PW n°4: Diffraction and interferences in Optics

A light beam going through a diffractive object (an opening or aperture) will be submitted to diffraction when the size of the object is close to the size of the wavelength. The opening acts as an ensemble of secondary sources that leads to a diffusion of the light in every direction. Due to the phase shift between the light originating from the different sources, one can observe in a plane of observation some regions where light amplitude add or cancel each other.

I. Diffraction in the Fraunhofer approximation

In Fraunhofer diffraction, we assume that the incident light comes from the infinite as well as for the plane of observation. The diffractive object is in the plane *(x'y')*. In the plane of observation *(x,y)*, the angular position for a point *M(x,y,z)* in the plane *Oxy* will be given by the angles $\sin\theta_x = x/r$ and $\sin\theta_y = y/r$.

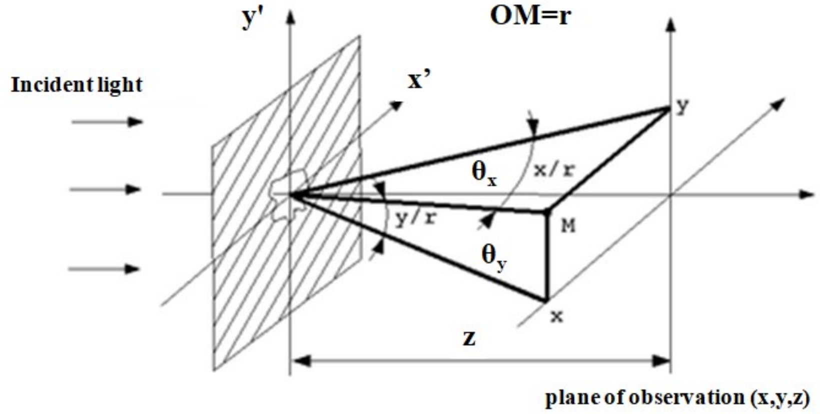


Figure 1.

A wave in the observation plane has the following complex form: $(r,t) = A(x',y')e^{i\omega t}e^{-i\vec{k}.\vec{r}}$, where $A(x',y')$ is the electric field amplitude emitted by a secondary source of the opening and the exponential part $e^{i\omega t}e^{-i\vec{k}.\vec{r}}$ traduces its phase evolution during the propagation. The spatial dependence $\vec{k}.\vec{r}$ inside the exponential can be written as $\vec{k}.\vec{r} = k\sin\theta_x\, x' + k\sin\theta_y\, y'$. By writing the amplitude of the wave vector $k = \frac{2\pi}{\lambda}$ we can express the spatial part of the wave as $e^{-i\vec{k}.\vec{r}} = e^{-2i\pi\left(\frac{x}{\lambda r}x' + \frac{y}{\lambda r}y'\right)}$. The electric field is thus written as $(r,t) = A(x',y')e^{i\omega t}e^{-2i\pi\left(\frac{x}{\lambda r}x' + \frac{y}{\lambda r}y'\right)} = A(x',y')e^{i\omega t}e^{-2i\pi\left(\frac{x}{\lambda r}x' + \frac{y}{\lambda r}y'\right)}$.

We assume that an elementary secondary source of the opening is located in a surface $dx'dy'$. Consequently the total electric field $E(x,y,z)$ at a point (x,y,z) of the observation plane, is the sum of the electric fields created by all elementary sources. It will be obtained by integrating over all elementary surfaces of the opening. Its expression is given as follows

$$E(x,y,z) = e^{i\omega t}\iint dx'dy'A(x',y')e^{-2i\pi\left(\frac{x}{\lambda r}x' + \frac{y}{\lambda r}y'\right)} = e^{i\omega t}\iint dx'dy'A(x',y')e^{-2i\pi(\nu_x x' + \nu_y y')} \quad (1)$$

where we introduce the new coordinates $\nu_x = \frac{x}{\lambda r} = \frac{\sin\theta_x}{\lambda}$ and $\nu_y = \frac{y}{\lambda r} = \frac{\sin\theta_y}{\lambda}$. With the last expression it is important to understand that the geometric shape of the opening will determine the intensity profile of the diffraction pattern. The intensity of the light on the observed plane will be determined as the the product between the electric field amplitude and its complex conjugate:

$$I \sim |E(x,y,z)E(x,y,z)^*| = \left| \iint dx'dy'A(x',y')e^{-2i\pi(v_x x' + v_y y')} \right|^2$$

FT is the fourier transform of the electrical field

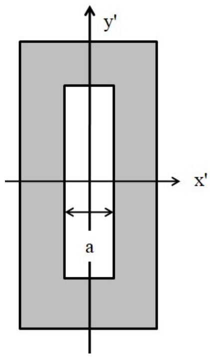
II Diffraction phenomena

The diffraction corresponds to the dispersion of the light due to the material limitation of the waves. This phenomenon appears when light encounters an obstacle of the same size then the wavelength. Mathematical formulations will be needed to describe and determine the physical dimensions of the encountered obstacles.

