Physics-L2 Electromagnetism

Approximative program

1) **Introduction and basics**
2) **Electric field calculations**
3) **Gauss theorem**
4) **Electric dipoles**

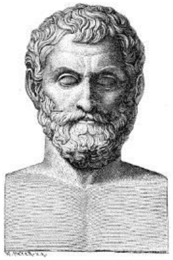
Chap 1: Electrostatics
Chap 2: Magnetostatics
Chap 3: Time-dependent regime-Induction phenomena
Chap 4: Maxwell equations
Chap 5: Dielectric media and applications
Chap 6: Conducting media and applications
Chap 7: Magnetic media and applications

week	Magistral lectures
1	**Electrostatics**
2	**Electrostatics**
3	**Electrostatics**
4	**Electrostatics**
5	Magnetostatics
6	Magnetostatics
7	Induction
8	Induction
9	Maxwell equations
10	Maxwell equations
11	Dielectric media
12	Dielectric / Metallic media
13	Metallic Media
14	Magnetic media

Electrostatics-L2
Introduction and Basics

1) Electrostatics devices **2) Electric charges** -Ponctual charges -Continuous charges distributions **3) Electric forces and Electric fields** - The Coulomb law -The electric field - Field lines **4) Electric potential and energy**-Work of an electric force-Electric potential -Equipotential lines **5) Electric field created by superposition of charges** -Two electric charges: Shape of the field lines-N electric charges-Continuous charges distribution **6) Symmetries of the electric field**

Static Electricity



Thales
625-547 BC

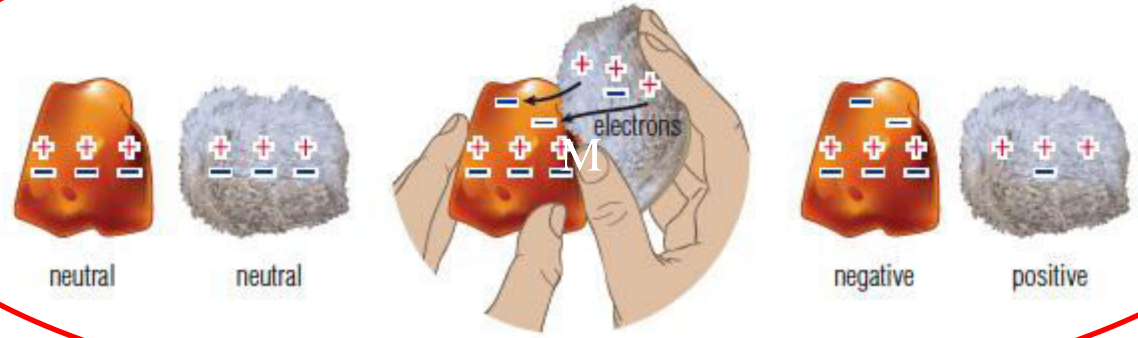
<u>Amber</u>
From ancient greek:
ἤλεκτρον elektron:

Electrisation by friction

Old greek hand

After rubing amber with, it can attract light objects

Modern explanation



Two types of electricity

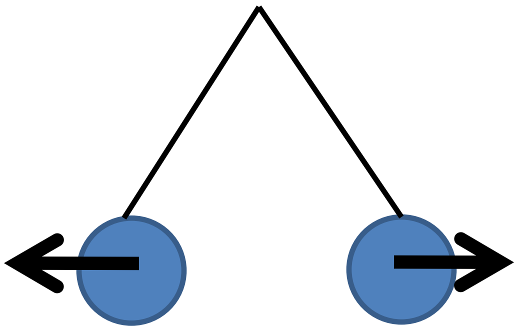
Electricity is seen as a fluid

Charles Du Fay
1698-1739

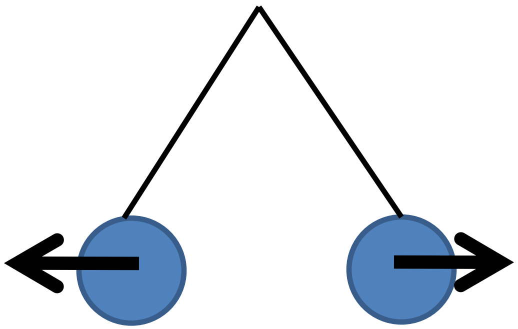
Resineous electricity:
obtained after <u>friction with **amber**</u>

Vitreous electricity:
obtained after <u>friction with **glass**</u>

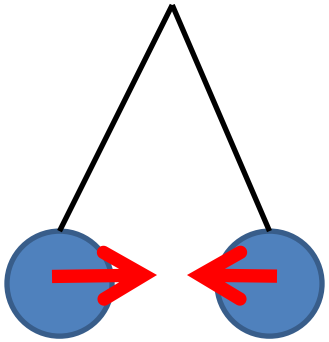
2 pendulums
rubbed with amber
REPULSION



2 pendulums
rubbed with glass
REPULSION



1 pendulum rubbed with glass and
1 pendulum rubbed with amber
ATTRACTION

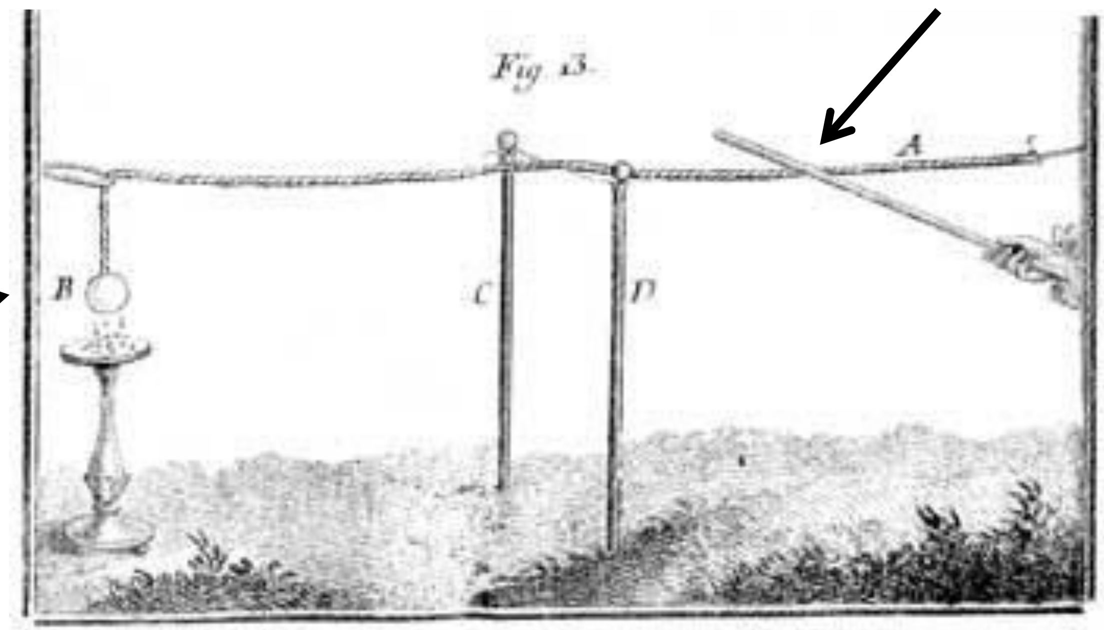


Propagation of electricity: conductor-insulator

Stephen Gray
1666-1736

- One can propagate electricity
- existence of materials that propagate electric fluids and others not.

Electrisation by contact with a glass stick

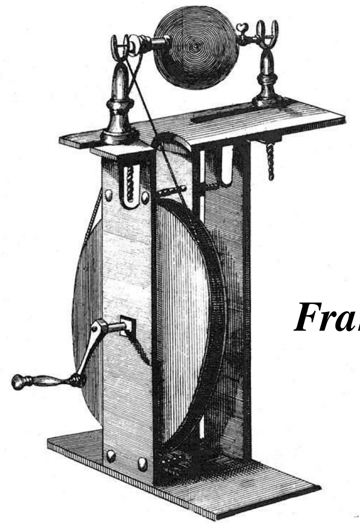


The electric fluid propagates until the sphere because it attracts little papers

Eletrostatic devices

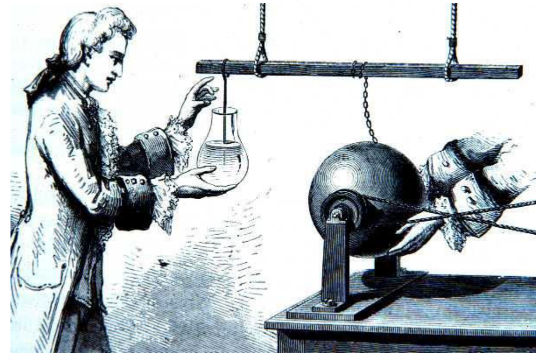
1706

Electrisation by contact with a rotating sulfur sphere

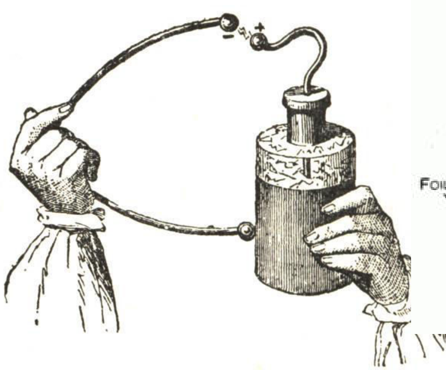


Francis Hauksbee
1666-1713

1746: University of Leiden
Pieter van Musschenbroek (1692-1761)



Leiden Jar (bottle)….first capacitor
Deliver electric discharge



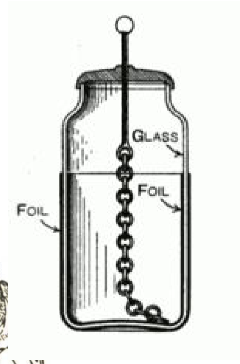


Fig. 213. Leyden Jar with Discharger.

Negative and Postives charges

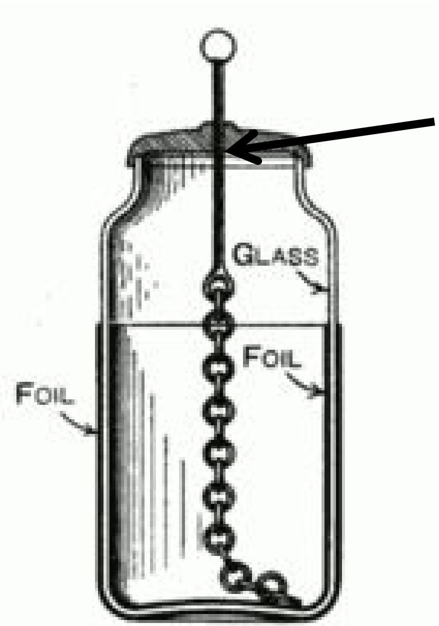
Vitreous electricity and resineous electricity represent the same fluid at different pressures, the circulation of electricity put the two states at equilibrium.

Benjamin Franklin
1706-1790

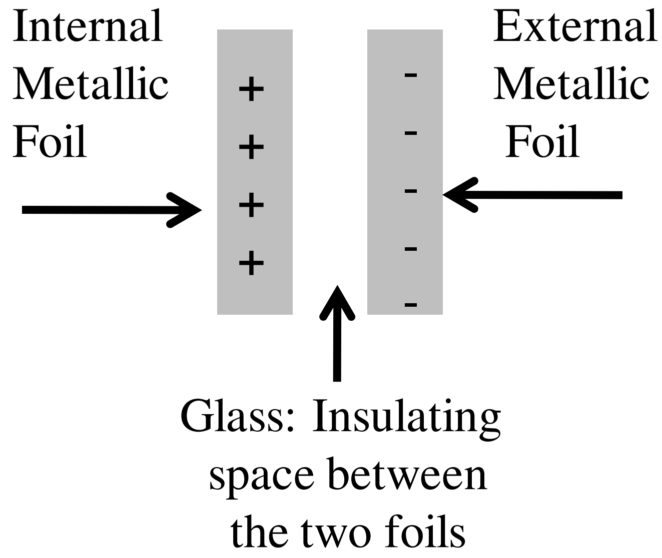
Lightning rod

- **Charge conservation**
- **Phenomena acting at distance**
- **Explanation of Leiden jar phenomena**

Interpretation of Leyden Jar



Metallic Chain propagates electric perturbation to the internal foil



- Possibility to **close the circuit** and to induce an **electric discharge !!**

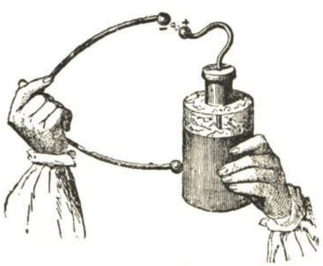
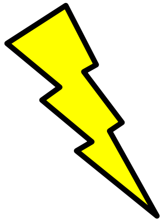
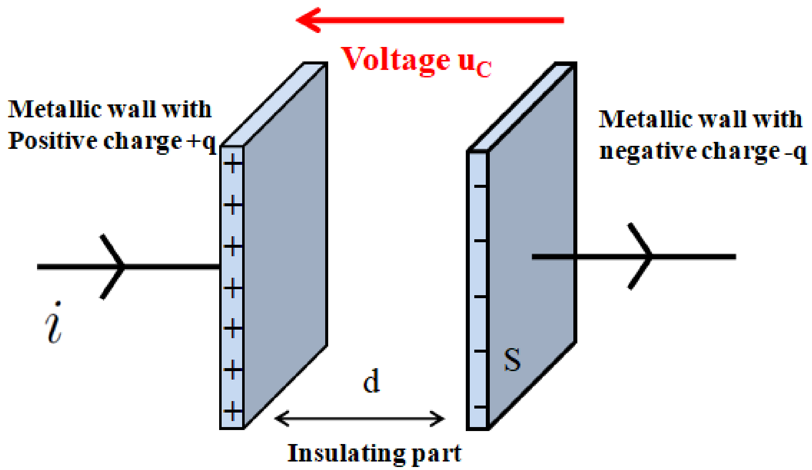


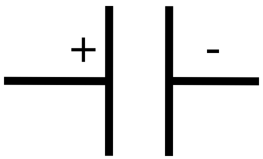
Fig. 213. Leyden Jar with Discharger.



Quick Modern description: Principle of a capacitor

- Can accumulate and stock electric charges:





Electric symbol

- Described by its Capacitance C (or capacity) in Farad (Coulomb/Volt):

$$q = C u_C$$

$$C = \frac{\epsilon_0 S}{d}$$ (if insulating part is the vaccum)

C: from µF to pF

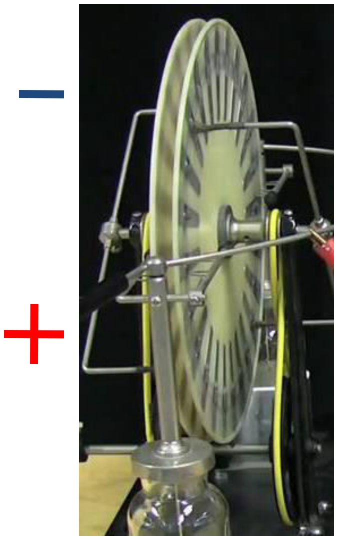
- Relation between current and voltage

$$\boxed{i = \frac{dq}{dt} = C\frac{du_C}{dt}}$$

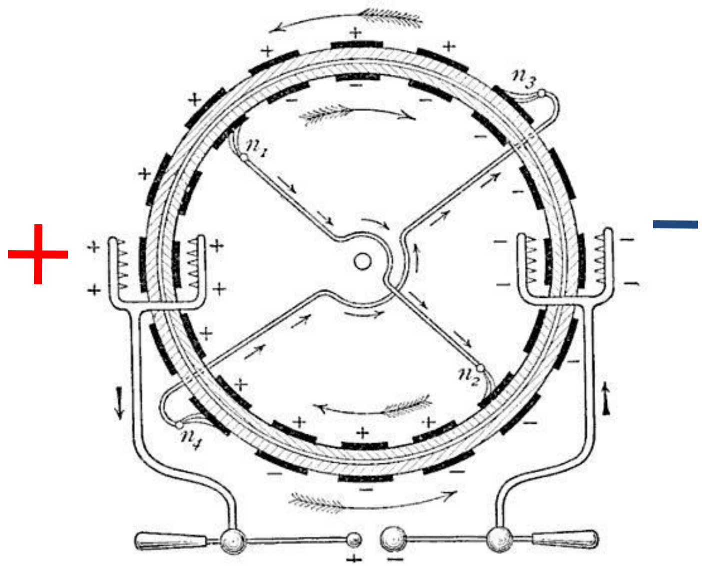
Wimshurst Machine (1882)

• Two wheels rotating in opposite direction in contact with metallic brushes.

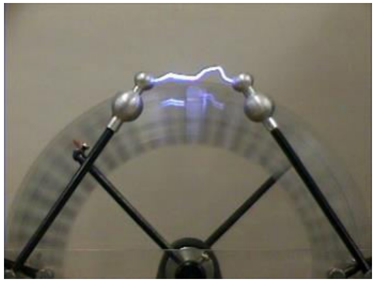
• Creation of negative charge induces apparition of a a positive charge on the other side of the brush



• By the weels rotation, charges are transported and collected



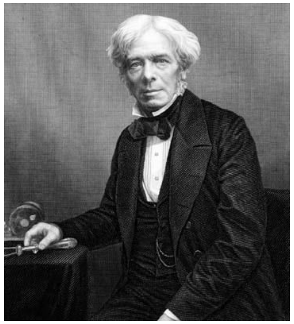
When electric charges become to high: lightning



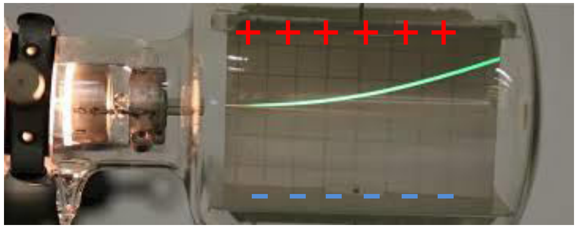
You Might watch
https://www.youtube.com/watch?v=Zilvl9tS0Og

Electric force…and Electric field…. And use to drive electric charges !!

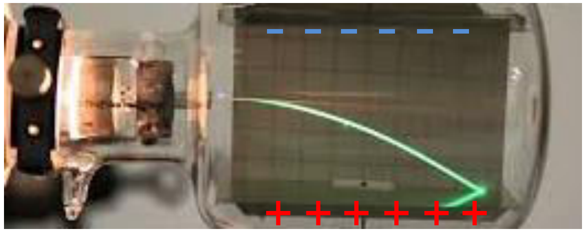
Charles Coulomb
1736-1806



Michael Faraday
1791-1867

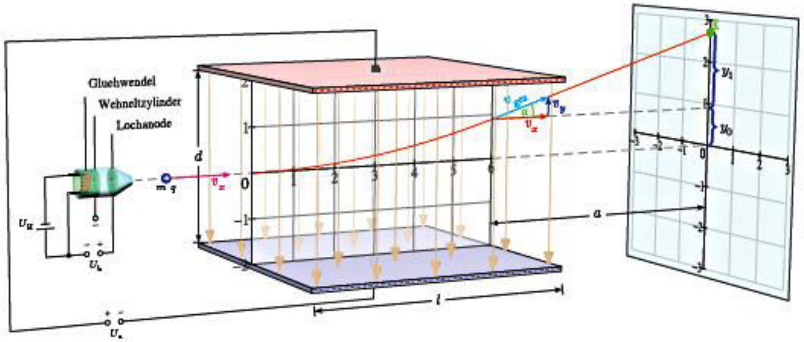


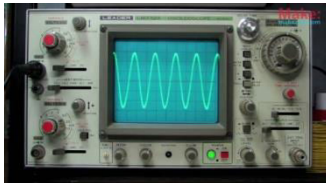
$\uparrow \vec{F}$



$\downarrow \vec{F}$

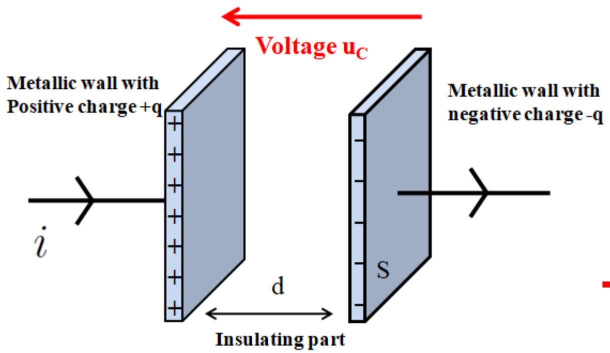
$$\vec{F} = q\,\vec{E} \qquad q<0$$

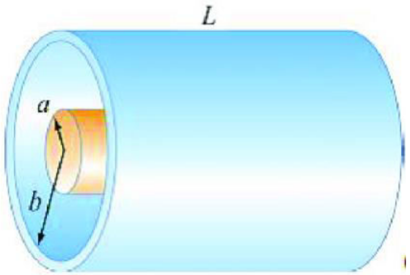




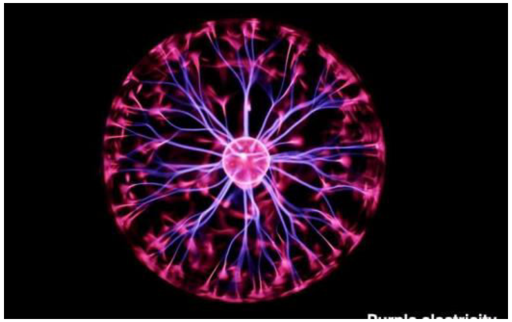
Important question of that Electrostatic lecture….

How is the Electric field created by the electrostatic devices ????





$\vec{E}$??

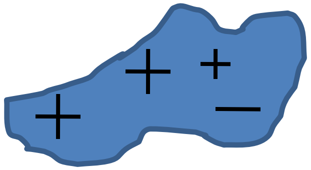


How can we connect it to capacitance C ?? Voltage ?? Energy ??

A neutral body get electrization and acquires an electric charge Q (in coulomb C)

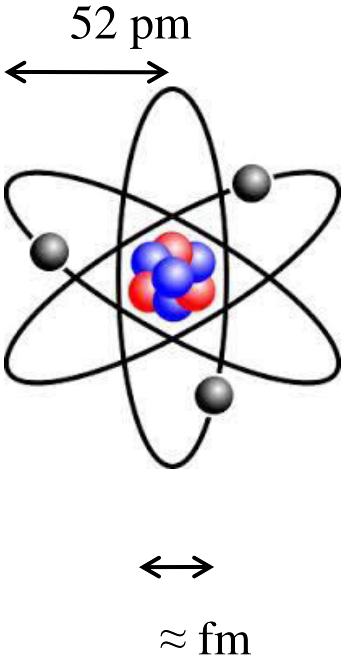
$$Q = \pm Ne$$

N is an integer



e= elementary charge: $e = 1.602 \times 10^{-19}\mathrm{C}$

Elementary particles



Electron:
$$\begin{aligned} m_e &= 9.11 \times 10^{-31}\ \mathrm{kg} \\ q_e &= -e = -1.602 \times 10^{-19}\ \mathrm{C} \end{aligned}$$

Proton:
$$\begin{aligned} m_p &= 1.67 \times 10^{-27}\ \mathrm{kg} \\ q_p &= +e = 1.602 \times 10^{-19}\ \mathrm{C} \end{aligned}$$

Neutron
$$\begin{aligned} m_n &= 1.67 \times 10^{-27}\ \mathrm{kg} \\ q_n &= 0\ \mathrm{C} \end{aligned}$$

Electrons are responsible of the electrization: there are lighter than protons and also protons can not escape from the strong interaction in the nucleus.

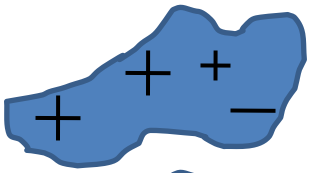
$$m\overrightarrow{a} = q\overrightarrow{E}$$

Dynamical properties: $a \approx qE/m$ with $|q_p|=|q_e|=q$

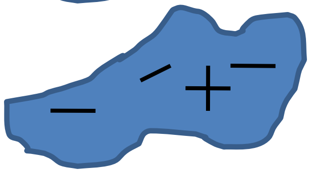
$$\frac{q}{m_{e-}} >> \frac{q}{m_p}$$

Stronger for electrons

Body charged **positively: lack of electrons**



Body charged **negatively: excess of electrons**



Electrons are responsible of the electrization: there are lighter than protons and also protons can not escape from the strong interaction in the nucleus.

$$m\,\overrightarrow{a} = q\,\overrightarrow{E}$$

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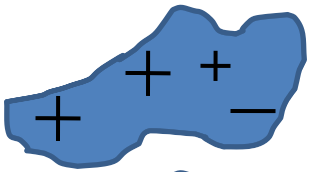
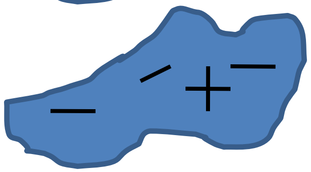
$$\frac{q}{m_{e-}} >> \frac{q}{m_p}$$

Stronger for electrons

Body charged **positively: lack of electrons**



Body charged **negatively: excess of electrons**



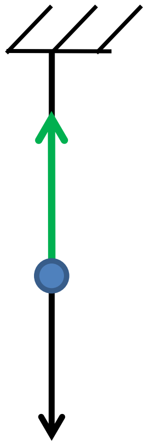
In friction experiment, the electric charge Q is usually between **milliCoulomb (mC) and microCoulomb (µC)**

How many electrons ?

$Q=10^{-3}$ C gives $N= Q/(-e) = 6.25\ 10^{15}$ electrons

$Q=10^{-6}$ C $N=6.25\ 10^{12}$ electrons

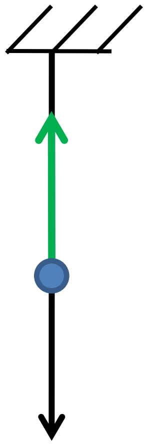
Electrostatic pendulum



$$\overrightarrow{W} + \overrightarrow{T} = \overrightarrow{0}$$

Equilibrium

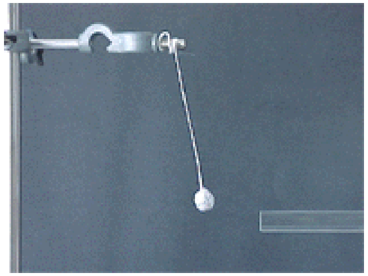
Electrostatic pendulum



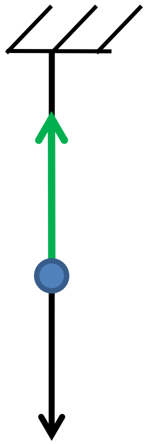
$$\overrightarrow{W} + \overrightarrow{T} = \overrightarrow{0}$$

Equilibrium

Electrostatic force with a charged stick



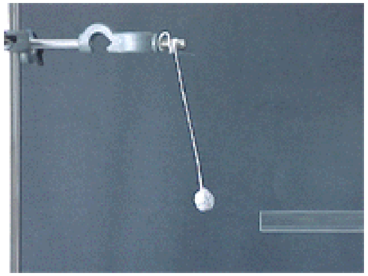
Electrostatic pendulum

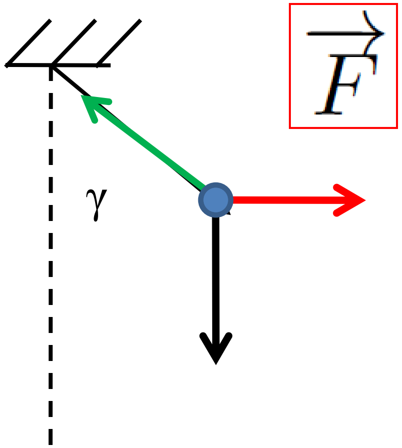


$$\overrightarrow{W} + \overrightarrow{T} = \overrightarrow{0}$$

Equilibrium

Electrostatic force with a charged stick





$$m\overrightarrow{a} = \overrightarrow{F} + \overrightarrow{W} + \overrightarrow{T} = \overrightarrow{0}$$

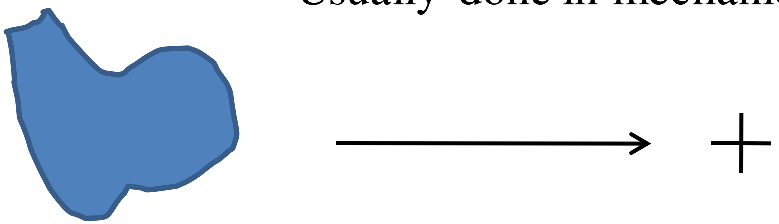
New equilibrium position

$$\begin{cases} -T\sin\gamma + F = 0 \\ T\cos\gamma + mg = 0 \end{cases}$$

$$\boxed{F = mg\tan\gamma}$$

Approximation of ponctual point or charge

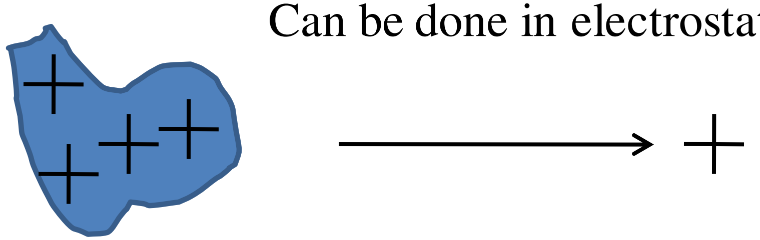
Usually done in mechanics



We assume the body being a material point (usually the center of mass) where all the mass stands

Can be done in electrostatics……

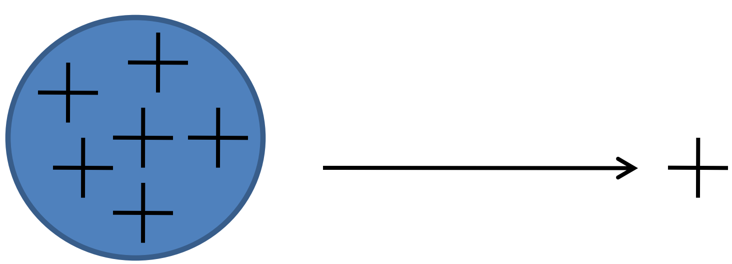
Q



Q

electrostatics of ponctual charge

Q

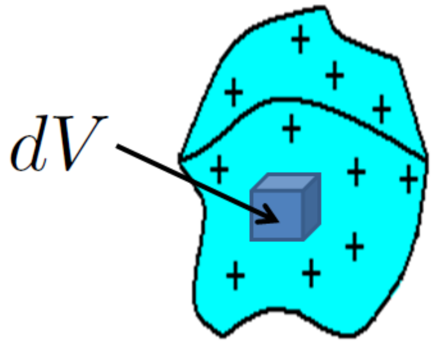


Q

Valid if charge distribution is spherical and size of object small compared to distance of « observation »

Volume

Is made of infinitesimal volume $dV=dxdydz$ of electric charge dq.



$$\rho = \frac{dq}{dV}$$

$$dq = \rho dV$$

With ρ the (**volumic**) electric charge density in C/m^3:

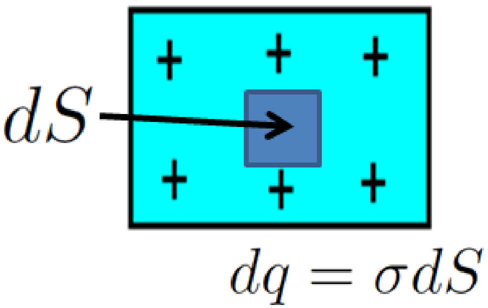
Total charge Q is obtained by

$$Q = \iiint \rho(M)dV \underset{\uparrow}{=} \rho \iiint dV = \rho V$$

If ρ is uniform

Surface

Is made of infinitesimal surface $dS=dxdy$ of electric charge dq.



$$\sigma = \frac{dq}{dS}$$

With σ the (**surface**) electric charge density in C/m^2:

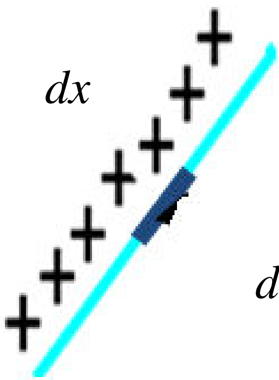
Total charge Q is obtained by

$$Q = \iint \sigma(M) dS = \sigma \iint dS = \sigma S$$

If σ is uniform

Lenght

Is made of infinitesimal length *dx* electric charge *dq*.



$dq=\lambda dx.$

$$\lambda = \frac{dq}{dx}$$

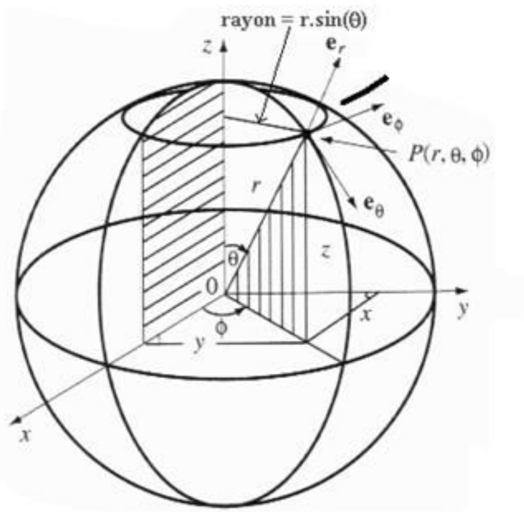
With λ the electric charge density

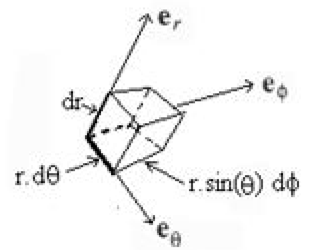
Total charge Q is obtained by

$$Q = \int \lambda(M)dx = \lambda \int dx = \lambda L$$

If λ is uniform

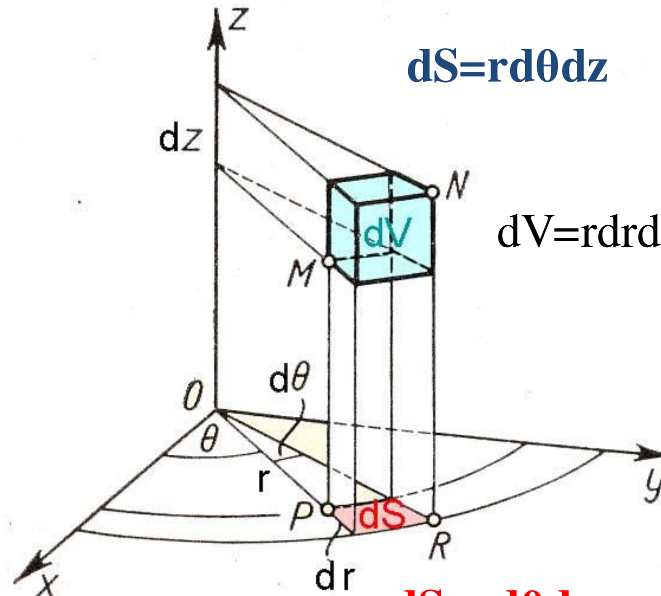
Infinitesimal volumes and surfaces cylindric and polar coordinates





$dV = dr\,dS$

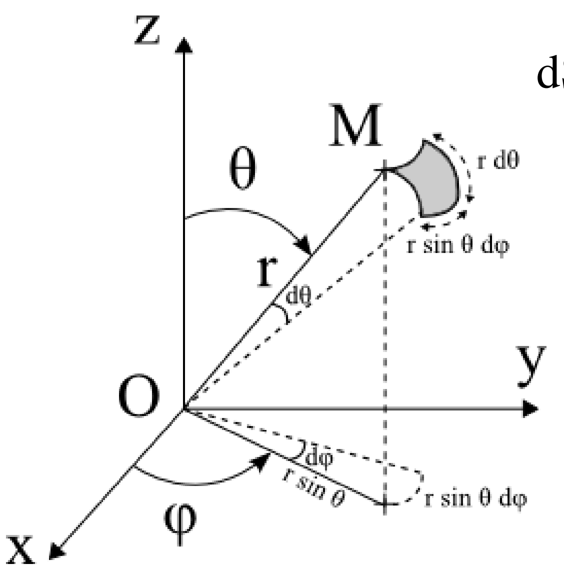
$= r^2 dr \sin\theta d\theta\ d\varphi$



$dS = r d\theta dz$

$dV = r dr d\theta dz$

$dS = r d\theta dr$



$dS = r d\theta\ r\sin\theta d\varphi$

$= r^2 \sin\theta d\theta\ d\varphi$

Advices
Review L0: Tutorial of second semester
Mathematical tools for Physics

Electrostatics-L2

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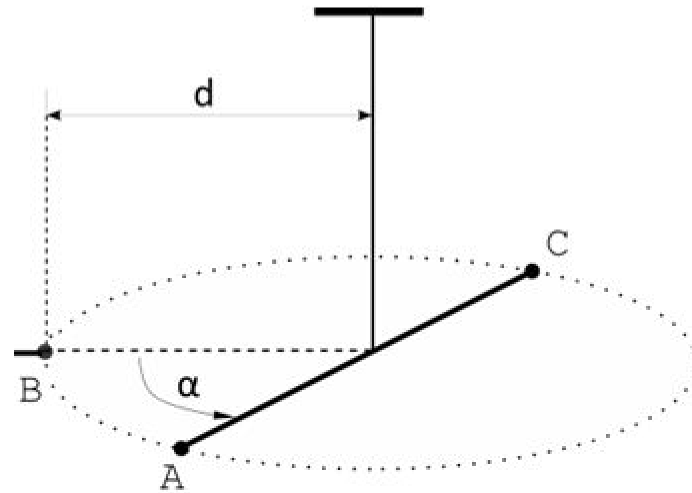
- The Coulomb law -The electric field - Field lines **4) Electric potential and energy**-Work of an electric force- Electric potential -Equipotential lines **5) Electric field created by superposition of charges** -Two electric charges: Shape of the field lines-N electric charges-Continuous charges distribution **6) Symmetries of the electric field**

Charles Coulomb
1736-1806

Parameters that influence the electric force

Coulomb balance: device composed with a torsion balance able to measure weak electric forces.

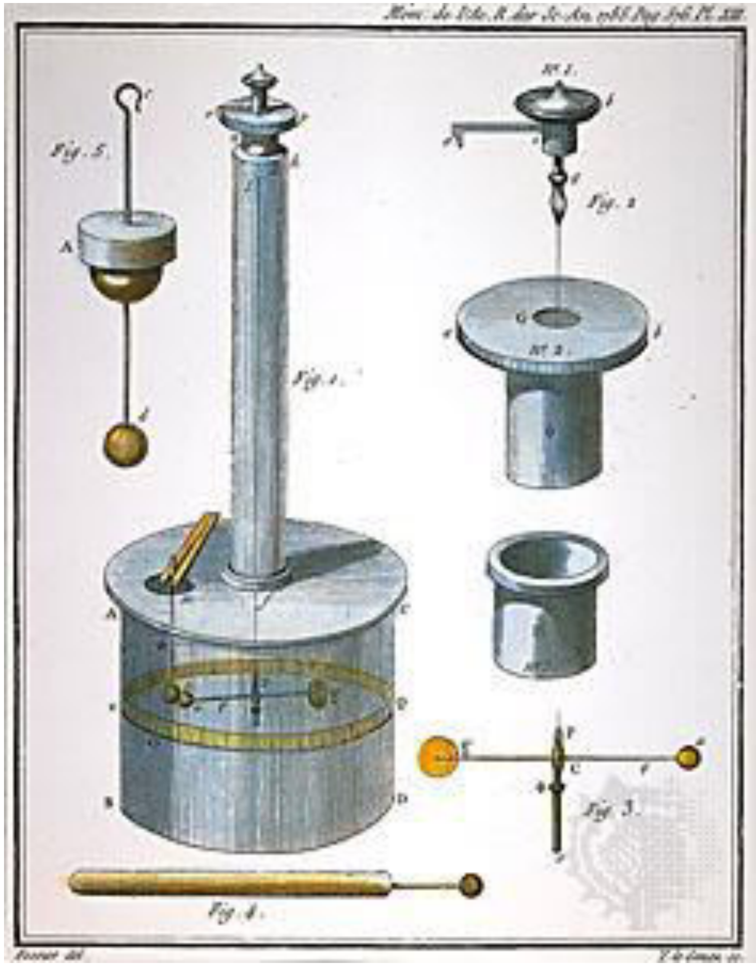


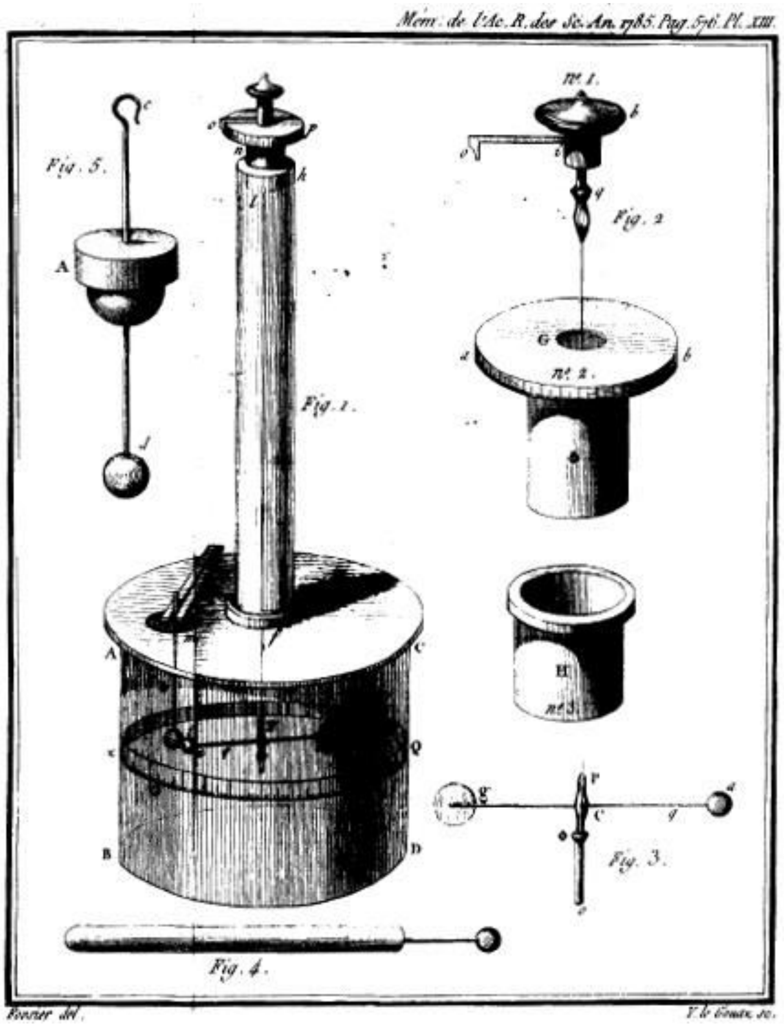
$$\boxed{F \approx \frac{q_1 q_2}{r^2}}$$

Repulsion of two spheres after electrisation of neutral sphere A by charged sphere B.

