UFAZ-L1-Electrostatics

Part 3: Gauss theorem

Statement:

$$\oiint \vec{E}.\overrightarrow{dS} = \frac{Q_{int}}{\varepsilon_0}$$

Exercise 1: Infinite charged wire

Remember the expression of the electric field at a distance z of an infinite wire having an uniform lineic electric charge distribution $\lambda > 0$.
Recover this result using the symmetries and the Gauss theorem.

Exercise 2: Infinite charged plane

Remember the expression of the electric field at a distance z of an infinite plane having an uniform surface electric charge distribution $\sigma > 0$.
Recover this result using the symmetries and the Gauss theorem.

Exercise 3: Infinite charged cylinder

1) We consider an infinite cylinder of radius R having a uniform volumic charge density ρ. Using Gauss theorem determine the value of the electric field at a distance z from the axis of the cylinder for $z < R$ and then for $z > R$. Deduce in each case the electric potential V.

2) We assume now that the infinite cylinder has a uniform charge distribution σ only on its surface. Determine the expression of the electric field and the electric potential inside and outside the cylinder respectively for $z < R$ and then for $z > R$

Exercise 4 : Charged Sphere

1) We consider a sphere of radius R having a uniform volumic charge density ρ. Using Gauss theorem determine the value of the electric field at a distance r from the sphere center for $r < R$ and then for $r > R$. Deduce in each case the electric potential V.

2) We assume now that the sphere has a uniform charge distribution σ only on its surface. Determine the expression of the electric field and the electric potential inside and outside the sphere respectively for $z < R$ and then for $z > R$.

Part 4: Some applications

Exercise 1: Electric potential and electric field in the dipolar approximation

We consider a dipole made of charges +q and –q separated by a distance a. In a two dimension Cartesian system *Oxy*, the negative charge -q is located at position N (-a/2 ; 0) and the positive charge q at position ¨P (a/2; 0).

1) What is the expression of the electric potential at a point M of coordinates M(x,y) ?

2) Show that $\|\overrightarrow{PM}\|=\sqrt{PO^2 + OM^2 + 2\overrightarrow{PO}.\overrightarrow{OM}}$ and deduce the expression of $\|\overrightarrow{NM}\|$.

3) The dipolar moment is defined as $\vec{p} = q\overrightarrow{NP}$ and we note $\overrightarrow{OM} = \vec{r}$. We work within the dipolar approximation assuming that the distance of observation r is large compared to the size of the dipole a: $a \ll r$. By performing a Taylor development at first order in a/r, show that the electric potential V(r), at a distance r can be approximated to the following expression :

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}.\vec{r}}{r^3}$$

4) Calculate the expression of the electric field using $\vec{\mathrm{E}} = -\vec{\nabla} V$. What is the difference between the field lines of a dipole in the dipolar approximation and an usual dipole?

One may use for that exercise:

- *Taylor development of* $(1+x)^\alpha$ *at first order :* $(1+x)^\alpha \approx 1 + \alpha x$
- *Gradient operator in polar coordinates:* $\vec{\nabla} f(r,\theta) = \frac{\partial f(r,\theta)}{\partial r}\overrightarrow{e_r} + \frac{1}{r}\frac{\partial f(r,\theta)}{\partial \theta}\overrightarrow{e_\theta}$