PW n°6: Propagation and interferences of water waves

1.Objectives of the experiments

Study the dispersion of water waves with a ripple tank. Deduce the surface tension. Referring to Huygens' principle and theory of interference and diffraction, we will explain experimental patterns.

2. Principles

2.1 About water waves

Mechanic waves need a material medium to propagate. When the medium is non dispersive the phase velocity $v = \omega/k$ is constant wathever the frequency $f$or angular frequency ω. In dispersive medium, the phase velocity depends on the frequency. In the case of water waves, the recoiling force on an oscillating (or better, orbiting) water particle is determined by its weight (gravity) and by the surface tension σ (capillary). The surface tension σ . This phenomena occurs usually at the interface of two different media for which microscopical organization of molecules can be different and thus induce forces on a length scale called capillary length (see for instance in Fig 1a, the example of the bending of a liquid surface at the interface between atmosphere, the liquid and a solid wall). For small depth, capillary effect are dominant while for larger depth the effect is mainly due to gravity. It is possible to
show that the phase velocity $v = \omega/k$ has the following expression:

$$v = \sqrt{\frac{g\lambda}{2\pi} + 2\pi\frac{\sigma}{\rho\lambda}}\sqrt{\tanh\left(\frac{2\pi h}{\lambda}\right)}$$

where $g = 9.81\ m/s^2$ is the gravitational acceleration, $\sigma = 0.07\ N/m$ the surface tension at the temperature of 321.5 Kelvins and $\rho = 1000$ kg.m^{-3} is the water density. We can rewrite the previous equation with the help of capillary length $l_c = \sqrt{\frac{\sigma}{\rho g}}$:

$$v = \sqrt{\frac{g}{2\pi\lambda}(\lambda^2 + (2\pi l_c^2)^2)}\sqrt{\tanh\left(\frac{2\pi h}{\lambda}\right)}$$

Consequently, it is possible to compare the capillary length l_c to the wavelength λ and to simplify the above expression. How is transformed the expression of the phase velocity:

- When gravity waves will be dominant meaning $\lambda \gg 2\pi l_c$?
- When capillary waves will be dominant meaning $\ll 2\pi l_c$?

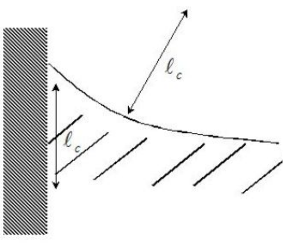

Fig 1.a Illustration of capillary effects

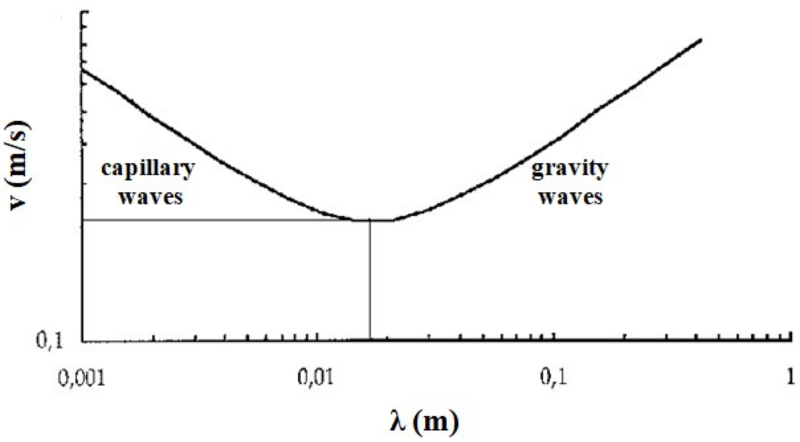

Figure 1b. Dispersion relation : phase velocity as a function of the wavelength And the illustration of the two domains

2.2 Apparatus

The water waves are generated in a wave trough (or tank) filled with water ; the bottom of the trough consists of a glass pane. To generate waves, the oscillations of a membrane, which are generated in the supply unit by variations in air pressure, are transmitted to the surface of the water via wave exciters. If the beam from a point-type lamp is shone through the wave trough, the wave crests act as collecting lenses to create bright lines on the observation screen; the wave troughs act as dispersing lenses to cause dark lines (see Fig 3). You can observe the wave propagation or you can display a stationary wave image with the help of a stroboscopic lamp that is synchronized with the frequency of the exciter membrane.