Coulomb: unit of electric charge

a) The Coulomb law





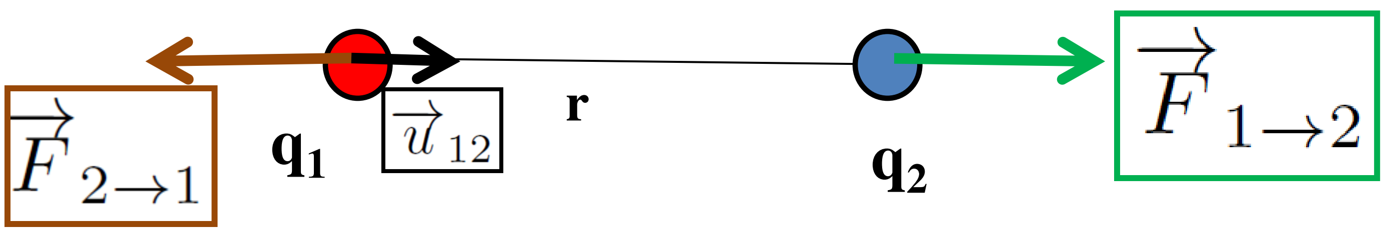
Electric Force

$$\overrightarrow{F}_{1\to 2} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\overrightarrow{u}_{12} = -\overrightarrow{F}_{2\to 1}$$

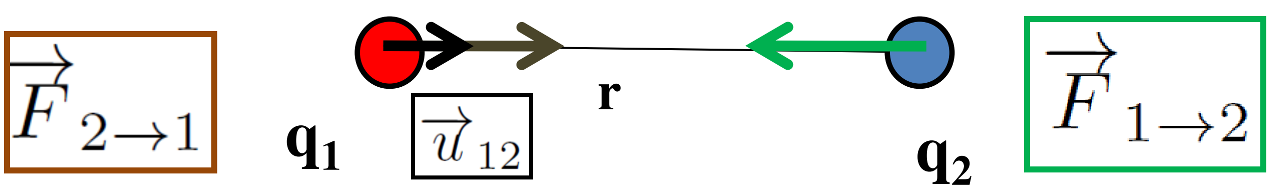
Coulomb Force

$$\epsilon_0 = 8.85 \times 10^{-12} \ \ F/m$$

Repulsion of electric charges: (q1,q2)>0 or (q1,q2)<0



Attraction of electric charges (q1<0, q2>0) or (q1>0 and q2<0)



Dielectric permittivity of vaccum $\epsilon_0 = 8.85 \times 10^{-12} \; F/m$

If medium is not the vaccum we use the substitution: $\epsilon_0 \longmapsto \epsilon_0 \epsilon_r$

ε_r (vaccum) =1
ε_r (air)=1.0001
ε_r (water)=80

Dielectric permittivity of vaccum $\epsilon_0 = 8.85 \times 10^{-12} \ F/m$

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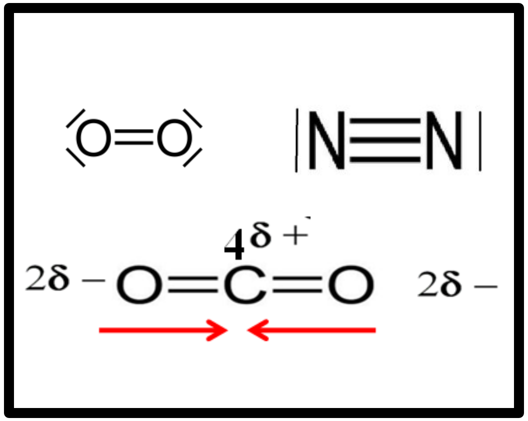
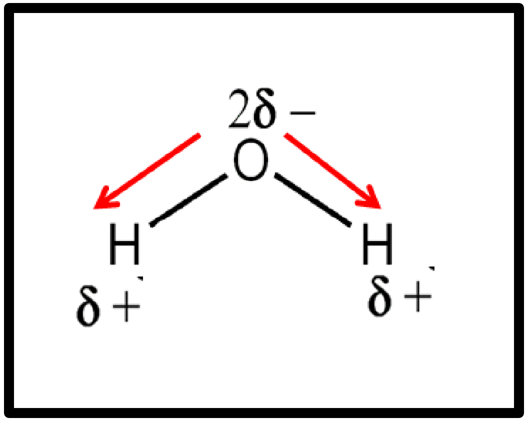
ε_r (vaccum) =1

ε_r (air)=1.0001

ε_r (water)=80

Due to the **polarisability of the medium (local presence of electric dipoles)**





ε_r indicates the sensibility to the action of an electric field

Exercises for tutorials

Exercise 1

1) The Hydrogen Atom is composed of a nucleus made of one proton ($m_p = 1.67 \times 10^{-2}$ kg and $q = e = 1.602 \times 10^{-1}$ C) and one electron ($m_{e-} = 9.11 \times 10^{-31}$ kg and $q = -e = -1.602 \times 10^{-19}$C) whose average distance from the nucleus is $d = 53$ pm. Calculate and compare the magnitude of the electric force and the gravitational force between the electron and the proton. We give $\varepsilon_0 = 8.85 \times 10^{-12}$ F.m^{-1} and G=6.67$\times 10^{-1}$ $m^3.kg^{-1}.s^{-2}$.

2) The size of an atomic nucleus made of protons and neutrons is in the order of magnitude of the femtometer. Calculate the electrostatic repulsion between two protons. Comments.

Exercise 2

Copper molar mass is $M = 63$ g/mol and its volumic mass is $\rho = 8{,}7$ g/cm^3. Calculate the number of atoms in 1 cm^3 of copper. We will assume that there is one free electron per copper atom. We give Na=6.023$\times10^{23}$ mol^{-1}.

1) What would be the electric charge of 1 cm^3 of copper if we remove $1/1000000$ of its free electrons?

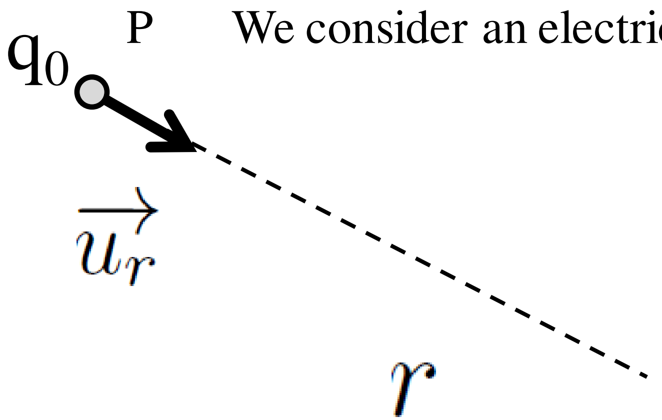
2) What would be the intensity of the electric force between two identical pieces of copper like described in the first question and separated from 1 meter?

Exercise 3

Two identical electrostatic pendulums of length $l = 0.3$ m and mass $m = 0.2$ g, attached in the same point are in contact at initial situation. We assume the radius of the spheres are negligible so that the initial angles made by the pendulums with the vertical is taken to be zero. We give them the same electric charge q. What is the value of that charge if the equilibrium distance between the two pendulums is $d = 10$ cm?

WHAT IS THE ELECTRIC FIELD ???

We consider an electric charge q0 at point P



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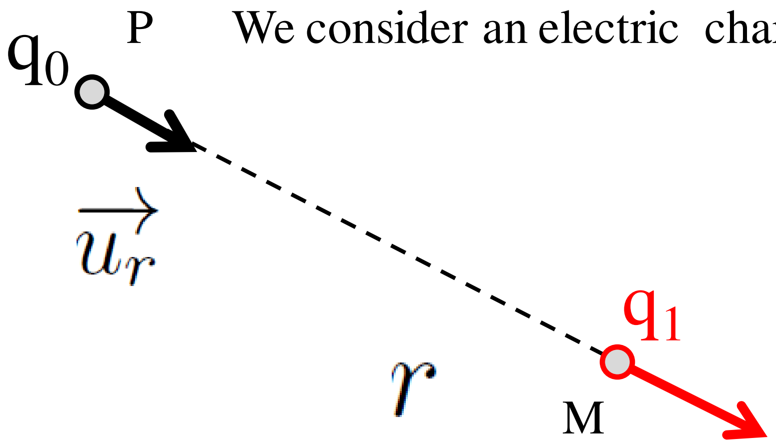
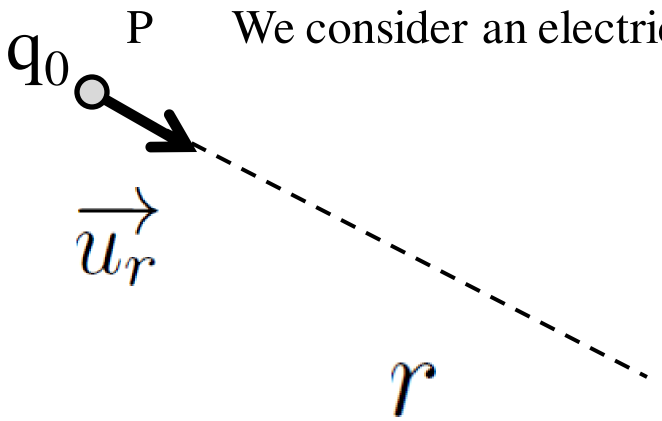


Then we put a charge q1 at M at a distance r from P, the force felt by q1 due to q0 is

$$\boxed{\overrightarrow{F_1} = \frac{q_0 q_1}{4\pi\epsilon_0 r^2}\overrightarrow{u_r}}$$

WHAT IS THE ELECTRIC FIELD ???

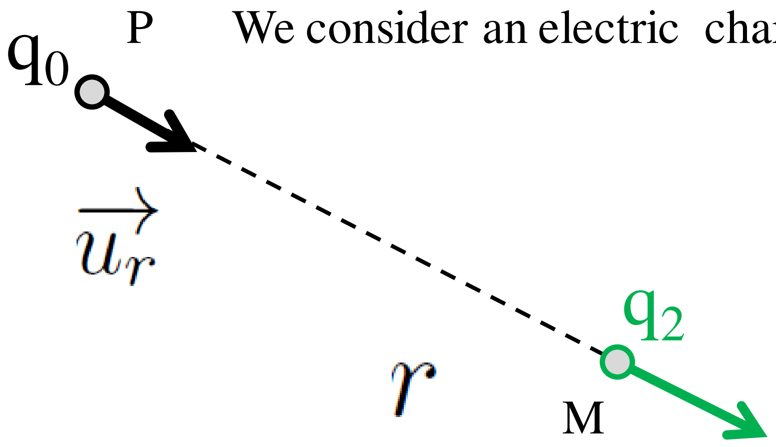


We consider an electric charge q0 at point P

We remove charge q1

WHAT IS THE ELECTRIC FIELD ???

We consider an electric charge q0 at point P

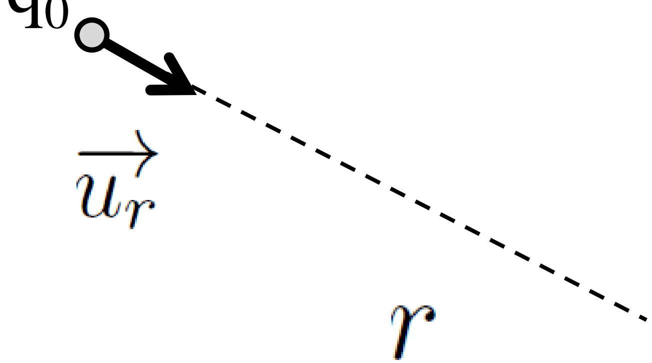


Then we put a charge q2 at M at a distance r from P, the force felt by q2 due to q0 is

$$\boxed{\overrightarrow{F_2} = \frac{q_0 q_2}{4\pi\epsilon_0 r^2}\overrightarrow{u_r}}$$

WHAT IS THE ELECTRIC FIELD ???

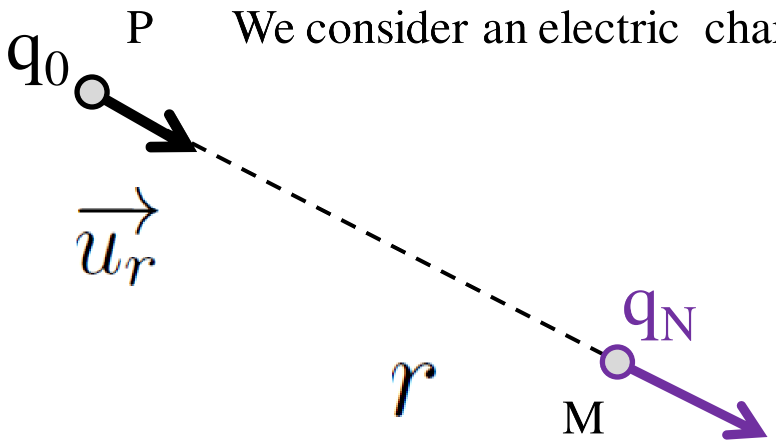
q_0 P We consider an electric charge q0 at point P



We remove charge q2

WHAT IS THE ELECTRIC FIELD ???

We consider an electric charge q0 at point P



Then we put a charge q3 at M at a distance r from P, we look the force felt by q3 due to q0.

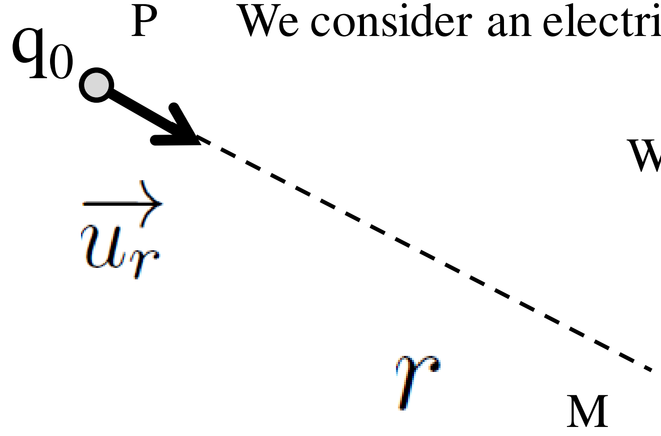
We remove charge q3……………until qN-1 and we remove qN-1.

Then we put a charge qN at M at a distance r from P, we the force felt by qN due to q0 is

$$\boxed{\overrightarrow{F_N} = \frac{q_N q_1}{4\pi\epsilon_0 r^2}\overrightarrow{u_r}}$$

WHAT IS THE ELECTRIC FIELD ???

We consider an electric charge q0 at point P



What is the invariant in that stupid experiment ?

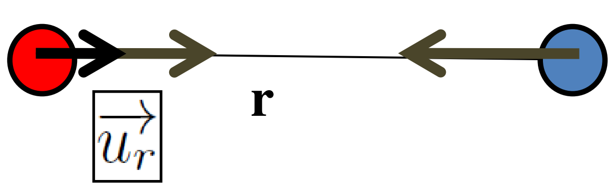
$$\boxed{\frac{\overrightarrow{F_1}}{q_1} = \frac{\overrightarrow{F_2}}{q_2} = \cdots = \frac{\overrightarrow{F_N}}{q_N} = \frac{q_0}{4\pi\epsilon_0 r^2}\overrightarrow{u_r} = \overrightarrow{E}}$$

It exist one quantity **induced by the existence of q0** that modifies the space-properties

Electric field

$$\boxed{\overrightarrow{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\overrightarrow{u_r}} = \overrightarrow{F}_{1\mapsto 2} = -\overrightarrow{F}_{2\mapsto 1}$$

$q_1>0$



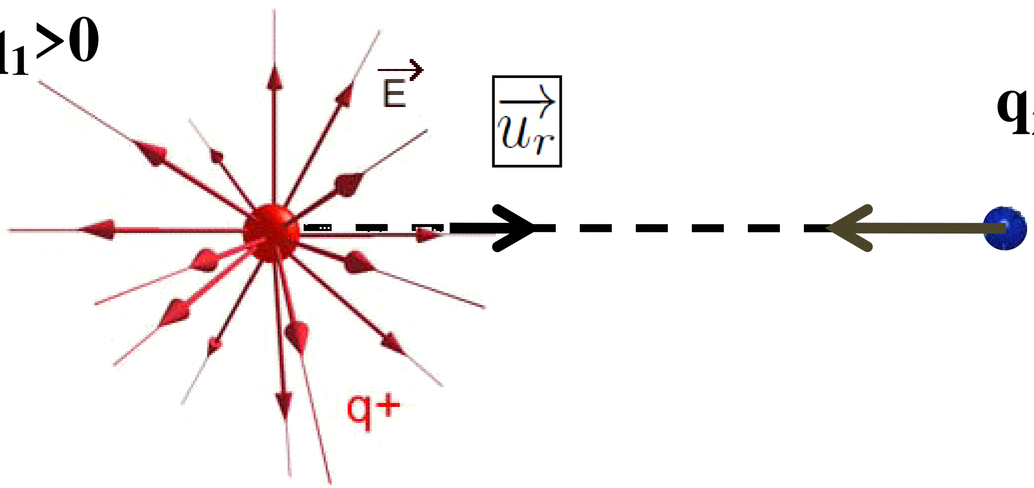
$q_2<0$

$$\boxed{\overrightarrow{E_1} = \frac{q_1}{4\pi\epsilon_0 r^2}\overrightarrow{u_r}}$$

Force felt by particule 2 due to electric field 1

$$\boxed{\overrightarrow{F}_{1\mapsto 2} = q_2\overrightarrow{E_1}}$$

$q_1>0$

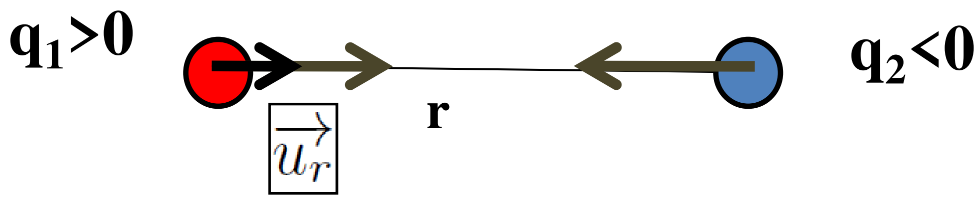


$q_2<0$

I.3) Electric Forces and Electric fields

Electric field

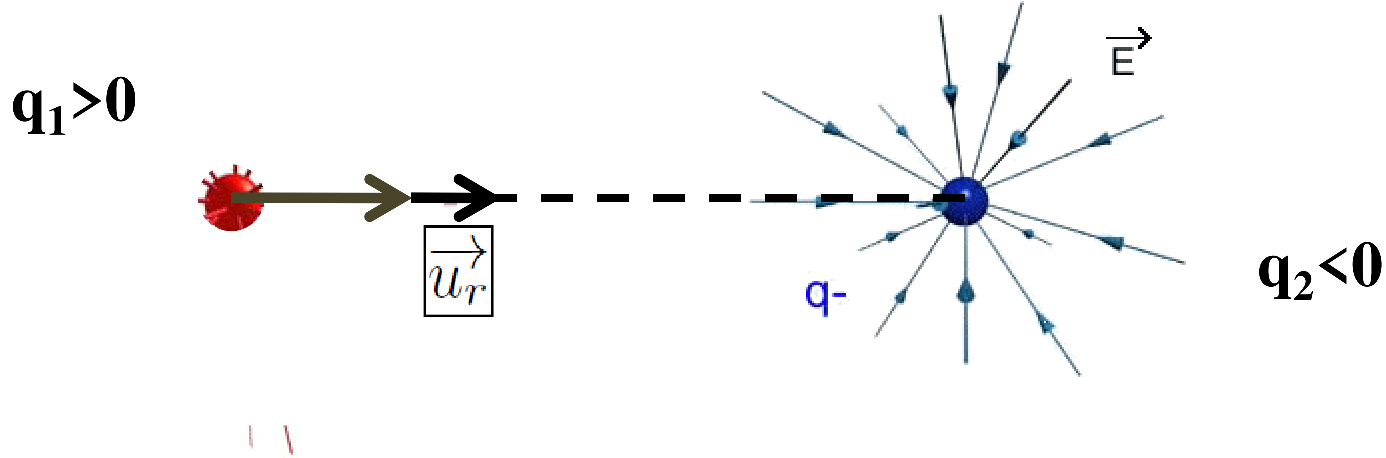
$$\boxed{\overrightarrow{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\overrightarrow{u_r}} = \overrightarrow{F}_{1\mapsto 2} = -\overrightarrow{F}_{2\mapsto 1}$$



Force felt by particule 2 due to electric field 1

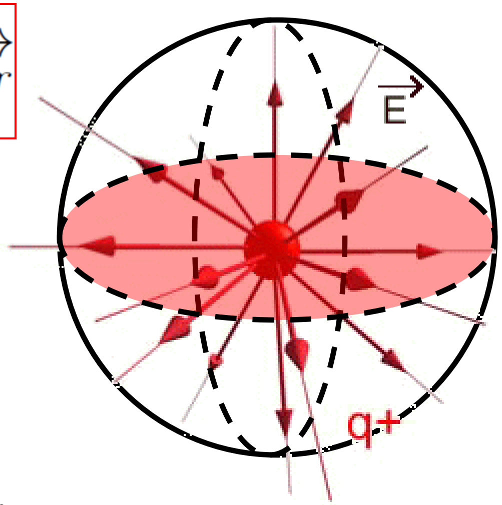
$$\boxed{\overrightarrow{F}_{2\mapsto 1} = q_1\overrightarrow{E_2}}$$

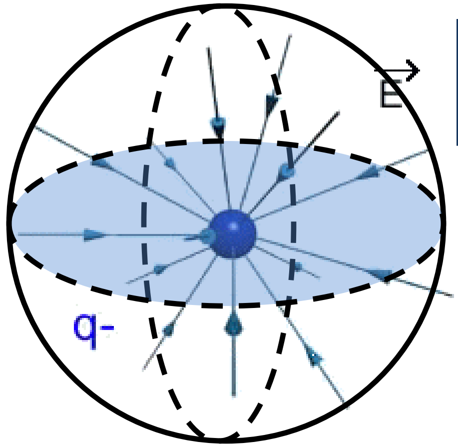
$$\boxed{\overrightarrow{E_2} = -\frac{q_2}{4\pi\epsilon_0 r^2}\overrightarrow{u_r}}$$



Electric field Lines

$$\overrightarrow{E_1} = \frac{q_1}{4\pi\epsilon_0 r^2}\overrightarrow{u_r}$$

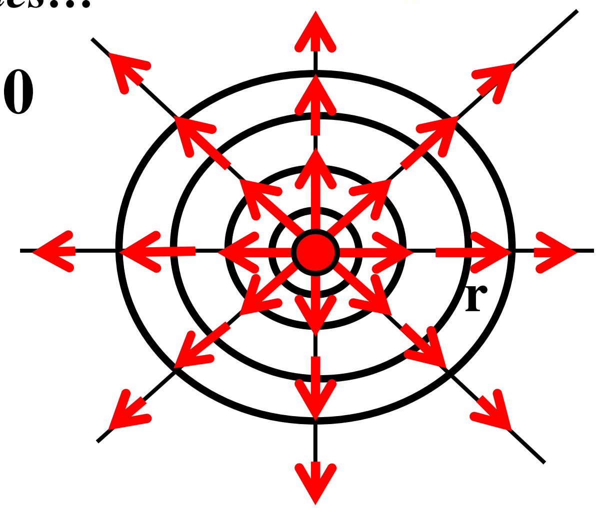


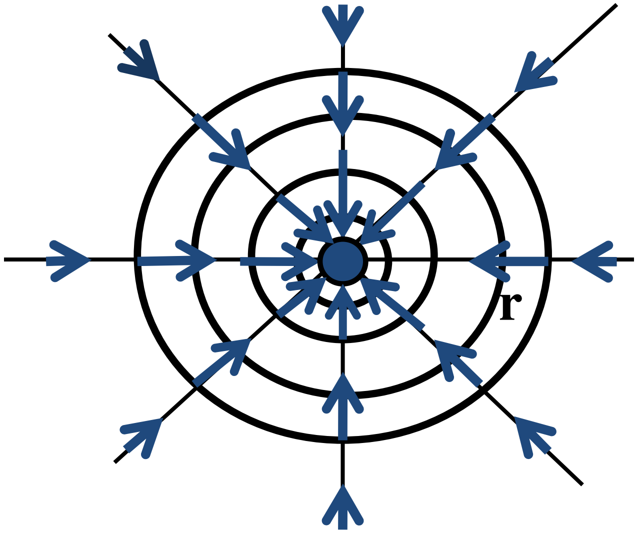


$$\overrightarrow{E_2} = -\frac{q_1}{4\pi\epsilon_0 r^2}\overrightarrow{u_r}$$

In the planes…

$q_1>0$





$q_2<0$

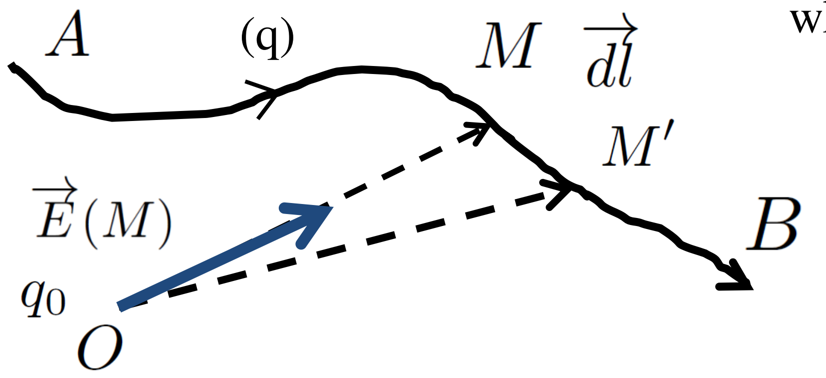
Electrostatics-L2

1) Electrostatics devices **2) Electric charges** -Ponctual charges -Continuous charges distributions
3) Electric forces and Electric fields - The Coulomb law -The electric field - Field lines

4) Electric potential and energy-

Work of an electric force- Electric potential -Equipotential lines **5) Electric field created by superposition of charges** -Two electric charges: Shape of the field lines-N electric charges-Continuous charges distribution **6) Symmetries of the electric field**

Work of an electric force



We move an electric charge q in a region where lives an electric field E(M)

$$\overrightarrow{dl} = \overrightarrow{OM'} - \overrightarrow{OM} = \overrightarrow{MM'}$$

Elementary work δW over distance dl (between M and M')

$$\delta W = \overrightarrow{F} \cdot \overrightarrow{dl} = q\overrightarrow{E} \cdot \overrightarrow{dl}$$

Work of an electric force

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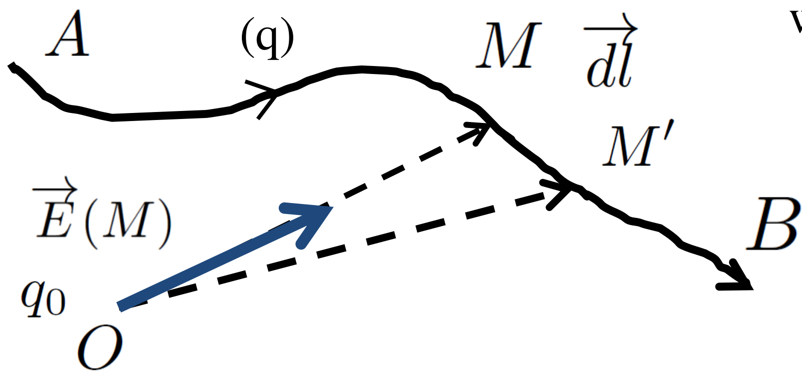

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$$\delta W = \overrightarrow{F} \cdot \overrightarrow{dl} = q\overrightarrow{E} \cdot \overrightarrow{dl}$$

Total work W over path AB

$$W_{AB} = \int_A^B \overrightarrow{F} \cdot \overrightarrow{dl} = q \underline{\int_A^B \overrightarrow{E} \cdot \overrightarrow{dl}}$$

Circulation of electric field along path AB

Relation with the energy

$$E = E_k + E_p = \text{const}$$

Conservation of energy

Relation with the energy

$$E = E_k + E_p = \text{const}$$

Conservation of energy

The total energy change is zero and the change of kinetic energy is the opposite of the change of potential energy

$$\Rightarrow \quad \Delta E = 0 = \Delta E_k + \Delta E_p = 0$$

$$\Leftrightarrow \quad \Delta E_k = -\Delta E_p$$

Relation with the energy

$$E = E_k + E_p = \text{const}$$ Conservation of energy

The total energy change is zero and the change of kinetic energy is the opposite of the change of potential energy

$$\Rightarrow \quad \Delta E = 0 = \Delta E_k + \Delta E_p = 0$$

$$\Leftrightarrow \quad \Delta E_k = -\Delta E_p$$

We know from kinetic energy theorem that the work of the system is equal to the change of kinetic energy

$$W = \Delta E_k \qquad \Leftrightarrow \quad W = -\Delta E_p$$

The work is the opposite of the change of Potential energy

Connection between electric field and potential energy

$$W_{AB} = \int_A^B \overrightarrow{F} \cdot \overrightarrow{dl} = q \int_A^B \overrightarrow{E} \cdot \overrightarrow{dl}$$

And with $W = -\Delta E_p$

Connection between electric field and potential energy

$$W_{AB} = \int_A^B \overrightarrow{F} \cdot \overrightarrow{dl} = q \int_A^B \overrightarrow{E} \cdot \overrightarrow{dl}$$ And with $$W = -\Delta E_p$$

We can connect the potential energy to a quantity called **electric potential**

$$\Delta E_p = -q \int_A^B \overrightarrow{E} \cdot \overrightarrow{dl}$$

$$= -q[-V(M)]_A^B = q[V(M)]_A^B = q(V_B - V_A)$$

Connection between electric field and potential energy

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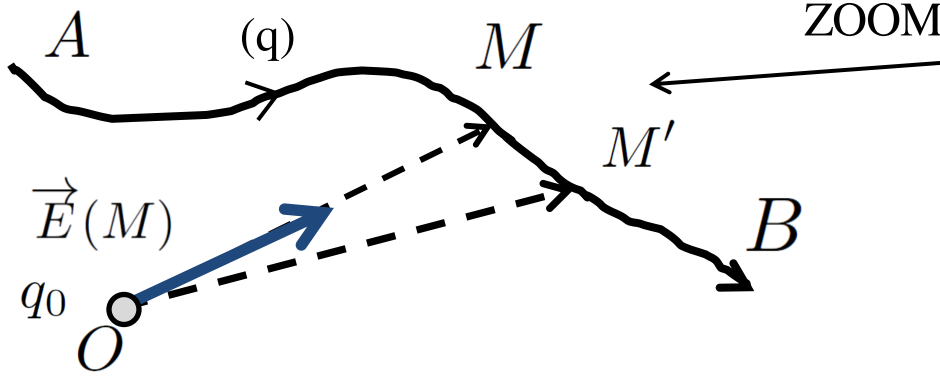
Defined as :

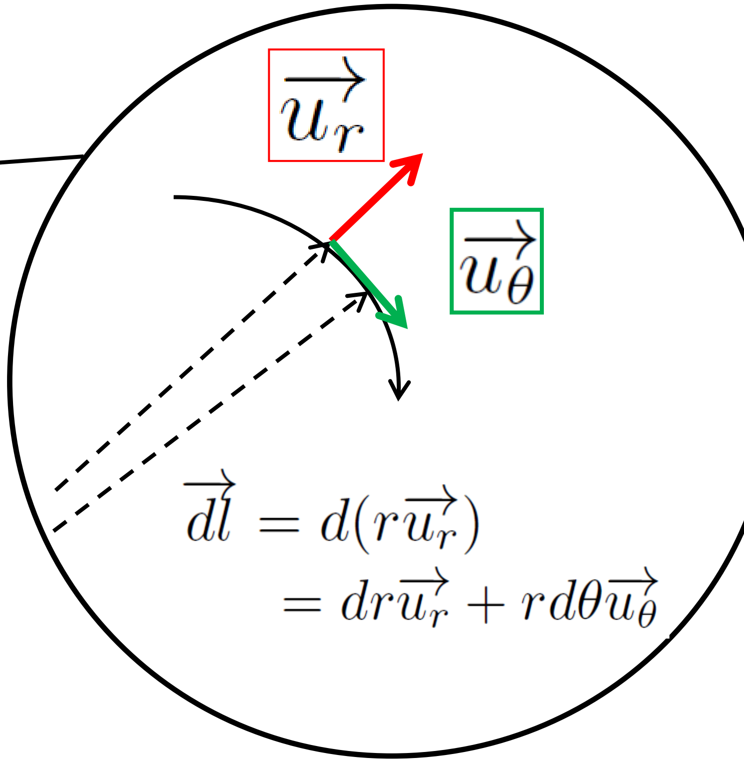
$$\boxed{[V]_A^B = -\int_A^B \overrightarrow{E} \cdot \overrightarrow{dr}}$$

Example:

Lets consider an electric field created by a charge q0.



$$\overrightarrow{E} = \frac{q_0}{4\pi\epsilon_0 r^2}\overrightarrow{u_r}$$



I.4) Electric potential and energy

Example:

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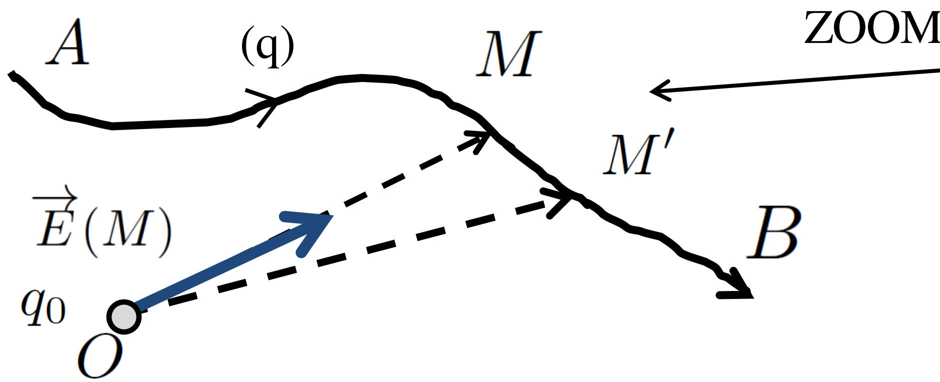

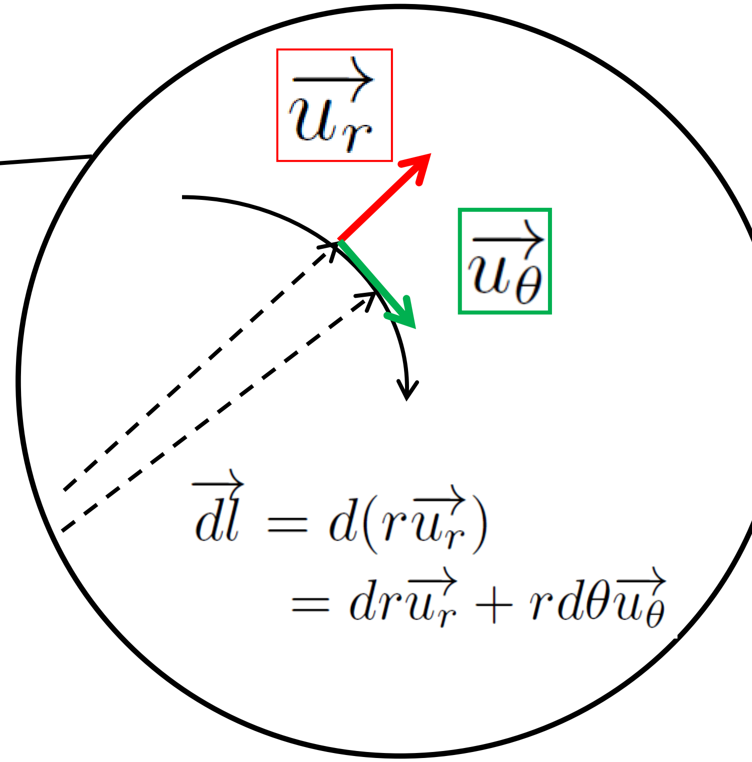

$$\vec{E} = \frac{q_0}{4\pi\epsilon_0 r^2}\vec{u_r}$$

$$\begin{aligned}
\int_A^B \vec{E}\cdot\vec{dl} &= \int_A^B \left(\frac{q_0}{4\pi\epsilon_0 r^2}\vec{u_r}\cdot dr\vec{u_r} + \frac{q_0}{4\pi\epsilon_0 r^2}\underbrace{\vec{u_r}\cdot rd\theta\vec{u_\theta}}_{\textbf{zero}}\right) \\
&= \int_A^B \frac{q_0 dr}{4\pi\epsilon_0 r^2} = \left[-\frac{q_0}{4\pi\epsilon_0 r} + \text{const'}\right]_A^B \\
&= -[V(r)]_A^B
\end{aligned}$$

Hello !! Potential created by an electric charge q0

$$\boxed{V(r) = \frac{q_0}{4\pi\epsilon_0 r} + \text{const}}$$

Unit: Volt

Usually we take the following convention to find the value of the constant:
At infinity, the effect of potential should be zero.

$$\lim_{r\to\infty} V(r) = 0 = \text{const}$$

However, in some case it could be different. A potential with non-zero value can be imposed at a given distance (for instance with capacitors)

Relation between Electric field and Electric Potential

$$\int \underline{\overrightarrow{E}} \cdot \overrightarrow{dr} = -[V(r)]$$

$$= - \int dV$$

But a total differential function dV can be written
With the help of the gradient operator

$$\begin{aligned} dV &= \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz \\ &= \underline{\overrightarrow{\mathrm{grad}}V} \cdot \overrightarrow{dr} = \overrightarrow{\nabla} V \cdot \overrightarrow{dr} \end{aligned}$$

