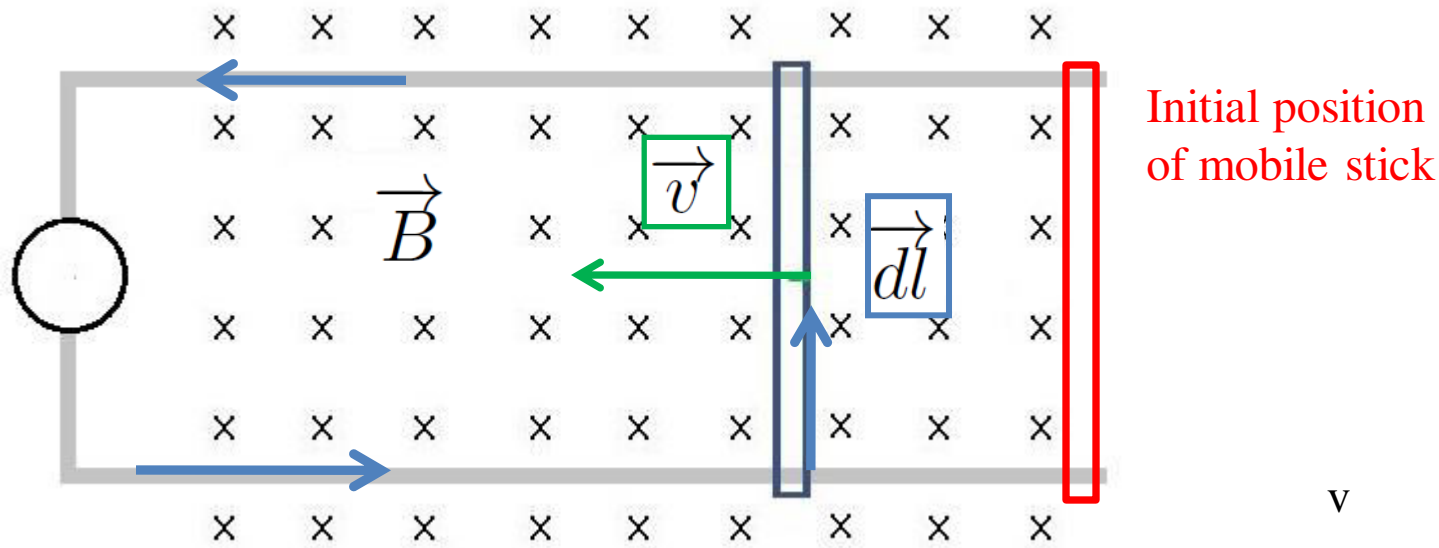
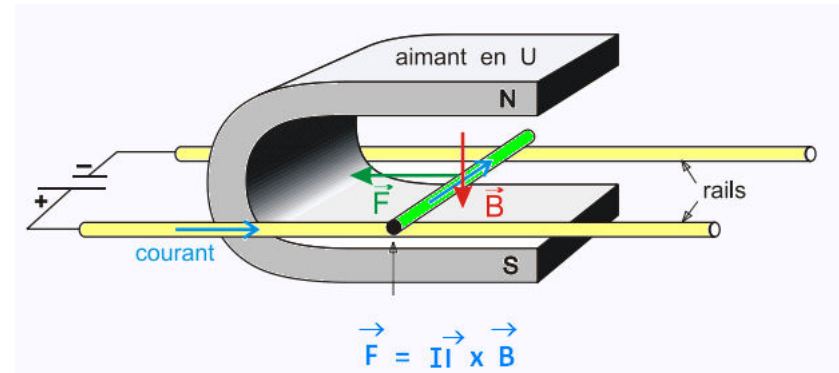
- We connect the « rail » to a generator being able to generate an electric current  $i$ .
- When turning the current on, nothing happens.
- However, when diving the system in a constant magnetic field, for instance the one created by a U-Magnet (*aimant en U*), it appears that the small stick (**in green**) starts to move.
- The reason is given by the action of the **Laplace force**.
- If we change the current direction, or the magnetic field direction, the force will be in the opposite direction.



$$\vec{F} = \int i d\vec{l} \wedge \vec{B}$$

## View from Samaliot

The mobile stick moves with a velocity  $v$   
 The  $\times$  represent direction of magnetic field

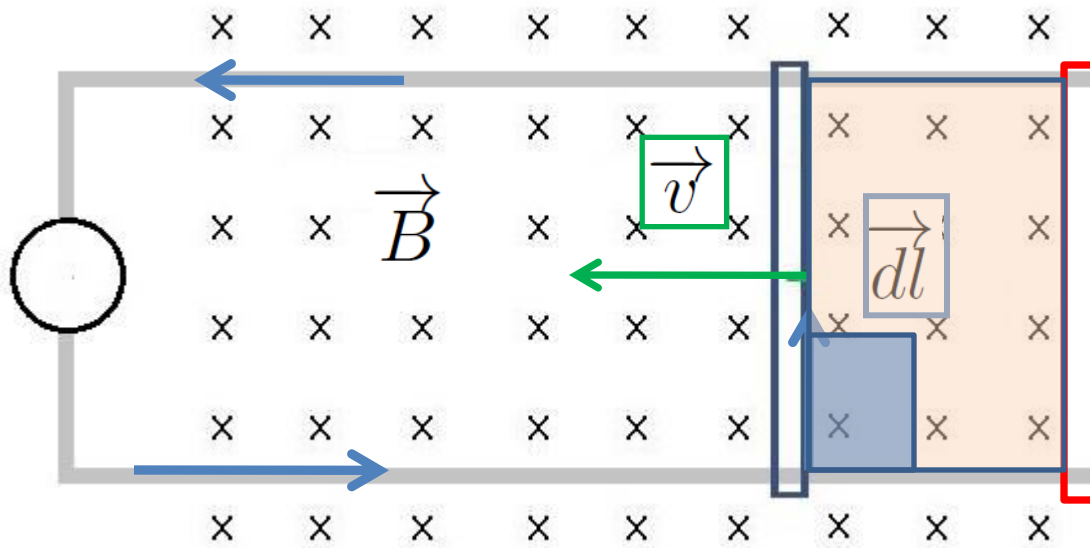
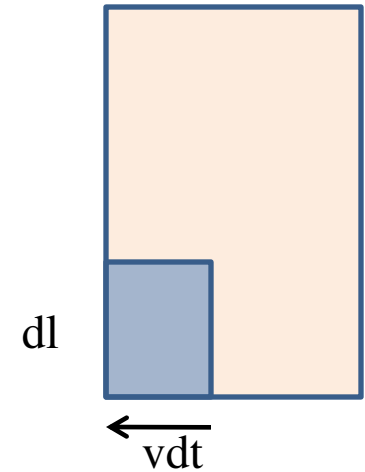


Direction of electric current

Surface covered by the mobile stick during the motion

Elementary surface covered during time  $dt$ : given by  $dS = vdt \cdot dl$

Oriented surface is  $\vec{dS} = (\vec{v} dt \wedge \vec{dl}) = dl v dt \vec{n}$



Initial position  
of mobile stick

Direction of electric current

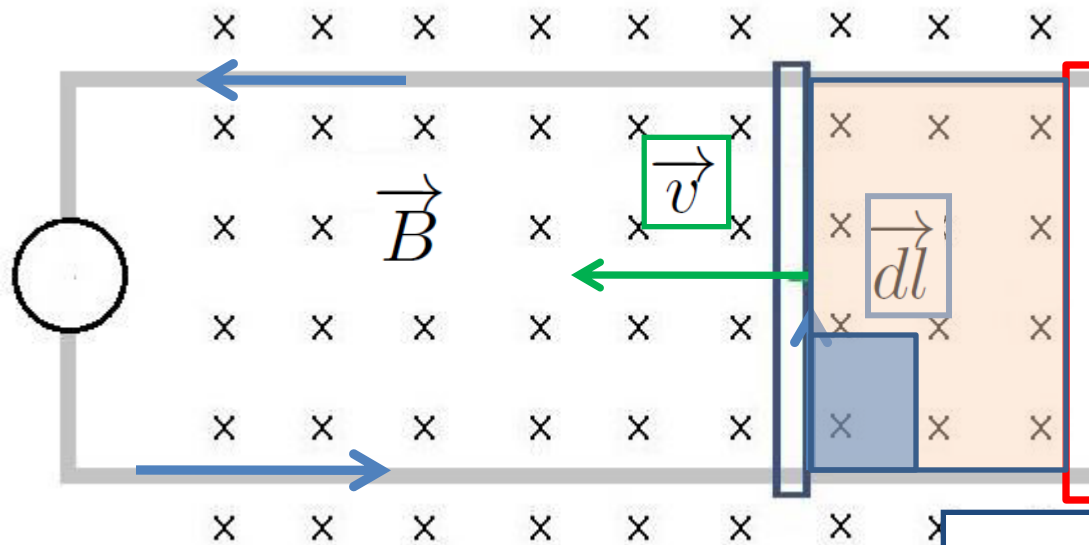
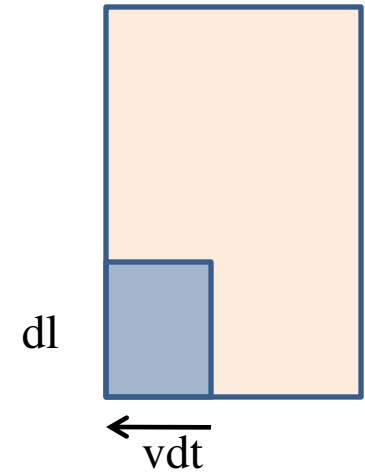
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Elementary Flux through  $dS$  is

$$\begin{aligned} \delta^2 \Phi &= \vec{B} \cdot (\vec{v} dt \wedge \vec{dl}) \\ &= \vec{dl} \cdot (\vec{B} \wedge \vec{v} dt) \end{aligned}$$