II.1 Diffraction by a slit of width a (1 dimension)

II.1.1 Calculations of the intensity profile

We consider a slit having a small width *a* along *O'x* axis and a size along *Oy'* large enough to be considered as infinite. The slit is centered on the origin with the opening going from$x' = -\frac{a}{2}$ to $x' = a/2$ in Ox axis. In the plane of the opening, the light can be transmitted only between $x' = -a/2$ and $x' = a/2$. The amplitude of the electric field is given by the following transmission function as:



Figure(s) 2.

$$A(x',y') = rect\left(\frac{x'}{a}\right) = A \begin{cases} 1 \; for \; |x'| < a/2 \\ 0 \quad elsewhere \end{cases}$$

The calculation of the total electric field

$$E(x) = e^{i\omega t} \int_{x'=-a/2}^{x'=a/2} A dx' e^{-2i\pi(v_x x')}$$

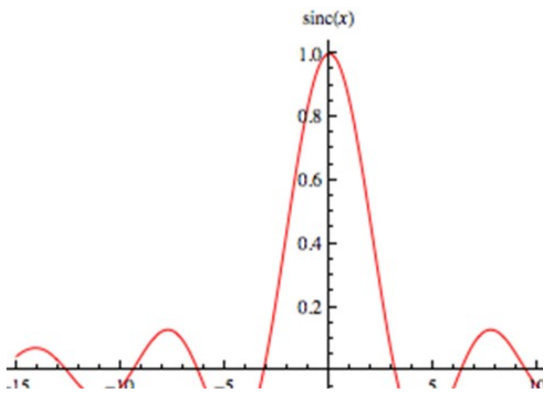
$$= e^{i\omega t} A \int_{x'=-a/2}^{x'=a/2} dx' e^{-2i\pi(v_x x')}$$

$$E(x) = \frac{e^{i\omega t} A}{-2i\pi v_x}\left(e^{-\frac{2i\pi v_x a}{2}} - e^{\frac{2i\pi v_x a}{2}}\right)$$

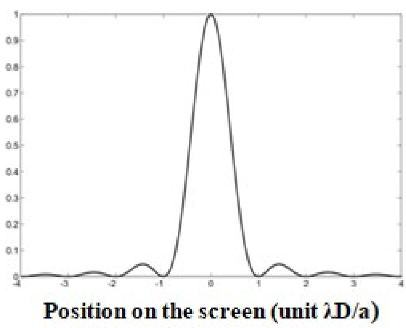
With the two exponential terms one can make appear a sine function which is then transformed into the sinus cardinal function $\mathrm{sinc}(x) = \frac{\sin(x)}{x}$:

$$E(x) = e^{i\omega t} A \frac{\sin(\pi v_x a)}{\pi v_x} = e^{i\omega t} A a sinc(\pi a v_x)$$

The intensity of the light will thus have the following form $I \sim A^2 a^2 sinc(\pi a v_x)^2$. The cardinal sinus is a 2π periodic function which has the behavior plotted just on the right. The succession of maxima and minima shows the periodicity. The intensity of the light is given by the square of this function. The abscise represented for it in the picture are given related to the screen position. When $\pi a v_x = \pi$ representing the first zero of the pattern, one has $a v_x = a\frac{\sin\ x}{\lambda} = 1$. For small angles one has $\sin\theta_x \approx \tan\theta_x \approx \theta_x \approx x/D$ where D is the distance between the slit and the observation screen, leading to $x = \lambda D/a$.



