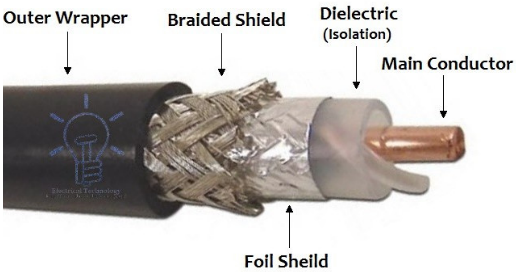
MAGNETOSTATICS

Tutorial n°1

MAGNETIC FIELDS FROM CURRENT DISTRIBUTIONS

Coaxial cable



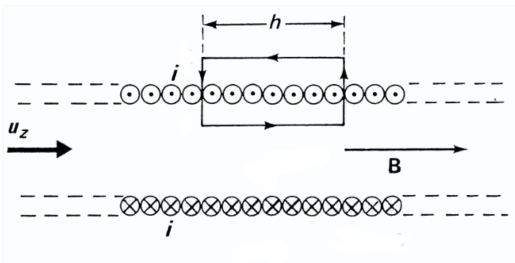
An infinite cylindrical coaxial cable is made up of (1) a cylindrical conducting copper core of radius R_1 through which a current of intensity I flows and (2) A braided shield, modelled by a hollow conducting cylinder located between radii $R_2 > R_1$ and $R_3 > R_2$. The shield has a current $-I$ by construction.

1. Use symmetry arguments to describe as precisely as possible the magnetic field in the regions $r \in [R_1, R_2]$ and $r > R_3$.
2. By using the Ampère's theorem, compute the magnetic field intensity in the same regions of space.
3. Assuming that the density of current $\vec{\jmath}$ is constant within the inner core, and directed parallel to the core axis, compute the relation between $|\vec{\jmath}\,|$ and I.

Electric wire with finite cross section

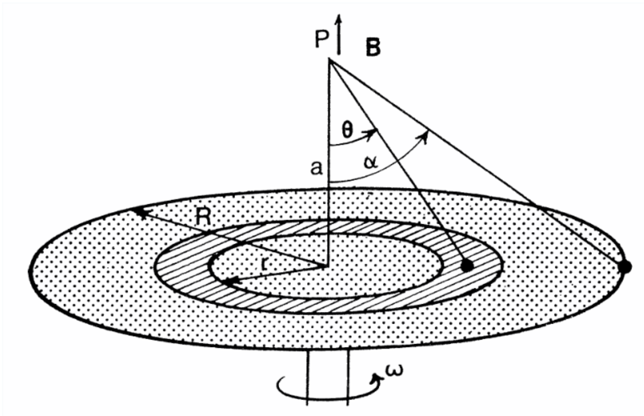
A real electric cable has a finite radius R. By assuming the current density a homogenous (i.e. constant in space) vector field $\vec{\jmath}$, compute the magnetic field outside and inside an infinite straight such wire. Why the field does not goes to zero as $1/r^2$ for $r \to \infty$ as in the Biot-Savart law ?

Ideal Solenoid from Ampère's law



Use the closed rectangular path of the figure to demonstrate the formula $\vec{B} = \mu_0 n i \vec{u}_z$ for the field inside the infinite solenoid (n is the number of current loops by unit length). One must assume that the external magnetic field is zero.

Convection currents : Rowland's experiment (1876)



In the figure above, a metallic disk of radius R is charged on its surface with a uniform charge density σ. It is put in rotation at a constant angular velocity ω around its axis.

1. Explain qualitatively why a magnetic field is created by the rotation of the disk.
2. Show that the magnetic field at $P(z = a)$ on the axis is given by (α is defined by $\tan \alpha = R/a$)

$$\vec{B}(P) = \mu_0 \sigma R \omega \sin^2\left(\frac{\alpha}{2}\right) \tan\left(\frac{\alpha}{2}\right)$$

3. Give an order of magnitude of the field created at the center of the disk (feed the formula with reasonable values, for σ think of a capacitor close to the breakdown electric field $3 \cdot 10^6$ V/m), and compare it with the magnitude of the terrestrial magnetic field in Baku $B_{\text{earth}} = 49\mu$T.