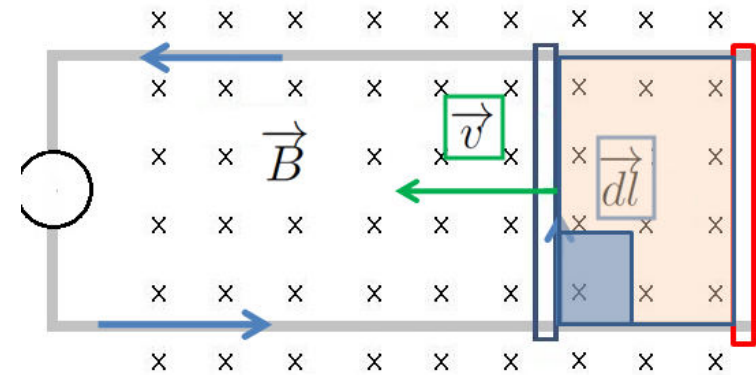
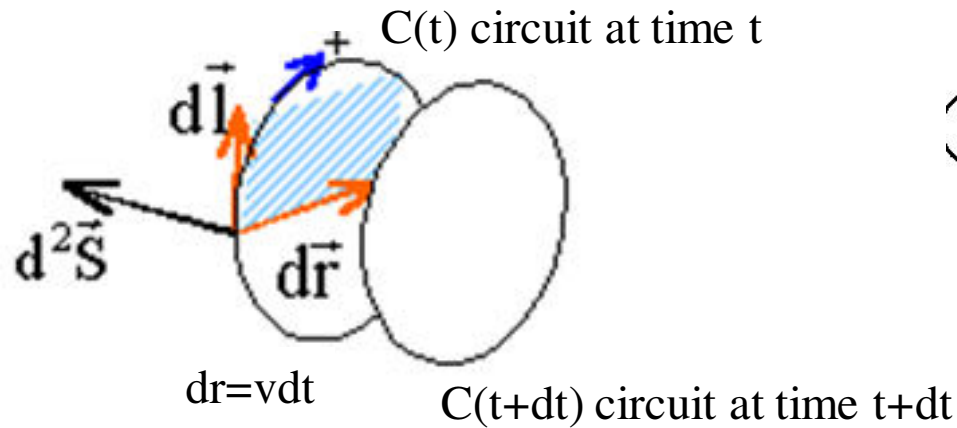
Initial position  
of mobile stick

And for the circuit

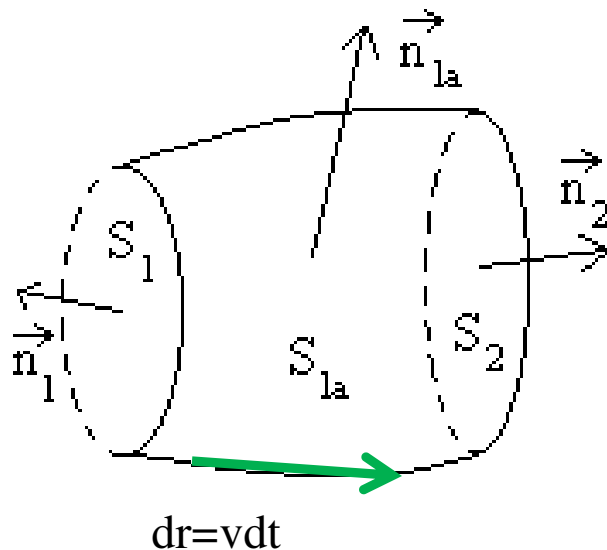
$$\delta \Phi = dt \oint \vec{dl} \cdot (\vec{B} \wedge \vec{v})$$

Direction of electric current

## Interlude: Flux through a moving circuit



$\Phi_{sla}$  is the  $\delta\Phi$  in



## Flux conservation

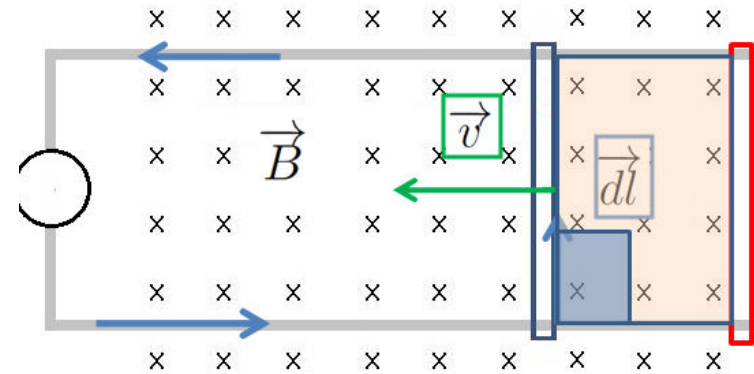
$$-\Phi_{S1}(t) + \Phi_{Sla} + \Phi_{S2}(t+dt) = 0$$

$$\Phi_{Sla} = \Phi_{S1}(t) - \Phi_{S2}(t+dt) = d\Phi$$

$$\delta\Phi = d\Phi$$

Finally we have

$$d\Phi = dt \oint \vec{dl} \cdot (\vec{B} \wedge \vec{v})$$



And by expressing the flux appears also an **electromotive force**

$$\begin{aligned} \frac{d\Phi}{dt} &= \oint \vec{dl} \cdot (\vec{B} \wedge \vec{v}) = \oint (\vec{B} \wedge \vec{v}) \cdot \vec{dl} \\ &= - \oint (\vec{v} \wedge \vec{B}) \cdot \vec{dl} = - \oint \vec{E}_m \cdot \vec{dl} = -e \end{aligned}$$

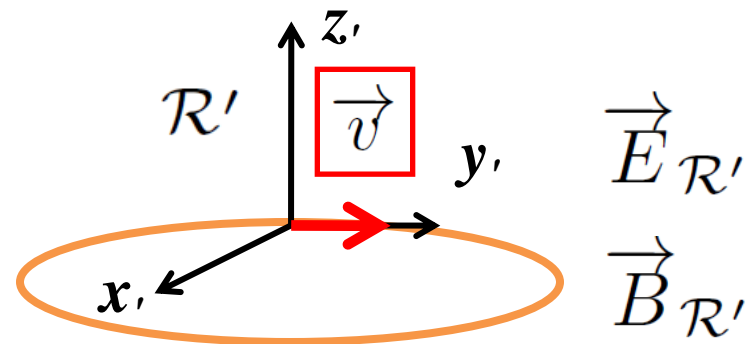
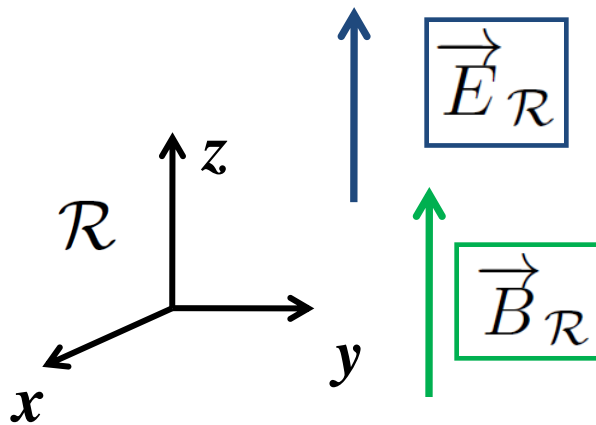
With the **electromotive field**

$$\vec{E}_m = \vec{v} \wedge \vec{B}$$

## Generalization of Lorentz electromotive field

Be a **fixed frame  $\mathcal{R}$**  in which we set an electric field (with a capacitor for instance) and a magnetic field (with coils or a magnet for instance)

Be a *moving circuit* attached to the **moving frame  $\mathcal{R}'$**  at velocity  $\vec{v}$