Relation between Electric field and Electric Potential

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Finaly

$$\boxed{\overrightarrow{E} = -\overrightarrow{\mathrm{grad}}V}$$

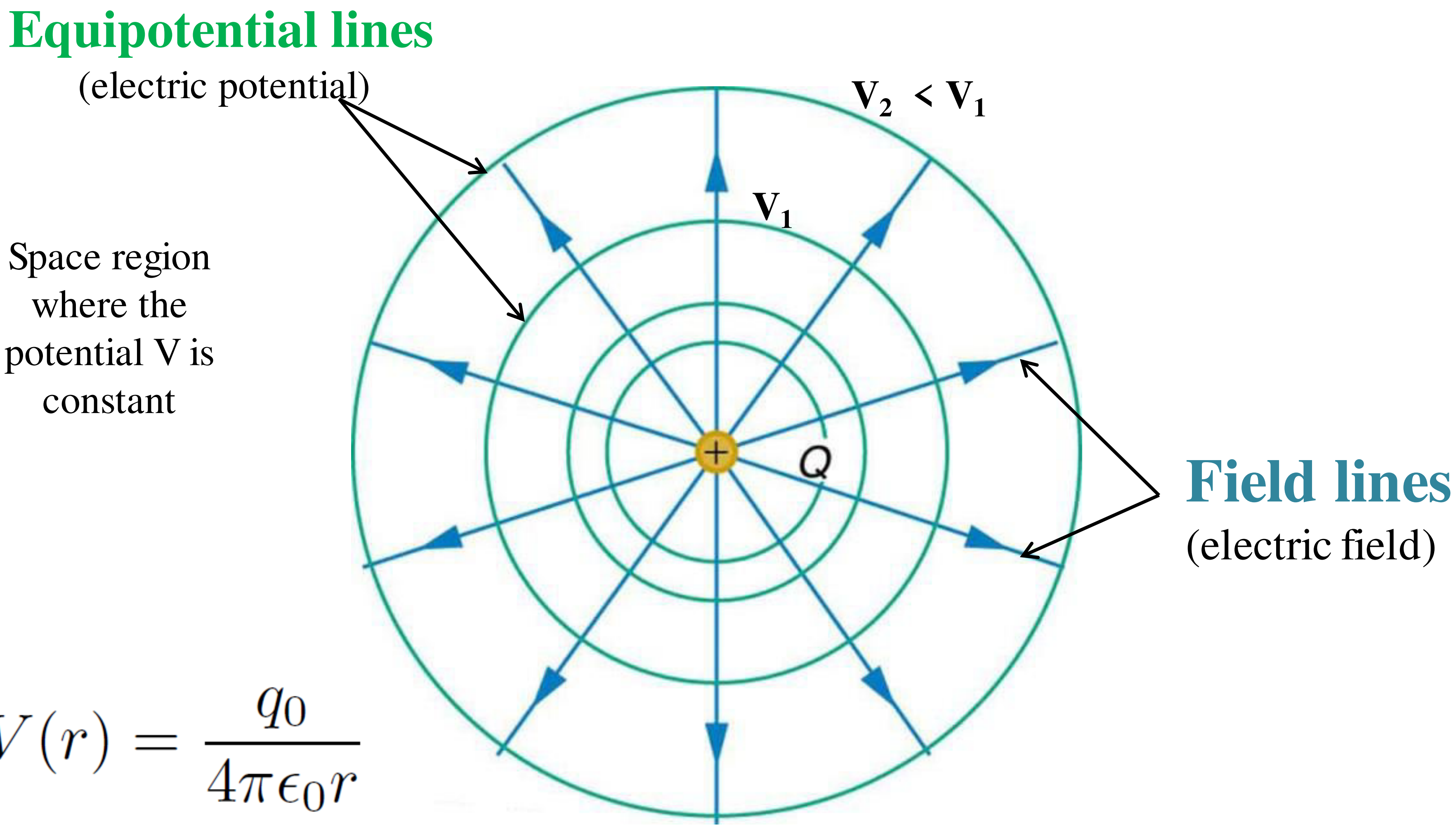
Electric Field

$\overrightarrow{E}$

Electric Potential

V

$$\boxed{\int \overrightarrow{E} \cdot \overrightarrow{dr} = -[V(r)]}$$



Space region where the potential V is constant

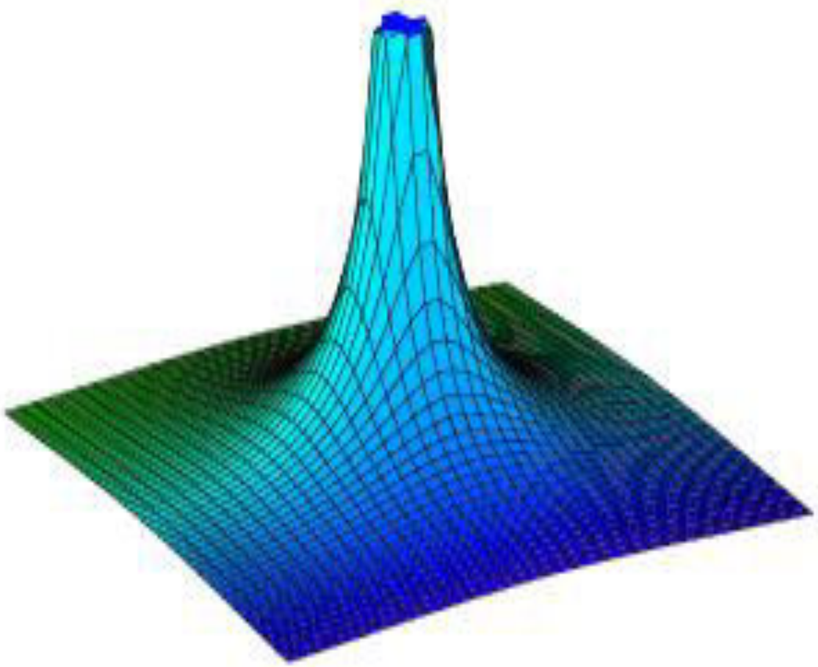
$$V(r) = \frac{q_0}{4\pi\epsilon_0 r}$$

Field Lines are perpendicular to equipotential lines

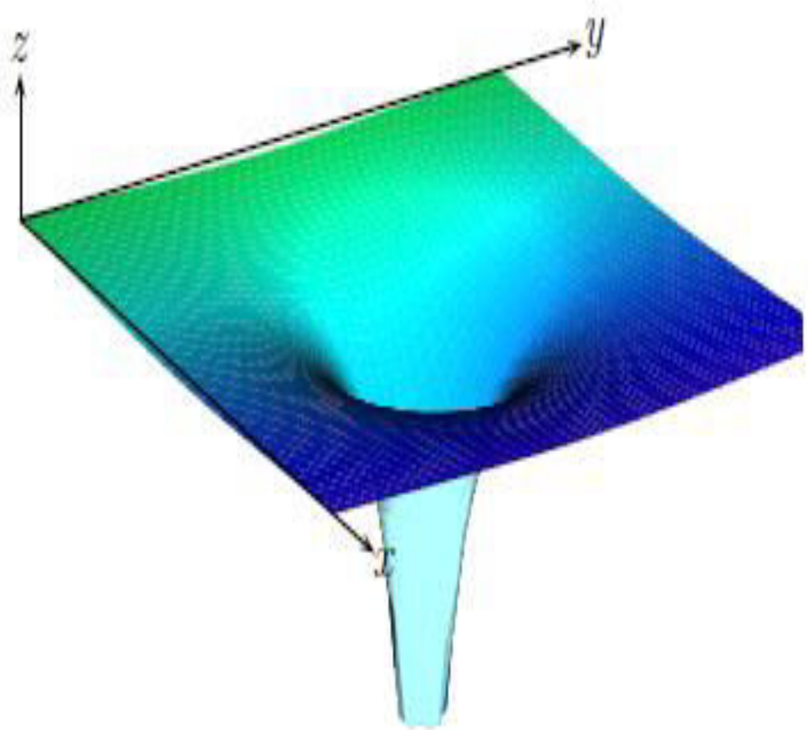
Topography of Electric potential V created by a ponctual charge

$$\boxed{V(r) = \frac{q_0}{4\pi\epsilon_0 r}}$$

q>0



q<0



Electrostatics-L2

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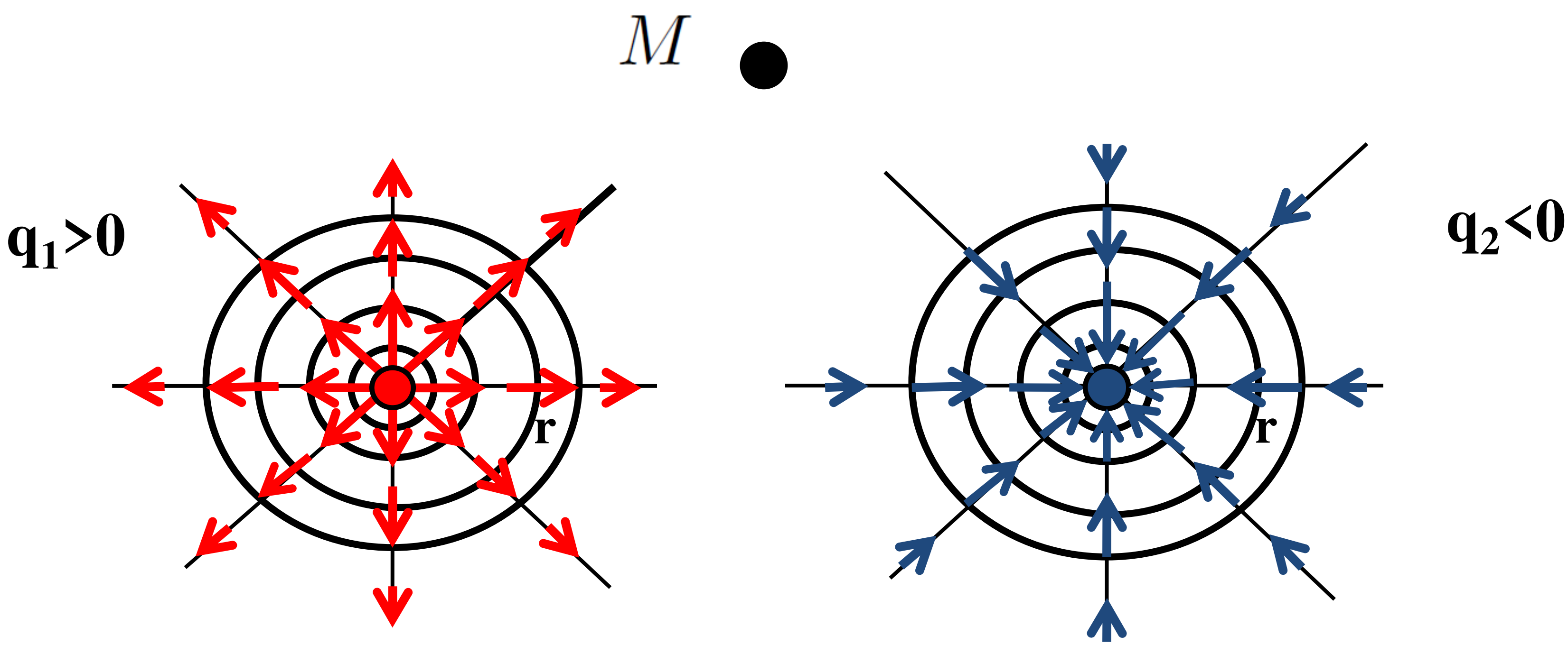
5) Electric field created by superposition of charges –

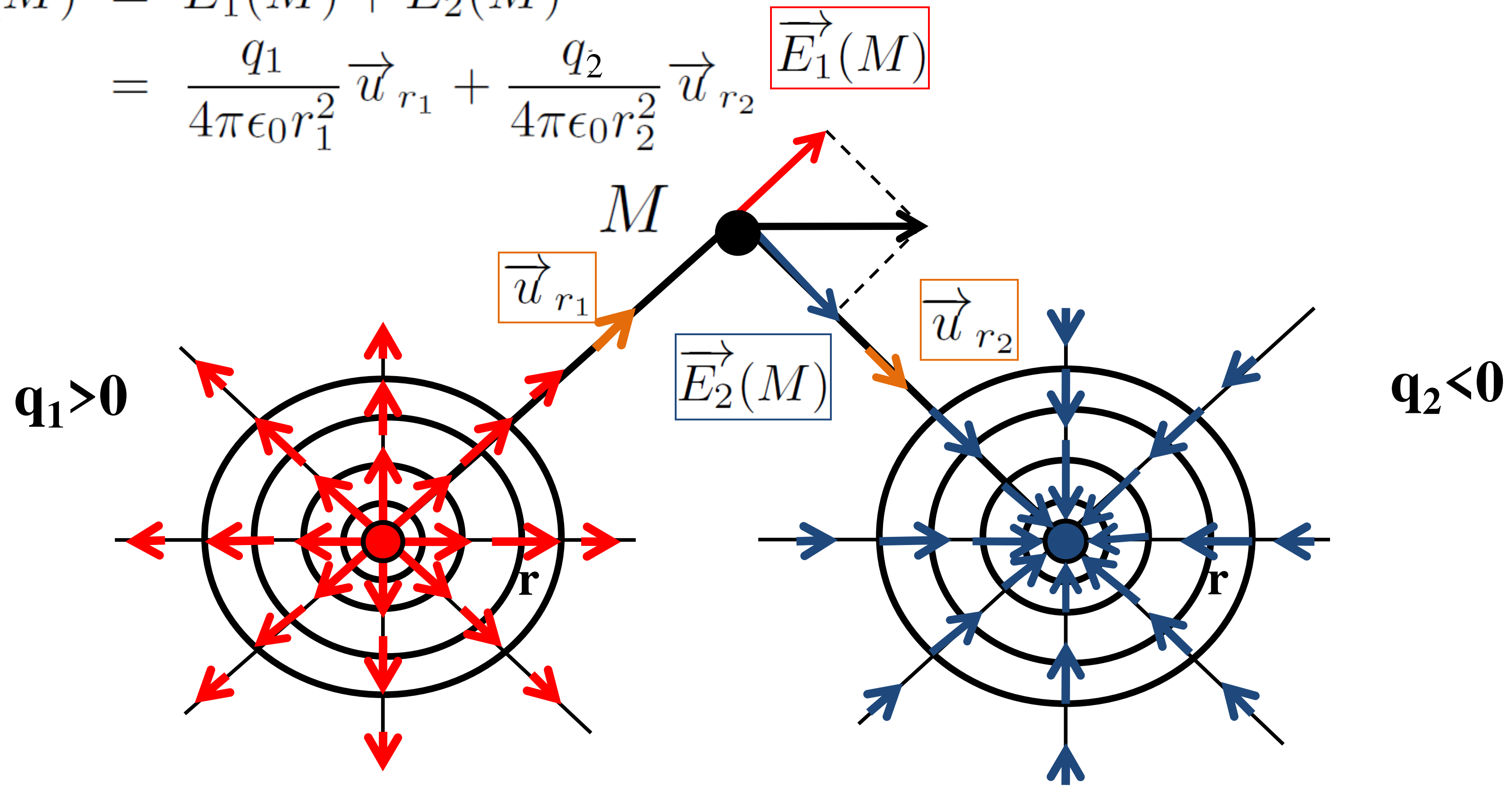
Two electric charges: Shape of the field lines-N electric charges-Continuous charges distribution **6) Symmetries of the electric field**

Superposition: electric field created by two charges

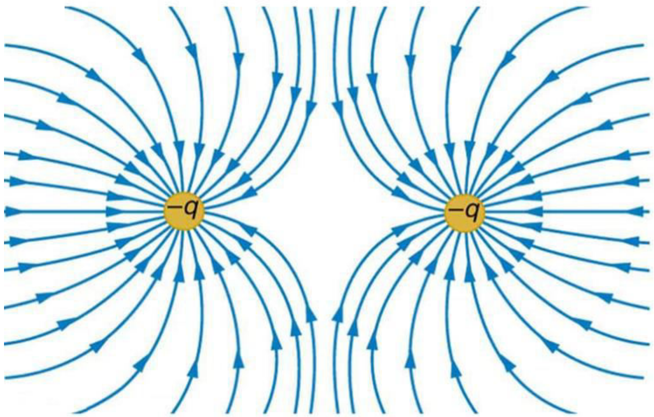


Superposition: electric field created by two charges

$$\begin{aligned}\overrightarrow{E}(M) &= \overrightarrow{E_1}(M) + \overrightarrow{E_2}(M)\\ &= \frac{q_1}{4\pi\epsilon_0 r_1^2}\overrightarrow{u}_{r_1} + \frac{q_2}{4\pi\epsilon_0 r_2^2}\overrightarrow{u}_{r_2}\end{aligned}$$

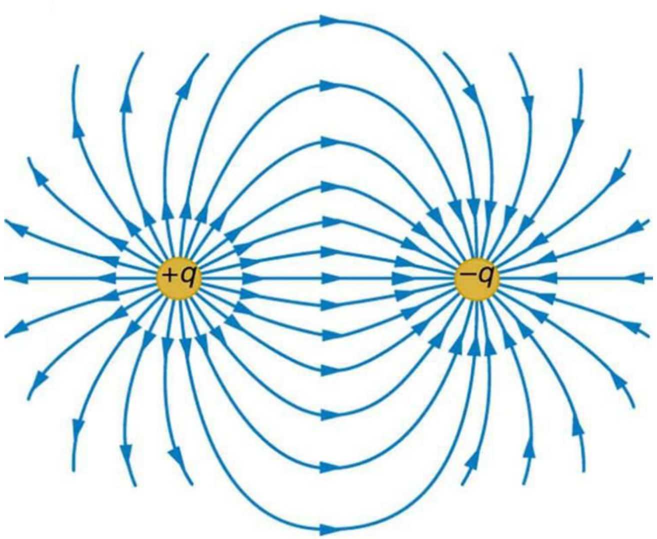


Electric field created by two charges



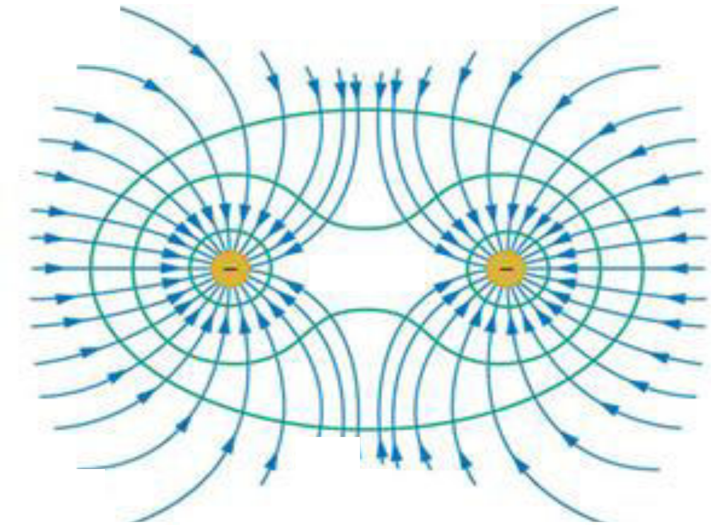
Field lines

q_1 and q_2 equal and <0



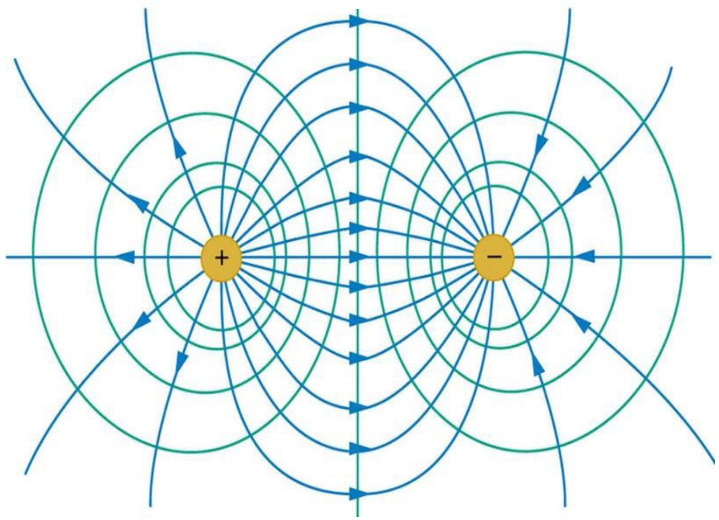
q_1 and q_2 opposite

Electric field created by two charges



Field lines
Equipotential lines

q_1 and q_2 equal and <0

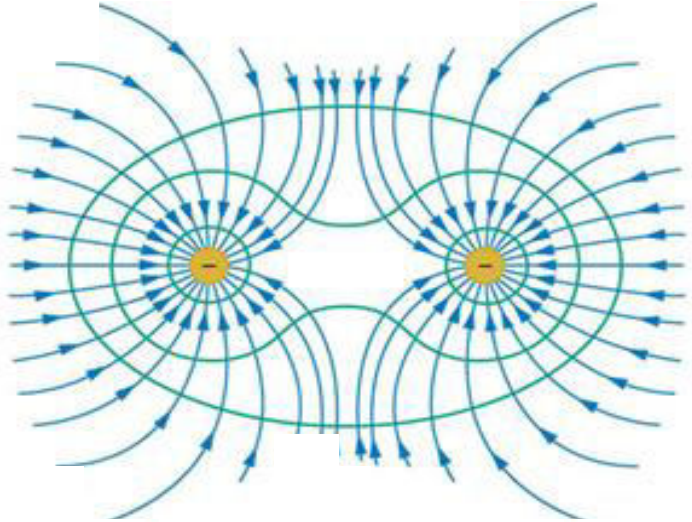


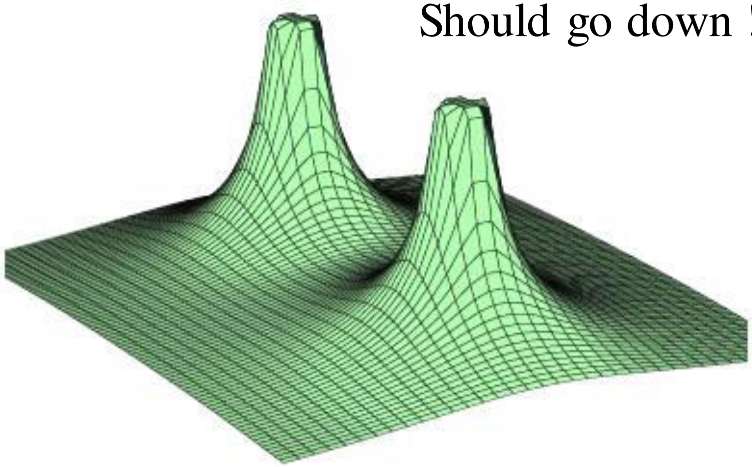
q_1 and q_2 opposite

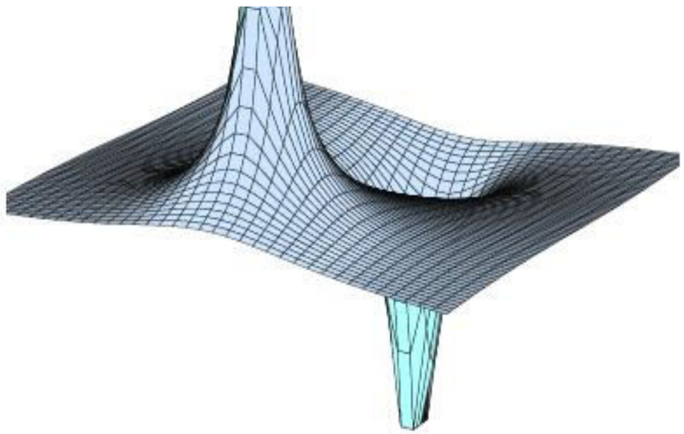
Electric field created by two charges

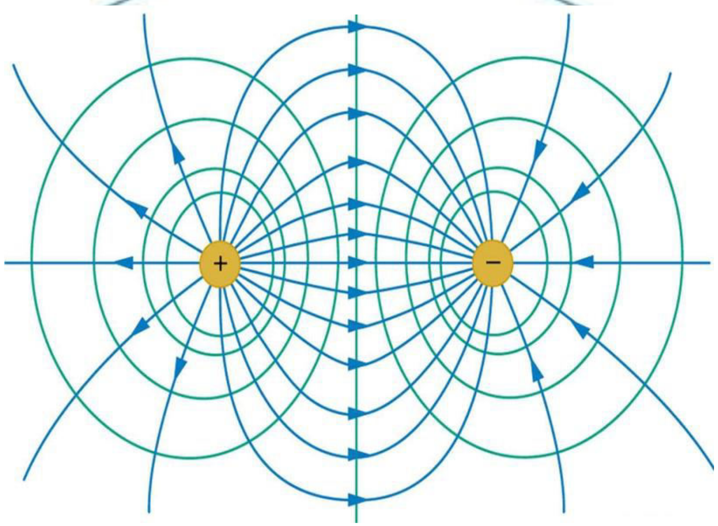
Topography of Electric potential V

!! (q>0 on map….)
Should go down !!

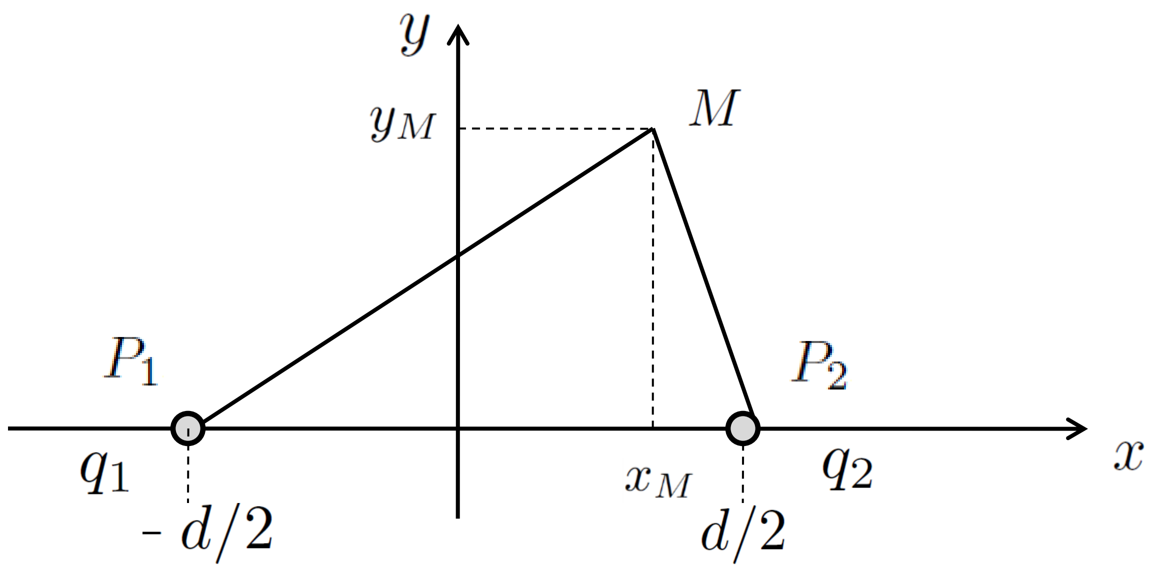








Challenging exercise to do at home



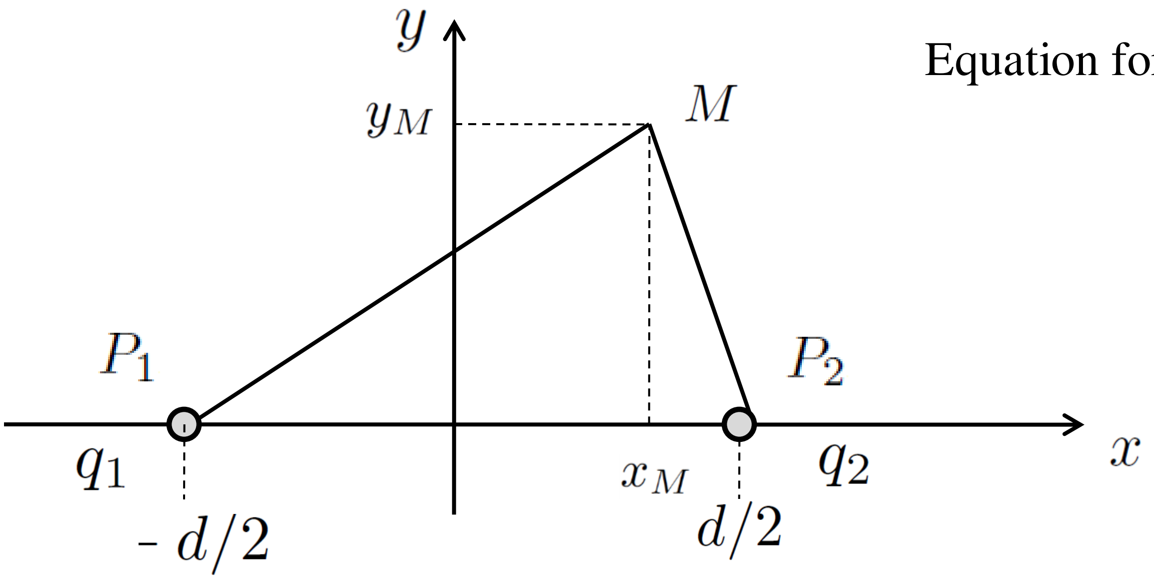
$$
\begin{aligned}
V(M) &= V_1(M) + V_2(M) \\
&= \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{P_1M} + \frac{q_2}{P_2M}\right) = \text{const}
\end{aligned}
$$

Potential at point M due to q_1 and q_2

With

$$
\begin{aligned}
P_1M &= \sqrt{(x_M - x_{P_1})^2 + (y_M - y_{P_1})^2} = \sqrt{(x+d/2)^+ y^2} \\
P_2M &= \sqrt{(x_M - x_{P_2})^2 + (y_M - y_{P_2})^2} = \sqrt{(x-d/2)^+ y^2}
\end{aligned}
$$

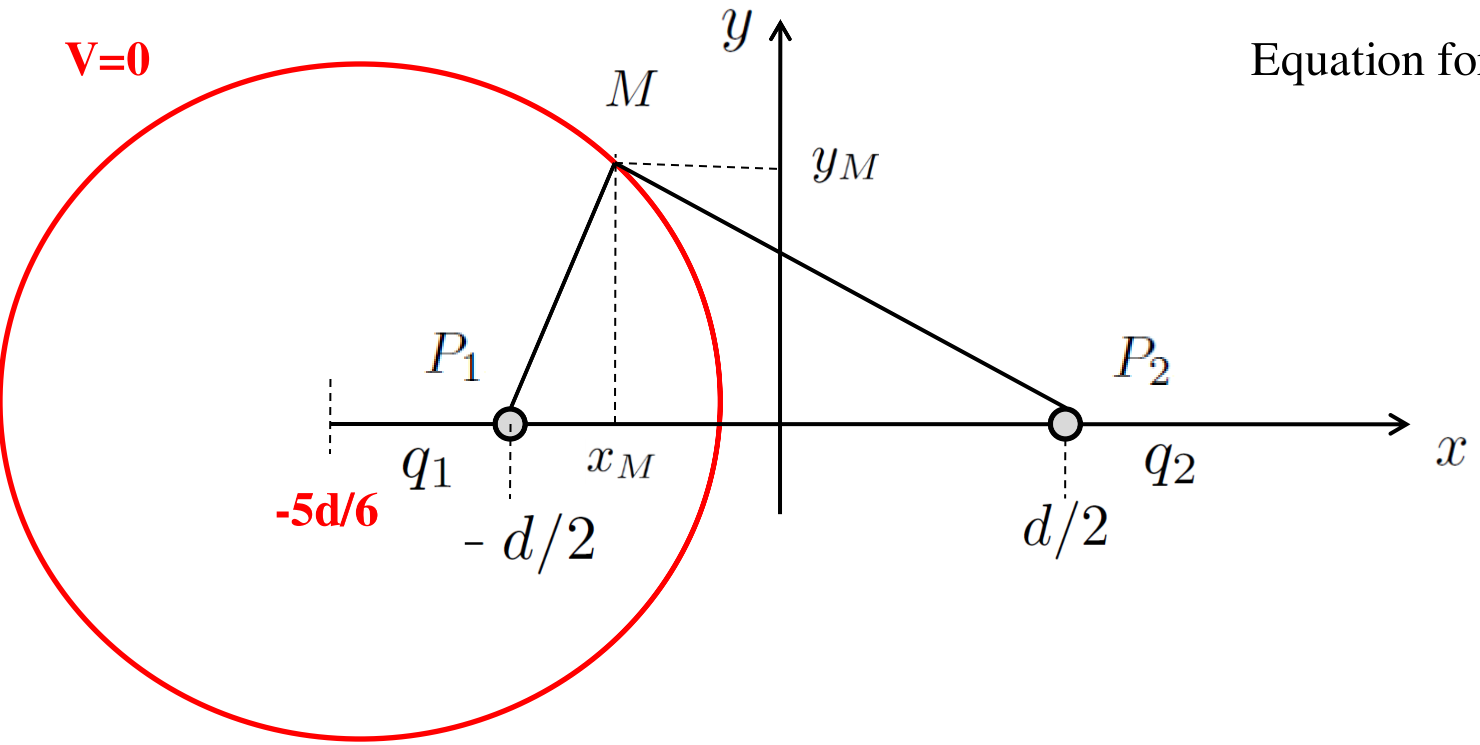
Challenging exercise to do at home



Equation for equipotential :

$$\boxed{V(M) = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{\sqrt{(x+d/2)^{+}y^2}} + \frac{q_2}{\sqrt{(x-d/2)^{+}y^2}}\right) = \text{ const}}$$

Challenging exercise to do at home



Equation for equipotential :

We put $\mathbf{q_1=-q}$ **and** $\mathbf{q_2=+2q}$: Show that the points that satisfy the **equipotential V=0**
Belong to a circle of radius R=2d/3 whose center is (x_c=-5d/6; y_c=0)

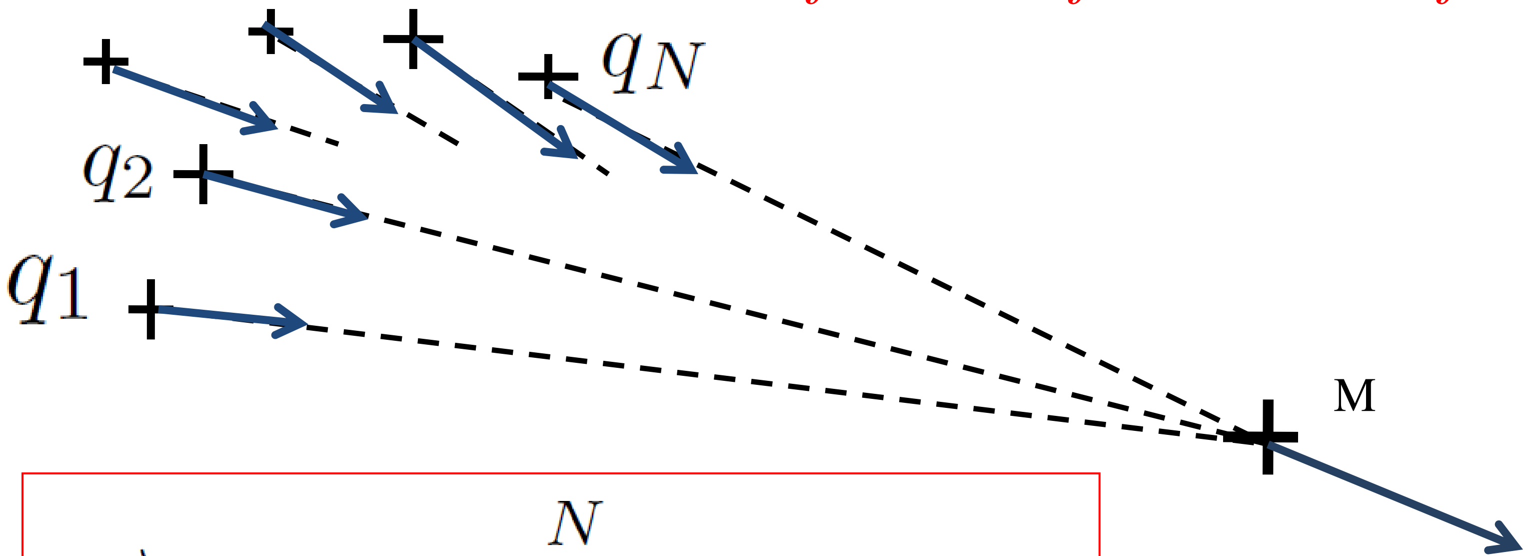
$$V = 0 \quad \Leftrightarrow \quad \left(x + \frac{5d}{6}\right)^2 + y^2 = \frac{4d^2}{9}$$

$$(x - x_c)^2 + (y - y_C)^2 = R^2$$

Equation of a circle

Superposition of N electric charges

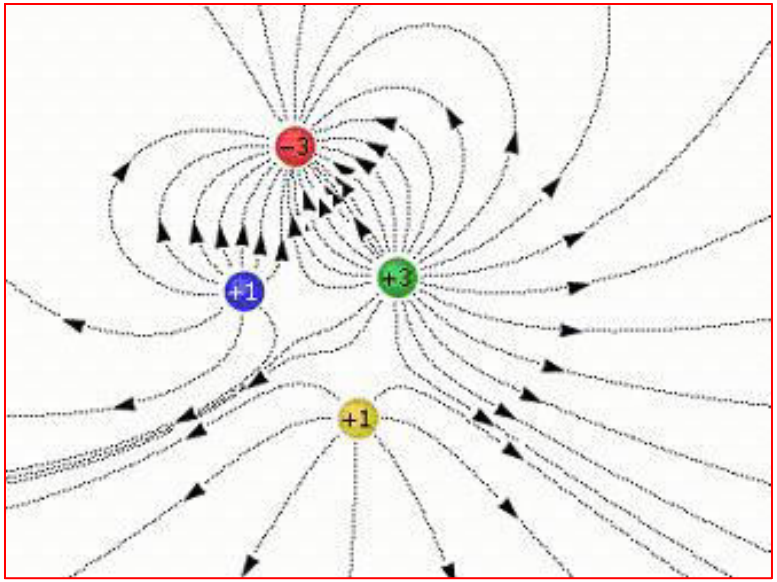
*If it is true for 2 it is true for N, no ??**

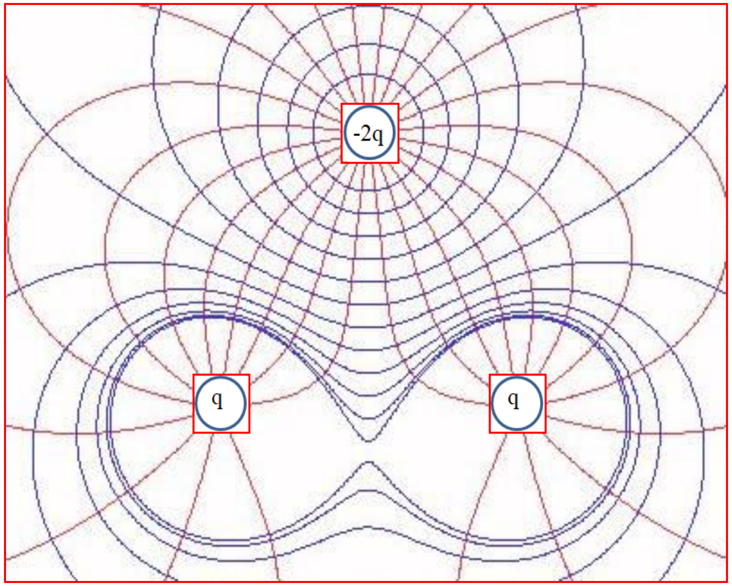


$$\boxed{\overrightarrow{E}(M) = \sum_{i=1}^{N} \frac{q_i}{4\pi\epsilon_0 r_i^2} \overrightarrow{u}_{r_i}}$$

* In reality, the interaction between the N charges modify the total electric field (out of program obviously and probably of your life which of course we do not wish)

So, it is a funny game to look to the **electric field lines** created by many charges





In tutorial….

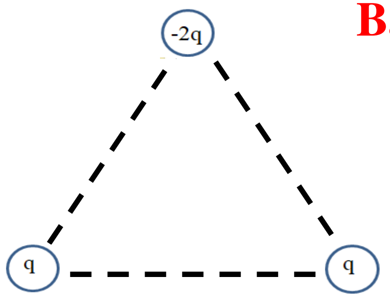
Exercise 4

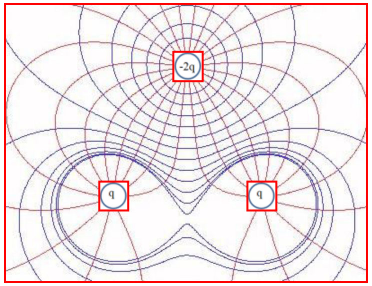
1) Draw the field lines of an electrostatic doublet: (-q ;+q) and then (+q ;+q).

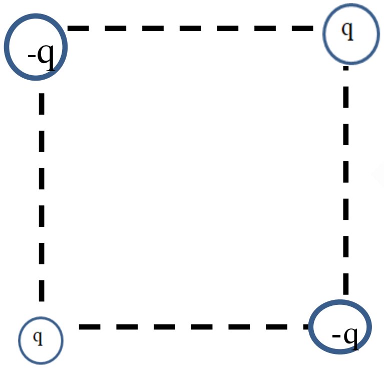
2) We consider an equilateral triangle with electric charges on its summits. We have three configuration (+q;+q;+q) , (+q;+q;+2q) and (+q,+q,-2q). Calculate the electric field in the center of the triangle for each case. Draw the field lines and determine the axis of symmetry.

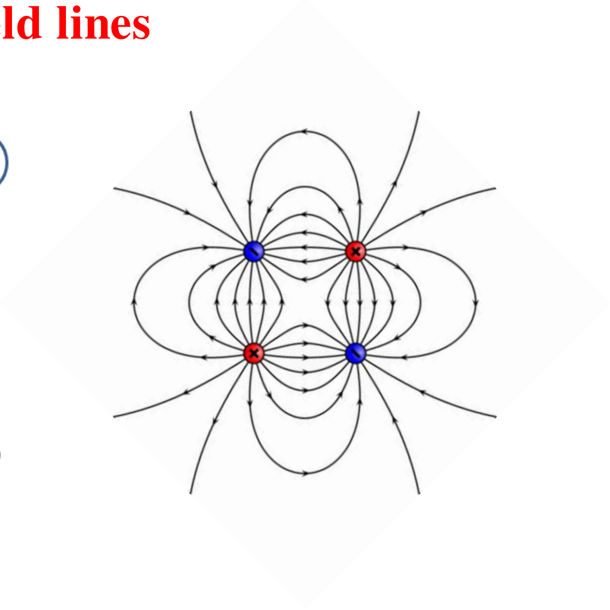
3) We consider four electric charges in the summits of a square ABCD of size a. Two are positive and two are negative. In the first configuration, the positive charges share the same side of the square (A and B) and in the second configuration, the positive charges occupy the diagonal (A and C). Draw the field lines and determine the axis of symmetry. For the second configuration, calculate the electric force felt by the charge in A and then the electric force in the center of the square when the electric charges are 0.4 μC, -0.8 μC and if the size of the square a is 10 cm.