Sinus cardinal function



Square of the Sinus cardinal function

The width of the first principal lob is given by the distance between the two first zeros of the cardinal sinus such as: $\Delta x = \frac{2\lambda D}{a}$. In terms of angular aperture between the slit and the screen the previous relation will be written as $\Delta\alpha = \frac{\Delta x}{D} = \frac{2\lambda}{a}$ .Therefore one can access to the size of the slit knowing the size of the first lob with following expression:

$$a = \frac{2D\lambda}{\Delta x}$$

Knowing the wavelength one can determine the size of the slit or vice et versa, knowing perfectly the aperture one can determine the wavelength of the light.

II.1.2 Experimental Part: measuring the width of the slit

Turn on the laser and send the light through the adjustable slit. The slit has a screw that permits to change the width of the opening. Observe the transmitted light on a screen or on the wall. Open the slit enough to have the width larger than the size of the the laser spot.

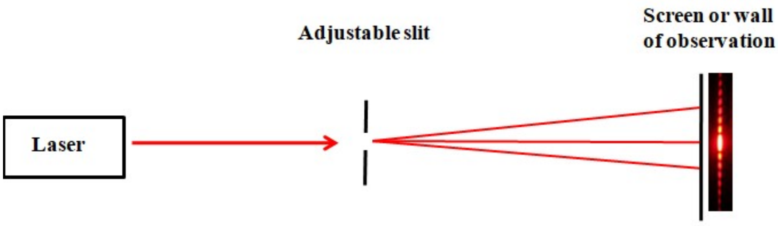


Figure 3

Then, reduce the width of the slit in order to obtain the diffraction pattern. How evolve the size of the pattern when changing the slit width. The wavelength of the laser is $\lambda = 632.8$ nm. Fix a configuration for the slit width and determine its size a by using the interference pattern obtained on the screen or on the wall.

II.1.3 Complementary apertures: Babinet theorem. Measuring the size of a hair

It is possible to demonstrate that the diffraction pattern given by a small slit is the same than the one given by a thin wire or line. At this step we will admit that result.
Take a hair and fix it on a lens holder. Reproduce the previous experiment with the hair. Observe the diffraction pattern and deuce the size of the hair.

Remark about a finite rectangular opening

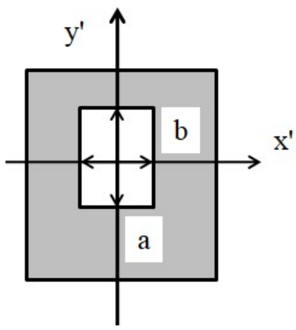
When the other size of the slit can not be consider as infinite the transmission function can now be written as:

$$A(x', y') = rect\left(\frac{x}{a}\right).rect\left(\frac{y}{b}\right)$$

for $x' \in [-\frac{a}{2}; \frac{a}{2}]$ and $y' \in [-\frac{b}{2}; \frac{b}{2}]$ where a and b are the width respectively in x and y from the rectangular opening. The calculations will lead to

$$E(x, y) = Ae^{i\omega t}ab\,.sinc(av_x).sinc(bv_y)$$

and the interference pattern will looks like as pictured on the right.



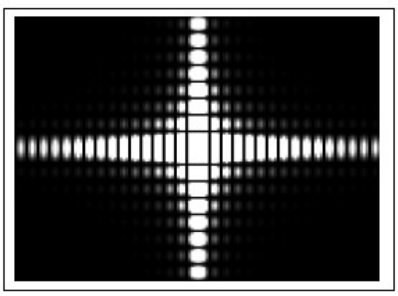


Figure 4.

II.2 Diffraction by a circular opening

II.2.1 Calculations of the intensity profile

A circular aperture of radius a possess the following transmission function in cartesian coordinates as $\tau(x',y') = 1$ for $x'^2 + y'^2 \le a^2$ and $\tau(x',y') = 0$ elsewhere. By introducing the polar coordinates (r,θ) with x= r cos(θ) and y= r sin(θ) the transmission can be written as $\tau(r,\theta) = 1$ for $r \le a$ and $\tau(r,\theta) = 0$ elsewhere. The electrical field in the observation plane can be rewritten as

$$E(r,\theta) = A\, e^{i\omega t} \int_0^r dr \int_0^{2\pi} d\theta\, e^{-2i\pi(r\cos(\theta)\nu_x + r\sin(\theta)\nu_y}$$

It is also necessary to introduce the polar coordinates in the Fourier plane with $\nu_x = \nu\cos(\psi)$ and $\nu_y = \nu\sin(\psi)$. One can introduce also the Bessel function J defined as:

$$J_0(s) = \frac{1}{2\pi}\int_0^{2\pi} e^{is\cos(\mu)}\, d\mu = \frac{1}{2\pi}\int_0^{2\pi} e^{is\sin(\mu)}\, d\mu$$