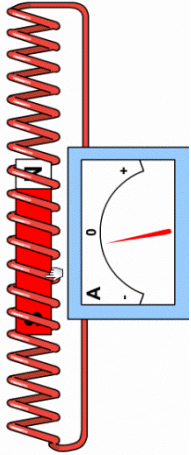


The electric field and the magnetic field in the moving frame are given by

$$\vec{E}_{\mathcal{R}'} = \vec{E}_{\mathcal{R}} + \vec{v} \wedge \vec{B}_{\mathcal{R}}$$

$$\vec{B}_{\mathcal{R}'} = \vec{B}_{\mathcal{R}}$$

# Electromagnetic Induction -L2



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iii) Lenz law

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i) example with « rail of Laplace ». Deformable circuit

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## 2) Theoretical description

## 3) Inductance and self inductance

## 4) Applications

i) Electric transformer

iii) Induction heating

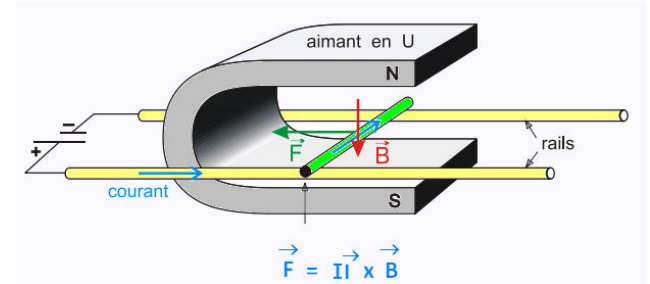
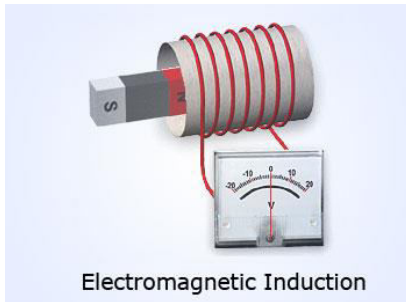
ii) Electro-mechanical conversion: DC and AC motors



## RESULTS ABOUT INDUCTION

**A time-dependent Magnetic field in a rigid circuit** AND a **constant magnetic field in a moving circuit** can generate an electromotive force  $e$ .

$$\frac{d\Phi}{dt} = -e = - \oint \vec{E}_m \cdot d\vec{l}$$



$$\vec{E}_m = - \frac{\partial \vec{A}}{\partial t}$$

Neumann electromotive field

$$\vec{E}_m = \vec{v} \wedge \vec{B}$$

Lorentz electromotive field

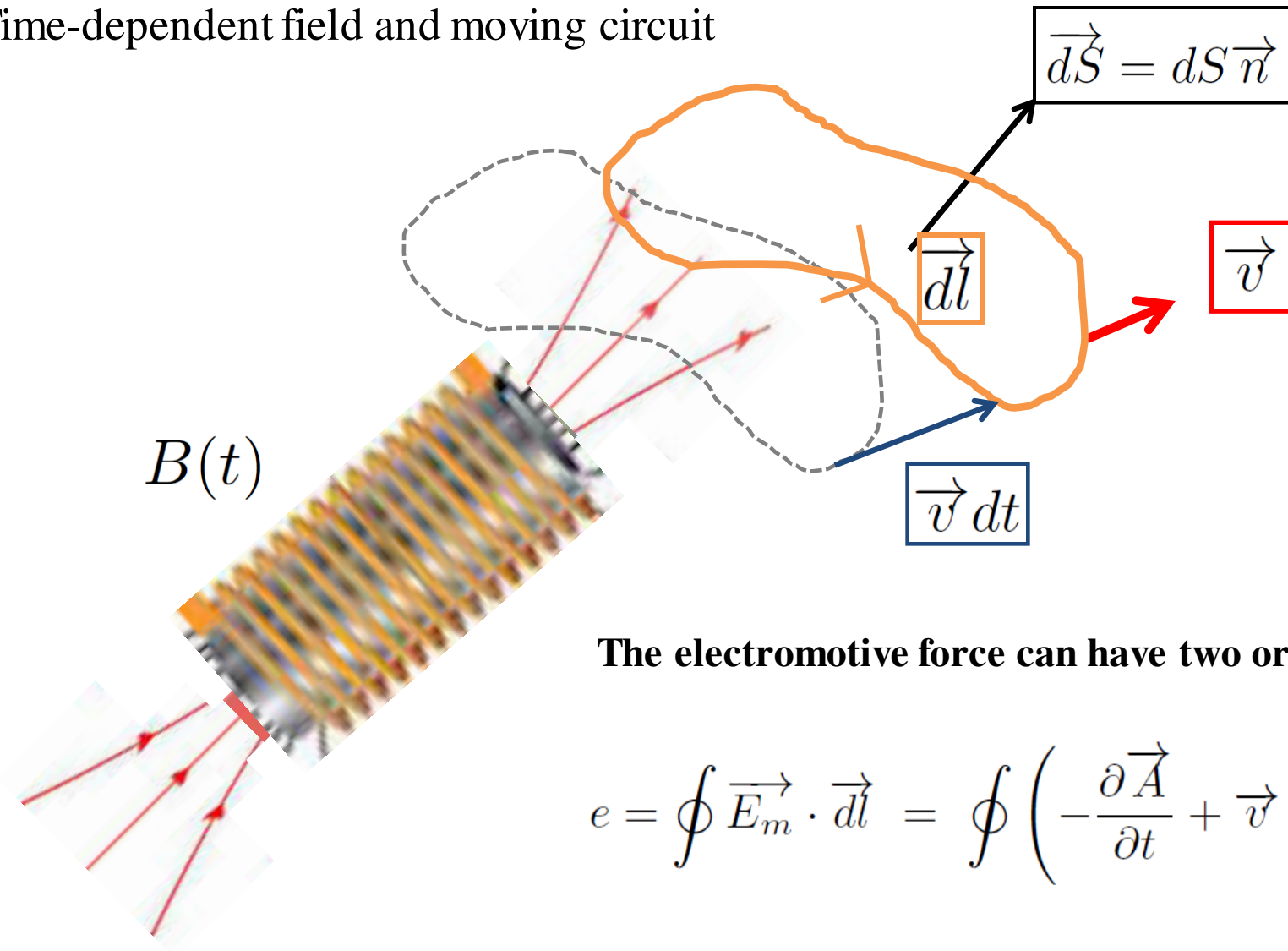
## RESULTS ABOUT INDUCTION

**A time-dependent Magnetic field in a rigid circuit** AND a **constant magnetic field in a moving circuit** can generate an electromotive force  $\mathcal{E}$ .

$$\vec{E}_m = -\frac{\partial \vec{A}}{\partial t} + \vec{v} \wedge \vec{B}$$

General case:

Time-dependent field and moving circuit

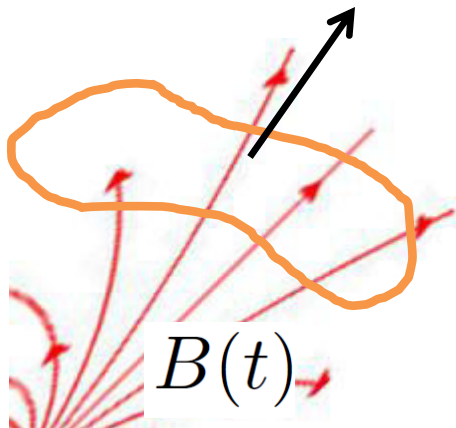


The electromotive force can have two origins

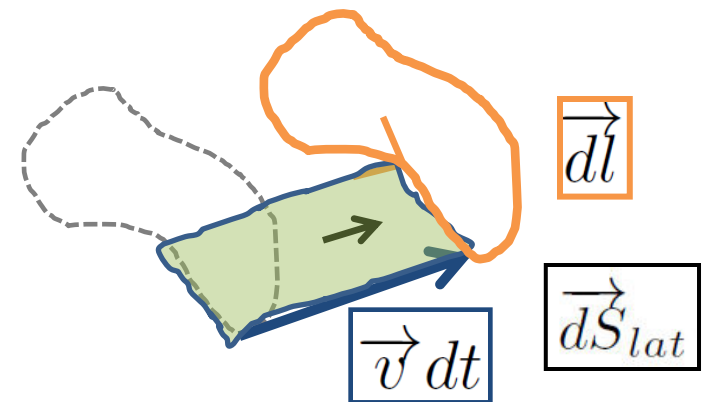
$$e = \oint \vec{E}_m \cdot d\vec{l} = \oint \left( -\frac{\partial \vec{A}}{\partial t} + \vec{v} \wedge \vec{B} \right) \cdot d\vec{l}$$

$$\begin{aligned}
 e &= \oint \vec{E}_m \cdot d\vec{l} = \oint \left( -\frac{\partial \vec{A}}{\partial t} + \vec{v} \wedge \vec{B} \right) \cdot d\vec{l} \\
 &= -\oint \frac{\partial \vec{A}}{\partial t} \cdot d\vec{l} + \oint \left( \vec{v} \frac{dt}{dt} \wedge \vec{B} \right) \cdot d\vec{l} \\
 &= -\frac{\partial}{\partial t} \iint \text{rot } \vec{A} \cdot d\vec{S} - \frac{1}{dt} \oint \left( \vec{v} dt \wedge d\vec{l} \right) \cdot \vec{B} \\
 &= -\frac{d}{dt} \iint \vec{B} \cdot d\vec{S} - \frac{1}{dt} \underbrace{\oint d\vec{S}_{lat} \cdot \vec{B}}_{\delta\Phi = d\Phi} \\
 &= -\frac{d\Phi^{\text{Neumann}}}{dt} - \frac{d\Phi^{\text{Lorentz}}}{dt}
 \end{aligned}$$