Basics geometric figures and….funny field lines

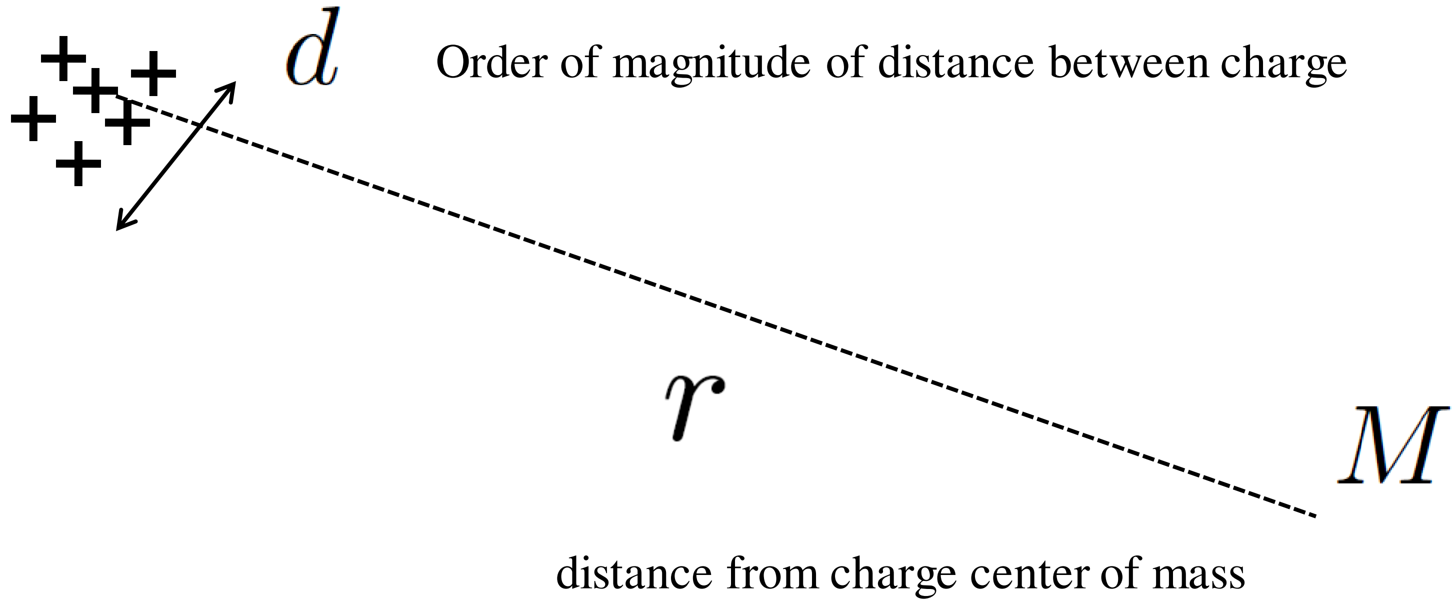




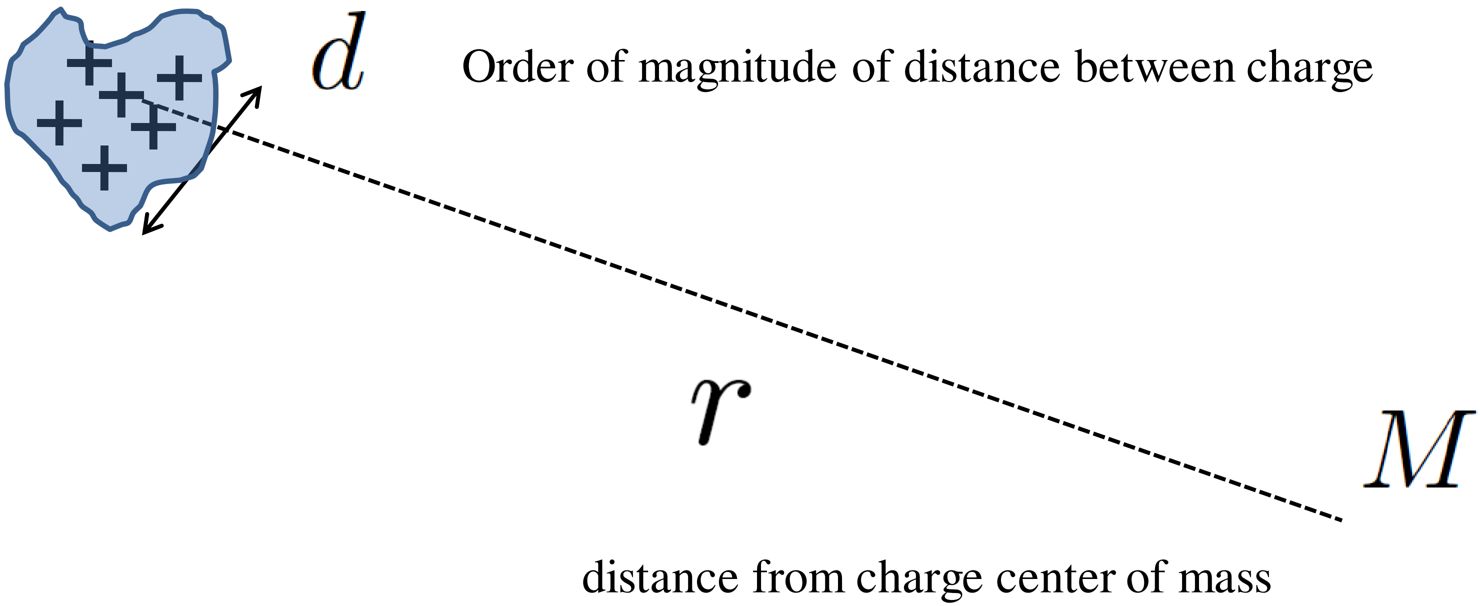




Calculation of the total field produced by charge distribution



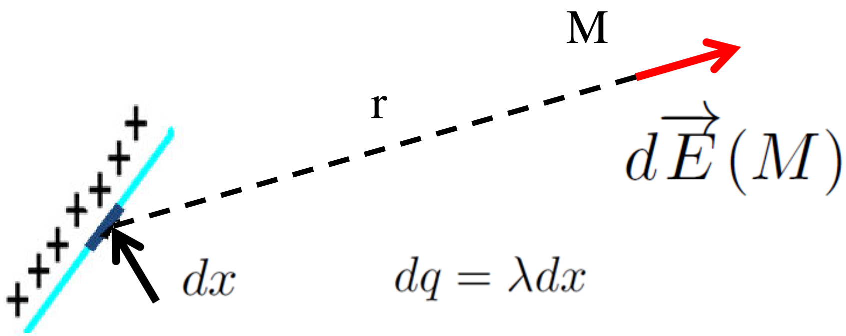
Calculation of the total field produced by charge distribution



If r >> d we can assume a <u>continuous charge distribution</u>

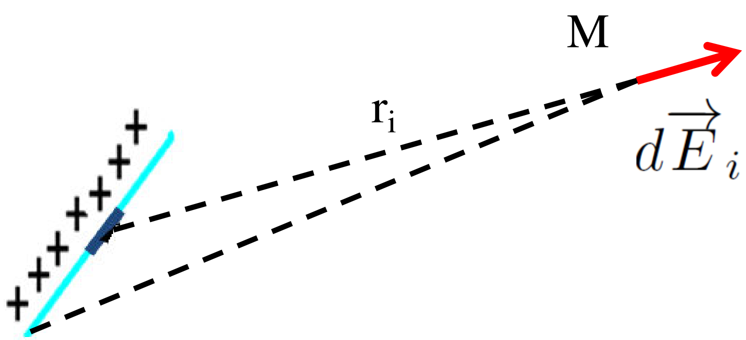
Calculation of the total field produced by charge distribution

1. We consider first elementary field produced by element dx of charge dq



$$d\overrightarrow{E}(M) = \frac{dq}{4\pi\epsilon_0 r^2}\overrightarrow{u_r}$$

2. Total field is the sum of the field created by all elements dqi

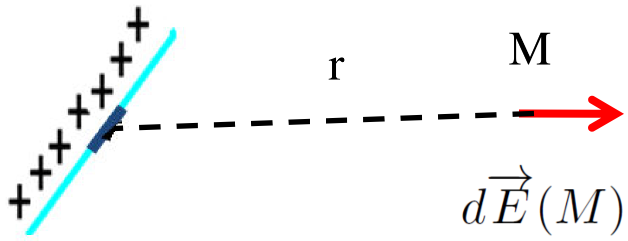


$$\overrightarrow{E}(M) = \sum_i d\overrightarrow{E}_i(M) = \sum_i \frac{dq_i}{4\pi\epsilon_0 r_i^2}\overrightarrow{u_{r_i}}$$

Can be written as an integral form

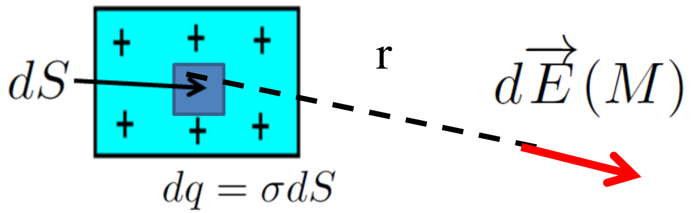
$$\overrightarrow{E}(M) = \int \frac{\lambda dx}{4\pi\epsilon_0 r^2}\overrightarrow{u_r}$$

ELECTRIC FIELD



For a lineic distribution we have an integral

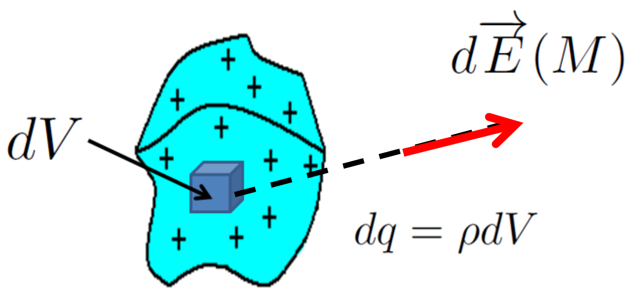
$$\vec{E}(M) = \int \frac{\lambda dx}{4\pi\epsilon_0 r^2}\vec{u_r}$$



For a surface distribution we have a double integral

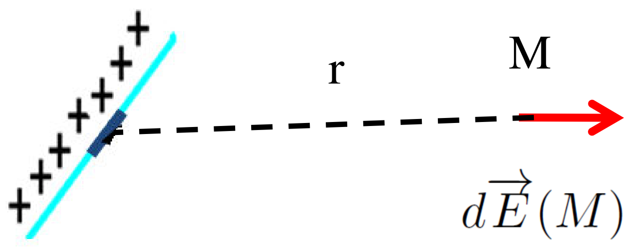
$$\vec{E}(M) = \iint \frac{\sigma dS}{4\pi\epsilon_0 r^2}\vec{u_r}$$

For a volumic distribution we have a triple integral



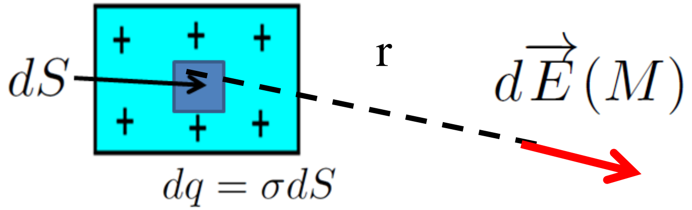
$$\vec{E}(M) = \iiint \frac{\rho dV}{4\pi\epsilon_0 r^2}\vec{u_r}$$

ELECTRIC POTENTIAL



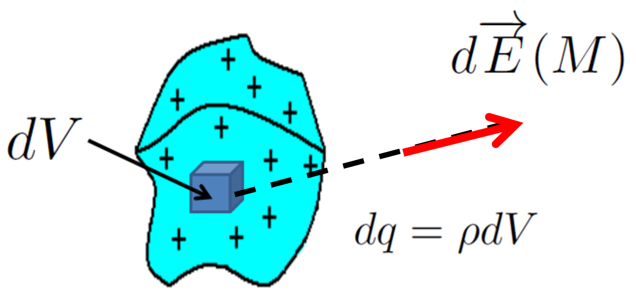
For a lineic distribution we have an integral

$$V(M) = \int \frac{\lambda dx}{4\pi\epsilon_0 r}$$



For a surface distribution we have a double integral

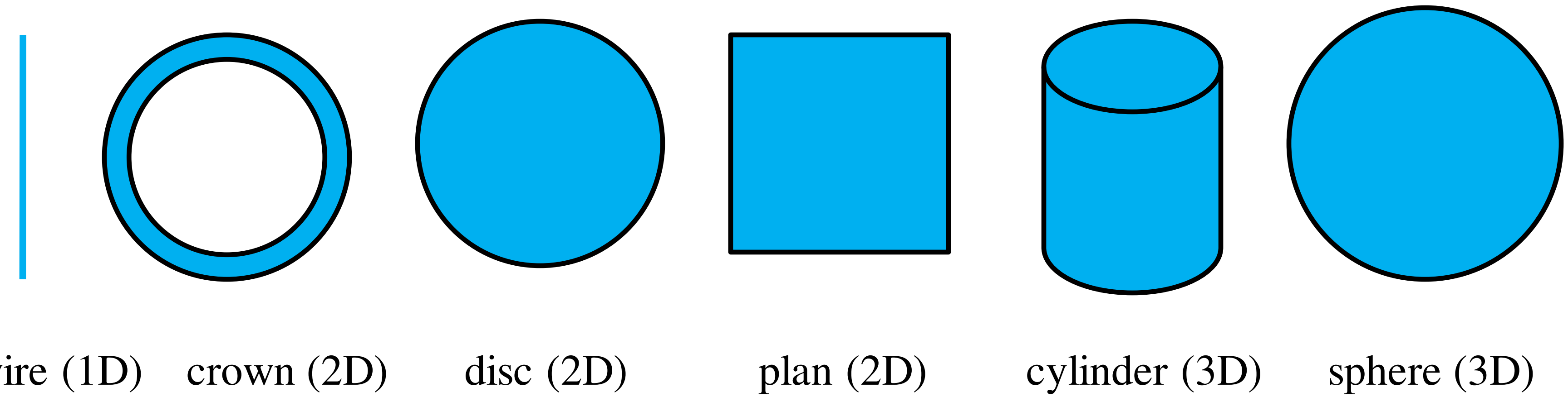
$$V(M) = \iint \frac{\sigma dS}{4\pi\epsilon_0 r}$$



For a volumic distribution we have a triple integral

$$V(M) = \iiint \frac{\rho dV}{4\pi\epsilon_0 r}$$

We will see during lecture the electric field and potential of some distributions



Method(s)

1) First calculation of electric field
   and then electric potential by integration

$$\boxed{\int_A^B \overrightarrow{E} \cdot \overrightarrow{dr} = V_A - V_B}$$

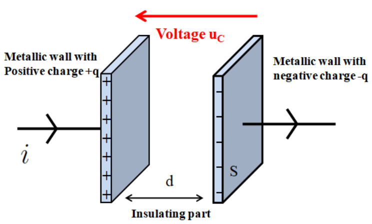
OR

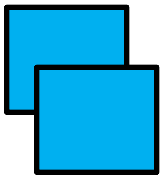
2) First calculation of electric potential
   and then electric field by derivation

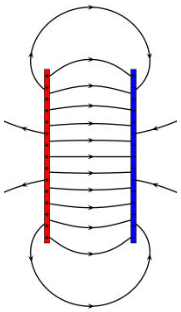
$$\boxed{\overrightarrow{E} = -\overrightarrow{grad}V = -\overrightarrow{\nabla}V}$$

Some Goal(s) of the Electrostatics chapter:
Given a electric charge distribution, how is distributed the electric field ?

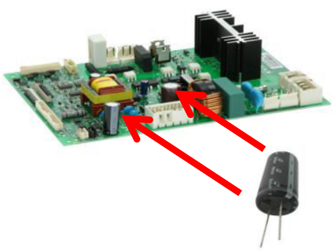
Example 1: The plane or cylindrical capacitor

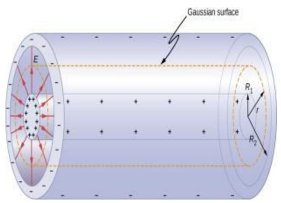


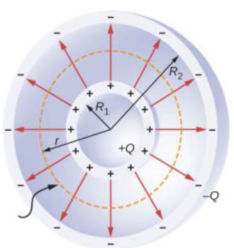




2 plans….

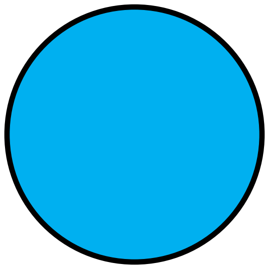






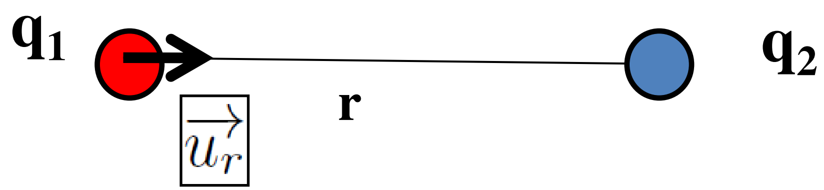
2 coaxial cylinders

Example 2: The electrostatics of Earth….



1 sphere…

The potential energy of two electric charges is given by



$$E_p = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

$$= q_1 \frac{q_2}{4\pi\epsilon_0 r} = q_1 V_2$$

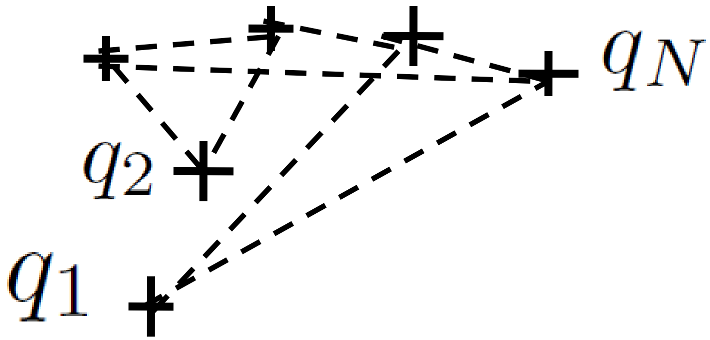
$$= q_2 \frac{q_1}{4\pi\epsilon_0 r} = q_2 V_1$$

$$\boxed{E_p = \frac{1}{2}\left(q_1 V_2 + q_2 V_1\right)}$$

Be careful, mistake in first version (between 1 and 2)

I.5) Energy of a superposition of charges

The potential energy of N electric charges…..a lot of interactions

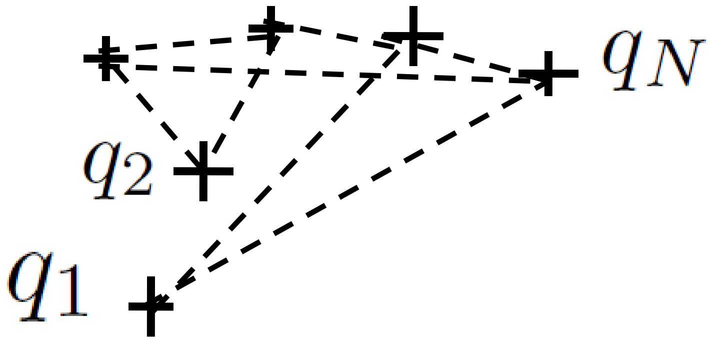


Let’s calculate the quantity…

$$q_1\,(0+V_2+V_3+\ldots+V_N)$$
$$+q_2\,(V_1+0+V_3+\ldots+V_N)$$
$$+q_2\,(V_1+V_2+0+\ldots+V_N)$$
$$\ldots.$$
$$+q_N\,(V_1+V_2+\ldots+V_{N-1}+0)$$

I.5) Energy of a superposition of charges

The potential energy of N electric charges…..a lot of interactions



Let's calculate the quantity…

$$
\begin{aligned}
&q_1\left(0+V_2+V_3+\ldots+V_N\right)\\
&+q_2\left(V_1+0+V_3+\ldots+V_N\right)\\
&+q_2\left(V_1+V_2+0+\ldots+V_N\right)\\
&\ldots.\\
&+q_N\left(V_1+V_2+\ldots+V_{N-1}+0\right)
\end{aligned}
$$

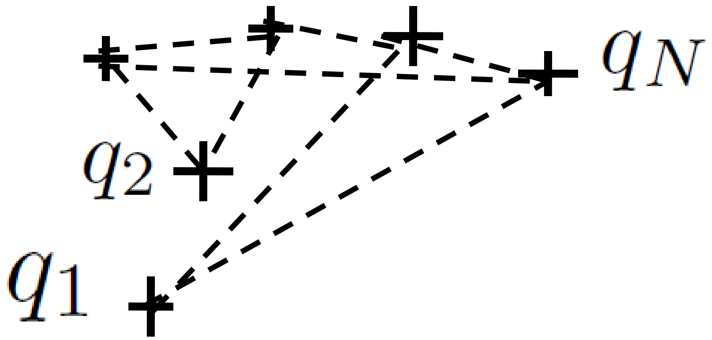
SUM

$$\sum_{i=1}^{N} q_i\left(\sum_{i\neq j}^{N} V_j\right)=2E_p$$

Because $q_iV_j=q_jV_i$

Each interaction is counted twice !

The potential energy of N electric charges…..a lot of interactions



Let's calculate the quantity…

$$q_1 \left(0 + V_2 + V_3 + ... + V_N\right)$$
$$+q_2 \left(V_1 + 0 + V_3 + ... + V_N\right)$$
$$+q_2 \left(V_1 + V_2 + 0 + ... + V_N\right)$$
$$....$$
$$+q_N \left(V_1 + V_2 + ... + V_{N-1} + 0\right)$$

SUM

$$\sum_{i=1}^{N} q_i \left(\sum_{i \neq j}^{N} V_j \right) = 2E_p$$

Because $q_iV_j=q_jV_i$

Each interaction is counted twice !

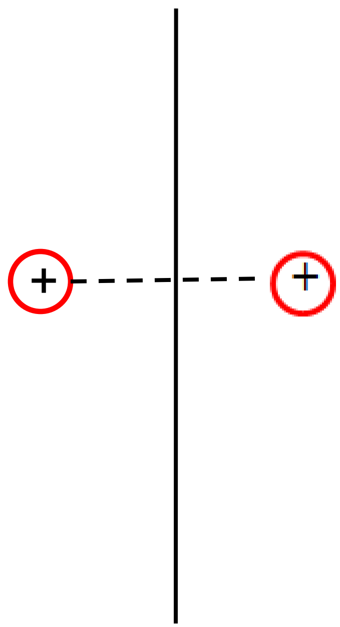
$$\boxed{E_p = \frac{1}{2} \sum_{i=1}^{N} q_i \left(\sum_{i \neq j}^{N} \frac{q_j}{4\pi\epsilon_0 r_{ij}} \right)}$$

Electrostatics-L2

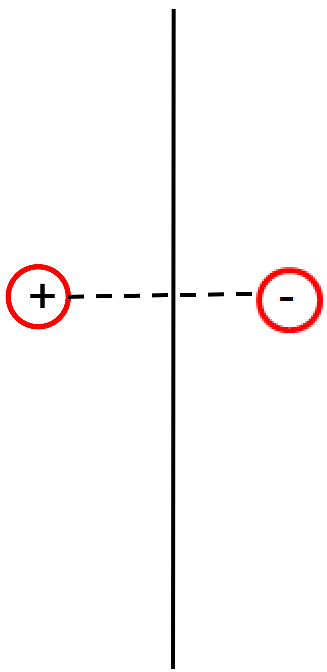
1) Electrostatics devices **2) Electric charges** -Ponctual charges -Continuous charges distributions
3) Electric forces and Electric fields - The Coulomb law -The electric field -Field lines
4) Electric potential and energy- Work of an electric force- Electric potential -Equipotential lines
5) Electric field created by superposition of charges –
Two electric charges: Shape of the field lines-N electric charges-Continuous charges distribution

6) Symmetries of the electric field

Symmetry of the electric charge distribution

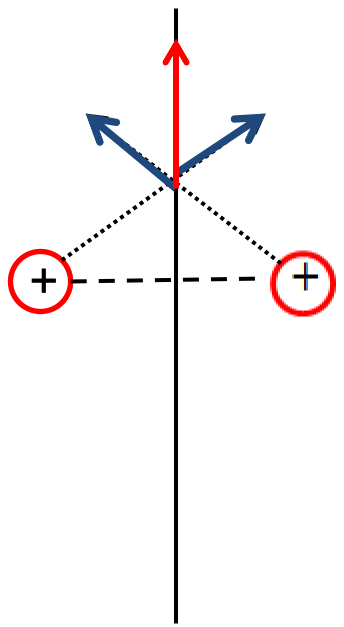


PLAN OF SYMMETRY



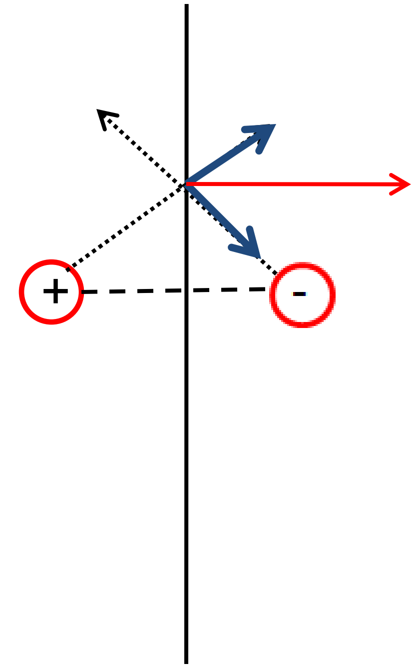
PLAN OF ANTI- SYMMETRY

Symmetry of the electric charge distribution



PLAN OF SYMMETRY

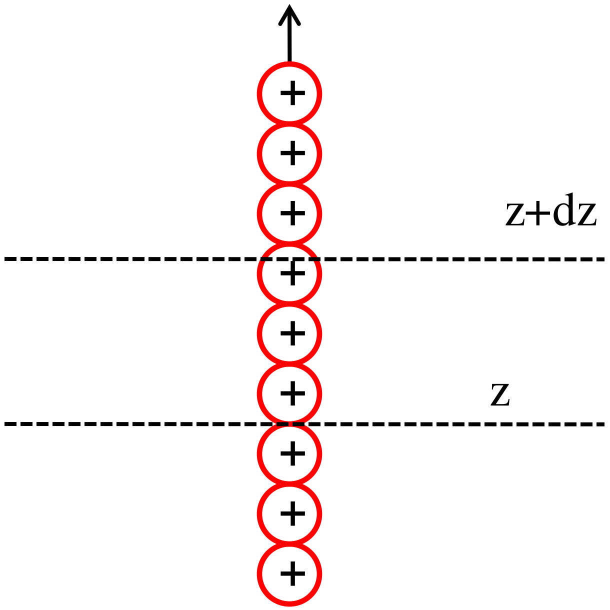
An electric field created in a point o f the plane of symmetry belongs to the plane

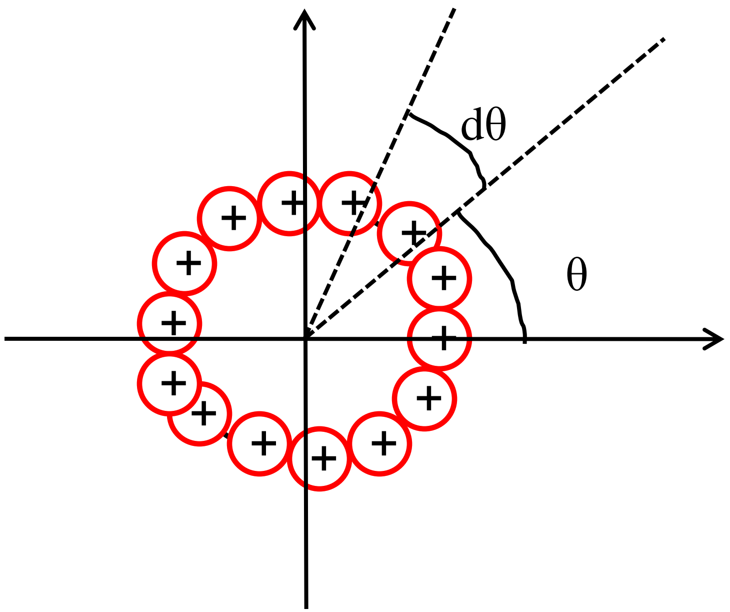


PLAN OF ANTI- SYMMETRY

An electric field created in a point of the plane of anti- symmetry is perpendicular to the plane

Symmetry of the electric charge distribution





SYMMETRY OF TRANSLATION

$\rho(x,y,z)=\rho(x,y)$

$\rho(z+dz)=\rho(z)$

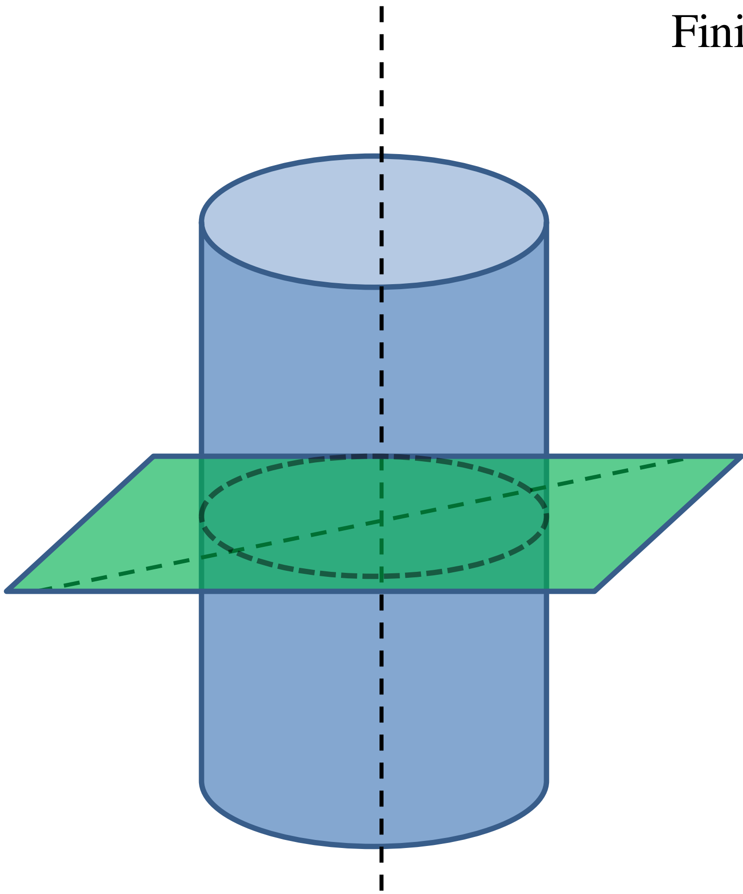
SYMMETRY OF ROTATION

$\rho(r,\theta)=\rho(r)$

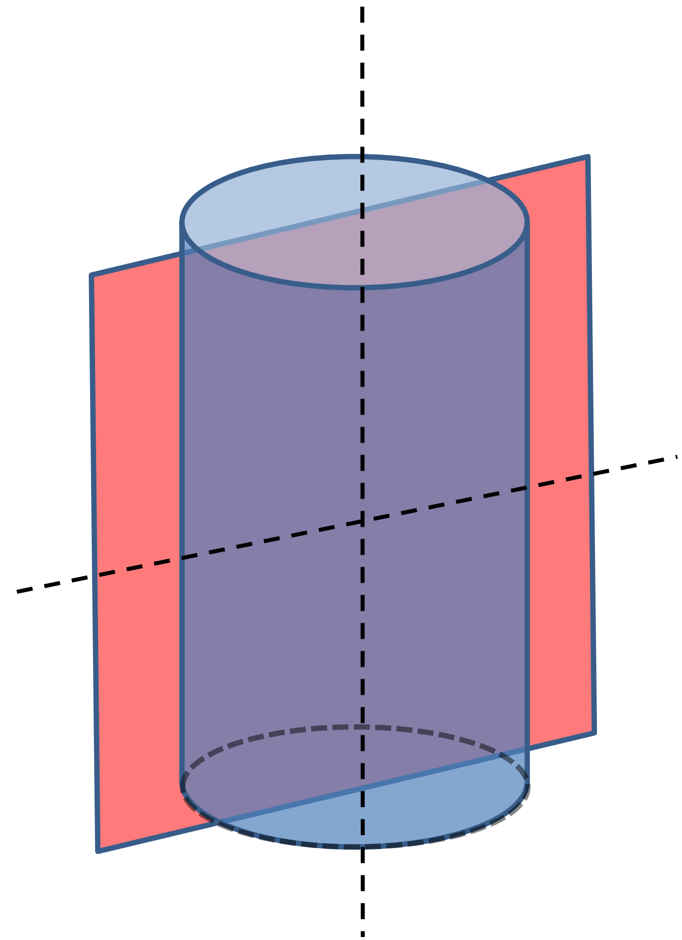
$\rho(\theta+d\theta)=\rho(\theta)$

Cylindrical symmetry

Finite cylinder



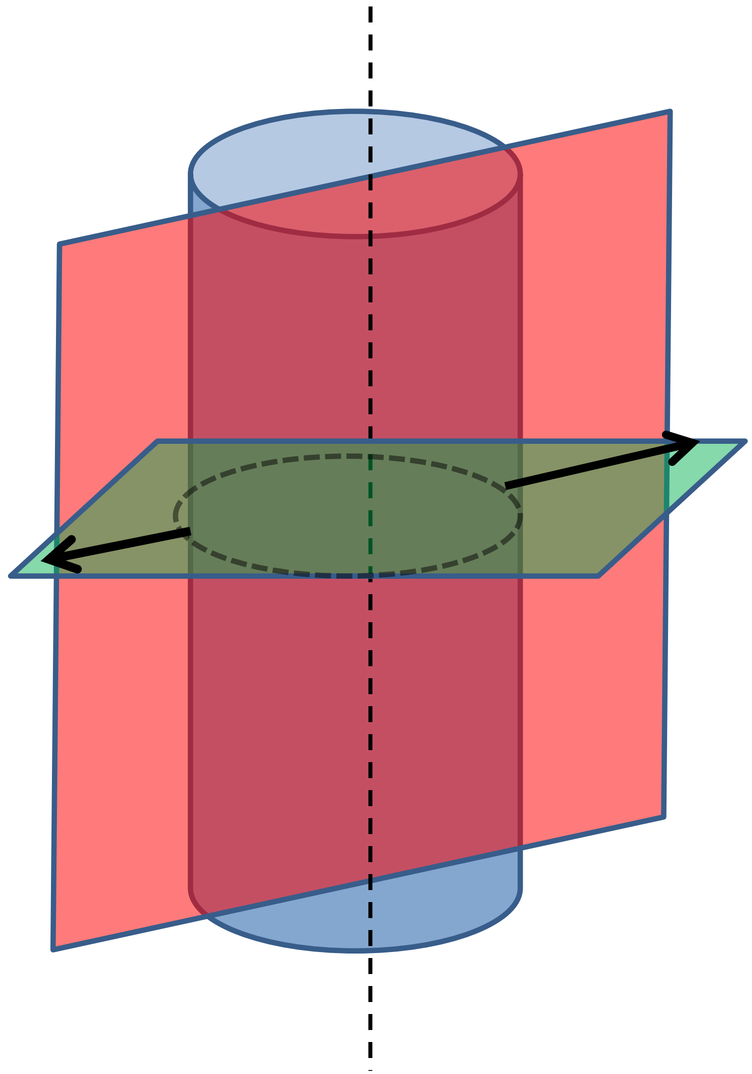
Plan in the middle of the cylinder



Any plan that goes through the center of base

Cylindrical symmetry

An electric field created in a point o f the plane of symmetry belongs to the plane



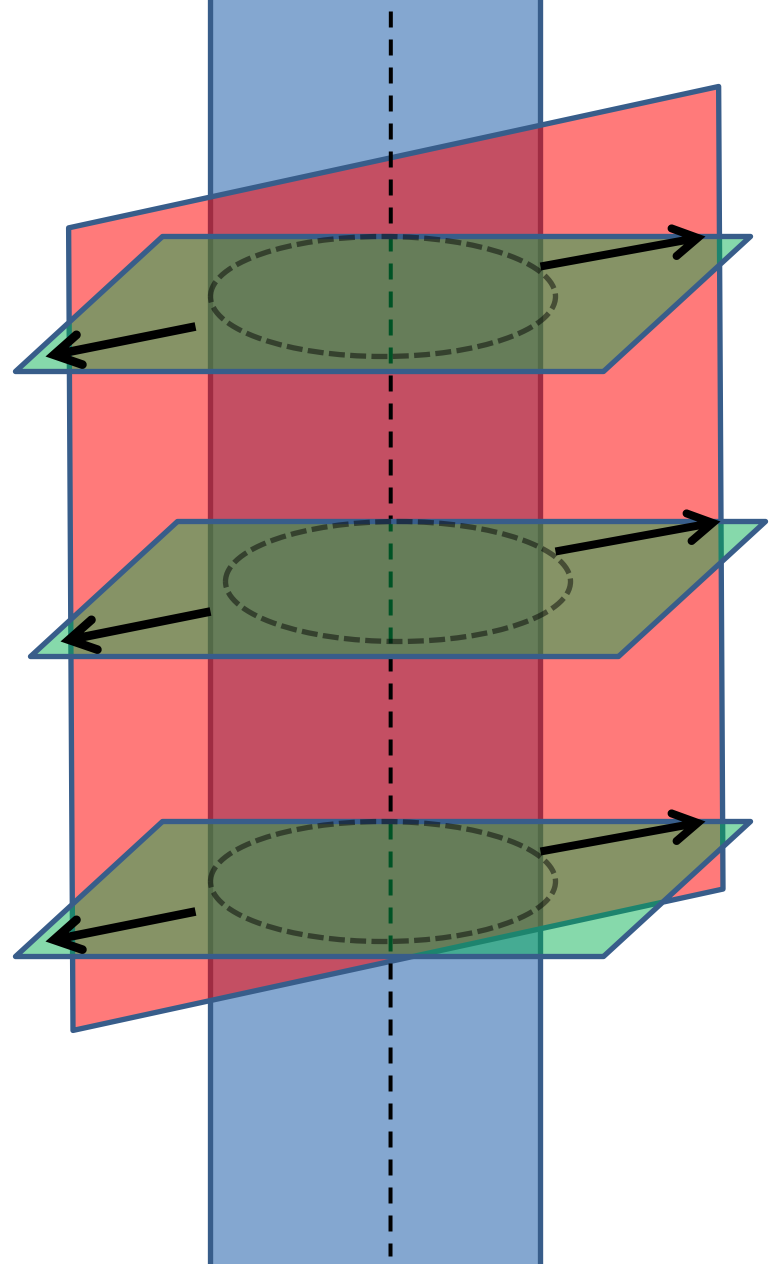
Cylindrical symmetry

Infinite cylinder

…..

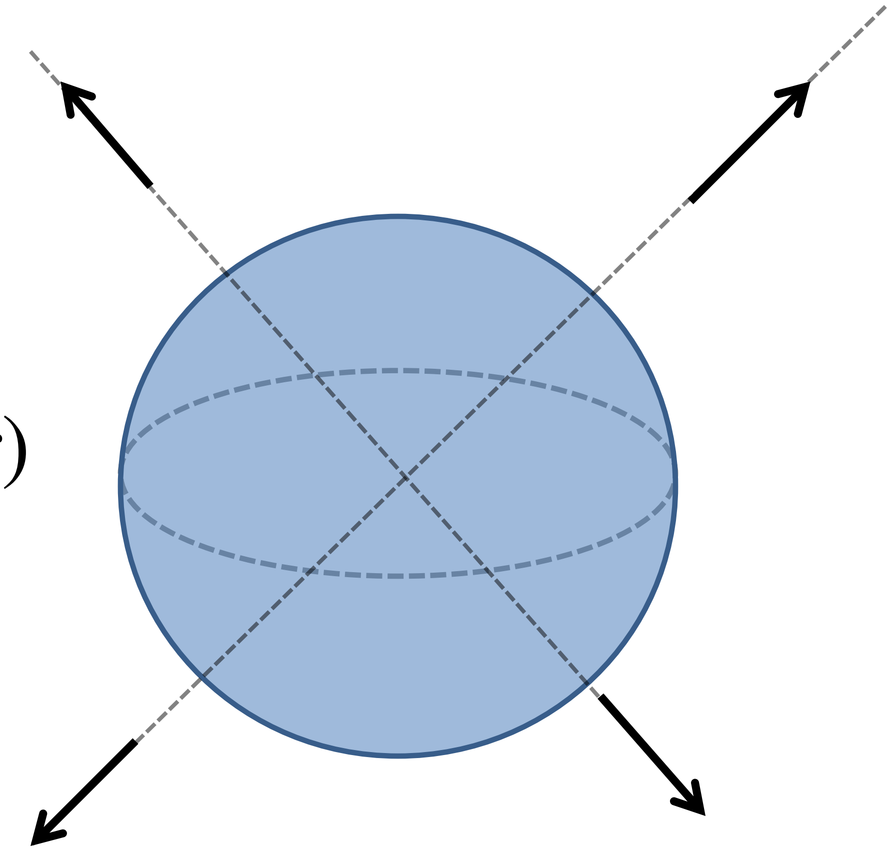
Infinity of *middle of cylinder*

$$F(r,\theta,z)=F(r)$$



Spherical symmetry

$$F(r,\theta,\varphi)=F(r)$$



Any axis that goes through the center

Physics-L2 Electromagnetism

Approximative program

1) Introduction and basics
2) Electric field calculations
3) Gauss theorem
4) Electric dipoles

Chap 1: Electrostatics
Chap 2: Magnetostatics
Chap 3: Time-dependent regime-Induction phenomena
Chap 4: Maxwell equations
Chap 5: Dielectric media and applications
Chap 6: Conducting media and applications
Chap 7: Magnetic media and applications

week	Magistral lectures
1	Electrostatics
2	Electrostatics
3	Electrostatics
4	Electrostatics
5	Magnetostatics
6	Magnetostatics
7	Induction
8	Induction
9	Maxwell equations
10	Maxwell equations
11	Dielectric media
12	Dielectric / Metallic media
13	Metallic Media
14	Magnetic media

Electrostatics-L2

2) Electric field calculations

1) **Electric field created by a charged electric wire-**
   Calculation of the electric field**-** Limit case of the infinite wire.**-**Analysis in terms of field lines
2) **Some examples of 2D electric charged structures-**Electric field and electric potential created by a crown and a disc**-** Limit case of the infinite charged plane**-**Analysis in terms of field lines
3) **Application to the plane capacitor-** Electric field and electric potential**-**Capacitance and energy.

Exercise 1: Electric field created by a charged wire

Exercise 1: Electric charged wire

We consider a wire of size 2a and a negligible section with a uniform lineic density of charge λ.

1) Show that the electric field at a point M that belongs to the system symmetry axis at a distance z from the wire is written as:

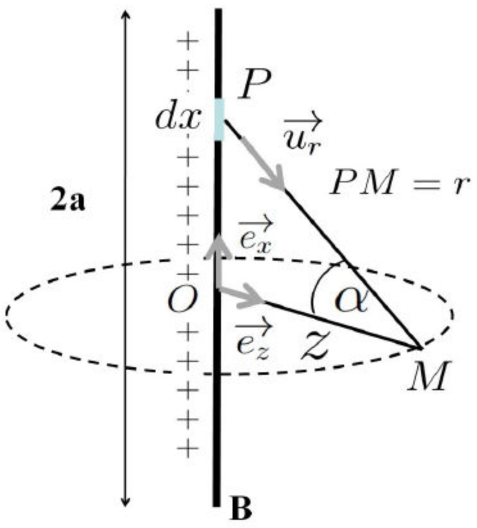
$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0} \frac{a}{z\sqrt{z^2 + a^2}} \overrightarrow{e_z}$$

2) Check that the obtained formula is homogenous to an electric field.

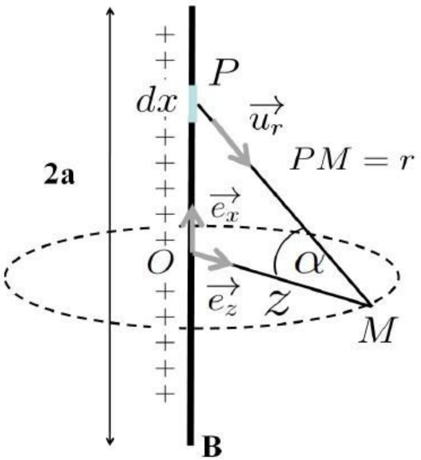
3) What is the expression of the electric potential V(M) ?

4) How is transformed the previous results when the wire is considered as infinite?

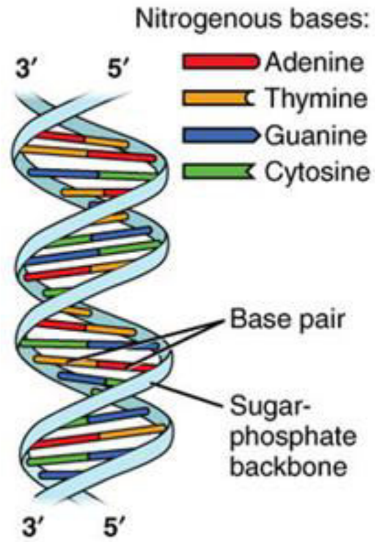
5) Discuss the orientation of the field lines.

