Exercise 1: Magnetic field created by an infinite wire of current

We consider an electric current i propagating in an electric wire. We assume that the wire can be considered as infinite in the region where we study its action on point M located at a distance z from the wire axis. With the Biot and Savart law show that the magnetic field produced by the current at a distance z from the wire is given by

$$\vec{B} = \frac{\mu_0 i}{2\pi z}\overrightarrow{u_\varphi}$$

where $\overrightarrow{u_\varphi}$ is an orthoradial vector and μ_0 is the magnetic permeability. What is the difference compared to the orientation of an electric field created by an infinite wire having static charge distribution?

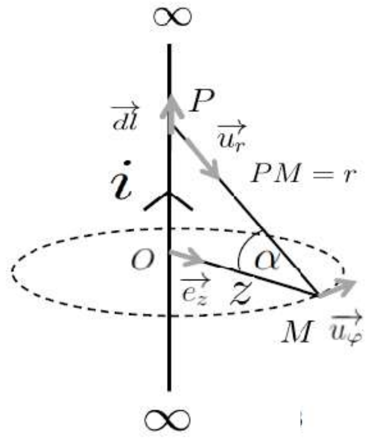

Figure 1.

Exercise 2: Magnetic field created by a loop of current

We consider a loop of current i of radius R. Show that the magnetic field created by the loop of current on axis Δ at a distance z is given by

$$\vec{B} = \frac{\mu_0 i}{2R}(\sin\alpha)^3\overrightarrow{e_z}$$

Discuss the magnetic field lines orientations with the help of the ones obtained with the infinite wire of current.

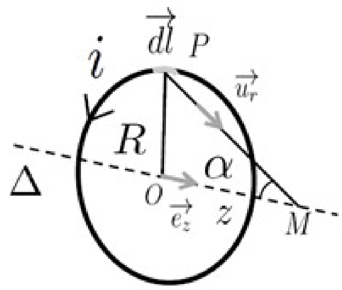

Figure 2.

Exercise 3: Magnetic field created by a finite and infinite solenoid

A solenoid is a helicoidal coil (a succesion of N loops of current). Use the result of Exercise 2 to show that the magnetic field produced by an infinite solenoid of length L is given by

$$\vec{B} = \frac{\mu_0 i}{2}(\cos\alpha_1 - \cos\alpha_2)\overrightarrow{e_z}$$

where $n = N/L$ is the lineic loop density. Show that in the case of an infinite solenoid, the magnetic field becomes $\vec{B} = \mu_0 n i\,\overrightarrow{e_z}$.

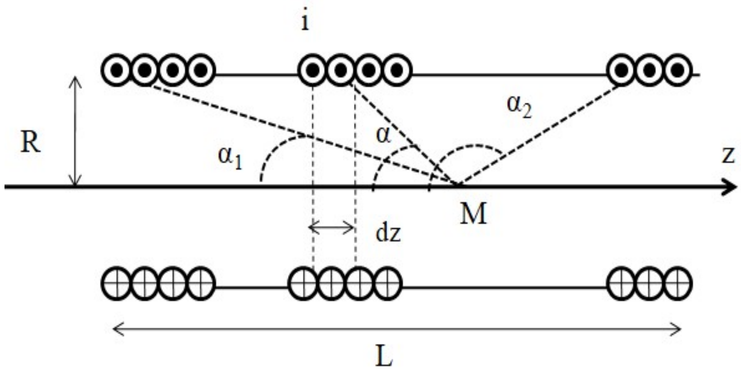

Figure 3.

What is the field produced by a solenoid of length L=10 cm made of 500 loops receiving a current of 1 ampere? Compare to the value of the Earth magnetic field.