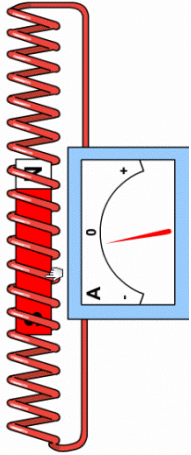
$$d\vec{S} = dS \vec{n}$$



$$\frac{d\Phi}{dt} = -e$$



# Electromagnetic Induction -L2



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## 2) Theoretical description

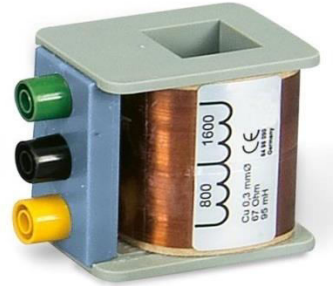
## 3) Inductance and self inductance

## 4) Applications

i) Electric transformer

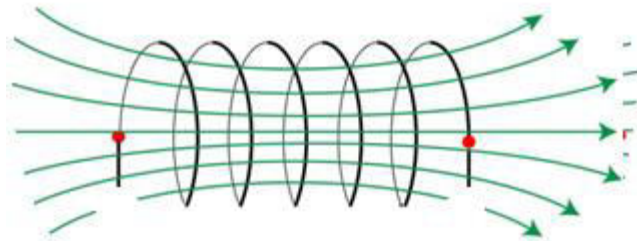
iii) Induction heating

ii) Electro-mechanical conversion: DC and AC motors



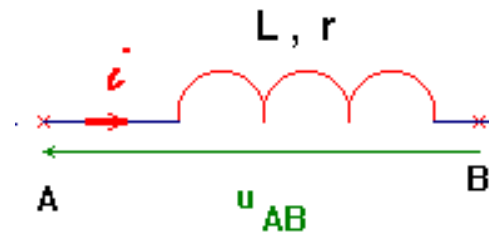
## Phenomena of self-induction: 1832

When a current propagates in a coil, the current generates a magnetic field leading to a flux through the coil itself.



$$\vec{B} = \mu_0 n i \vec{e}_z$$

$$\Phi = BS$$

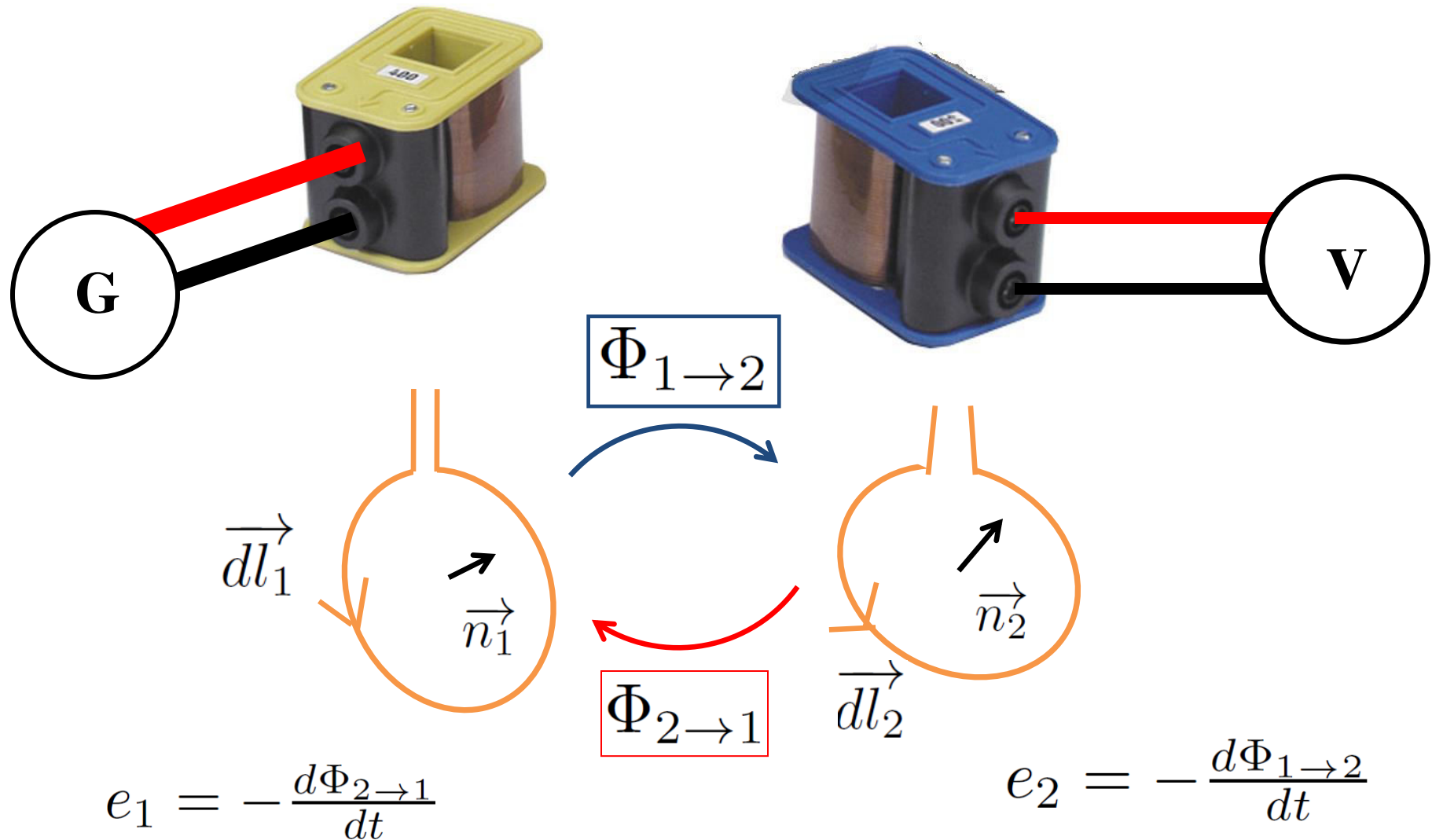


**Joseph Henry**  
1797-1868

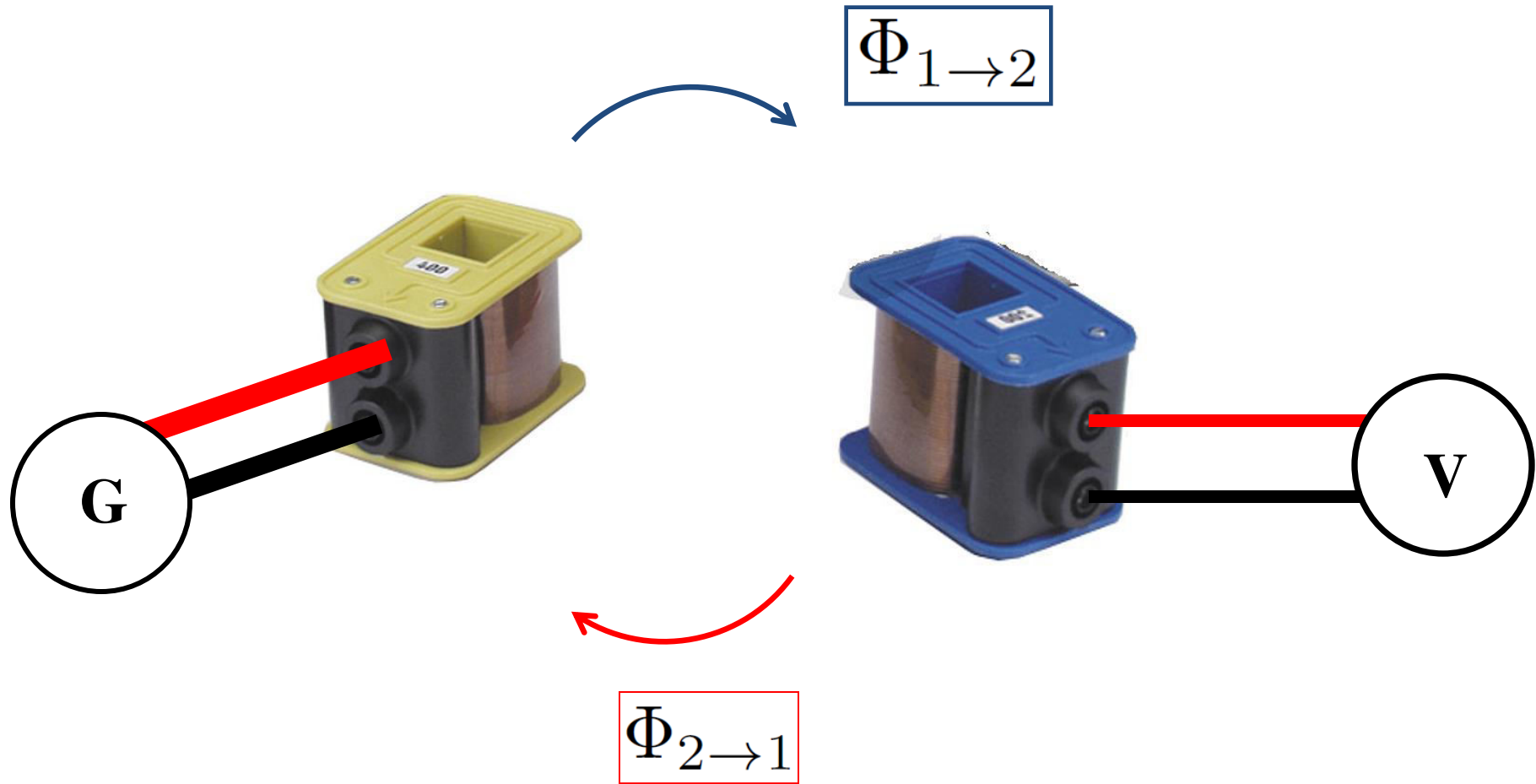
Consequently appears a negative electromotive force