Figure 3a. General view of the wave tank

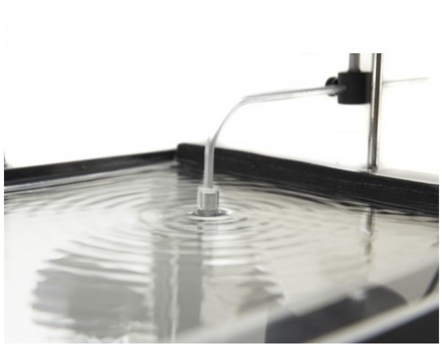

Figure 3b. Top view with mechanical source

2.3 Huygens principle in water waves

Each point in a wave front can be considered as the starting point of a "wavelet", or secondary wave, which propagates with the same velocity and wavelength of the original wave. The envelope of all wavelets is the new wave front. In the experiment it is possible to use a punctual source or to create a planar source to generate planar wave fronts. Figure 4 illustrate such phenomena.

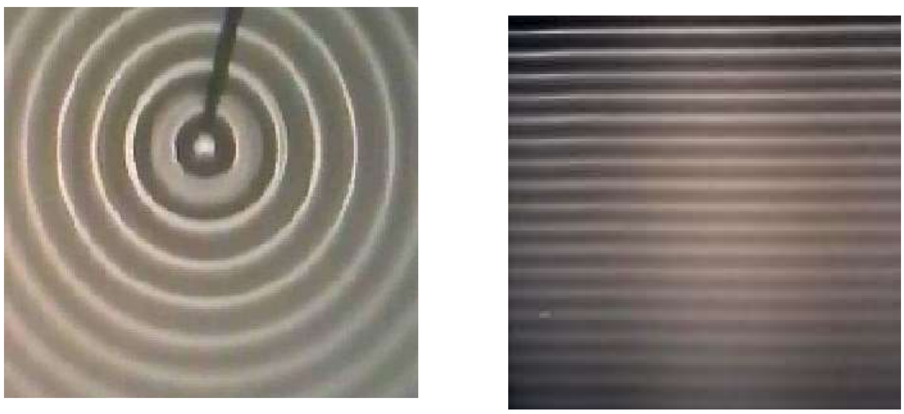

Figure 4. Propagation of plane or spherical waves

2.4 Refraction of plane or circular waves at a straight obstacle

Water waves are reflected at obstacles. According to Huygens' principle, we can see the reflected waves at the envelope of the wavelets formed at the obstacles. The reflexion at straight obstacle corresponds to the optical phenomena of reflection of light at a plane mirror. In figure 5 one can see the reflection of a plane wave at a straight obstacle. As for optical phenomena, the incident angle (with the normal to the interface) has the same value then the reflected angle. For circular wave, the wave reflected from the wall appears to emanate as a spherical wave from an image point on the other side of the wall. The center of the reflected circular waves is the "mirror" point of the exciter (image of the source).

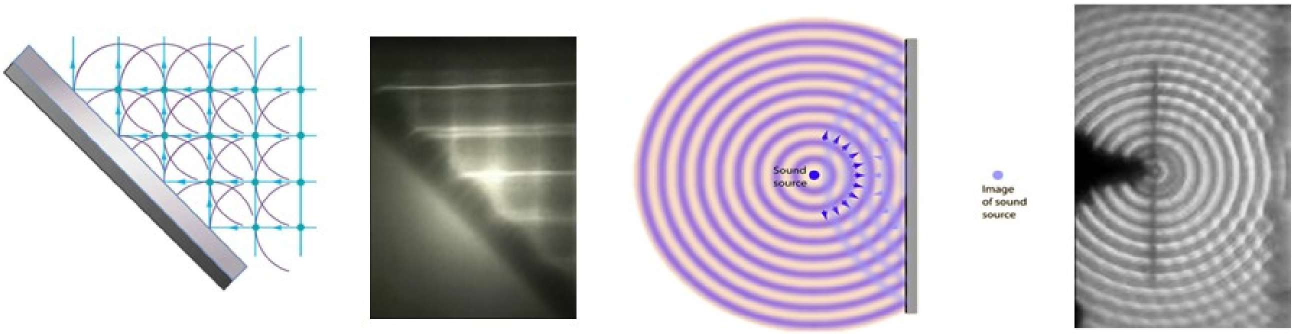

Figure 5. Reflection of planar and circular waves

2.5 Interferences

Interference is the superposition of two coherent waves. At some space regions, the amplitude of the total signal can be amplified, attenuated or even canceled out entirely. The interference phenomenon at a particular location will depend on the phase shift between the two interfering waves. When the waves have the same frequency and the same phase at origin, this phase shift between the two waves is written $\Delta\Phi = \frac{2\pi}{\lambda}\delta$ where δ is the "path difference" between the two waves at this location.