Exercise 4: Uniform magnetic field obtained with Helmholtz coils

We consider first a loop (a thin coil) like depicted in Exercise 2. Show that the magnetic field created by the current i onto *Oz* axis at distance z is given by

$$\vec{B}(x) = \frac{\mu_0\, i}{2}\frac{R^2}{(R^2+z^2)^{\frac{3}{2}}}\overrightarrow{e_z}.$$

1) The amplitude of the magnetic field is given by $B(z)$. Show that we can write $B(z) = B(0)\left(1+\frac{z^2}{R^2}\right)^{\alpha}$ and give the expression of $B(0)$ and α. Give the expression of $B(\pm R/2)$ and $B(\pm R)$.
2) Calculate the second-order derivative $B''(z)$ and show its value is zero for $z = \pm R/2$. Then draw the shape of the graph of $B(z)$.

To obtain a uniform magnetic field, one can use Helmholtz coils (Fig 5). That device is composed of two coils C_1 and C_2 separated by a distance h and located respectively at distance $-h/2$ and $h/2$. The magnetic fields produced by C_1 and C_2 can be written as $B_1(z) = B(z - h/2)$ and $B_2(z) = B(z + h/2)$ where $B(z)$ function is given in question **1)** or **2)**.

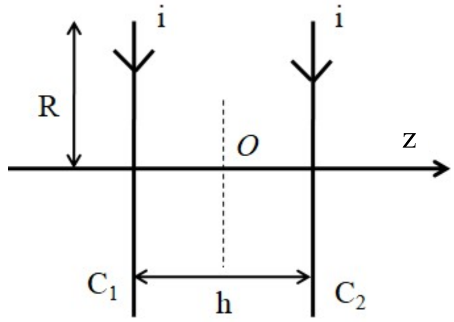

Fig 4.

3) Using a Taylor development at second order, show that the total field onto the y-axis can be given by:

$$B_1(z) + B_2(z) = 2B(h/2) + z^2 B''(h/2) + o(z^3).$$

4) What should be the distance h between the two coils C_1 and C_2 to make the second order in the previous formula equal to zero? Calculate the value of the field at O between C_1 and C_2 and compare to the one in the center of each coil. Draw the shape of the total magnetic field. Comments.

Exercise 5: Ampere theorem. Application to the magnetic field created by a wire

An "infinite" rectilinear wire of radius a, traversed by a current I, is stretched vertically in the $\vec{e_z}$ direction.
1) Study the symmetries, apply the Ampere Theorem and determine the expression of the magnetic field outside the wire at a distance r from the axis
2) Magnetic Field inside the wire: The cylindrical cable, of vertical axis, of radius a, of height "infinite", is traversed by a uniform current density $\vec{j} = j\vec{e_z}$ supposed uniform. Determine the expression of the magnetic field inside the wire.
3) Check the continuity of B at $r = a$. Draw the complete graphic $B(r)$.
4) Redo the complete study considering a surface current density on the wire surface only.

Exercise 6: Hall effect: how to measure a magnetic field ?

The figure shows a piece of silver with a width $z_1 = 11.8$ mm and thickness $y_1 = 1.023$ mm carrying a current $I = 120$ A in the Ox direction. The piece is submitted to a constant magnetic field $\vec{B} = B\vec{e_y}$ of magnitude 0.95 T.

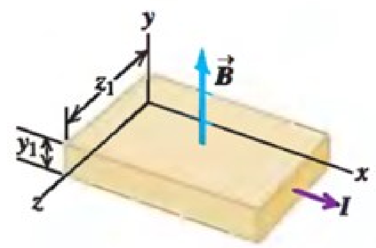

Figure 6.

1) Show qualitatively that the motion of the charges carriers leads to excess charges on the faces $z = z_1$ and $z = 0$. Where is the negative excess of charges?

2) Assuming that these two faces are equivalent to a capacitor, how is oriented the electric field in the material?
3) We assume that the piece is long in either direction parallel to x so that a stationary regime is set, and electrons move parallel to $\vec{e_x}$. What is the relation between ΔV, B and I where ΔV is the the potential difference between the faces $z = 0$ and $z = z_1$?
4) With one free electron per atom, what is the voltage difference given by a voltmeter plugged between the two faces?