$$U_L = -e = L \frac{di}{dt}$$

**Mutual inductance:** coupling between two coils L1 and L2:

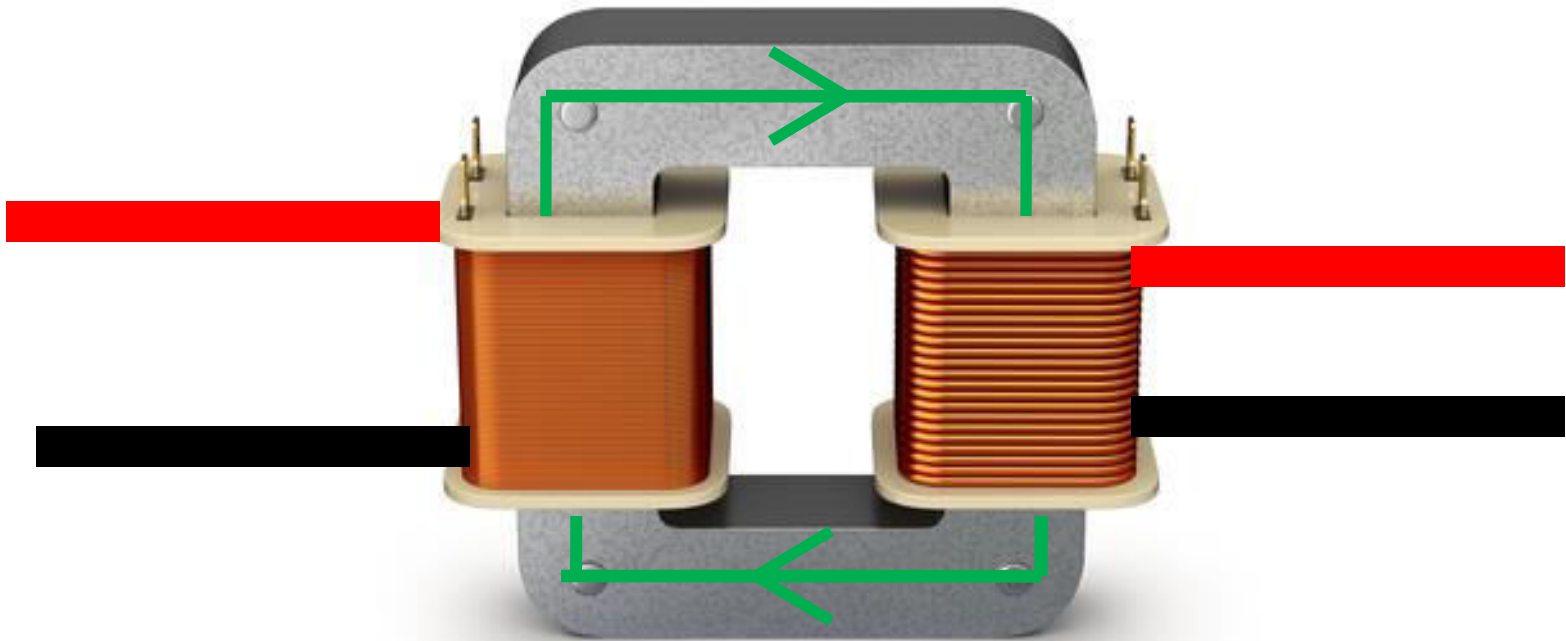


How to optimize the Fluxes ??

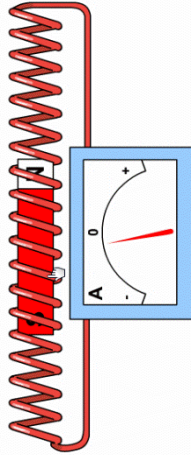




Use of a ferromagnetic core that will kept and drive the magnetic field lines from The first coil to the second one



# Electromagnetic Induction -L2



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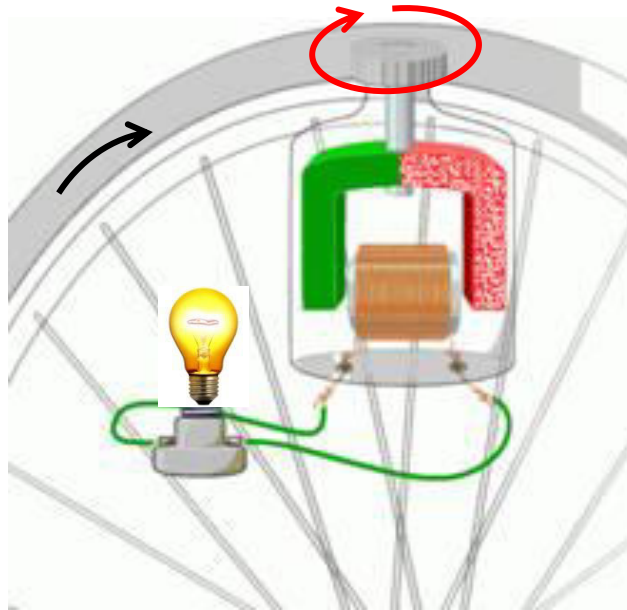
iii) Induction heating