For a path difference that is an entire multiple of the wavelength $\delta = n\,\lambda$ where $n = 0, 1, 2, \ldots$ the oscillations of the individual waves are in phase and the wave superposition has its maximum amplitude. They are called **constructive interferences.** For path difference that is a half entire multiple of the wavelength $\delta = (n + \frac{1}{2})\,\lambda$ where $n = 0, 1, 2, \ldots$the oscillations of the individual waves are in phase opposition, so that theirs amplitude cancel each other out and the wave superposition has zero amplitude (assuming both sources have the same amplitude). **They are called destructive interferences**.

It is possible to show that the space regions where the path differences is constant δ =const are hyperbolas (see Figure 6.). Their location can be described by the angle, which they form with the center axis between the two punctual sources.

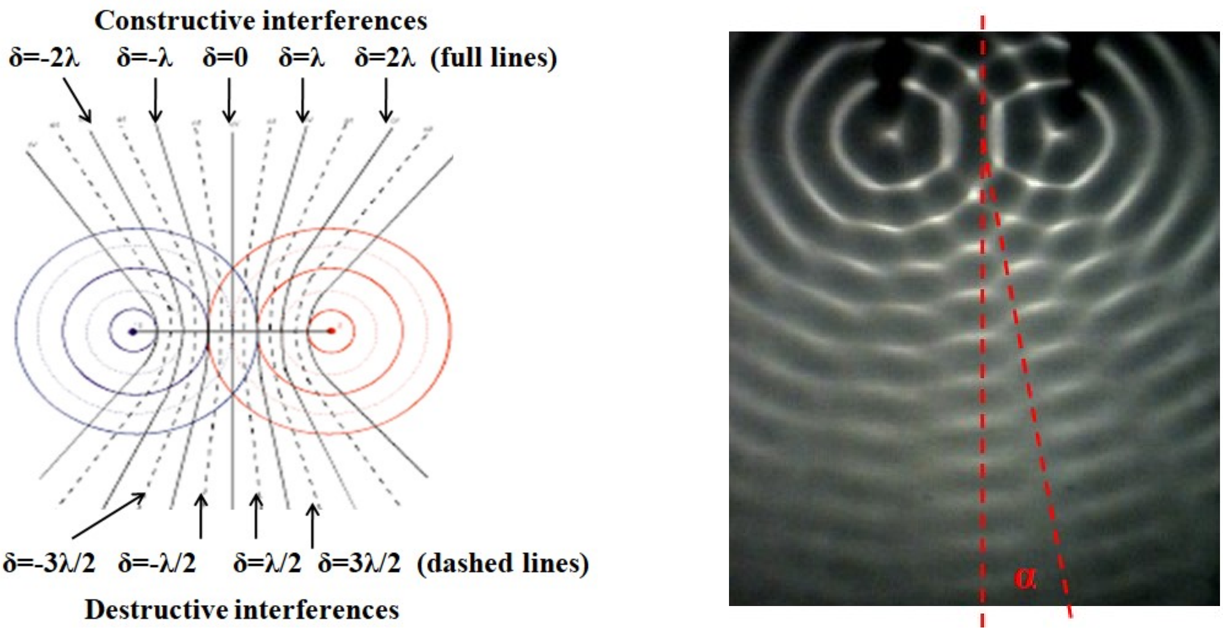

Figure 6. Interference pattern obtained with two punctual sources.

2.6 Diffraction

The waves behind an obstacle or a slit (an opening) do not only propagate in their original direction. Each point in the plane of the slit, as well as the edges of the slit or obstacle acts as point-type excitation center of circular waves, giving rise to wavelets which are superposed on each other. In the process, the amplitudes are reinforced at certain points (creation of maxima), and at others attenuated or even canceled out entirely (creation of minima). The interference phenomenon at any point is a function of the phase shift of the interfering waves at that point. Behind a small slit whose width is smaller than the wavelength, circular waves are emitted. If the slit is somewhat wider than the wavelength, minima are formed from the sides in addition to the main maxima. When a slit with a width significantly greater than the wavelength is used, the straight waves pass unchanged between the edges of the slit. In this zone, the wavelets have no phase differences with respect to one another.