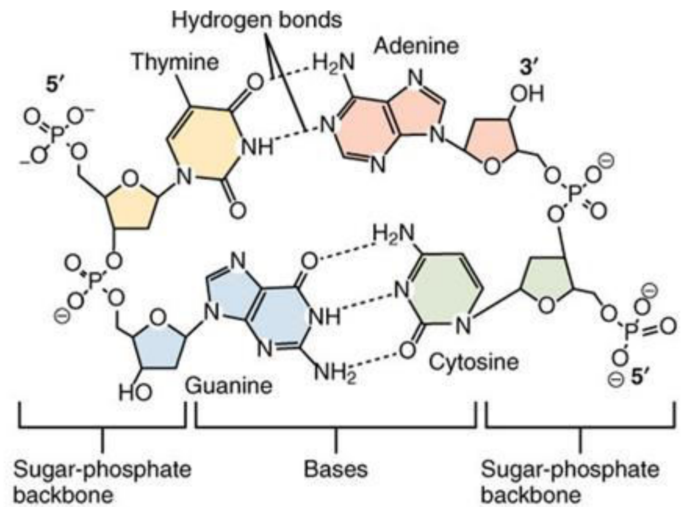


Application in Biology-Biophysics



0.34 nm

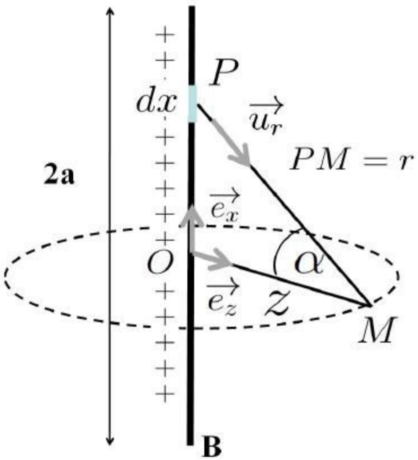


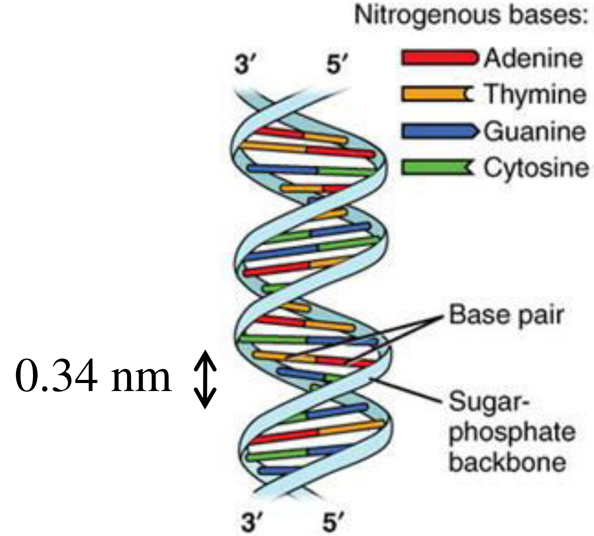


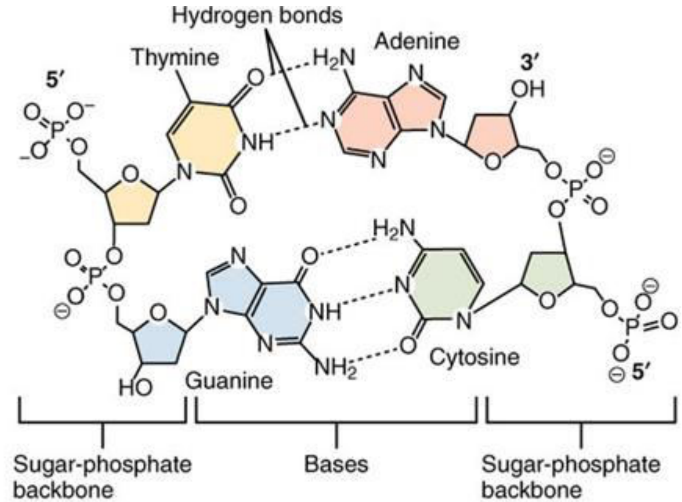
Phosphorus atoms with negative charge

-e at altitude of each base pair

Application in Biology-Biophysics



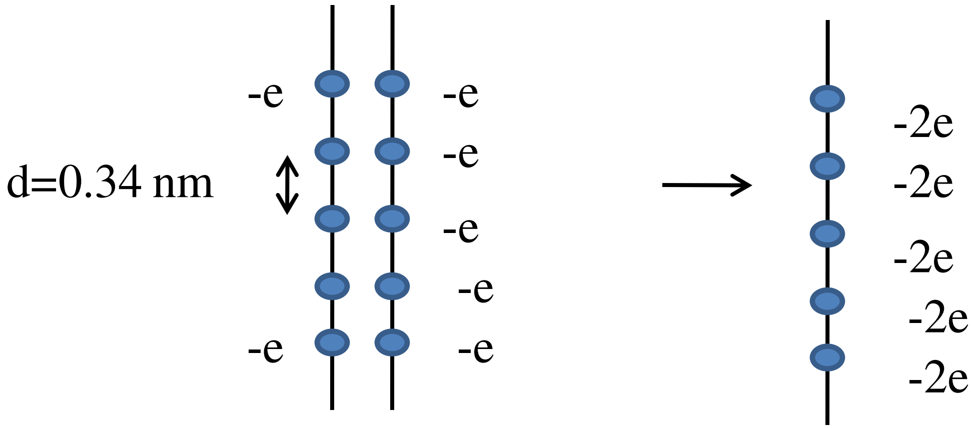




Phosphorus atoms with negative charge

-e at altitude of each base pair

Modelisation with Physics

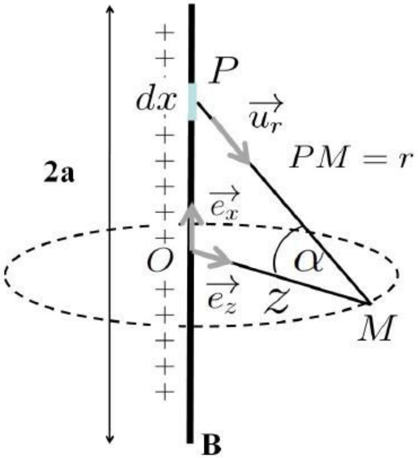


$\lambda = -2e/d$

$= -2.\ 1{,}6\ 10^{-19} / 3.4\ 10^{-10}$

$= 1.06\ 10^{-9}$ C/m

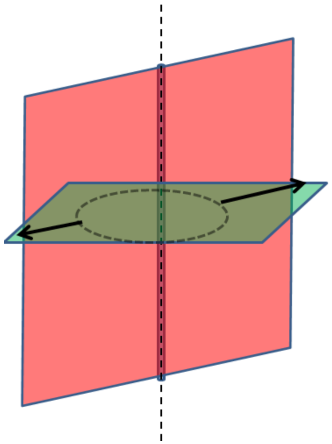
After Calculations using integration

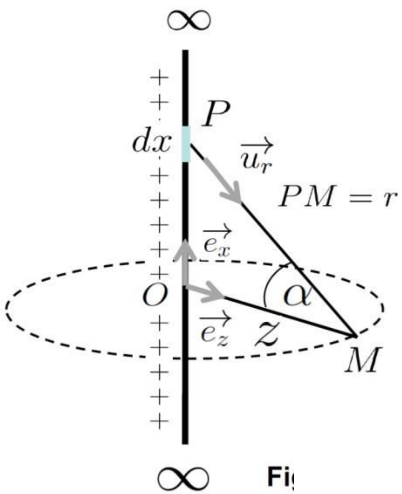


Expression of the electric field in the plane cutting the middle of the stick of size 2a:

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0}\frac{a}{z\sqrt{z^2+a^2}}\overrightarrow{e_z}}$$

The electric field vector is perpendicular to the axis given by the wire



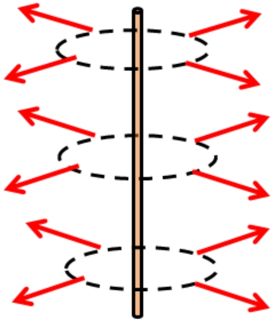


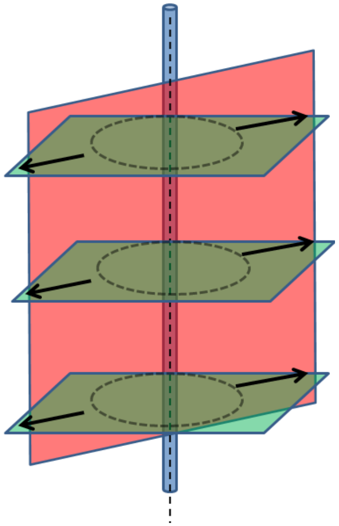
Limit case of infinite wire (with 2a >> z):

It exists an infinity of symmetry planes

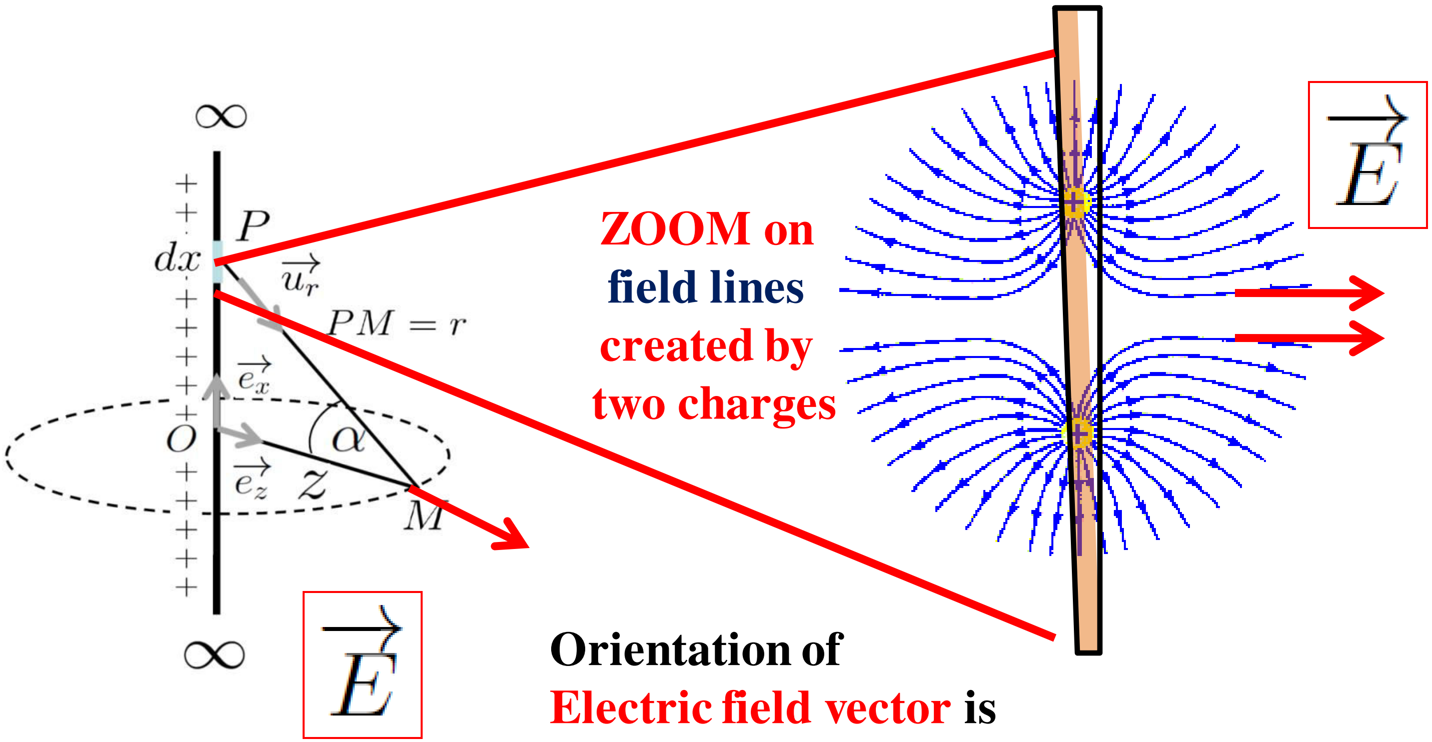
The electric field is perpendicular to the wire everywhere

$$\boxed{\overrightarrow{E} = \frac{\lambda}{2\pi\epsilon_0 z}\overrightarrow{e_z}}$$



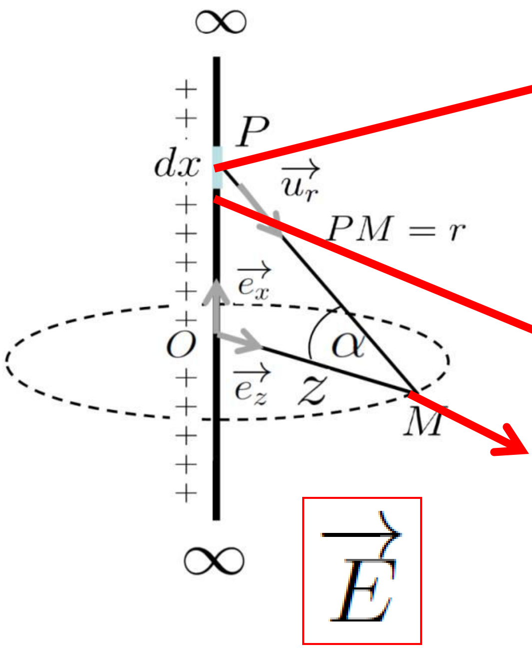


Interpretation in terms of Field lines !!

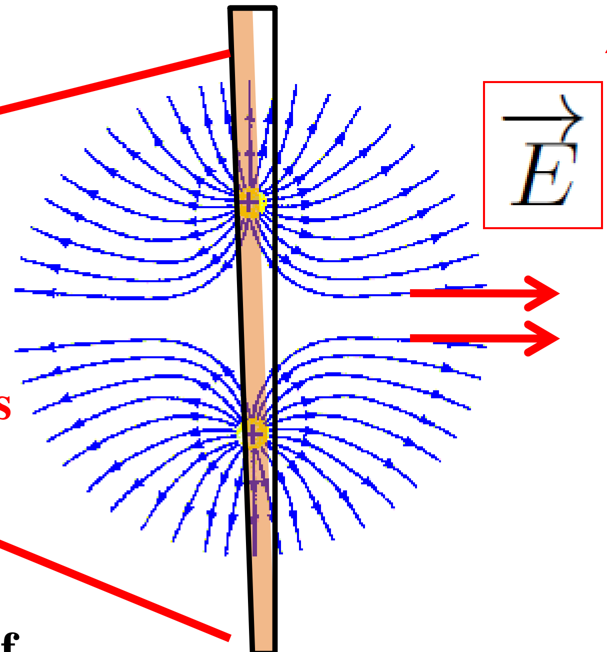


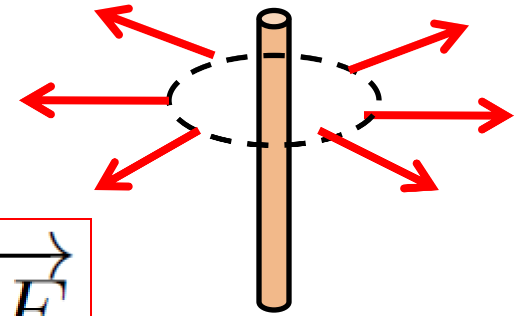
Orientation of
Electric field vector is
parallel to the field lines

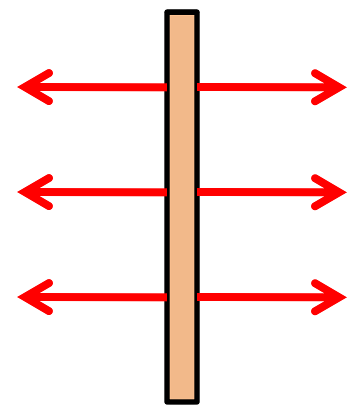
Interpretation in terms of Field lines !!



ZOOM on field lines created by two charges



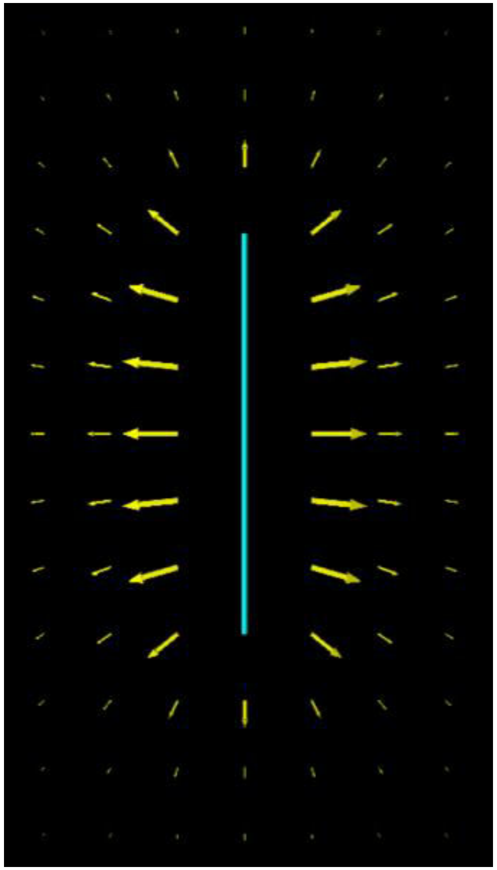


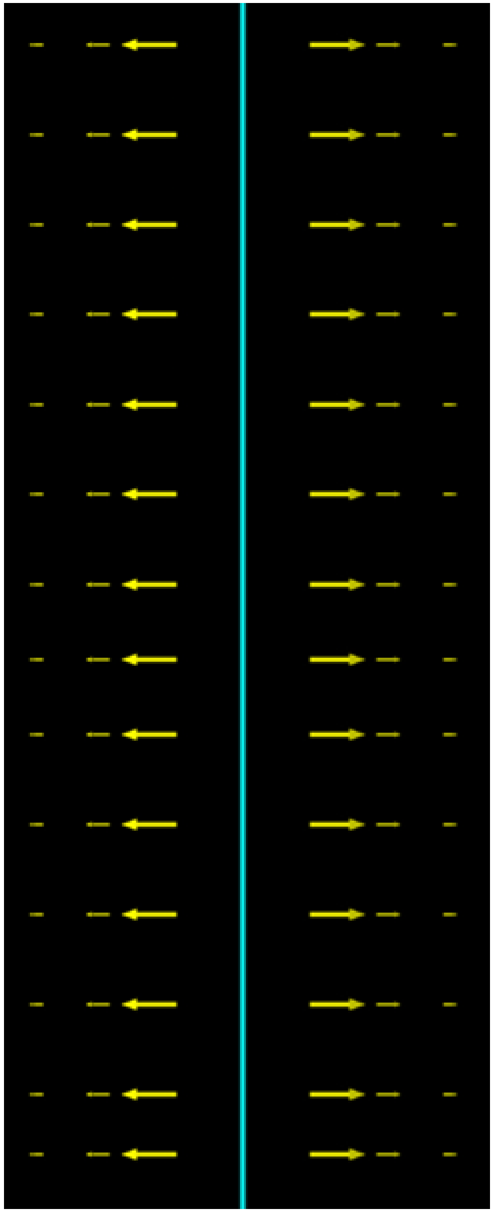


Orientation of Electric field vector is parallel to the field lines

Summary of Electric Field Lines

Finite wire





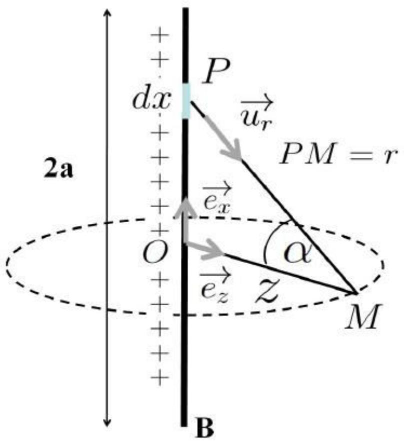
Infinite wire

$$\overrightarrow{E} = \frac{\lambda}{2\pi\epsilon_0 z}\overrightarrow{e_z}$$

1/z dependence

About **the electric potential** at distance z in the middle of the wire: **two methods of calculation**

Finite wire



1) By integrating the electric field

$$\int \vec{E} \cdot \vec{dr} = -[V(r)]$$

with $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0} \frac{a}{z\sqrt{z^2 + a^2}} \vec{e_z}$

and dr =dz

2) By calculating the potential created by all small elements

$$V(z) = \int_{-\alpha_0}^{\alpha_0} \frac{\lambda dx}{4\pi\epsilon_0 PM}$$

Both cases are not easy to achieve. We give the results for your information….

$$V(z) = \frac{\lambda}{4\pi\epsilon_0} \ln\left[\frac{\sqrt{a^2+z^2}+a}{\sqrt{a^2+z^2}-a}\right]$$

Easier for infinite wire

The integration of electric field is easy

$$V(z) = \int \vec{E} \cdot dz\vec{e_z} = -\frac{\lambda}{2\pi\epsilon_0} \int \frac{dz}{z}$$
$$= -\frac{\lambda}{2\pi\epsilon_0} \ln z + \text{ const}$$

But the result is problematic because we obtain *ln z* with *z* a distance and what is inside the *ln* should be a number….

So we define a given z_0 for which the potential is zero $V(z_0)=0$

$$V(z_0) = -\frac{\lambda}{2\pi\epsilon_0} \ln z_0 + \text{ const}$$

We can find a value for the constant and we obtain finally for V(z)

$$\text{const} = \frac{\lambda}{2\pi\epsilon_0} \ln z_0 = -\frac{\lambda}{2\pi\epsilon_0} \ln \frac{1}{z_0}$$

$$V(z) = -\frac{\lambda}{2\pi\epsilon_0} \ln \frac{z}{z_0}$$

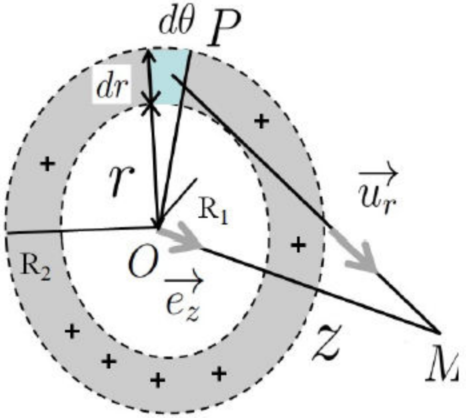
Electrostatics-L2

2) Electric field calculations

1) **Electric field created by a charged electric wire-** Calculation of the electric field**-** Limit case of the infinite wire.**-**Analysis in terms of field lines
2) **Some examples of 2D electric charged structures-** Electric field and electric potential created by a crown and a disc**-** Limit case of the infinite charged plane**-**Analysis in terms of field lines
3) **Application to the plane capacitor-** Electric field and electric potential**-**Capacitance and energy.

Exercise 2: electric charged crown, disc and plane

We consider a crown of internal radius R_1 and external radius R_2 and center O, having a uniform surface density of charge σ.



1) Calculate the expression of the electric field vector produced at a point M in the symmetry axis of the system at a distance z from O. Deduce the expression of the electric potential V(z).

2) How are modified the results if the crown is transformed into a disc of radius R?
3) Deduce that the electric field vector created by a charged infinite plane of electric density σ at a distance z is:

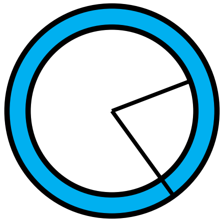
$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\overrightarrow{e_z}$$

4) Give the expression of the electric potential V(z).

Steps of calculations

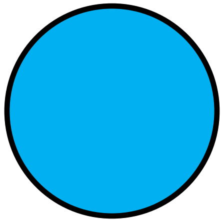
Crown

R_1, internal radius
R_2 external radius



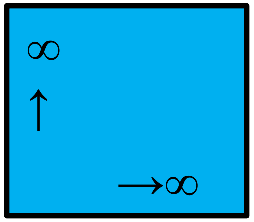
$$\overrightarrow{E} = \frac{\sigma z}{2\epsilon_0}\overrightarrow{e_z}\left(\frac{1}{\sqrt{R_1^2+z^2}} - \frac{1}{\sqrt{R_2^2+z^2}}\right)$$

Disc
$R_1 \rightarrow 0$



$$\overrightarrow{E} = \frac{\sigma z}{2\epsilon_0}\overrightarrow{e_z}\left(\frac{1}{z} - \frac{1}{\sqrt{R_2^2+z^2}}\right)$$

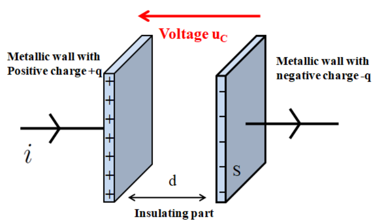
Infinite Plan
$R_1 \rightarrow 0$
$R_2 \rightarrow \infty$

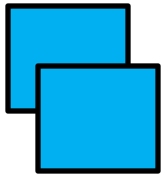


$$\overrightarrow{E} = \frac{\sigma}{2\epsilon_0}\overrightarrow{e_z}$$

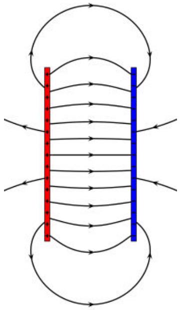
Interests

Example 1: The plane capacitor



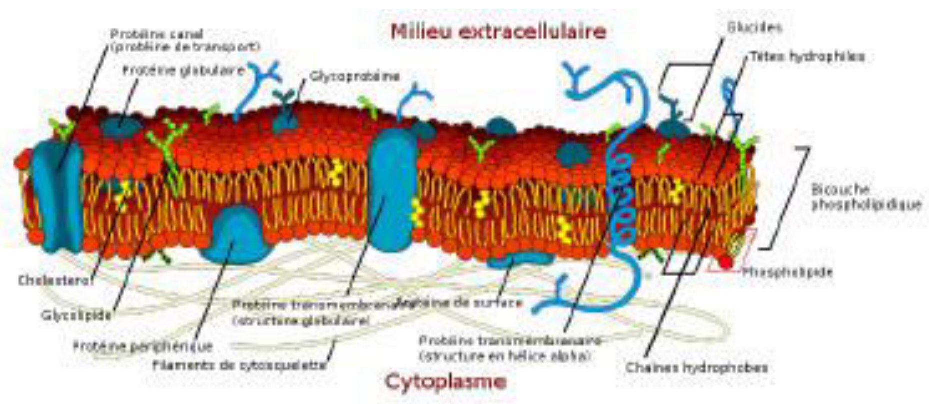


2 plans….



Will be seen in details….

Example 2: Biological membrane



Membrane is made of phospholipids that can be neutral or charged negatively. Assuming 1 over 10 molecules is charged and its surface is S=60 Angström squared $(10^{-10}\,\mathrm{m})^2$ we can estimate the electric surface density (2 membranes)

$\sigma = Q/S = -2e/(10\ S) = -2*1.6\ 10^{-19}/10\ /\ 60\ 10^{-20}$
$= -32/600 \approx -0.053\ \mathrm{C/m^2}$

Results in the case of infinite plane

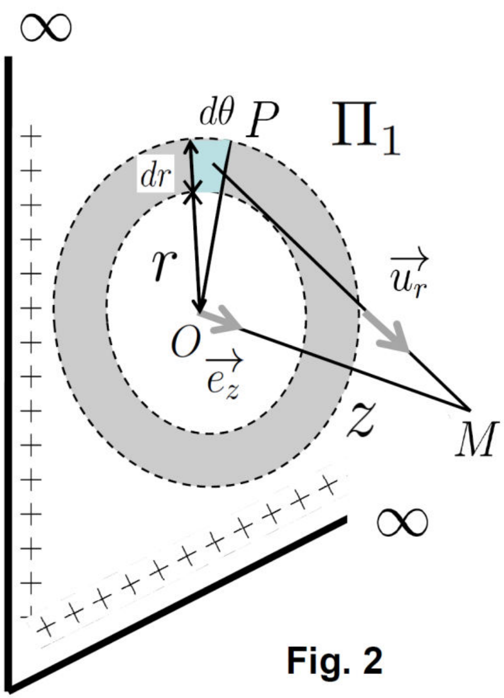
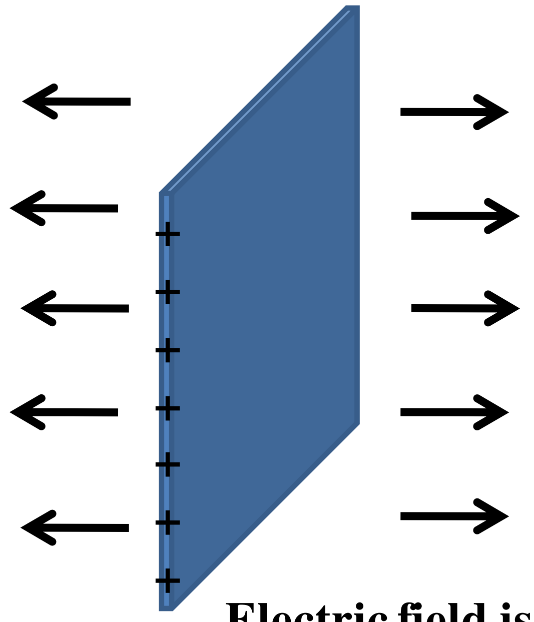


Fig. 2

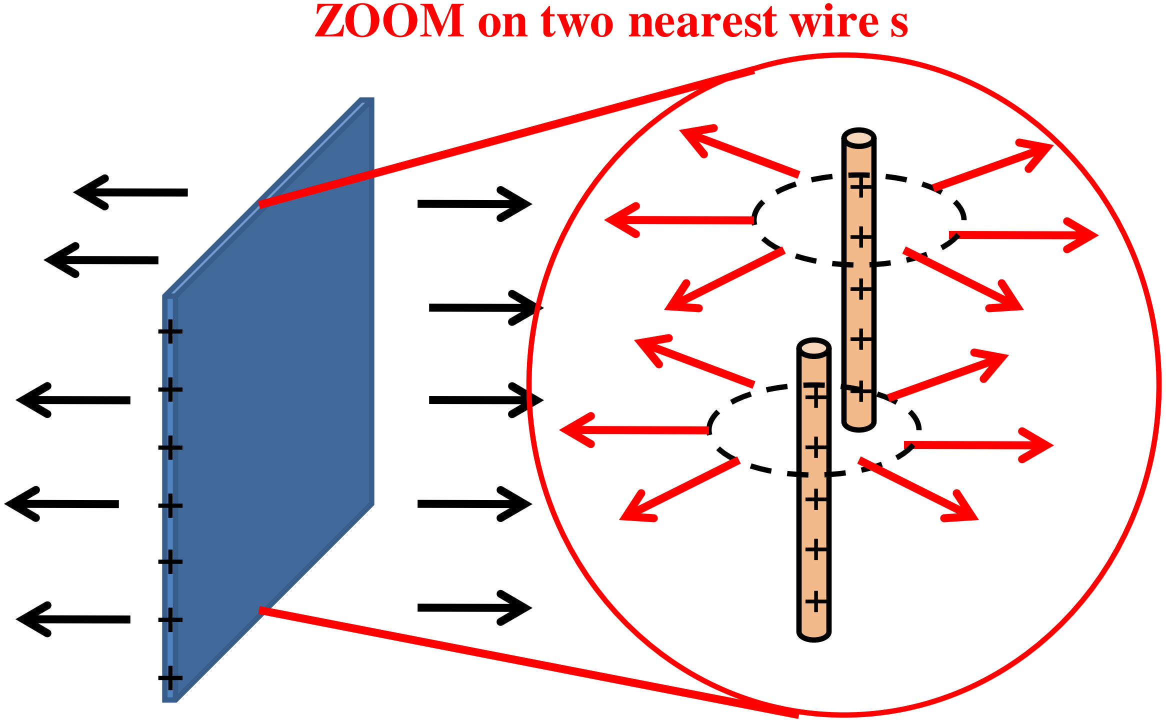


$$\boxed{\overrightarrow{E} = \frac{\sigma}{2\epsilon_0}\overrightarrow{e_z}}$$

Electric field is constant and perpendicular to the plane

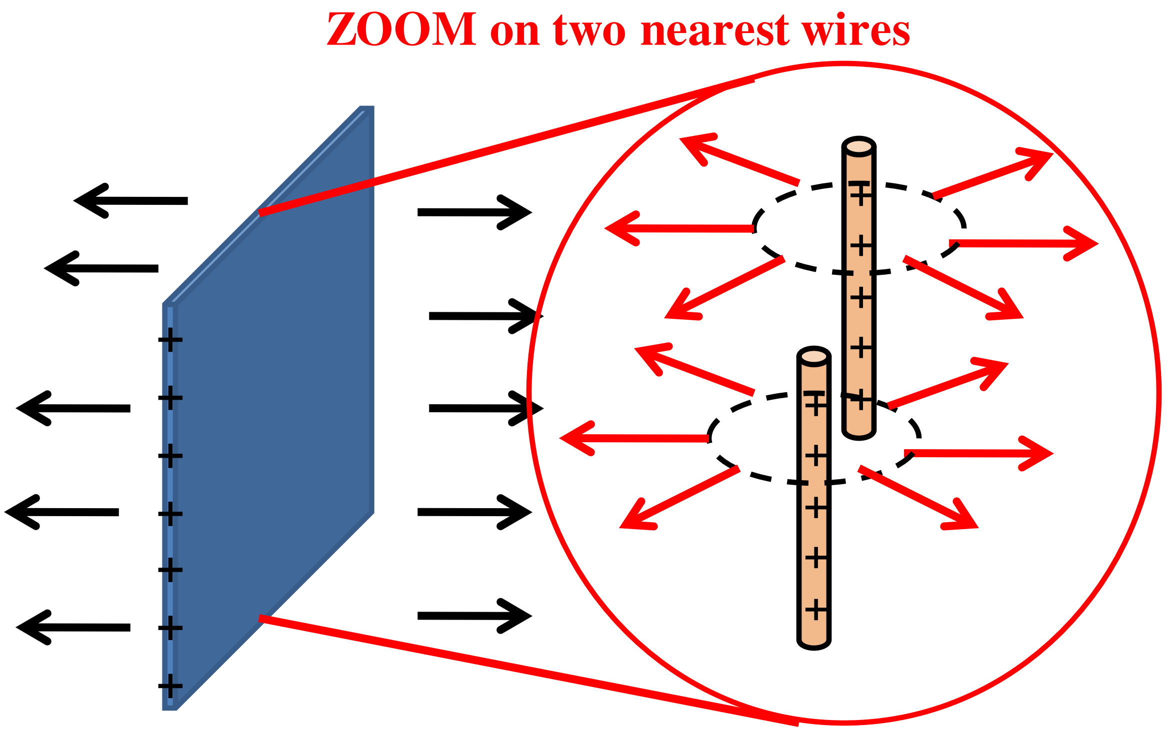
Interpretation in terms of field lines:

plane is an infinite sum of infinte wires

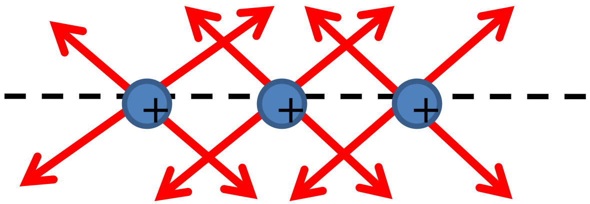


Interpretation in terms of field lines:

plane is an infinite sum of wires



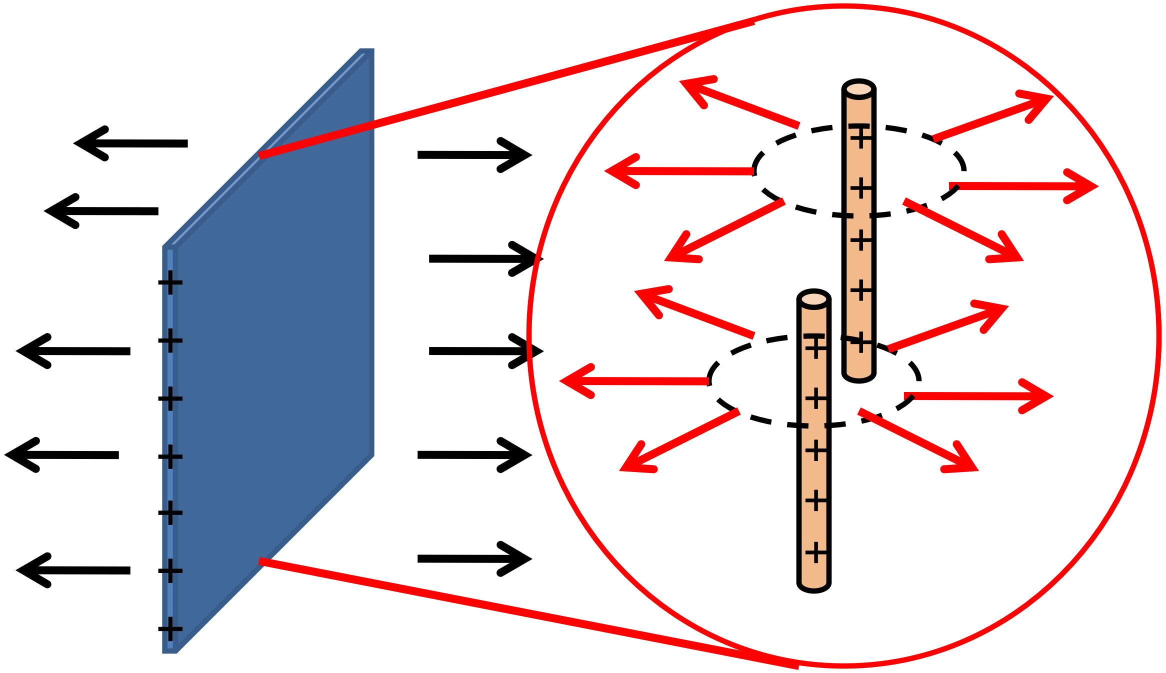
Vision from the top
to the bottom (3 wires)



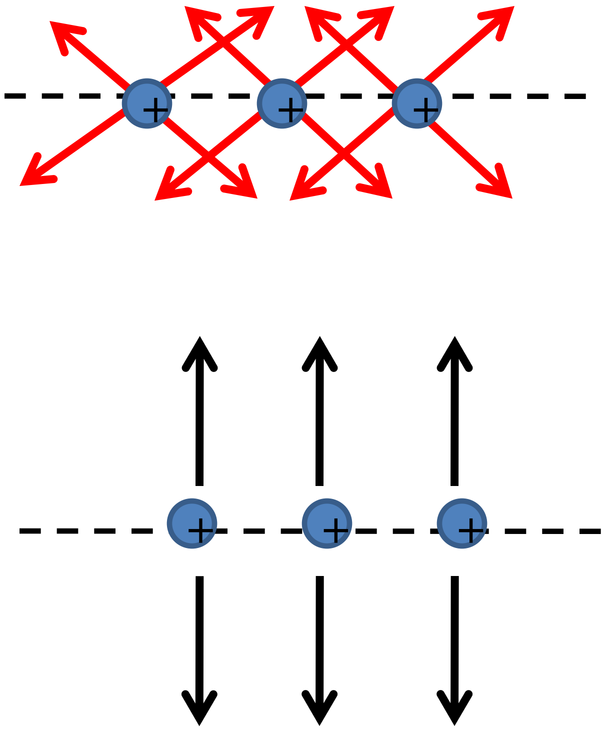
Interpretation in terms of field lines:

plane is an infinite sum of wires

ZOOM on two nearest wires



Vision from the top to the bottom (3 wires)



Electrostatics-L2

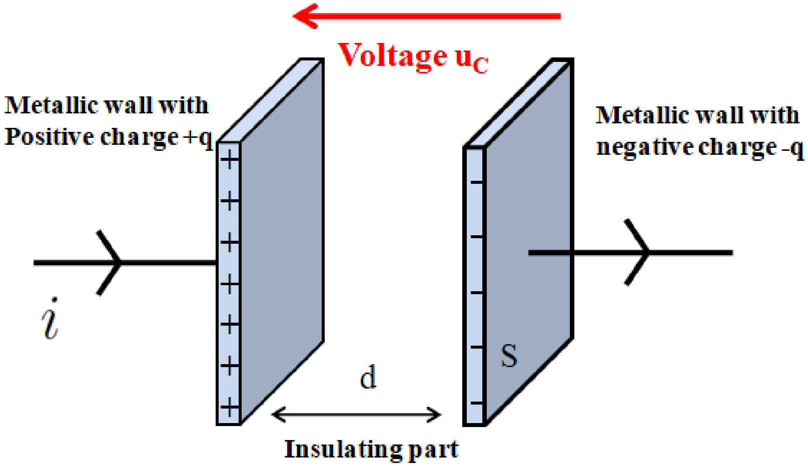
2) Electric field calculations

1) **Electric field created by a charged electric wire-**
   Calculation of the electric field- Limit case of the infinite wire.-Analysis in terms of field lines

2) **Some examples of 2D electric charged structures-**
   Electric field and electric potential created by a crown and a disc- Limit case of the infinite charged plane-Analysis in terms of field lines

3) **Application to the plane capacitor-**
   Electric field and electric potential-Capacitance and energy.

Application to plane capacitor



Plane Capacitor can be….