ii) Electro-mechanical conversion: DC and AC motors; Loud speaker

## Dynamo / Electric generator

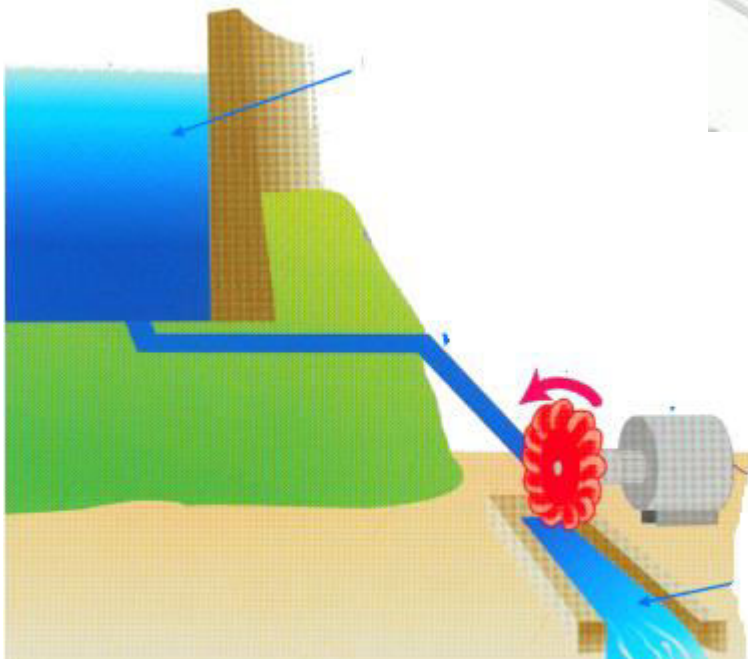


Rotation of the wheel leads to the rotation of the magnet

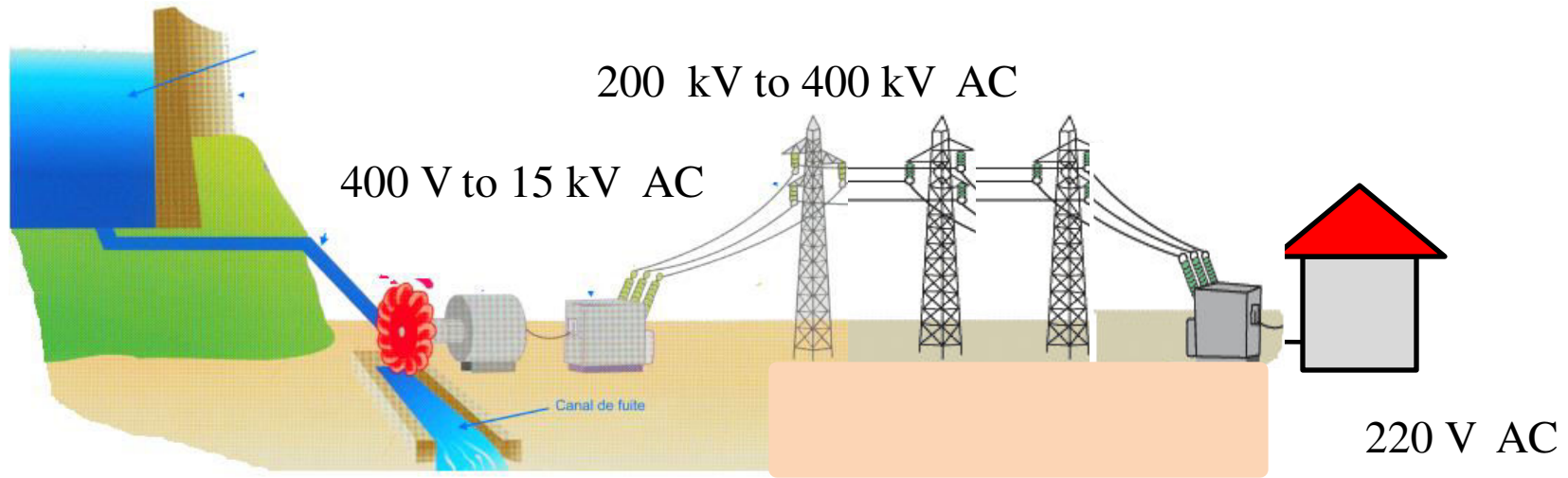


A time depending  
Magnetic flux is created  
In the coil and it generate  
A voltage used to make  
Shining a lamp