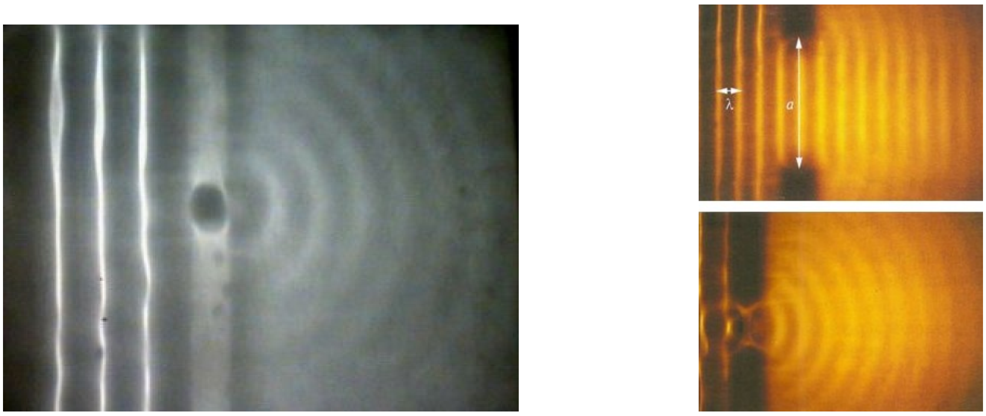

Figure 7. Diffraction phenomena observed when the size of the opening is smaller than the size of the wavelength

3. Manipulations and Experiment

3.1 Dispersion of water waves

Measuring a wavelength. Generate planar waves with the appropriate exciter. Switch on the stroboscope and generate a stationary wave image by synchronizing the system. Play with the frequency and with the amplitude to see how evolve the image and the contrast. Read the frequency of the exciter on the multimeter. Measure the distance between two wave fronts on the observation screen. To optimize your measurement, measure the distance over many wavelength and divide it by the number of wavelength you have counted. Be sure to take the image scale into consideration to determine the actual wavelength. To determine the factor conversion between the real object and its image you can measure the real size of a glass stick and then the size of its image on the screen. Apply it to the measurement of your wavelength.
Then determine the phase velocity.

Velocity as a function of the wavelength. Try to put the wave exciter at the interface air/water (avoid to dive it in the tank). For a given water height, determine the wavelength and the plot phase velocity for a range of frequencies going from 12,75 Hz to about 35 Hz.
Use origin software to build the graph $v = f(\lambda)$ illustrating the phase velocity as a function of the wavelength.
Is water a dispersive medium? Are gravity waves or capillary waves the dominant process in that case? With the obtained results, to deduce the value of the surface tension σ.

3.2 Reflection of water waves at a straight obstacle

Connect the exciter for plane waves and put the reflecting barrier in the middle of the wave trough, at an angle of 45° to the exciter. Using a paper, sketch the position of the reflecting barrier, the directions of propagation and the spacing of the wave fronts. Draw the axis of incidence and measure the incident and reflected angles. Measure and compare the wavelength in both zones. Change the orientation of the incident waves by turning the reflecting barrier. Measure and compare the incident and the reflected angle for each position. Repeat the experiments with circular waves by connecting the point-type exciter instead of the straight waves exciter.

3.3 Two-beam interferences of water waves

Connect two points-type exciter.Observe the position and number of interference maxima and minima along the axis joining the two exciters. (You may take time to set up parameters in order to observe a clear image). Sketch the excitation centers and the interference hyperbolas on a paper. Measure the wavelength λ, the distance between the two sources d and the directions at which the interference minima appear (angular position of the first destructive interference). Be sure to take the image scale into consideration to determine the actual wavelength. Reduce the distance between (if possible) and repeat the experimental steps. Compare the two interference patterns.

Then, for a given distance, change the frequency of the exciters and compare the interference patterns. How evolves the angular position when changing distance d or wavelength λ

3.4 Diffraction of water waves

Connect the exciter for straight waves and set it up parallel to the obstacle (take two stick and build with them a large opening (or slit)), at a distance of approximately 5 cm. Observe the wave image behind the slit . Bring together the two sticks in order to reduce the slit width so that the width is less than the wavelength. Observe the wave image behind the slit again. Make some comments. Repeat the experiment with different frequencies. Are the diffraction phenomena described in 2.6) confirmed by experiment ?