….the sum of two charged and

opposites planes separated from distance d

- Described by its Capacitance C (or capacity) in Farad (Coulomb/Volt):

$$q = Cu_C$$

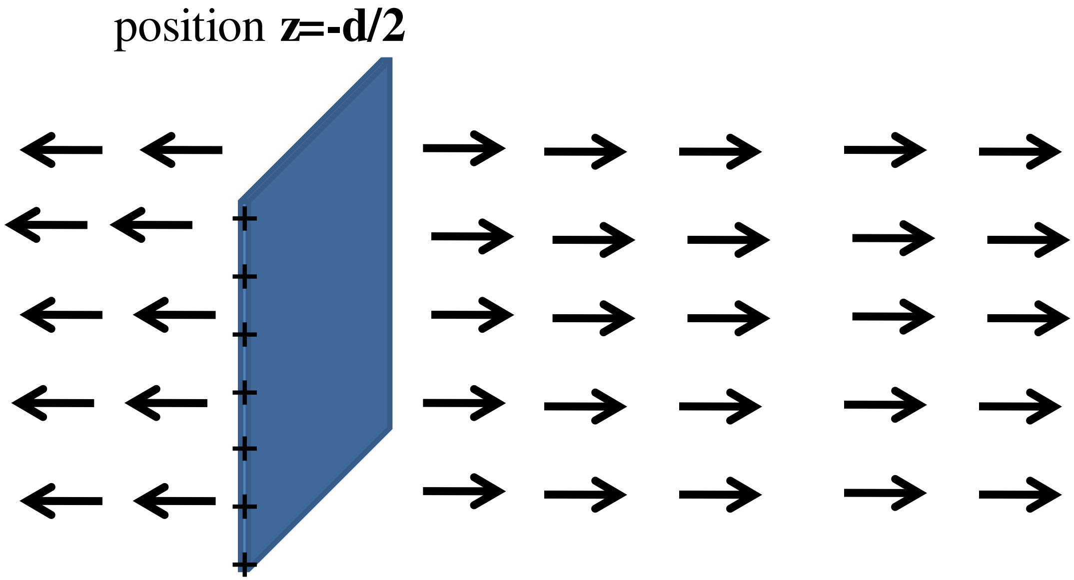
$$C = \frac{\epsilon_0 S}{d}$$ (if insulating part is the vaccum)

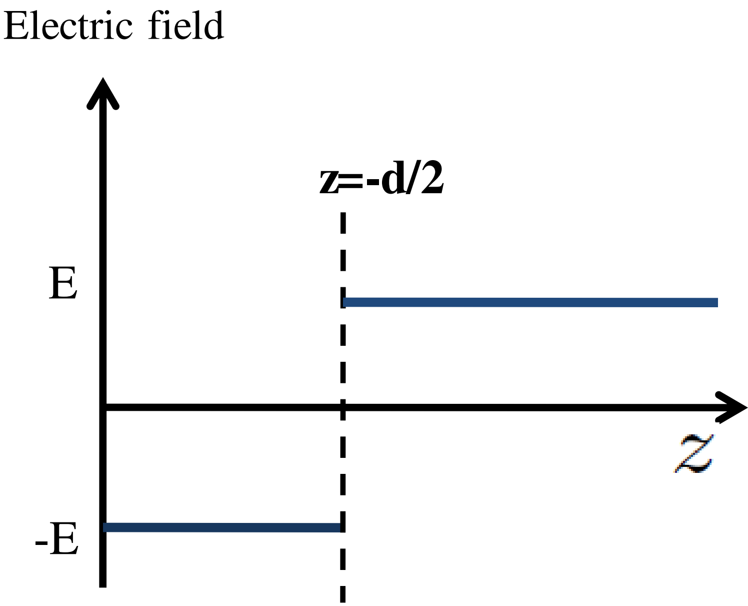
- Relation between current and voltage

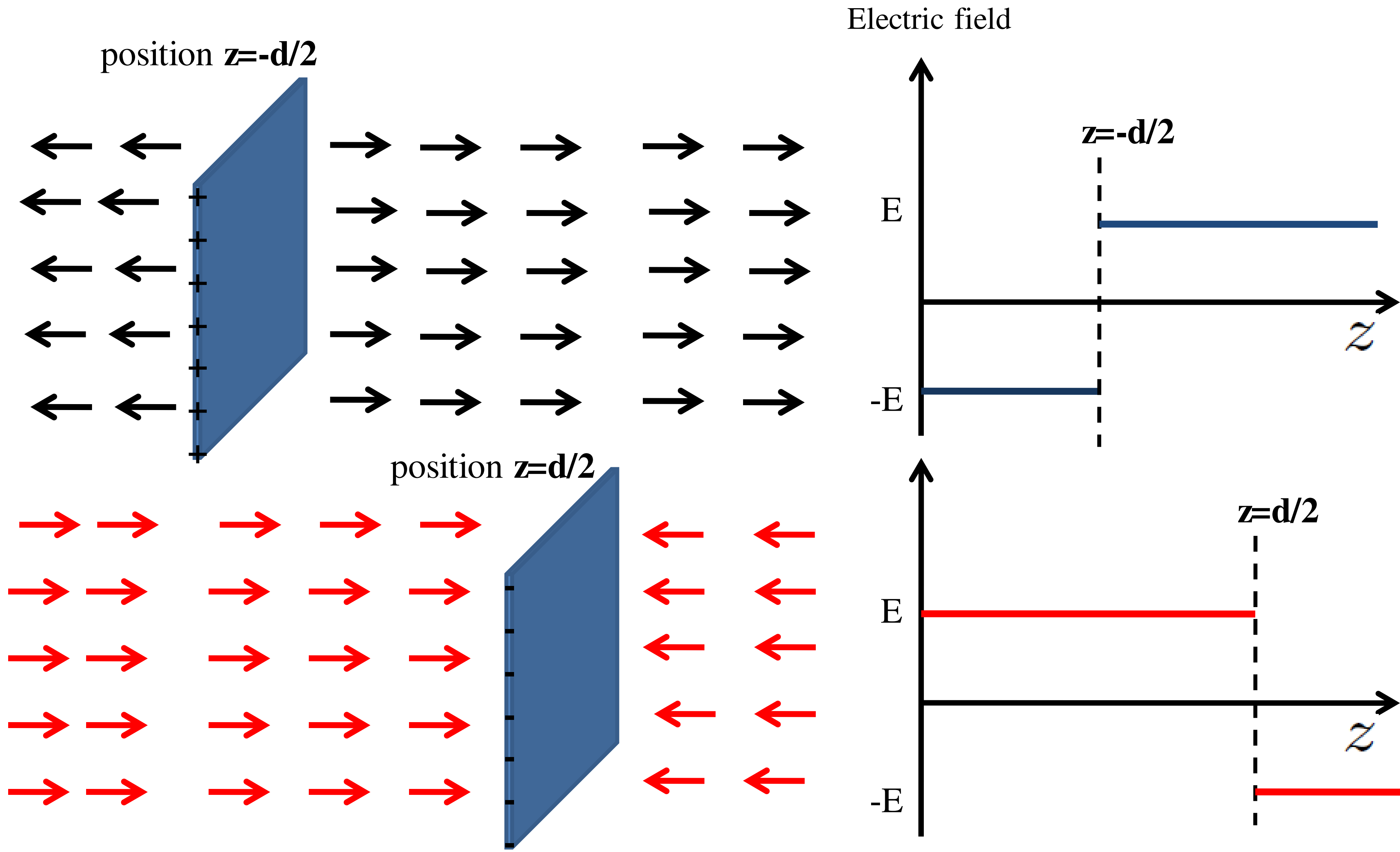
$$\boxed{i = \frac{dq}{dt} = C\frac{du_C}{dt}}$$

C: from µF to pF (cylindrical and plane Capacitors)

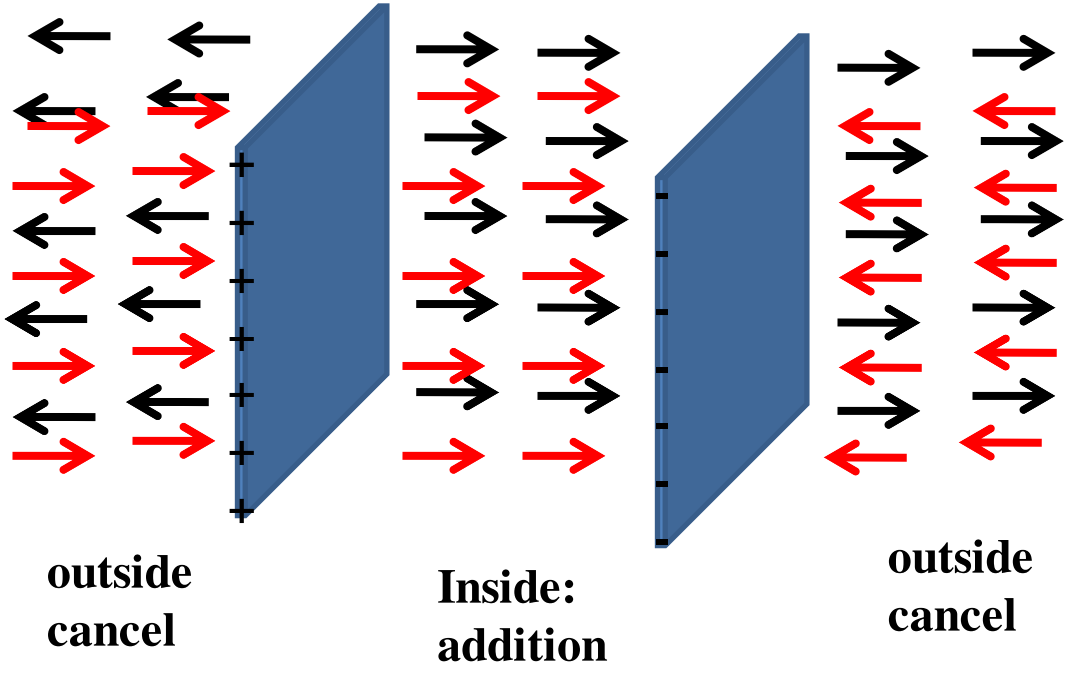
II.3) Application to plane capacitor

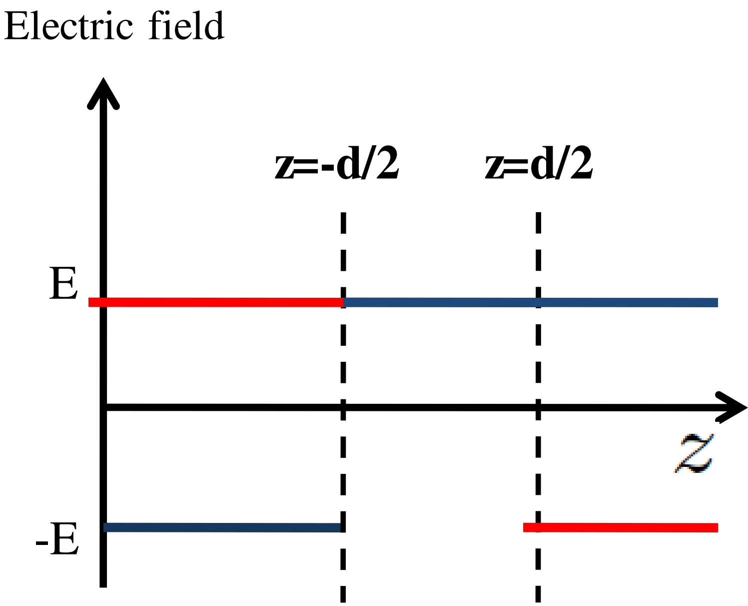




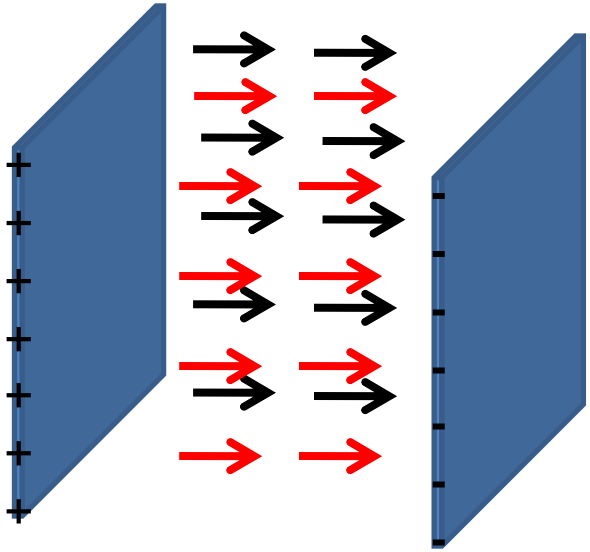

Electric field
position z=-d/2
z=-d/2
E
-E
z
position z=d/2
z=d/2
E
-E
z

We superpose the two configurations





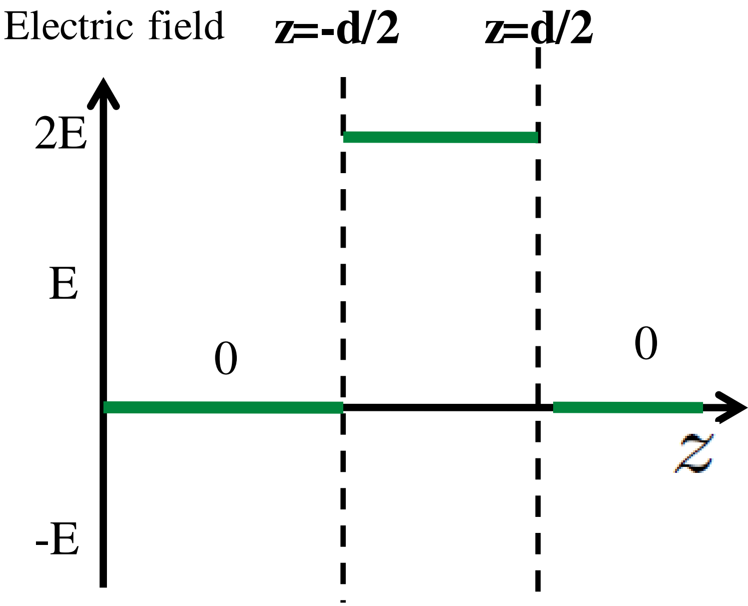
We superpose the two configurations



outside
cancel

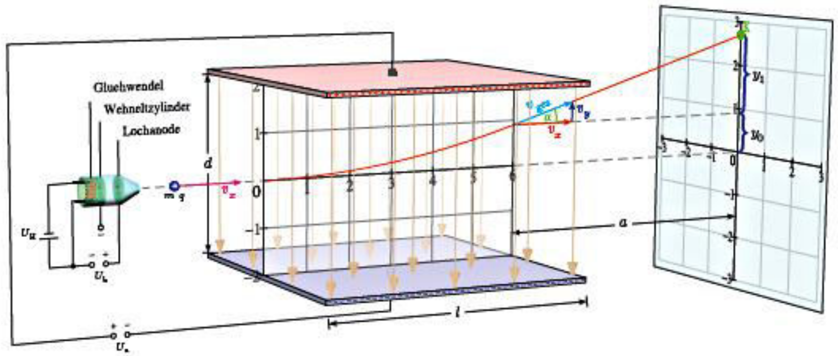
Inside:
addition

outside
cancel

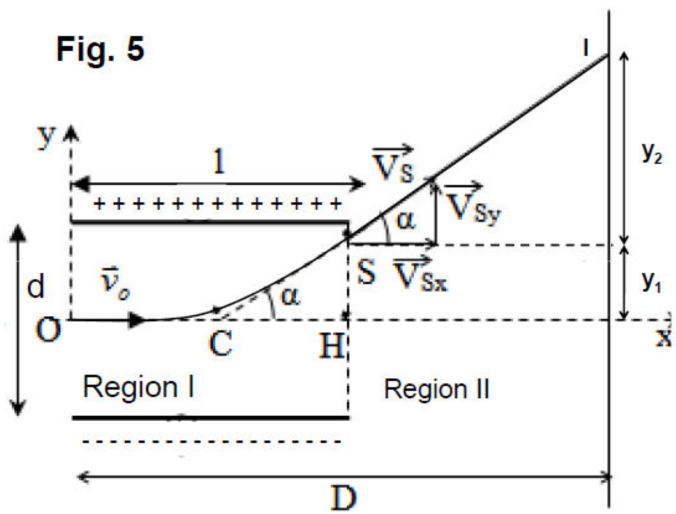


We obtain a capacitor:
electric field is only INSIDE !

Two charged plates: creation of uniform electric field



Device used to accelerate and/or deviate charged particle

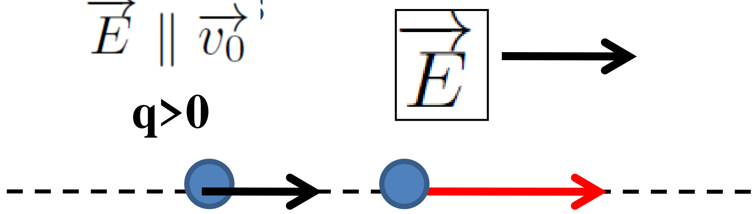


II.3.c) Electron dynamics in a constant and uniform electric field

Motion equation with electric field along positive x axis
(contrary than previous slide where electric field is along y axis)

Accelerates (and deviates) charge

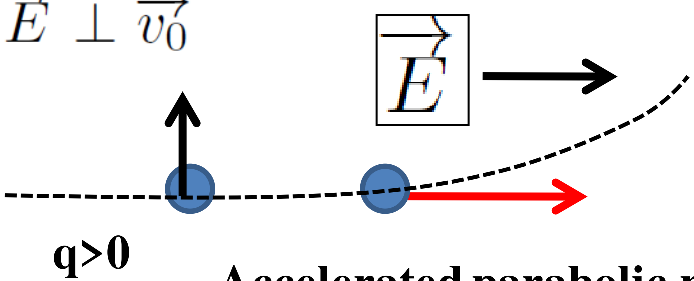
$\overrightarrow{E} \parallel \overrightarrow{v_0}$;



$$x = \frac{qEt^2}{2m} + v_0 t$$

Accelerated rect motion (1D)

$\overrightarrow{E} \perp \overrightarrow{v_0}$



$$x = \frac{qEt^2}{2m}$$

$$y = v_0 t$$

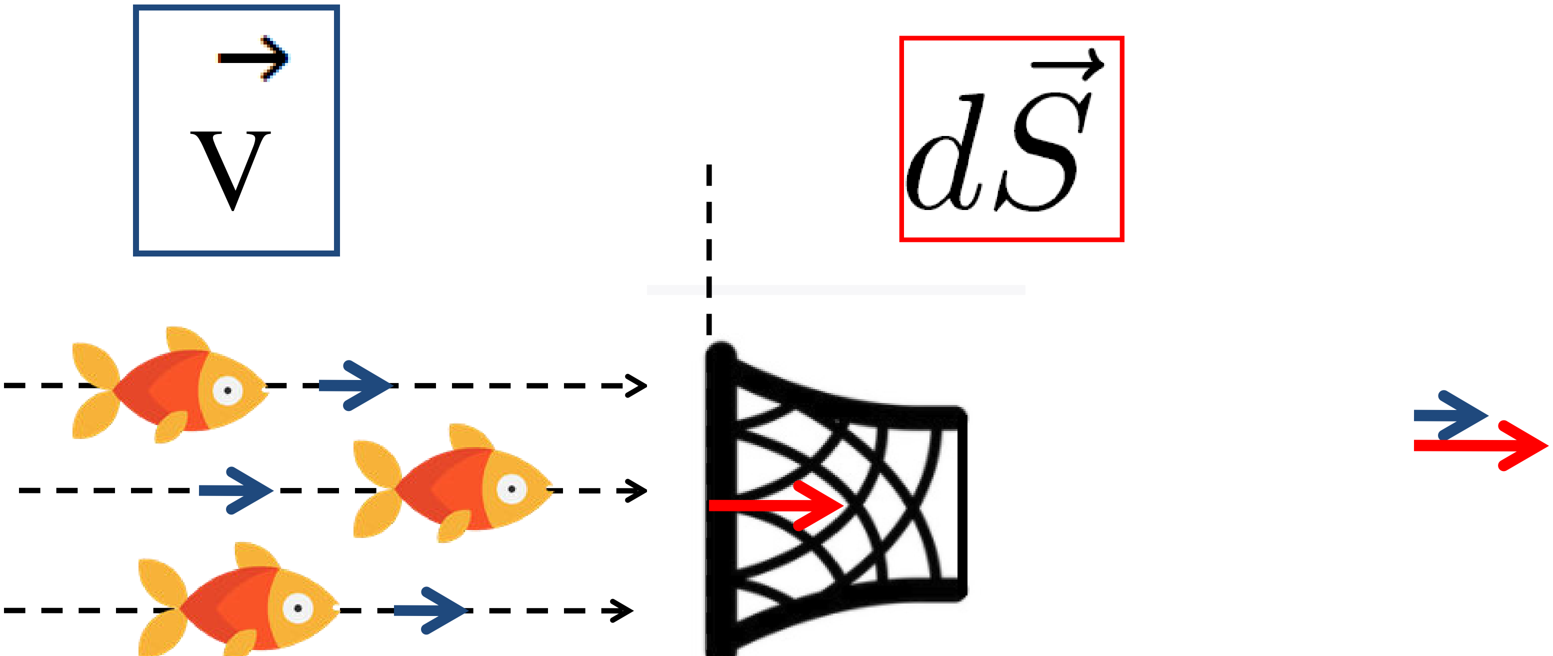
Accelerated parabolic motion (2D)

Karl Friedrich Gauss
1777-1865

Electrostatics-L2

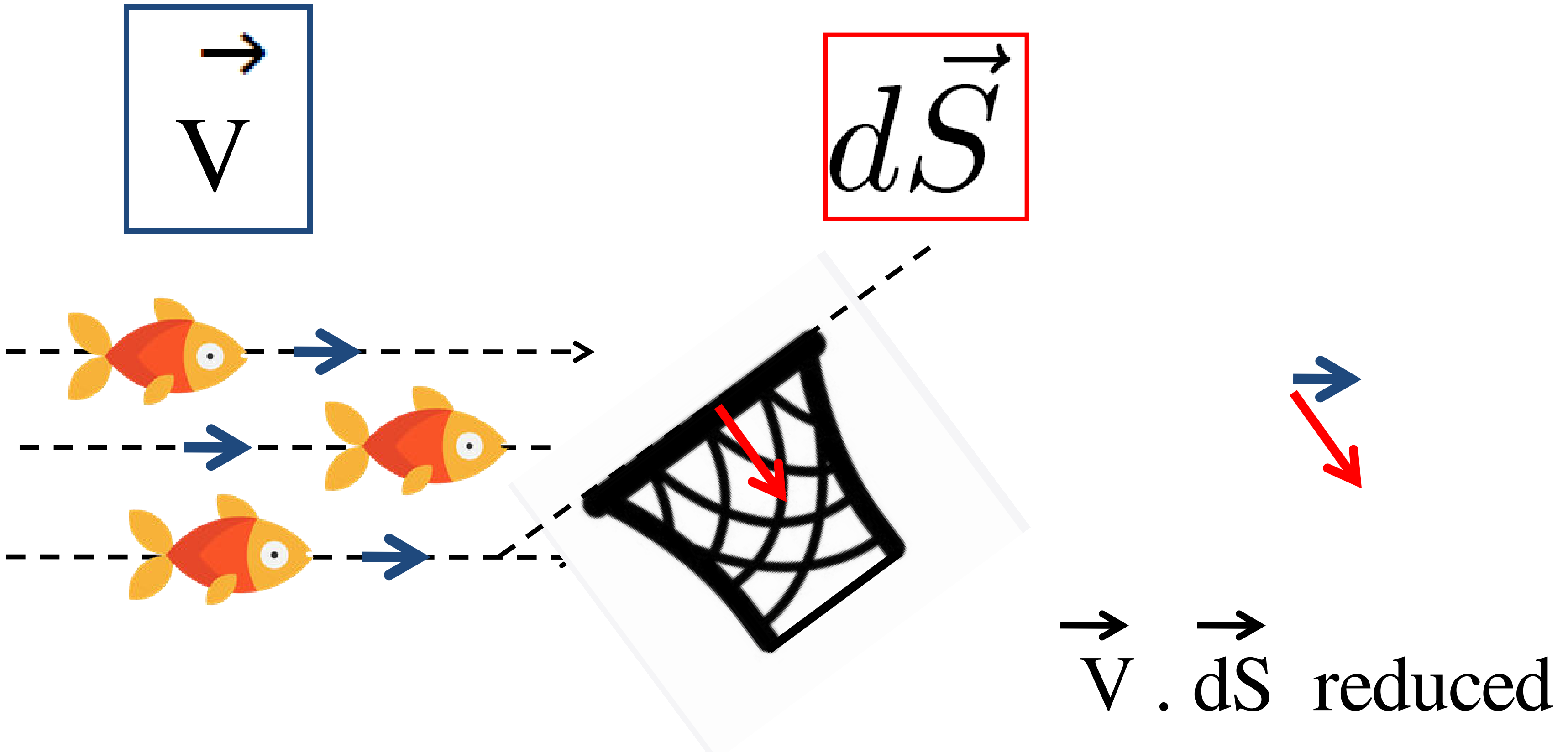
1) Need of Geometry-Notion of vector flux**-** Solid Angle **2) Statement of Gauss theorem** **3) Direct applications-**Electric fields created by an infinite wire and by an infinite plane**-by an** empty and full charged cylinder**-by an** Empty and full sphere **4) Examples:- Cylindrical Capacitors 5) Earth as a Capacitor**

III.1a) Notion of vector flux



$\vec{V} \cdot \vec{dS}$: maximum

Flux of $\vec{V}$ through surface $\vec{dS}$ is maximum

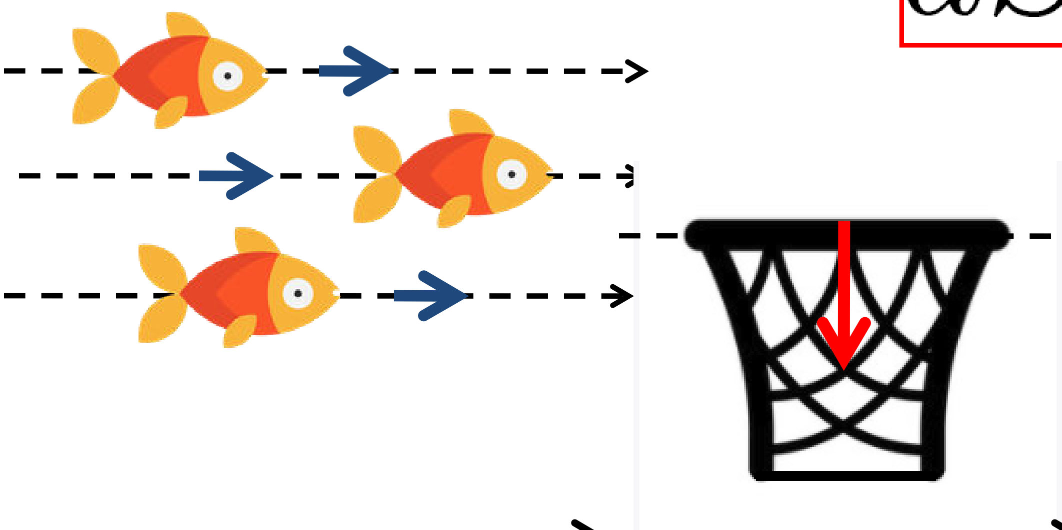


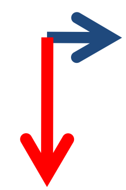
Flux of $\vec{V}$ through surface $\vec{dS}$ is reduced

III.1a) Notion of vector flux

$\vec{V}$

$d\vec{S}$

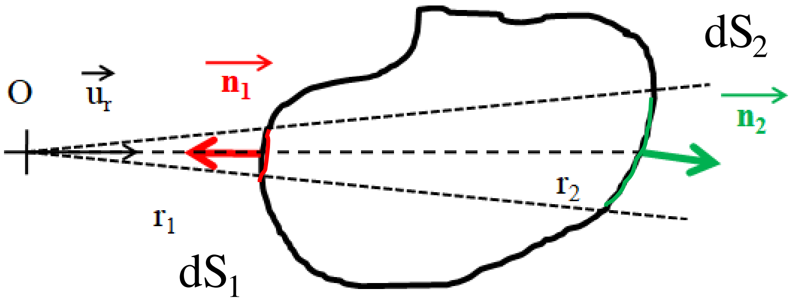




$$\vec{V} \cdot \vec{dS} = 0$$

Flux of $\vec{V}$ through surface $\vec{dS}$ is zero !!

Point O is **outisde** the closed surface

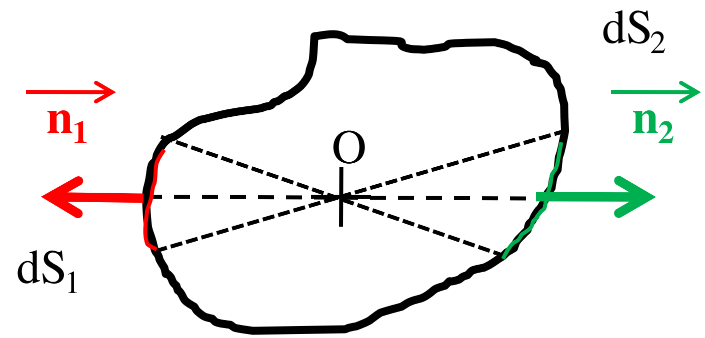


$$d\Omega_1 = \frac{\vec{u}_r . \vec{dS}_1}{r_1^{\,2}} \qquad d\Omega_2 = \frac{\vec{u}_r . \vec{dS}_2}{r_2^{\,2}}$$

$d\Omega_1 = - d\Omega_2$ solid angles are opposite

$\iint d\Omega = 0$ total contribution is zero.

Point O is **inside** the closed surface



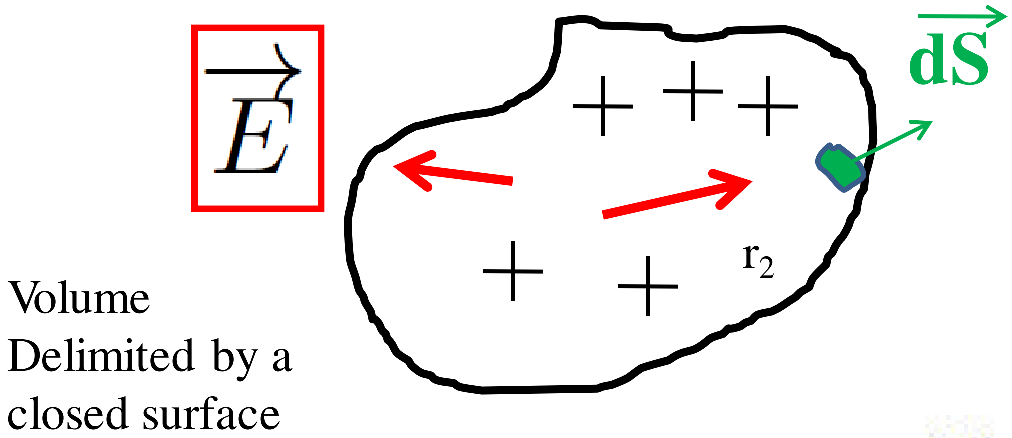
We assume spherical elementary surface for each dS

$$dS = r^2 \sin\theta d\theta \, d\varphi$$

$$\iint d\Omega = \iint \frac{\vec{u}_r . \vec{dS}}{r^2} = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi = 4\,\pi$$

The solid angle for all space is $4\,\pi$

The flux of electric field crossing a closed surface is equal to the electric charge inside the volume delimited by the closed surface divided by espilon0



$$\oiint \overrightarrow{E} \cdot \overrightarrow{dS} = \frac{Q_{\text{int}}}{\epsilon_0}$$

Tutorial 2: Exercises 1 and 2 have been done in Lecture

Part 3: Gauss theorem

Statement:

$$\oiint \vec{E}.\overrightarrow{dS} = \frac{Q_{int}}{\varepsilon_0}$$

Exercise 1: Infinite charged wire

Remember the expression of the electric field at a distance z of an infinite wire having an uniform lineic electric charge distribution $\lambda > 0$.
Recover this result using the symmetries and the Gauss theorem.

Exercise 2: Infinite charged plane

Remember the expression of the electric field at a distance z of an infinite plane having an uniform surface electric charge distribution $\sigma > 0$.
Recover this result using the symmetries and the Gauss theorem.

Tutorial 2: Exercises **3 and 4 during the two next Tutorial sessions**

Exercise 3: Infinite charged cylinder

1) We consider an infinite cylinder of radius R having a uniform volumic charge density ρ. Using Gauss theorem determine the value of the electric field at a distance z from the axis of the cylinder for $z < R$ and then for $z > R$. Deduce in each case the electric potential V.

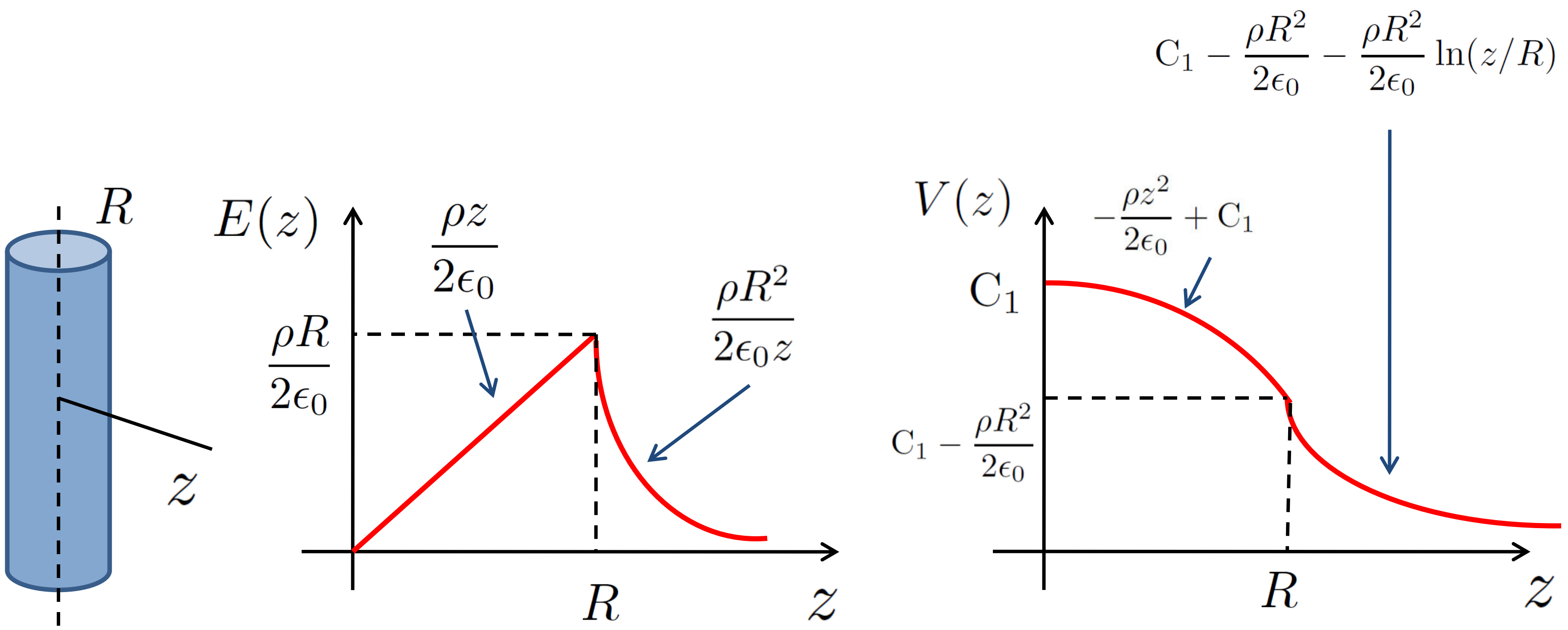
2) We assume now that the infinite cylinder has a uniform charge distribution σ only on its surface. Determine the expression of the electric field and the electric potential inside and outside the cylinder respectively for $z < R$ and then for $z > R$

Exercise 4 : Charged Sphere

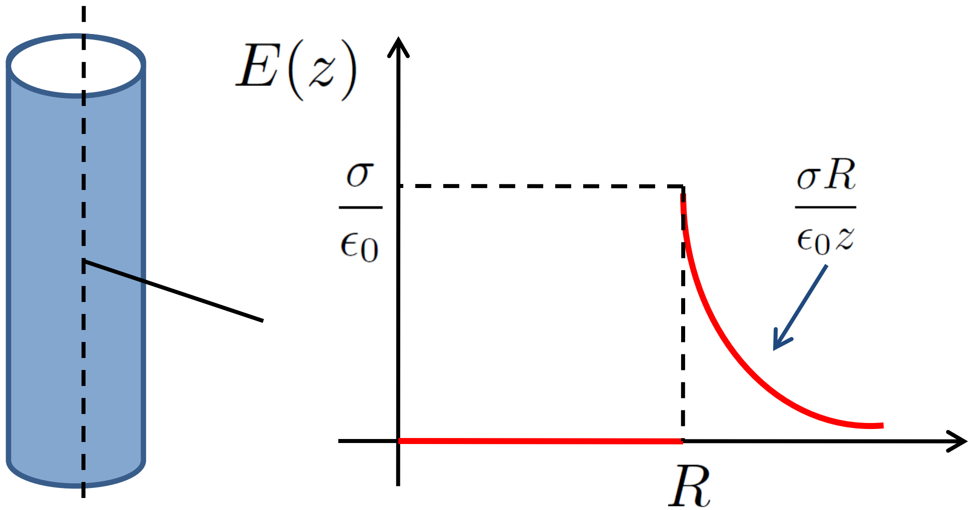
1) We consider a sphere of radius R having a uniform volumic charge density ρ. Using Gauss theorem determine the value of the electric field at a distance r from the sphere center for $r < R$ and then for $r > R$. Deduce in each case the electric potential V.

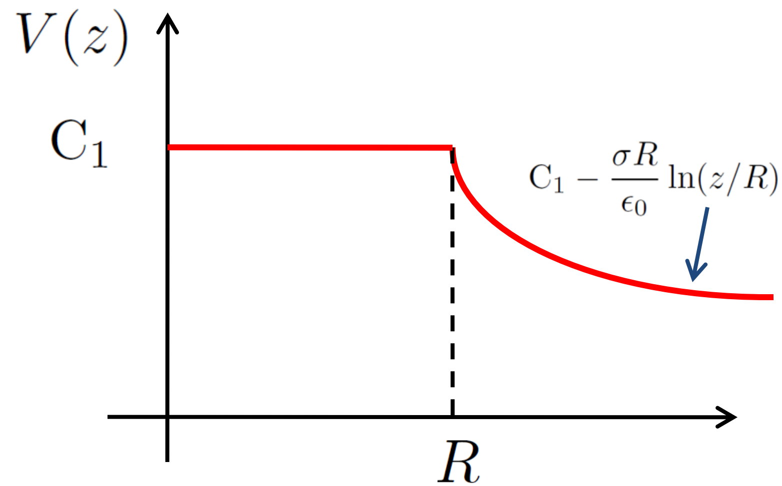
2) We assume now that the sphere has a uniform charge distribution σ only on its surface. Determine the expression of the electric field and the electric potential inside and outside the sphere respectively for $z < R$ and then for $z > R$.