## Electric central

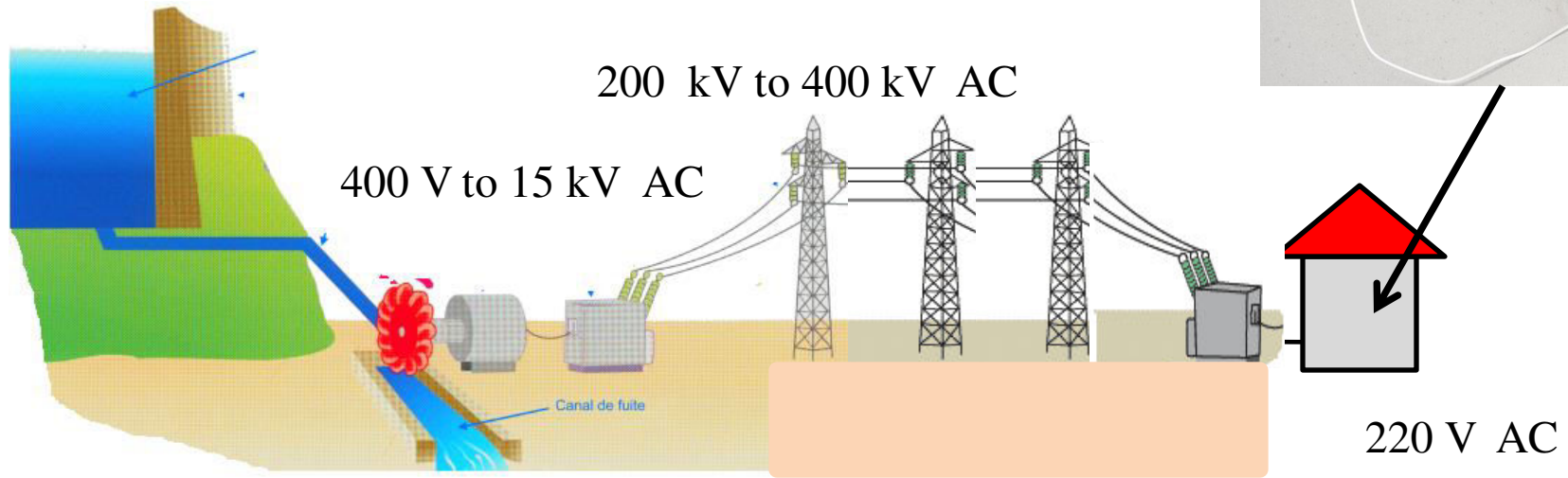


## Dynamo / Electric generator



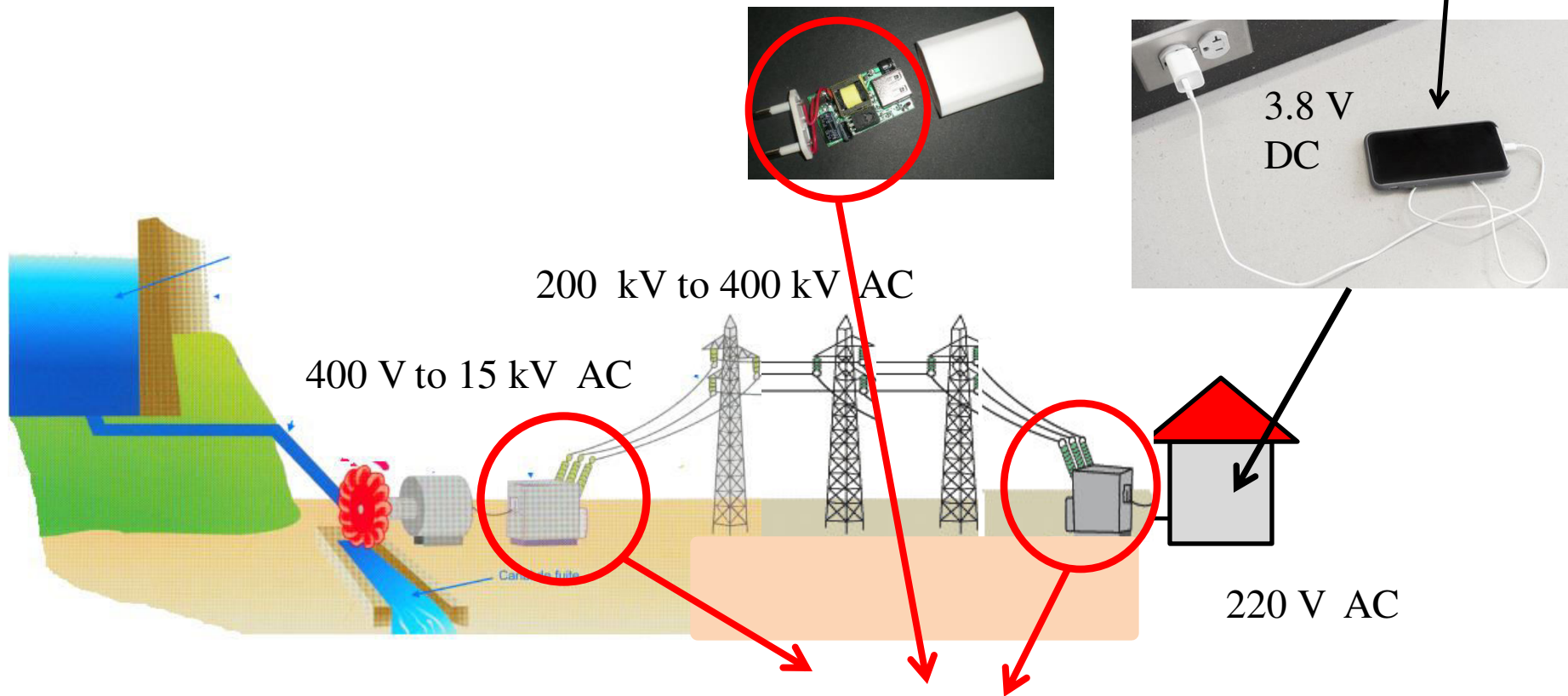
## Dynamo / Electric generator

New drug of Homo sapiens sapiens



## Dynamo / Electric generator

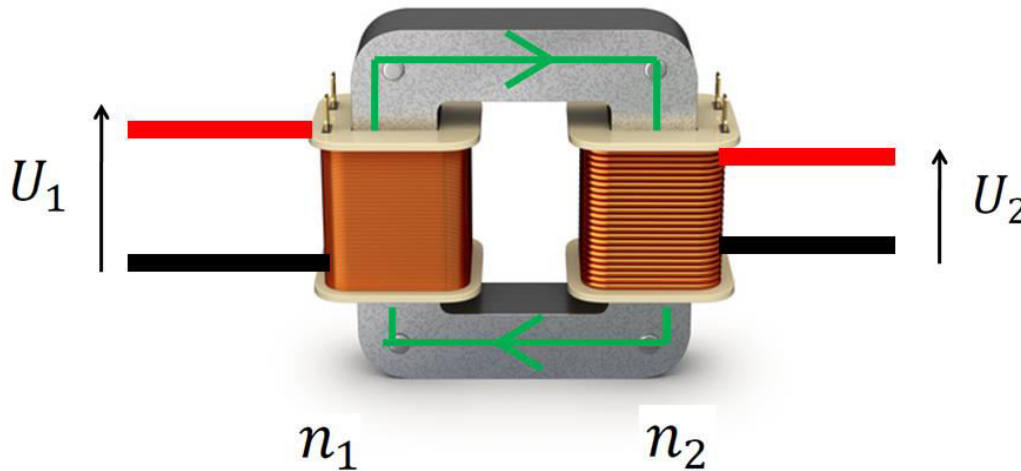
New drug of Homo sapiens sapiens



**Electric Trasnformer**

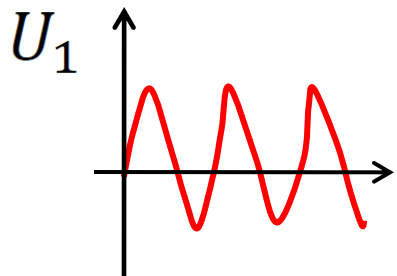
## Principle of an electric transformer

Increasing or decreasing an output voltage

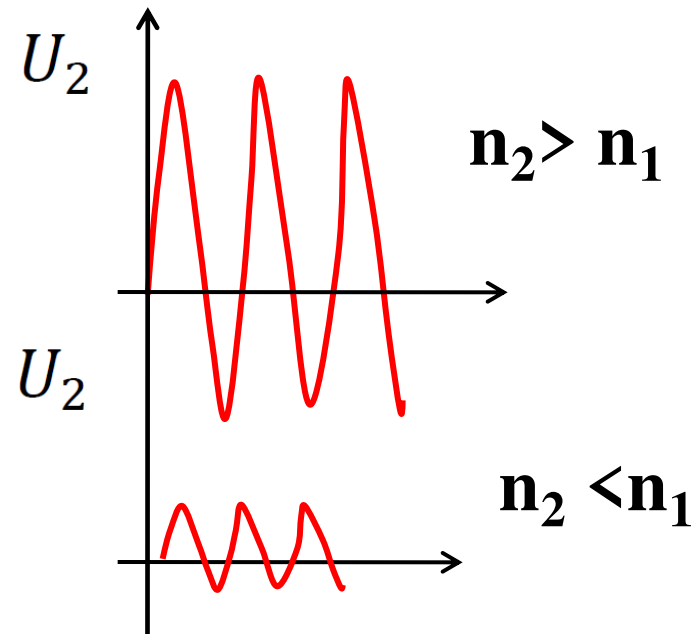


Ideal transformer

$$\frac{U_2}{U_1} = \frac{n_2}{n_1}$$

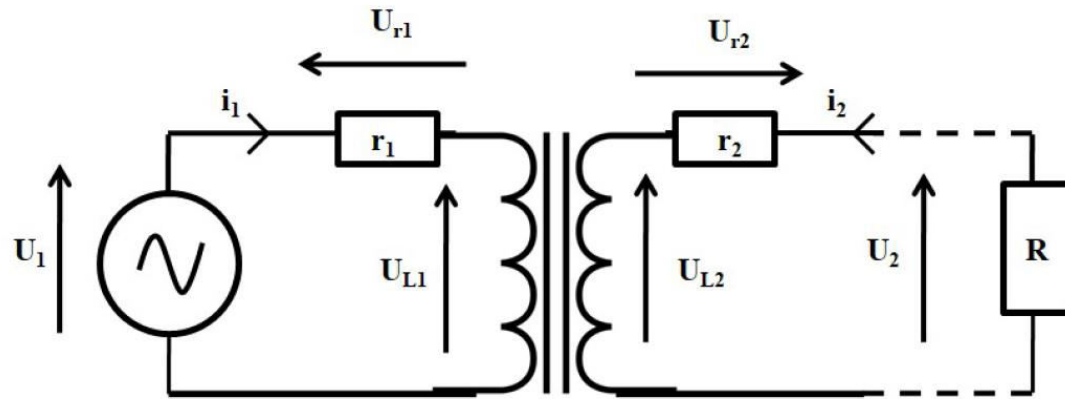


$$U_2 = \frac{n_2}{n_1} U_1$$





## PW n° 2: Study of an electric transformer

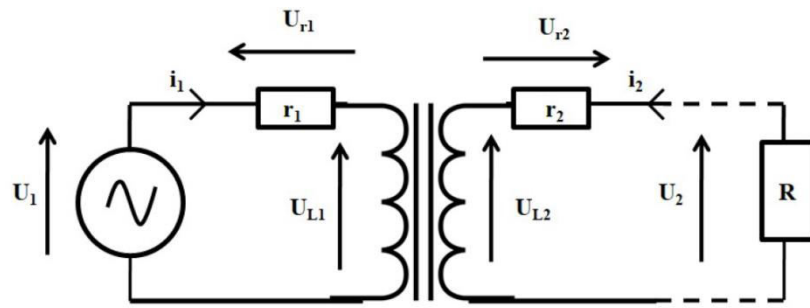


$$U_1 = r_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$U_2 = r_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$



## Transformer yield



Input power:

$$P_1 = U_1 i_1$$

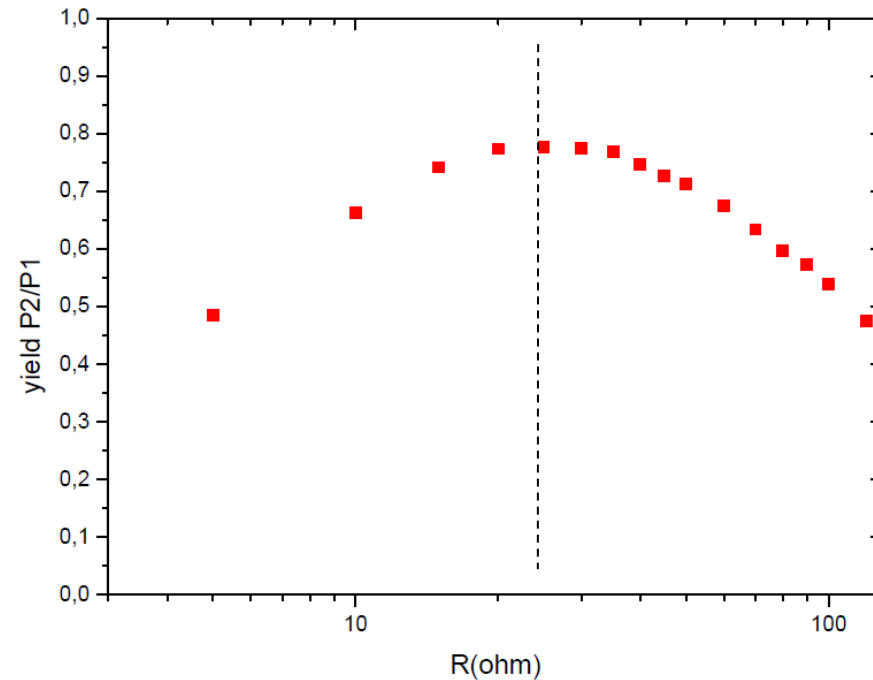
Output power:

$$P_2 = U_2 i_2$$

Yield

$$\eta = \frac{P_2}{P_1}$$

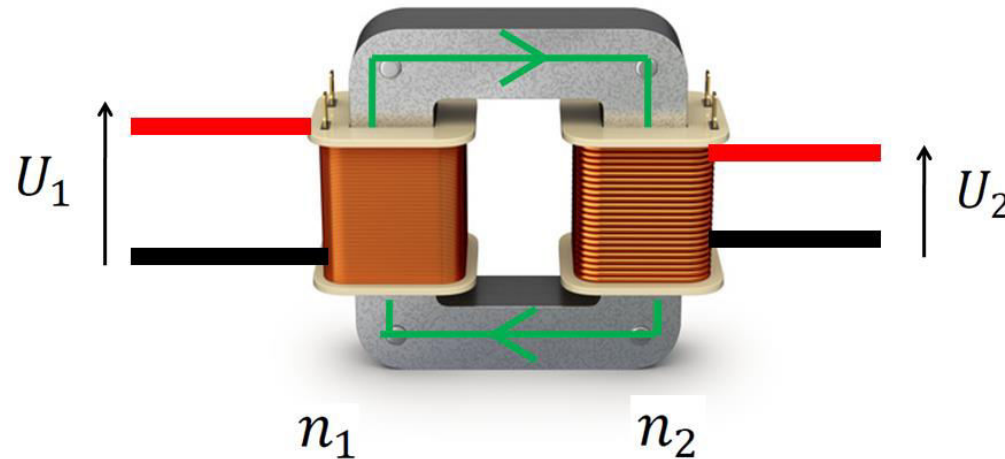
Results from our Lab (anonymous teacher)



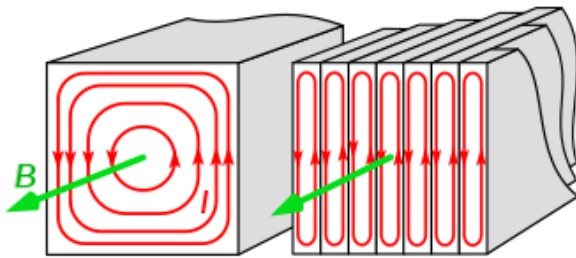
Usually, transformers have a yield of 95%

The losses are mainly due to the i) **joule effect** (copper losses of electric elements)  
ii) the **iron losses (magnetic origin)**

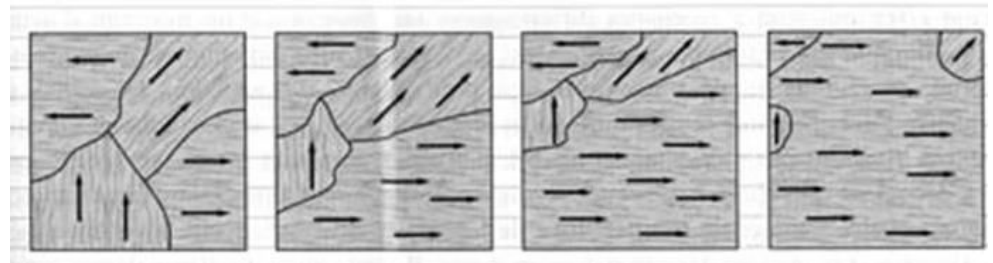
## About Iron losses



The magnetic field propagating in the magnetic material (and conductor) induces also an electromotive force leading to the creation of surface currents INSIDE the magnetic core. These currents lose energy due to Joule effect. (*courants de Foucault* or *Eddy currents*)

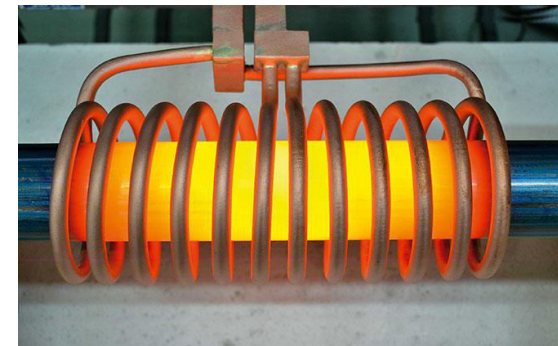
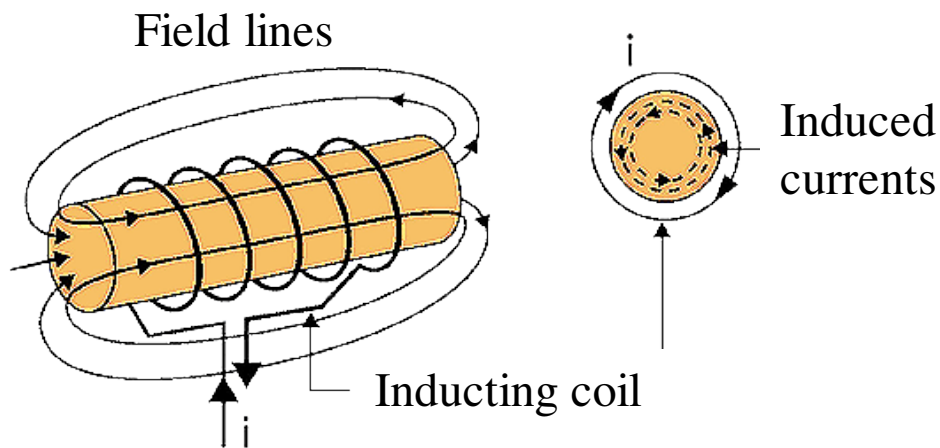


Friction between the walls of magnetic domains

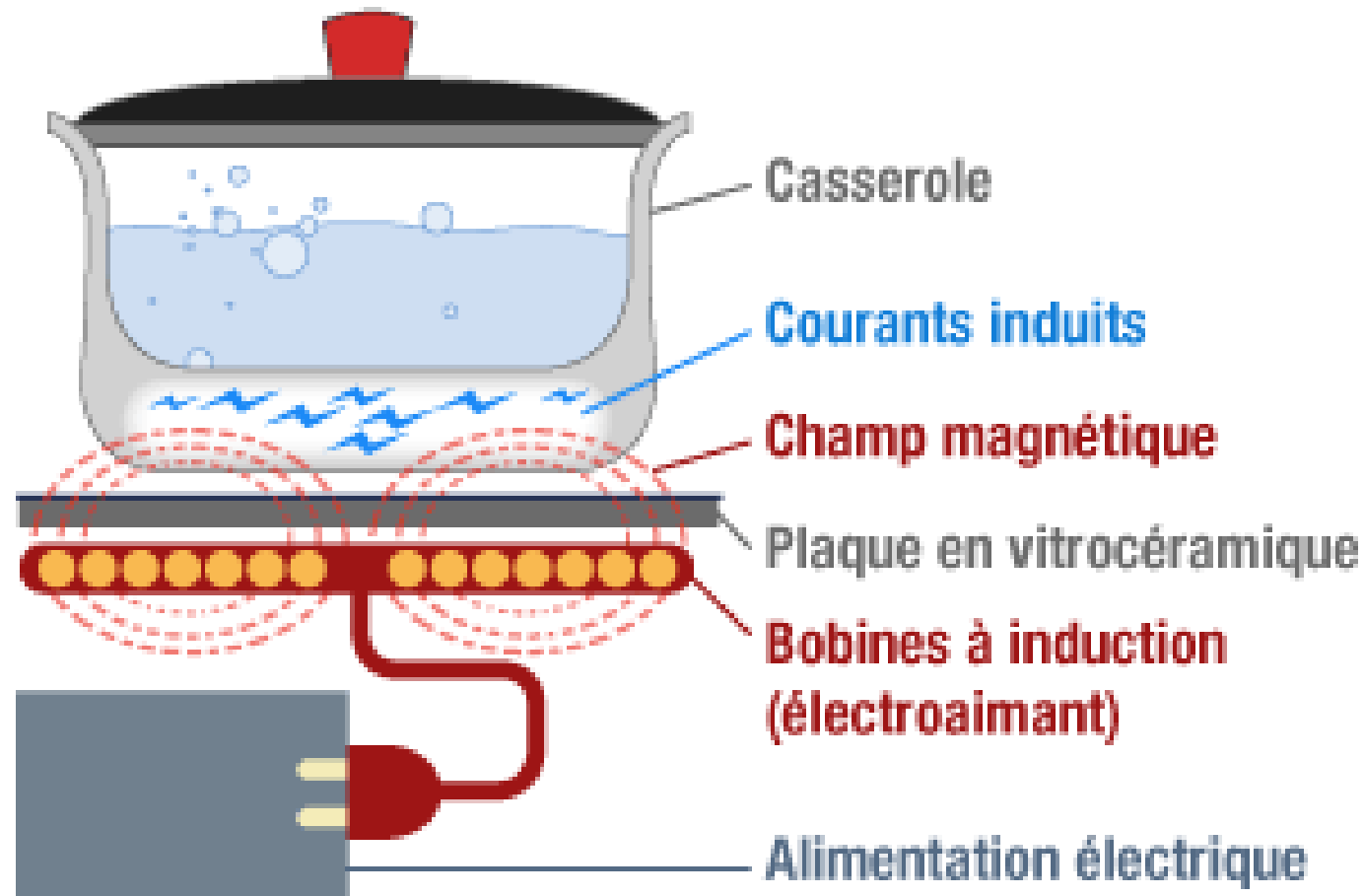


## Induction heating

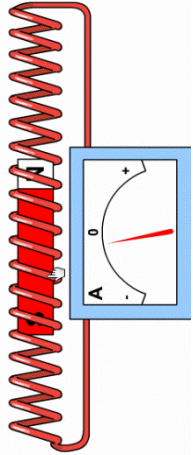
Foucault currents (I am subject also to mental sickness) can be used to increase temperature of a metal submitted to a magnetic field



## Induction heating



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## Electro-mechanical conversion