Cylinder charged in volume



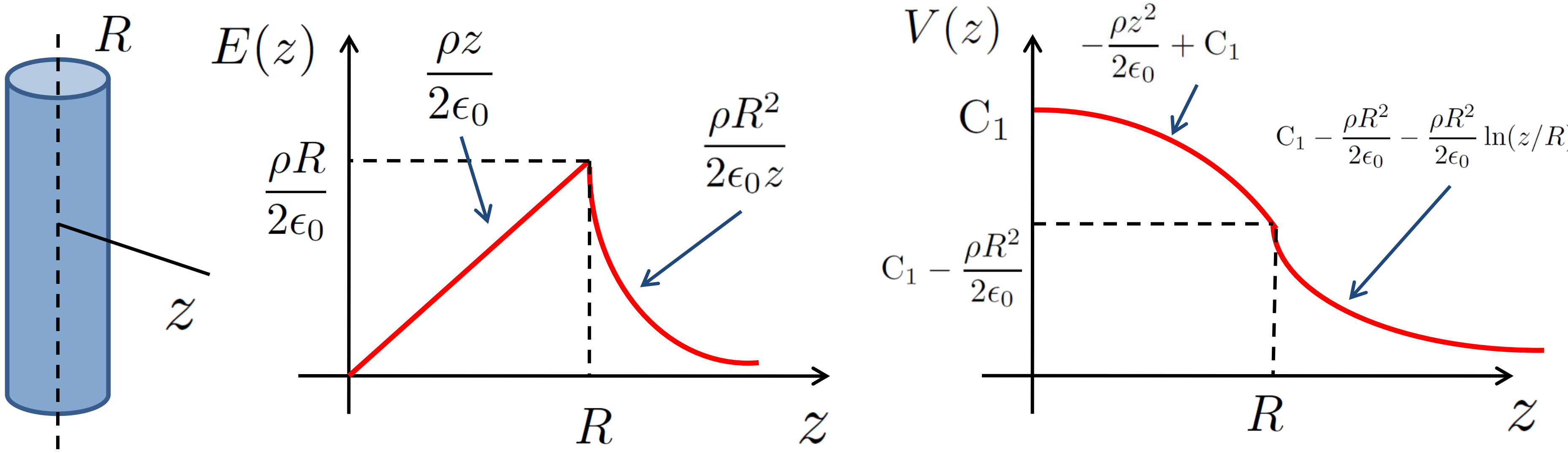
Cylinder charged in surface



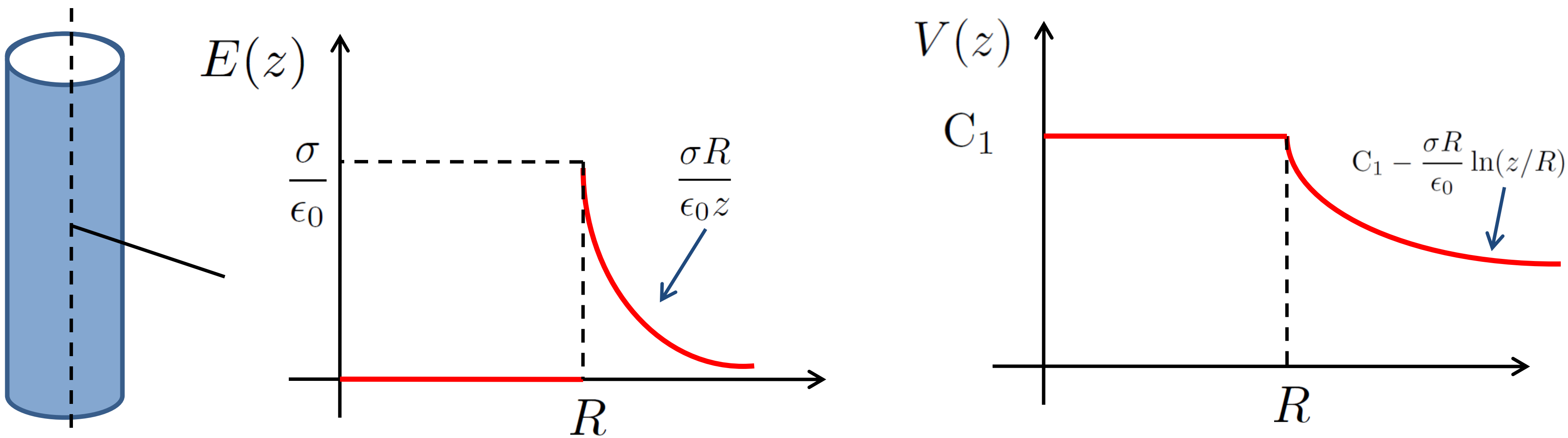


III.3a) Electric field created by an infinite cylinder charged in volume

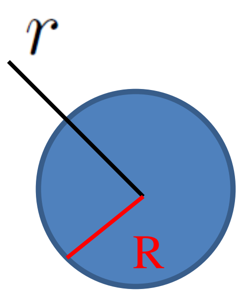
Cylinder charged in volume

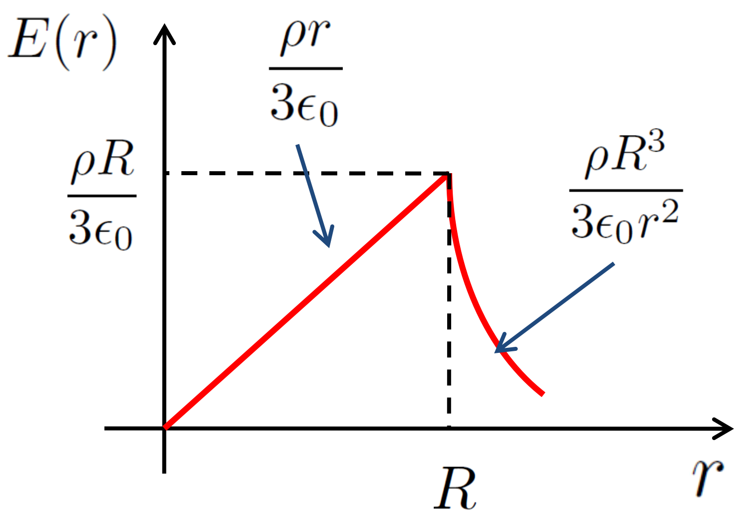


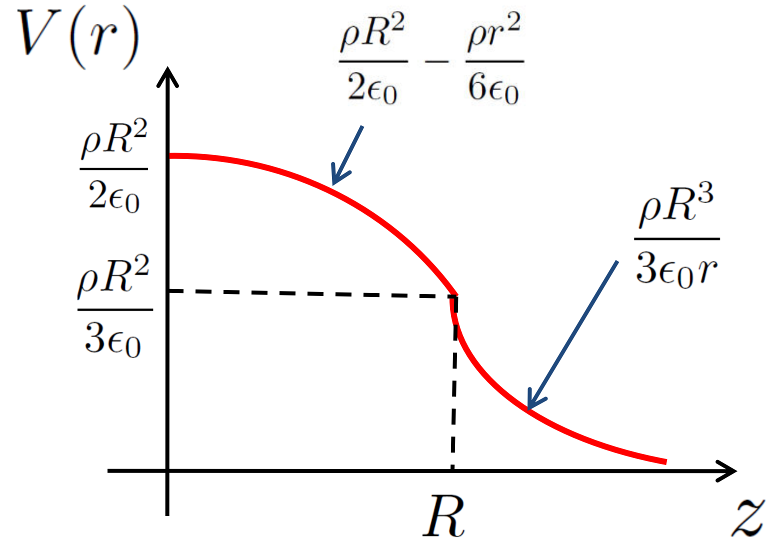
Cylinder charged in surface



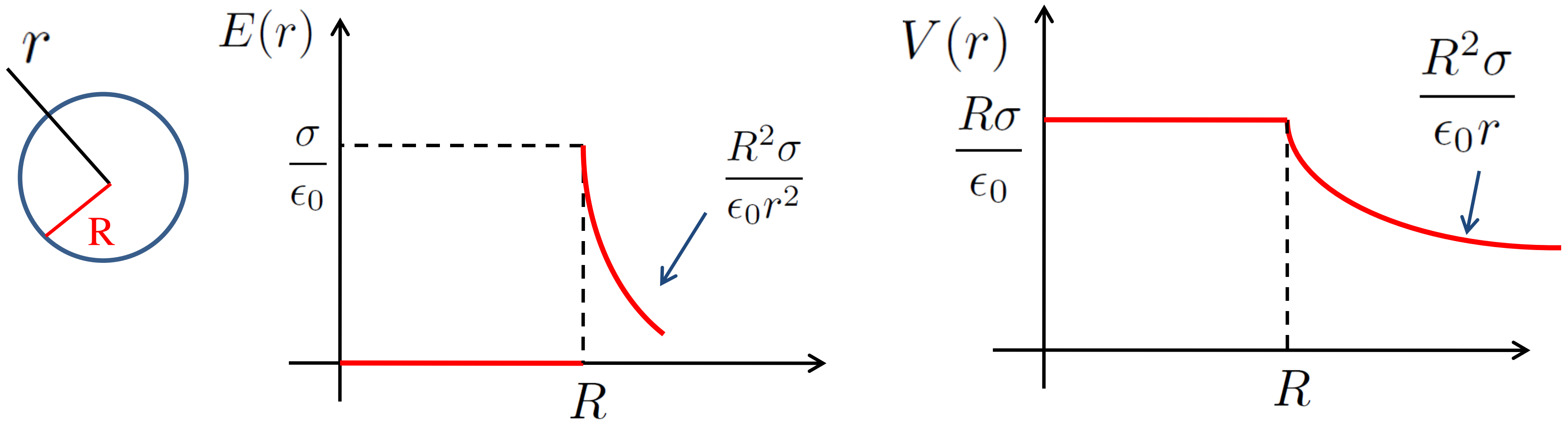
Sphere charged in volume



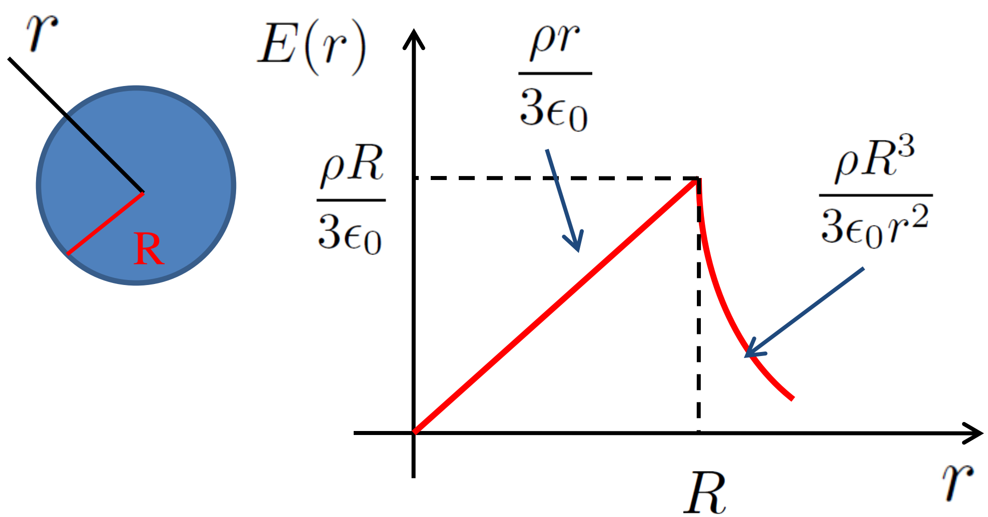


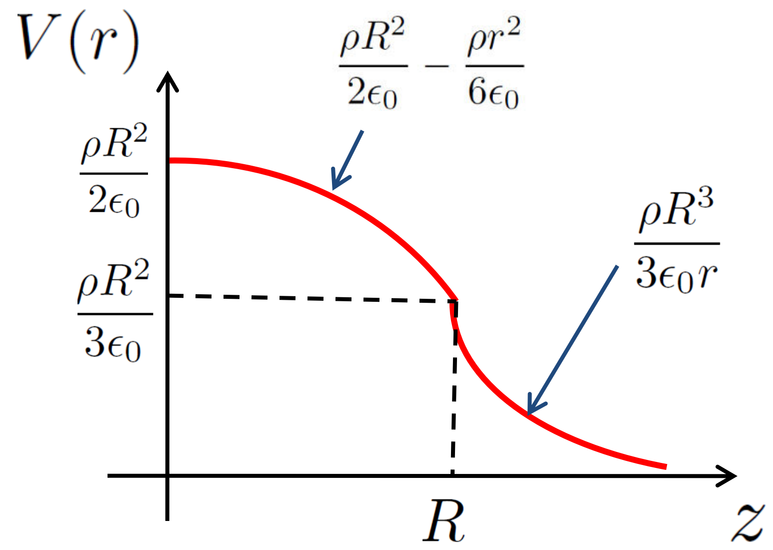


Sphere charged in surface

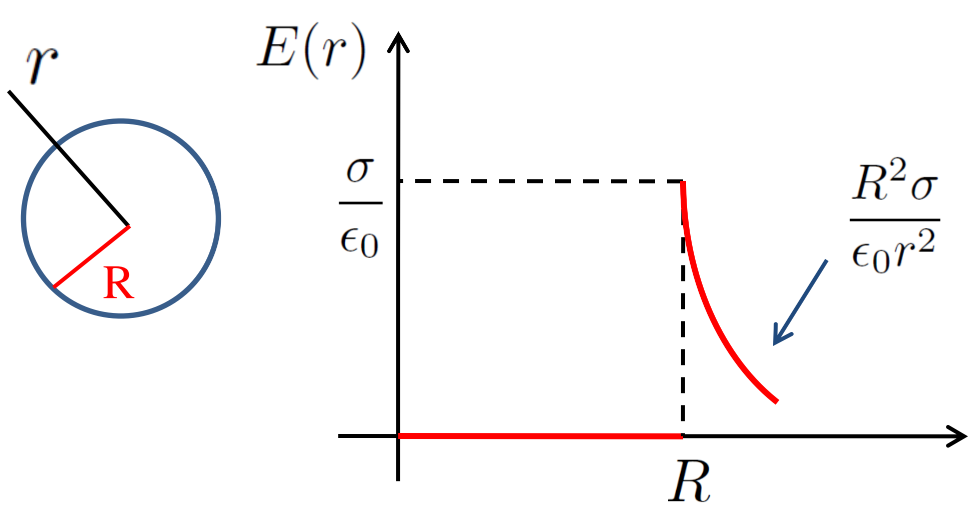


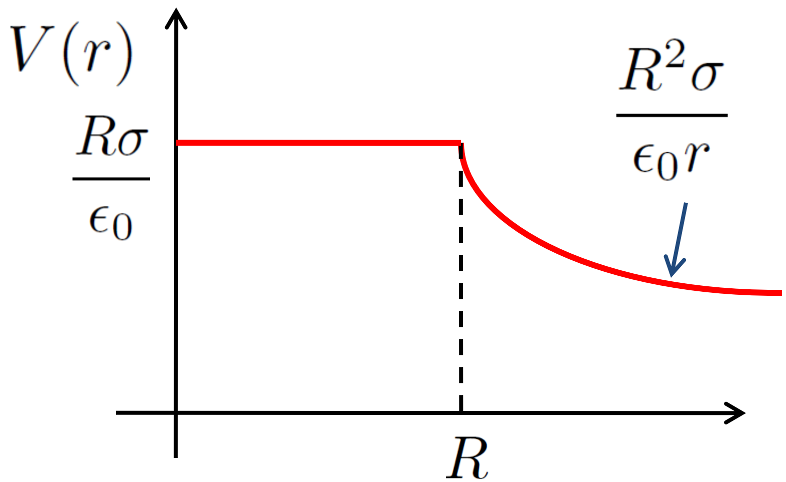
Sphere charged in volume





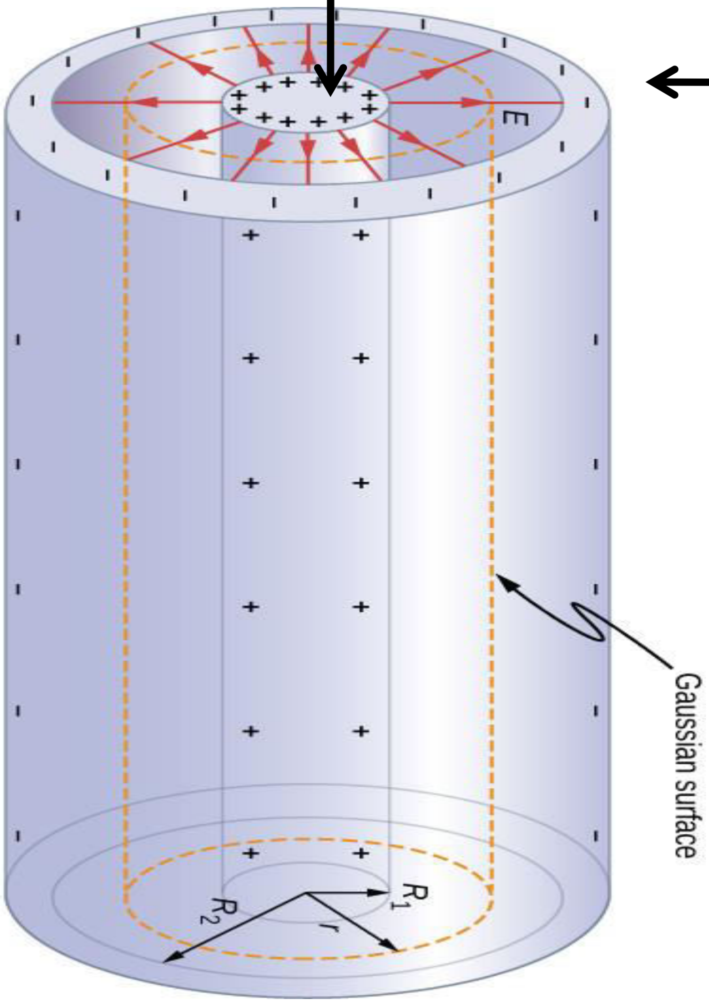
Sphere charged in surface



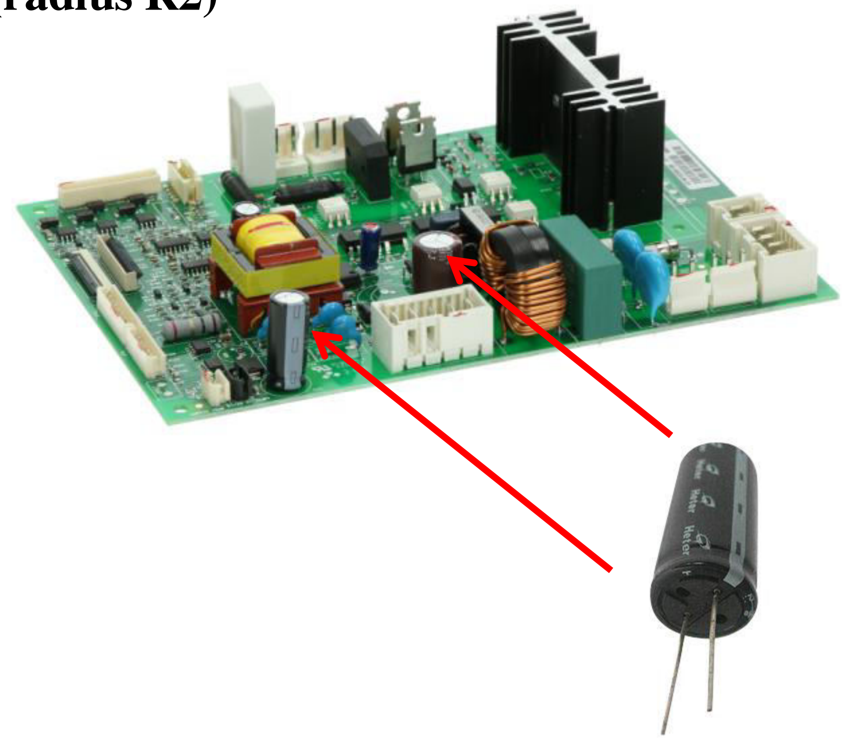


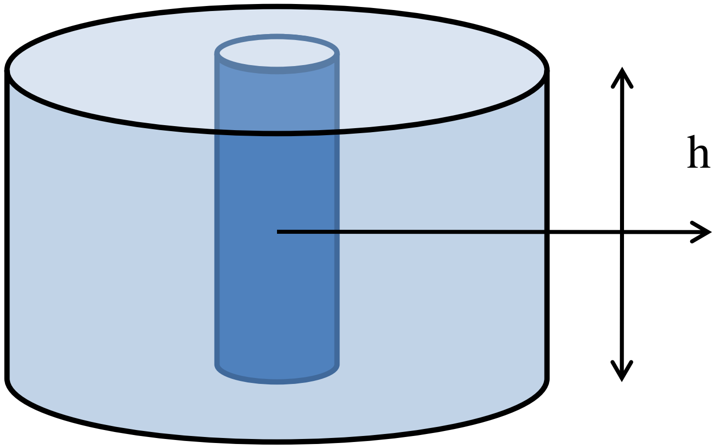
III.4) Cylindric Capacitor

Potential V_1 on internal cylinder (radius R1)



Potential V_2 on external cylinder (radius R2)





We assume cylinder charged on the surface with a negligible thickness

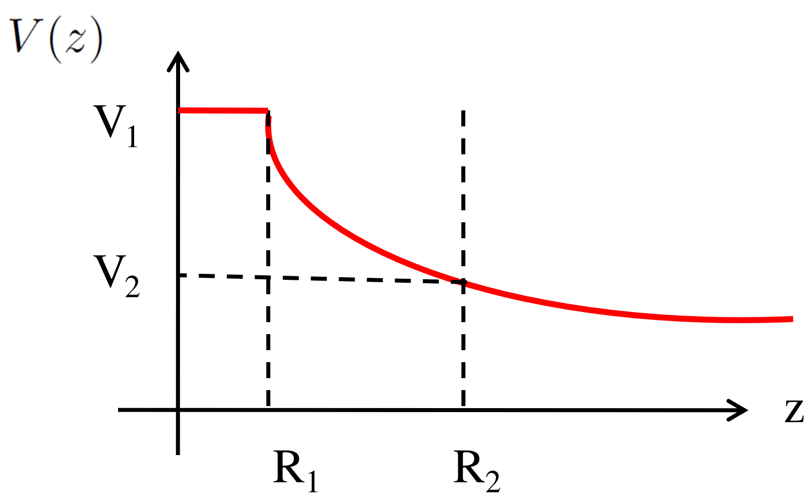
$$E(z) = \frac{\sigma R_1}{\epsilon_0 z}$$

Difference of Potential $\Delta V = V_1 - V_2$ between the internal cylinder and the external cylinder

$$[V]_1^2 = -\int_{R_1}^{R_2} E(z)dz = -\int_{R_1}^{R_2} \frac{\sigma R_1}{\epsilon_0 z} dz$$

$$= -\frac{\sigma R_1}{\epsilon_0}[\ln z]_{R_1}^{R_2} = \frac{\sigma R_1 \ln(R_1/R_2)}{2\epsilon_0}$$

$$= \frac{2\pi\sigma R_1 h \ln(R_1/R_2)}{2\pi h \epsilon_0} = \frac{Q \ln(R_1/R_2)}{2\pi h \epsilon_0}$$

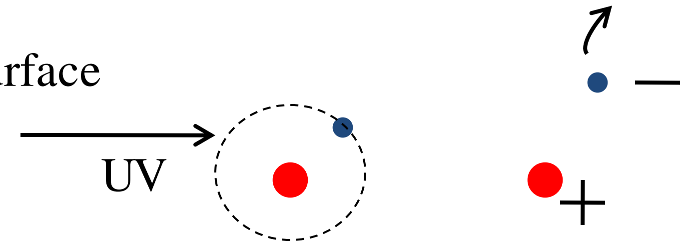


$$\boxed{\Delta V = \frac{Q}{C}} \qquad \boxed{C = \frac{2\pi h \epsilon_0}{\ln(R_1/R_2)}}$$

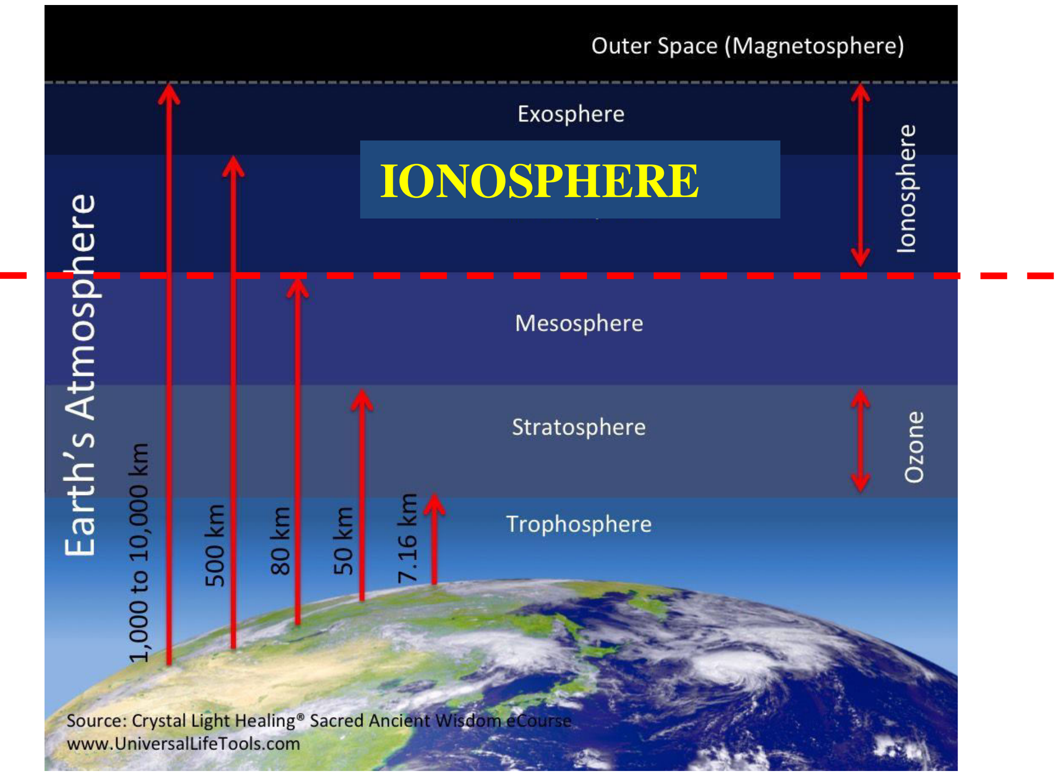
h=2cm
R2= 1 cm
R1= 5 mm

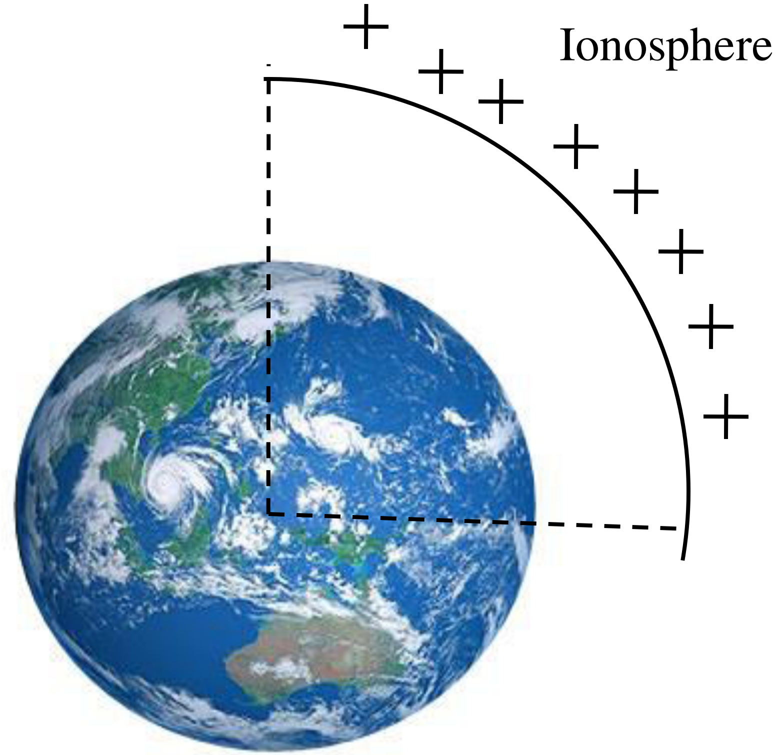
C=1.6 10^{-12} F

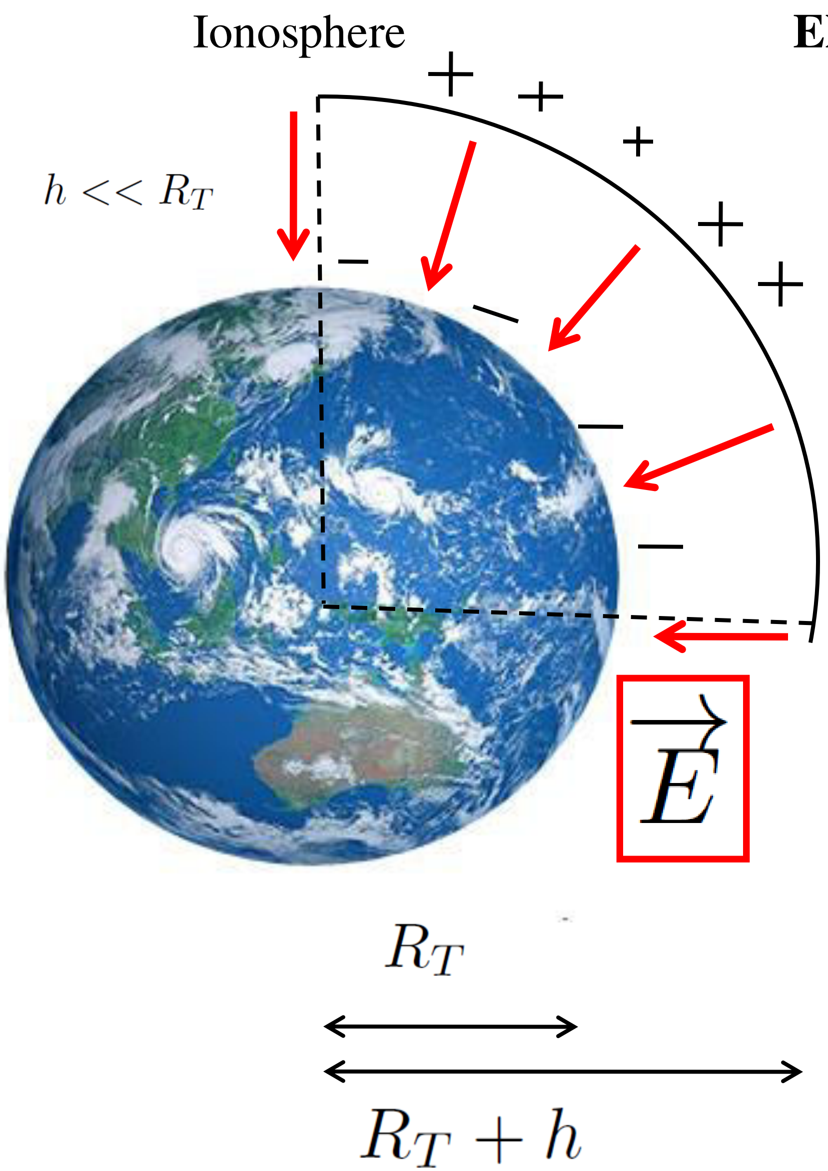
Ionisation processess at an altitude of 80 km from Earth surface



Pressure ≈ 2 Pascal. UV radiation collide with molecules
Electrons are ejected into space and positive ions remain







Electric Field and potential created by a sphere

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\vec{u_r} \qquad V(r) = \frac{Q}{4\pi\epsilon_0 r}$$

Potential difference between ground and ionosphere

$$\begin{aligned}\Delta V &= V(r = R_T) - V(r = R_T + h)\\ &= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_T} - \frac{1}{R_T + h}\right)\\ &= \frac{Q}{4\pi\epsilon_0}\frac{h}{R_T(R_T + h)}\\ &\approx \frac{Q}{4\pi\epsilon_0}\frac{h}{R_T^2}\end{aligned}$$

Earth capacitance

$$\Delta V = \frac{Q}{C} \quad \Rightarrow \quad \boxed{C = \frac{\epsilon_0 4\pi R_T^2}{h}}$$

Similar to $C = \frac{\epsilon_0 S}{d}$

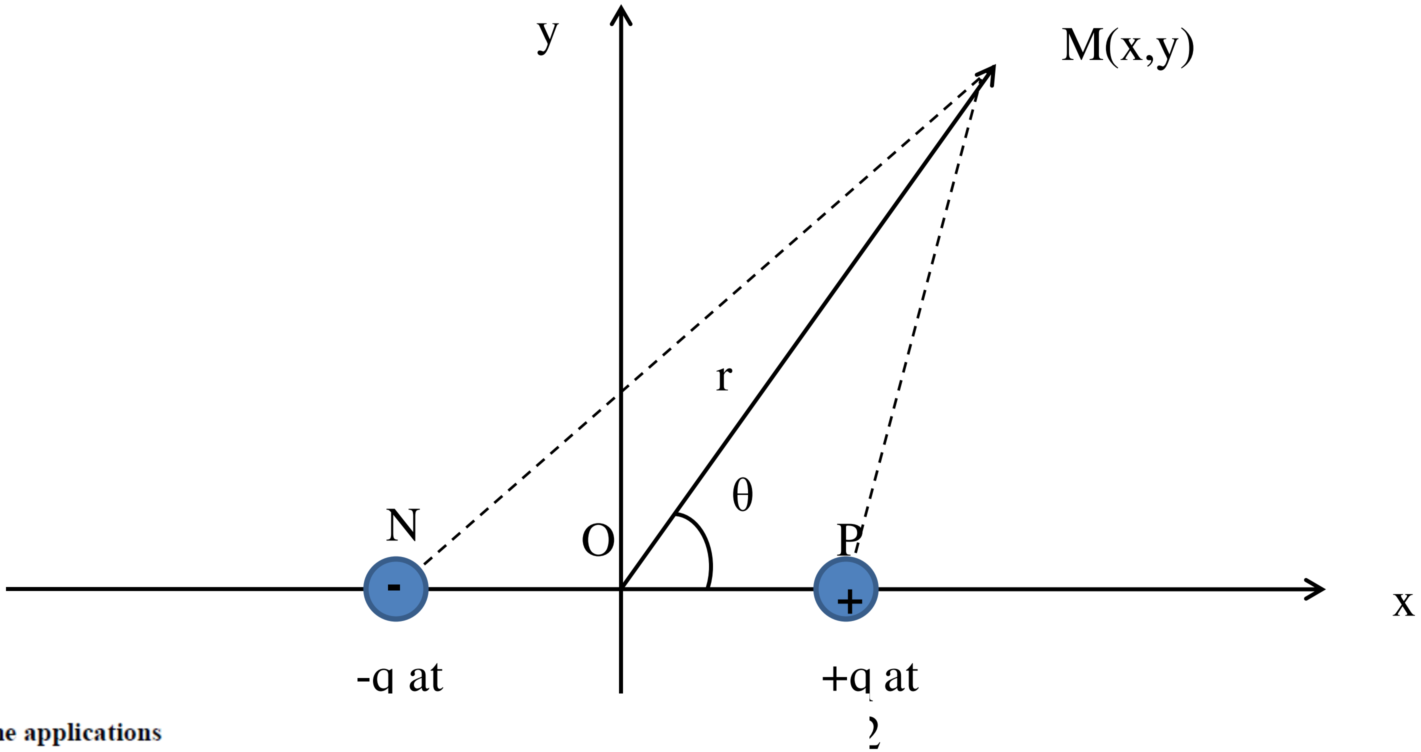
RT= 6371 km
h =80 km

C= 56,4 mF
value of Earth capacity !!

Electrostatics-L2

1) Electrostatic dipole- Potential and electric fields in the dipolar approximation- Molecules - Dipole-dipole interactions **2) Electrostatics of conductors**

IV.1.a) Potential and electric fields in the dipolar approximation



Part 4: Some applications

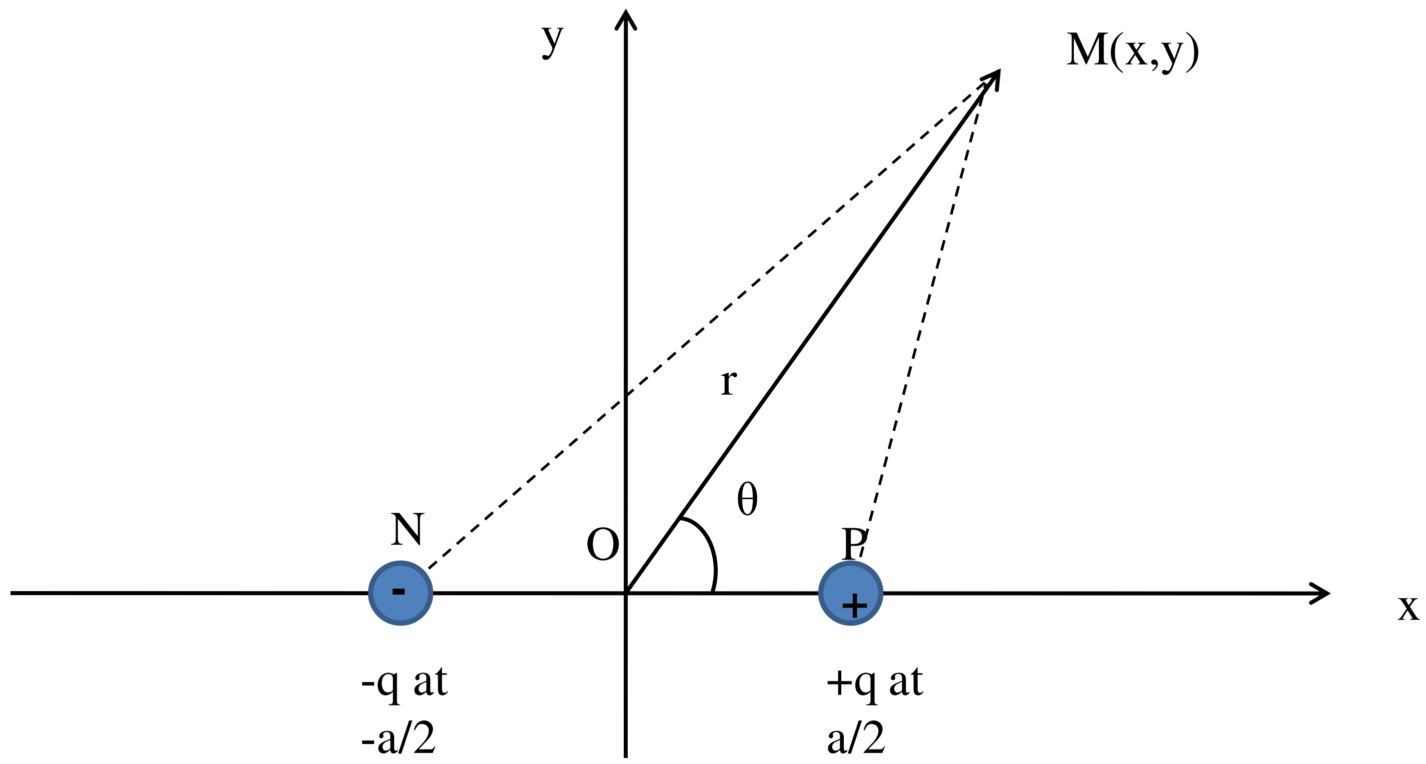
Exercise 1: Electric potential and electric field in the dipolar approximation

We consider a dipole made of charges +q and –q separated by a distance a. In a two dimension Cartesian system Oxy, the negative charge -q is located at position N (-a/2 ; 0) and the positive charge q at position P (a/2; 0).

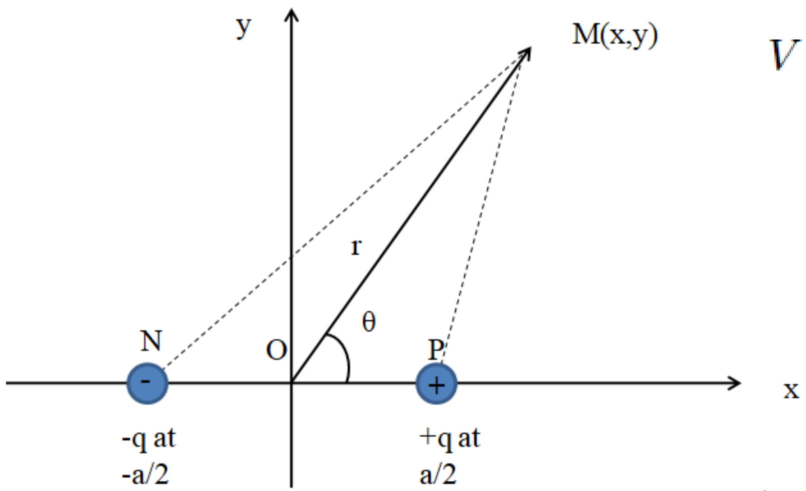
1) What is the expression of the electric potential at a point M of coordinates M(x,y) ?

2) Show that $\|\overrightarrow{PM}\| = \sqrt{PO^2 + OM^2 + 2\overrightarrow{PO}.\overrightarrow{OM}}$ and deduce the expression of $\|\overrightarrow{NM}\|$.

IV.1.a) Potential and electric fields in the dipolar approximation



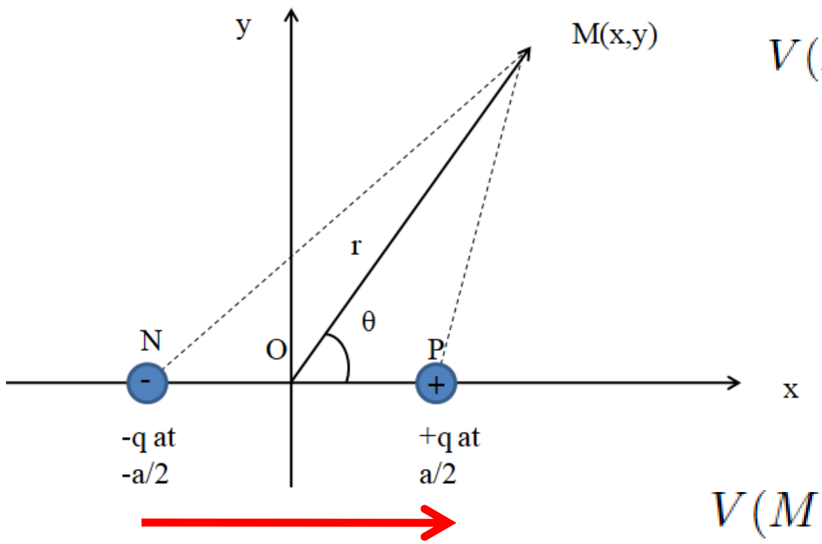
IV.1.a) Potential and electric fields in the dipolar approximation



$$V(M) = V_P(M) + V_N(M) = \frac{q}{4\pi\epsilon_0 PM} - \frac{q}{4\pi\epsilon_0 NM}$$

$$\begin{aligned} PM &= \sqrt{(x_M - x_P)^2 + (y_M - y_P)^2} = \sqrt{(x - a/2)^+ y^2} \\ NM &= \sqrt{(x_M - x_N)^2 + (y_M - y_N)^2} = \sqrt{(x + a/2)^+ y^2} \end{aligned}$$

$$V(M) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{(x - a/2)^+ y^2}} - \frac{1}{\sqrt{(x + a/2)^+ y^2}}\right)$$



$$\boxed{\overrightarrow{p} = qa\overrightarrow{e_x}}$$

$$V(M) = V_P(M) + V_N(M) = \frac{q}{4\pi\epsilon_0 PM} - \frac{q}{4\pi\epsilon_0 NM}$$

$$\begin{aligned} PM &= \sqrt{(x_M - x_P)^2 + (y_M - y_P)^2} = \sqrt{(x-a/2)^{+}y^2} \\ NM &= \sqrt{(x_M - x_N)^2 + (y_M - y_N)^2} = \sqrt{(x+a/2)^{+}y^2} \end{aligned}$$

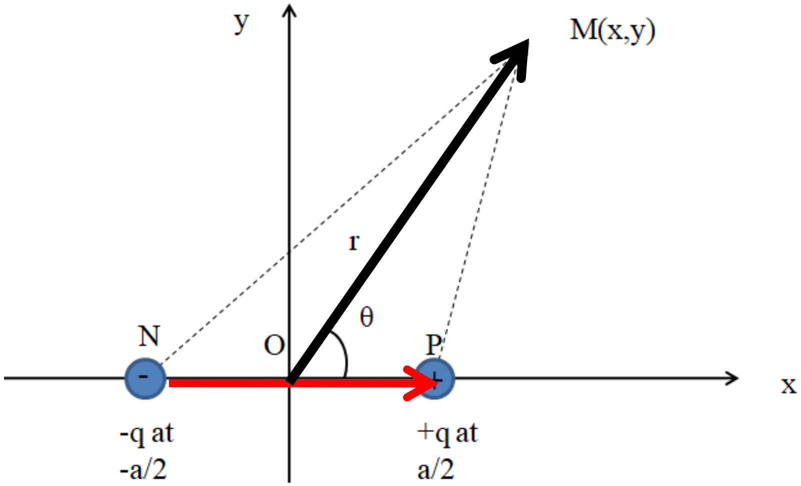
$$V(M) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{(x-a/2)^{+}y^2}} - \frac{1}{\sqrt{(x+a/2)^{+}y^2}}\right)$$

Working within the **dipolar approximation:** r>>a for which the distance of observation is large compared to the size of the dipole.

We will obtain in polar coordinates

Dipolar moment: $\overrightarrow{p} = q\overrightarrow{NP}$ $\quad \overrightarrow{p} = qa\overrightarrow{e_x}$

$$\boxed{V(M) = \frac{\overrightarrow{p} \cdot \overrightarrow{r}}{4\pi\epsilon_0 r^3}}$$



$$\boxed{V(M) = \frac{\overrightarrow{p} \cdot \overrightarrow{r}}{4\pi\epsilon_0 r^3}}$$

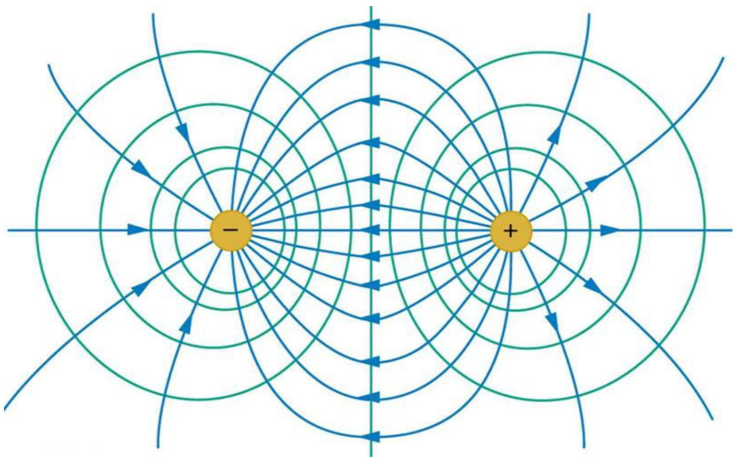
Calculation of the electric field

Using polar coordinates

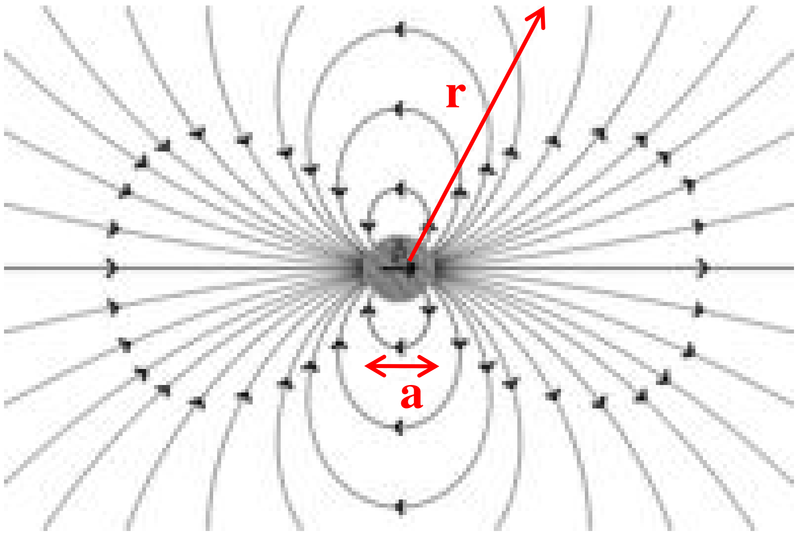
$$\overrightarrow{E} = -\overrightarrow{\text{grad}} V = \frac{1}{4\pi\epsilon_0 r^3}\left(3(\overrightarrow{p} \cdot \overrightarrow{e_r})\overrightarrow{e_r} - \overrightarrow{p}\right)$$

Field Lines

Normal Dipole



Dipole in the dipolar approximation: r>>a



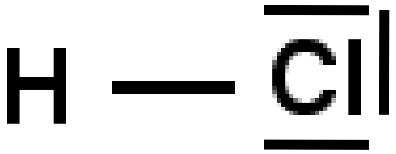
Not valid with approximated formula close to r≈a

Results obtained from potential

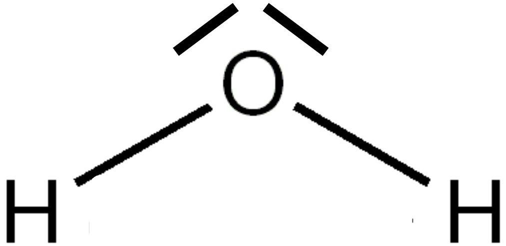
$$V(M) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{(x-a/2)^{+}y^2}} - \frac{1}{\sqrt{(x+a/2)^{+}y^2}}\right)$$

$$V(M) = \frac{\overrightarrow{p}\cdot\overrightarrow{r}}{4\pi\epsilon_0 r^3}$$

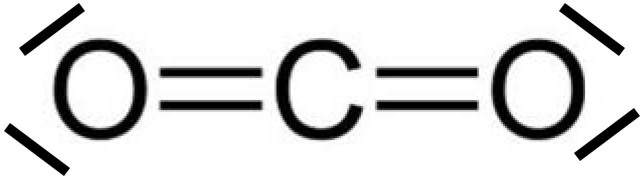
Hydrogen Chloride



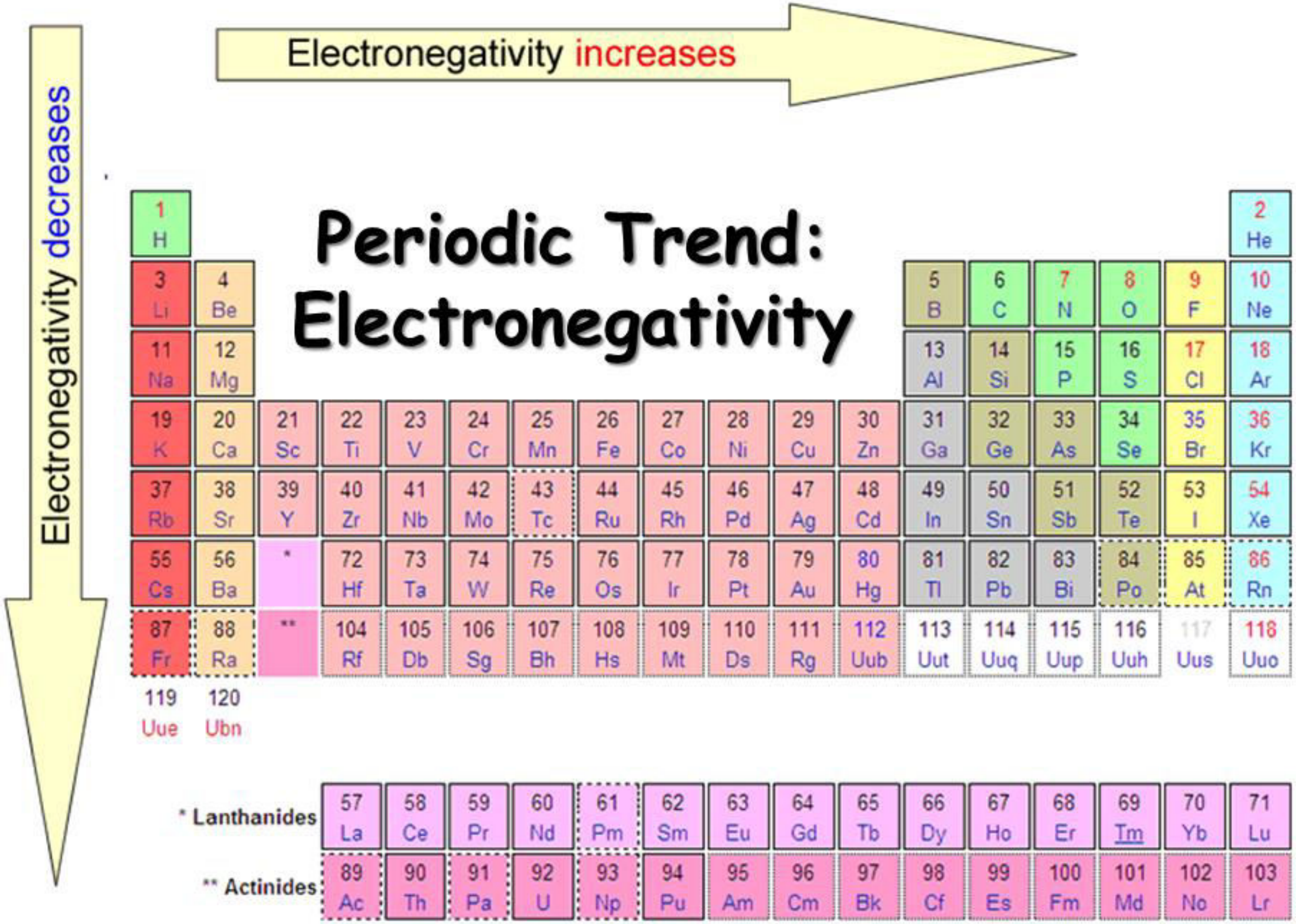
Water



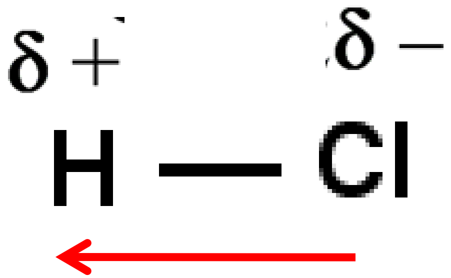
Carbone dioxide



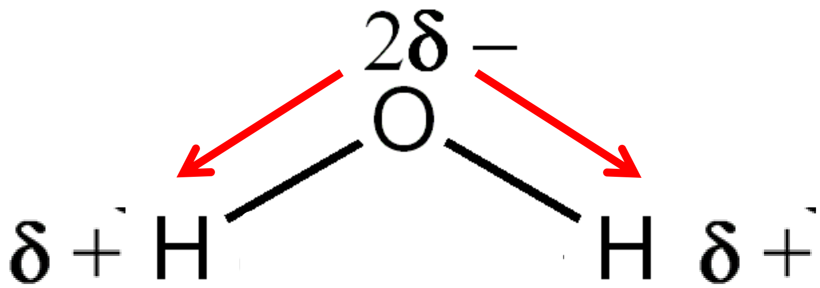
Even in neutral molecules
Non homogeneous repartition of electric
Charges due to electronegativity


Electronegativity increases
Electronegativity decreases
Periodic Trend:
Electronegativity
1 H
2 He
3 Li
4 Be
5 B
6 C
7 N
8 O
9 F
10 Ne
11 Na
12 Mg
13 Al
14 Si
15 P
16 S
17 Cl
18 Ar
19 K
20 Ca
21 Sc
22 Ti
23 V
24 Cr
25 Mn
26 Fe
27 Co
28 Ni
29 Cu
30 Zn
31 Ga
32 Ge
33 As
34 Se
35 Br
36 Kr
37 Rb
38 Sr
39 Y
40 Zr
41 Nb
42 Mo
43 Tc
44 Ru
45 Rh
46 Pd
47 Ag
48 Cd
49 In
50 Sn
51 Sb
52 Te
53 I
54 Xe
55 Cs
56 Ba
*
72 Hf
73 Ta
74 W
75 Re
76 Os
77 Ir
78 Pt
79 Au
80 Hg
81 Tl
82 Pb
83 Bi
84 Po
85 At
86 Rn
87 Fr
88 Ra
**
104 Rf
105 Db
106 Sg
107 Bh
108 Hs
109 Mt
110 Ds
111 Rg
112 Uub
113 Uut
114 Uuq
115 Uup
116 Uuh
117 Uus
118 Uuo
119 Uue
120 Ubn
* Lanthanides
57 La
58 Ce
59 Pr
60 Nd
61 Pm
62 Sm
63 Eu
64 Gd
65 Tb
66 Dy
67 Ho
68 Er
69 Tm
70 Yb
71 Lu
** Actinides
89 Ac
90 Th
91 Pa
92 U
93 Np
94 Pu
95 Am
96 Cm
97 Bk
98 Cf
99 Es
100 Fm
101 Md
102 No
103 Lr

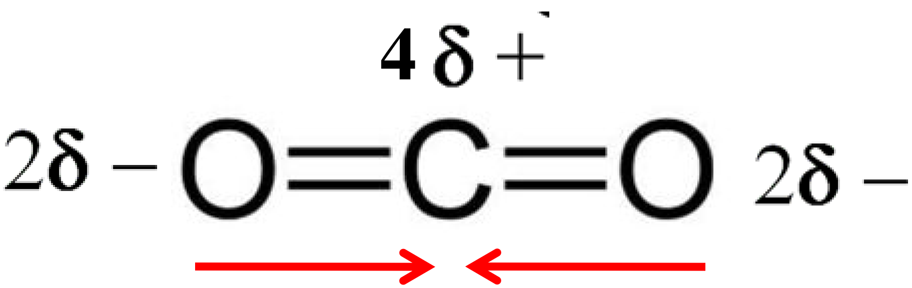
Hydrogen Chloride



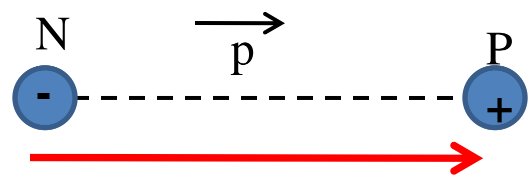
Water



Carbone dioxide



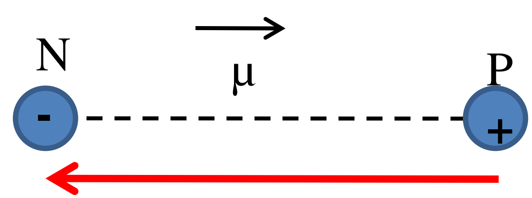
Existence of electrostatic dipoles



Defined from N to P in Physics

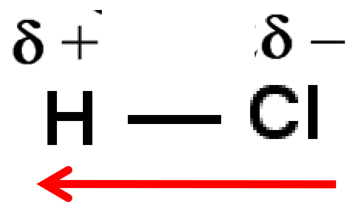
BUT

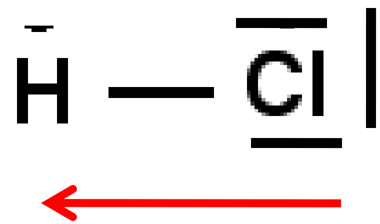
Defined from P to N in Chemistry…..



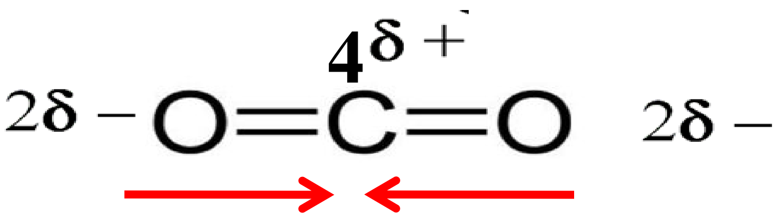
IV.1.b) Molecules

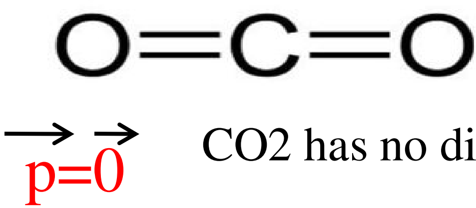
Hydrogen Chloride





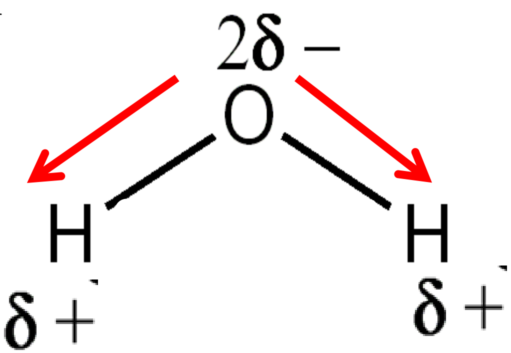
Carbone dioxide

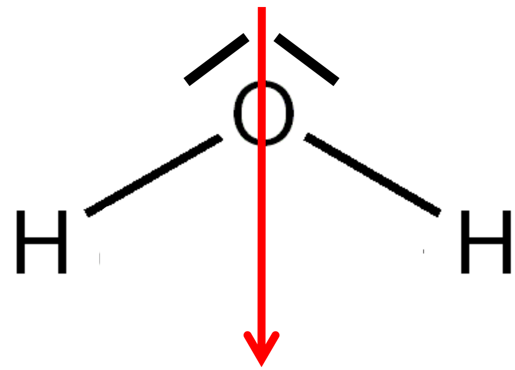




CO2 has no dipolar moment

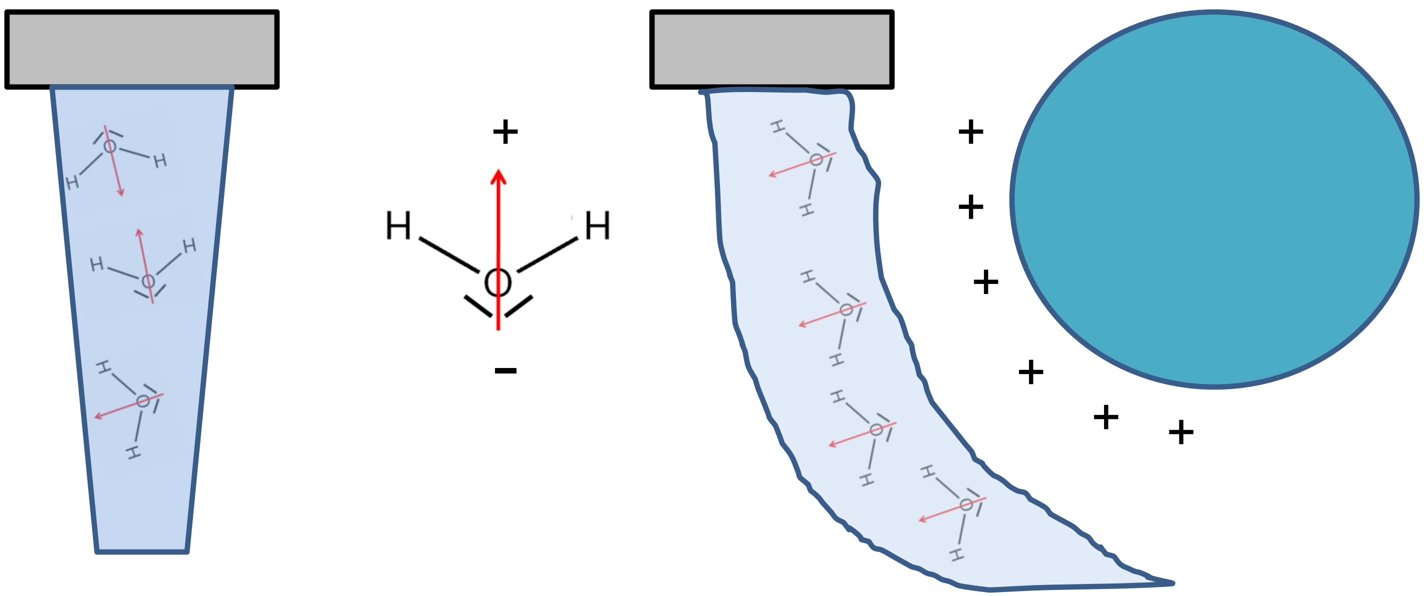
Water

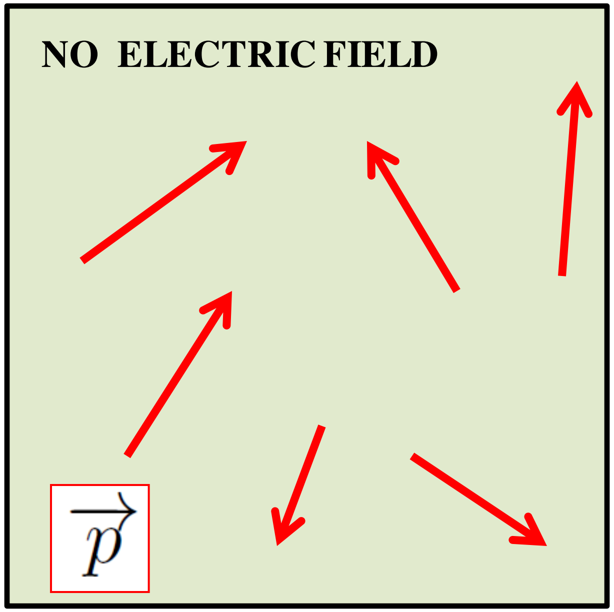


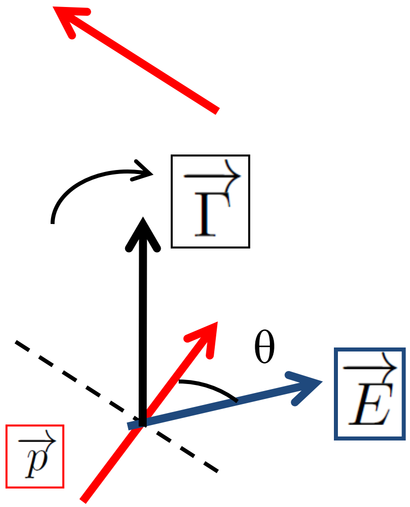


IV.1.b) Molecules

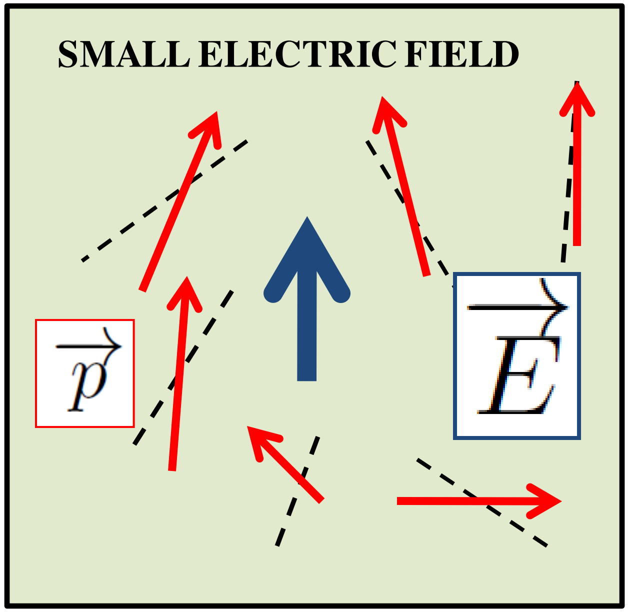
Deviation of a water flow by a rubbed object

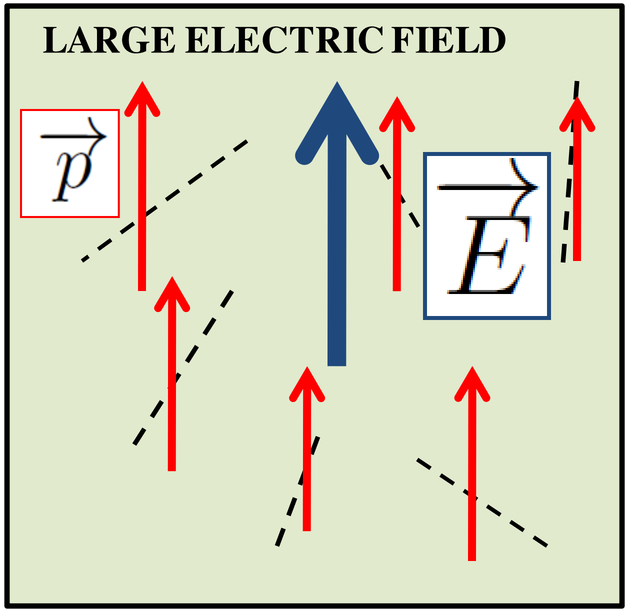






U = -p E cos θ





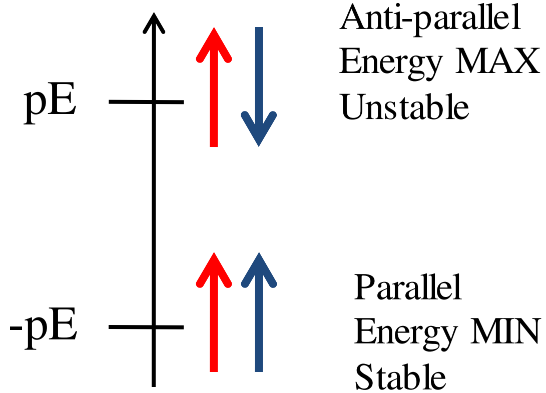
Dipole rotate in the direction of the field

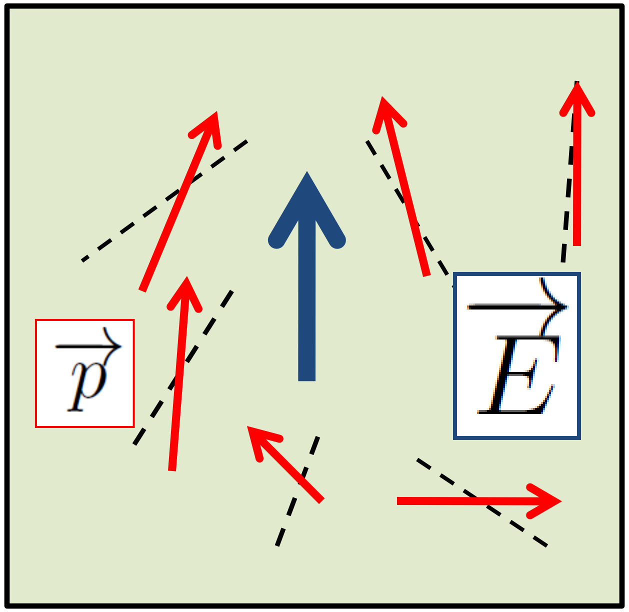
Torque of a force

$$\vec{\Gamma} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

Energy:





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Its amplitude is proportional to the applied electric field

$$\overrightarrow{P} = \epsilon_0 \chi \overrightarrow{E}$$

Dielectric succeptibility: χ depends on material

Total electric field is:

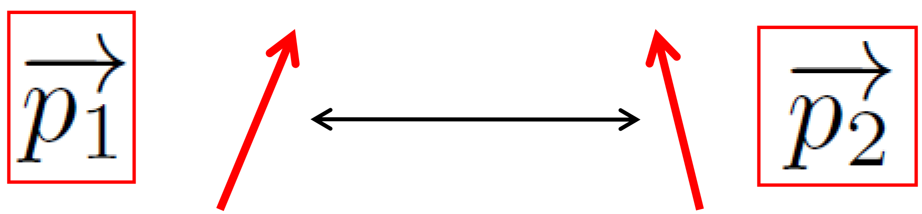
$$\begin{aligned}\overrightarrow{E}_{\mathrm{total}} &= \overrightarrow{E} + \frac{\overrightarrow{P}}{\epsilon_0} \\ &= \overrightarrow{E} + \chi\overrightarrow{E} \\ &= (1+\chi)\overrightarrow{E} = \epsilon_r \overrightarrow{E}\end{aligned}$$

$$\boxed{\overrightarrow{E}_{\mathrm{total}} = \epsilon_r \overrightarrow{E}}$$

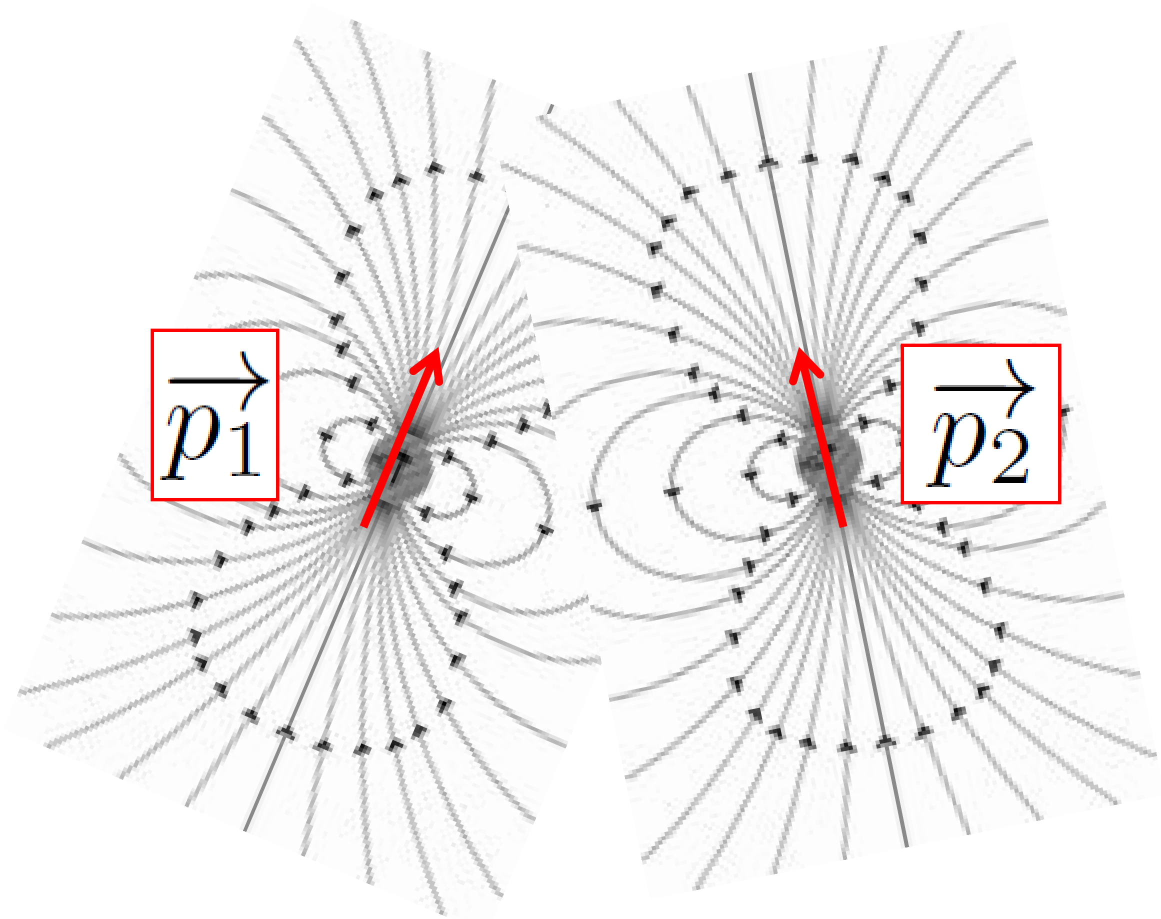
Dielectric permitivity ε_r

IV.1.c) Dipole-Dipole interaction

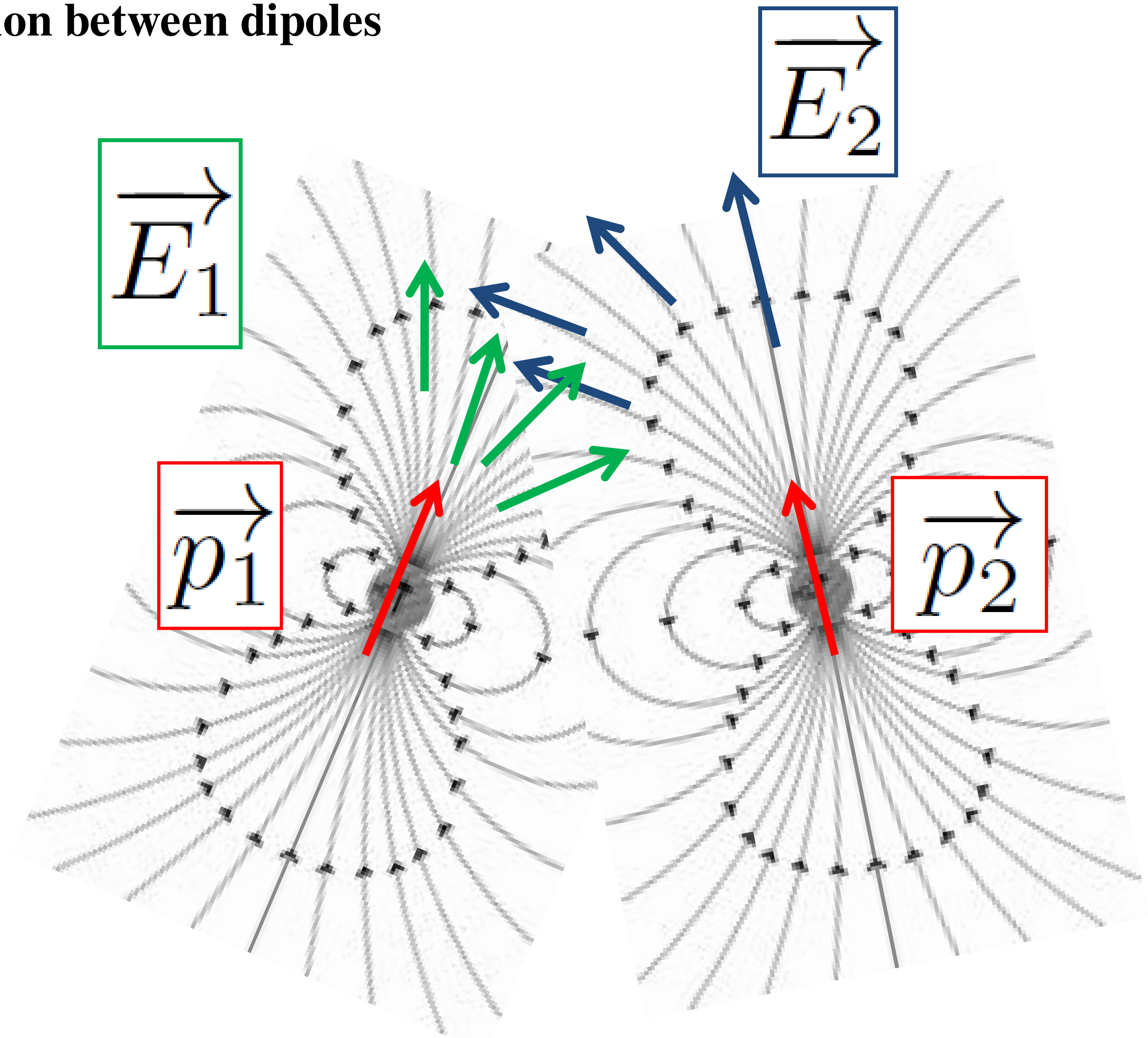
Interaction between dipoles



Interaction between dipoles



Interaction between dipoles

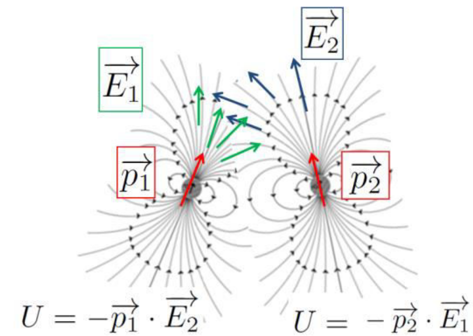


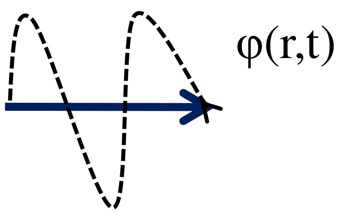
About dipole-dipole interactions

1) Permanent dipole-permanent dipole

2)Permanent dipole- induced dipole

3) Induced dipole- induced dipole

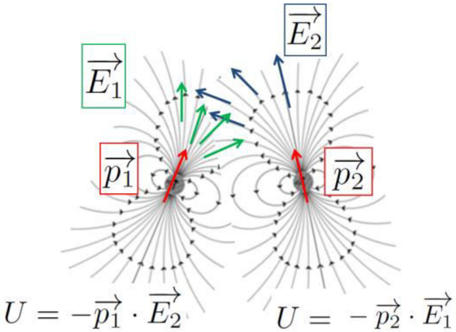




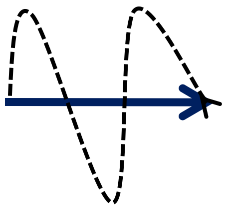
Electron is described by wave function φ(r,t), it modifies localy the electronic distribution and Induces a electric dipole

About dipole-dipole interactions

1) Permanent dipole-permanent dipole



2)Permanent dipole- induced dipole



φ(r,t)

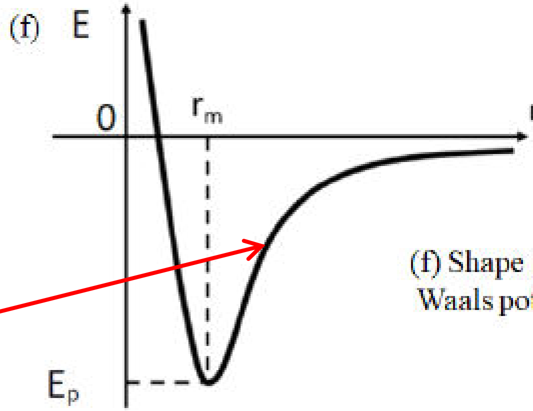
Electron is described by wave function φ(r,t), it modifies localy the electronic distribution and Induces a electric dipole

3) Induced dipole- induced dipole

VAN DER WAALS INTERACTIONS

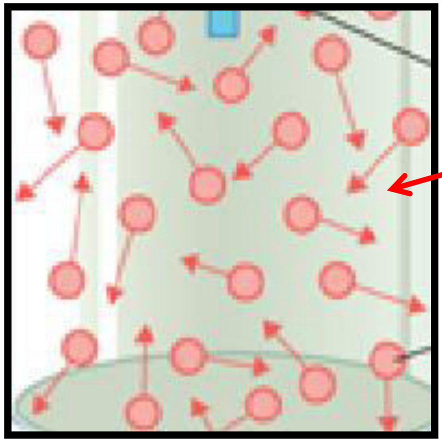
$$U_p \approx \frac{1}{r^6} \qquad \vec{F} = -\overrightarrow{\mathrm{grad}}\,U_p \approx -\frac{1}{r^7}\vec{u}$$

Attractive part of the potential



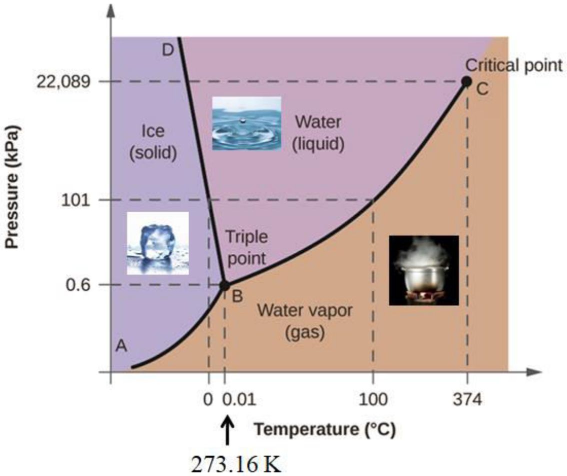
(f) Shape of the Van der Waals potential energy

Difference between **ideal gas and real gas** (described with Van der Waals interactions)



interaction between molecules

The interaction between molecules permit to have **Phase transitions**



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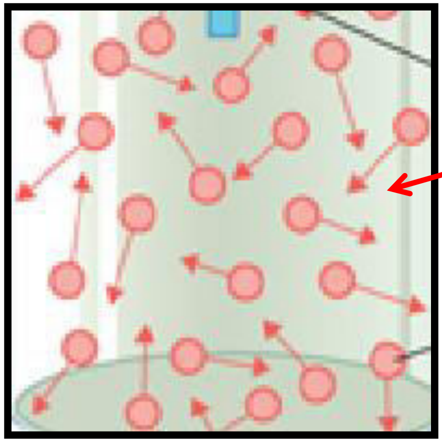

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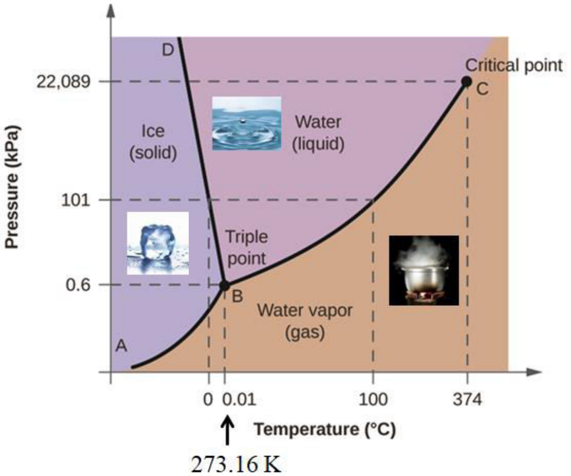


It will be a competition between the electrostatic interactions and the the thermal agitation

$$\mathcal{E} = \mathcal{E}_k^{\text{int}} + \mathcal{E}_p^{\text{int}}$$

$$< \mathcal{E} > = \frac{3k_BT}{2} + U_{\text{electrostat}}$$

U_{elec} VS U_{thermal}

Order VS Disorder

Gas is less ordered than liquids that are less ordered than solids

Important topic in Physics:

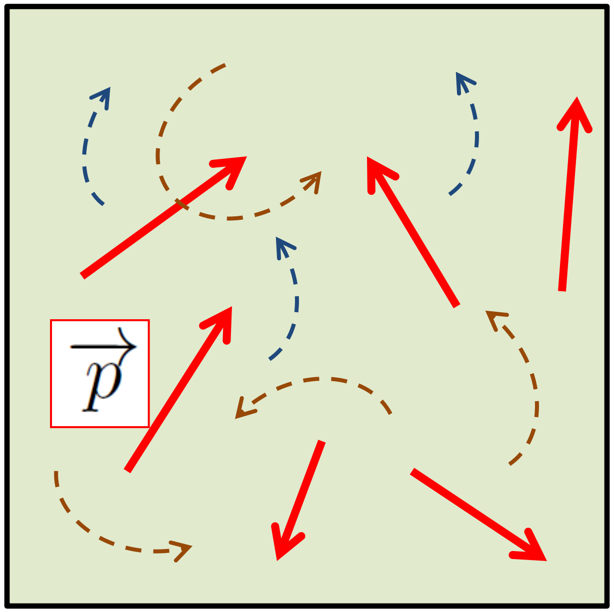
The Entropy /Potential Competition

Some physical effects are an illustration of the competition between two type of phenomena

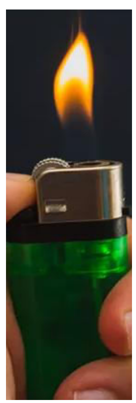
The ones originating from a **potential energy** that will induces forces and an **ordered behaver** (eletric, magnetic or gravitational ordering)

The ones originating from **the entropy** traducing a **random and chaotic situation** leading to a **disorderd behaver** (like thermal agitation or diffusion phenomena.

Without considering Phase transition, one can have a competition between:



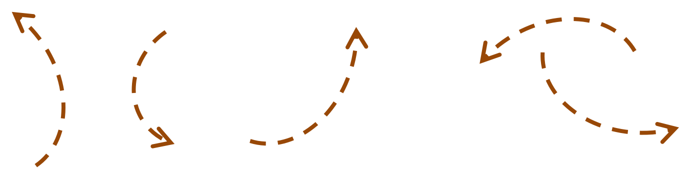
VS



Ordering imposed by the electric field:
It wants to impose a direction to the dipoles

$$U = -\overrightarrow{p} \cdot \overrightarrow{E}$$

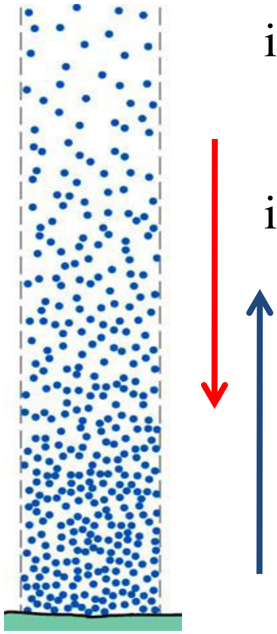
Disordering due to thermal agitation: it gives a random orientation to the dipole.



In statistical physics we measure the proability for the dipole to have the energy U with the Boltzmann factor

$$P \approx e^{-\frac{U}{kT}}$$

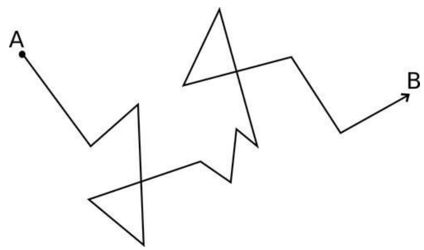
Why air-molecules do not fall on the ground ?

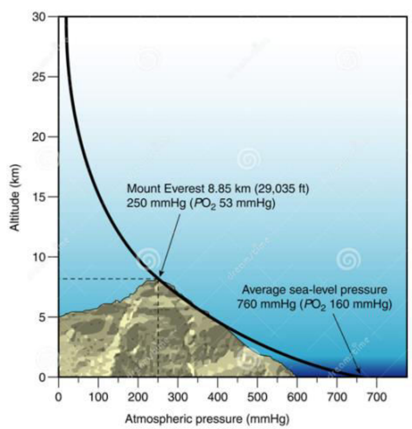


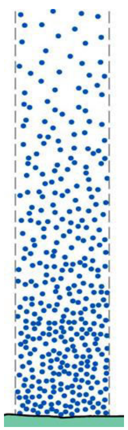
Because equilibrium between

i) The current due to the gravity. Force originating in the gravitational potential energy : Weight =- Grad (mgz) that **makes molecules falling on the ground.**

ii) The current due to **diffusion which is a random phenomena** that will send some molecules to the top.

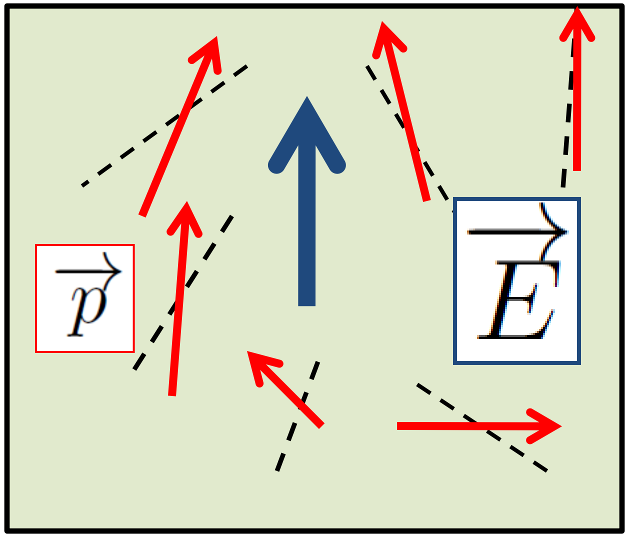






$$P(z) = P_0 \exp\left(-\frac{mgz}{kT}\right)$$

WHAT IS IMPORTANT FOR NEXT PART OF LECTURE



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