Hence, by realizing that: $r\nu\cos\theta\cos\psi + r\nu\sin\theta\sin\psi = r\nu\cos(\theta - \psi)$ and by writing $\mu = \theta - \psi$ and $s = -2\pi\nu r$ we can make appear the function $J_0(s)$ in the expression of the electric field as

$$E(r,\theta) = 2\pi A\, e^{i\omega t} \int_0^a rJ_0(-2\pi\nu.r)$$

The Bessel function J_0, is connected to the first J1 Bessel function as :

$$J_1(\omega) = \frac{1}{\omega}\int_0^{\omega} sJ_0(s)ds$$

Consequently the electric field finally reads

$$E(r,\theta) = A\, e^{i\omega t}\frac{a}{\nu}.J_1(2\pi\nu.a)$$

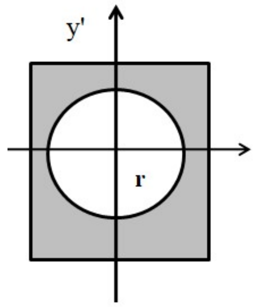
The intensity repartition will be given by

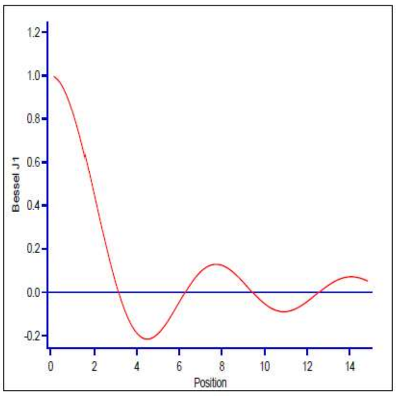
$$I \sim \left|A\,\frac{a}{\nu}.J_1(2\pi\nu.a)\right|^2$$

whose picture is given on the right. . It represents the diffraction of a circular opening of radius a. The central lob is usually called the Airy disk. To determine the size of the aperture (opening) it is necessary to use the table of the zero of the Bessel function:

Zeros of Bessel functions	
$2\pi a\nu_1 = 3.832$	$\nu_1 = 0.61/a$
$2\pi a\nu_2 = 7.015$	$\nu_2 = 1.118/a$
$2\pi a\nu_3 = 10.173$	$\nu_3 = 1.619/a$

Where we remember that $\nu = \frac{x}{D\lambda}$ is related to the angular opening.





J_1 Bessel function

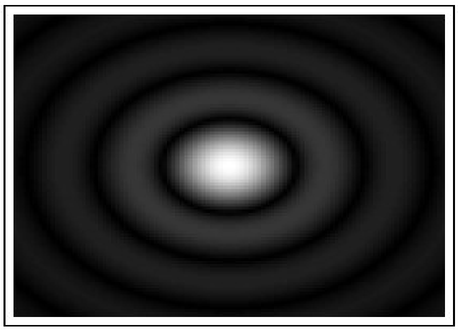


Figure 5.

Experimental part

In the diffraction kit box you can find a slide with holes having different sizes. By using the same method than previously, observe the diffraction pattern given by a hole and determine its radius a.

III Use of a CCD camera and polarizer

Figure 6.

In what follows we will record the optical signal with a **CCD camera (Charge-Coupled device)** that permits to transform an optical signal into an electrical one. The transformed signal will then be observed with an oscilloscope. The CCD camera is made of 2048 pixels having a size of 14 µm. The light recorded on that length is transformed into an electric signal seen with the temporal basis of an oscilloscope. On the oscilloscope, the spatial extremities of the captor

Send the laser light on the detector by having approximately 1,5 meter between them. The CCD camera is very sensitive to light. To illustrate the phenomena plug the two outputs of the CCD camera with the oscilloscope and turn on the oscilloscope. The signal output is plugged into CH1 terminal and the synchronize output into the CH2 terminal. The camera should also be fed with a 12 V power supply. Select the channel 1 of the oscilloscope and by playing with the time-sensitivity button you should observe the all signal recorded by the detector. The signal is saturated. If you cut the optical signal with your hand, the amplitude on the oscilloscope screen should decrease.
Measure the time duration corresponding to the size of the all 2086 pixels in order to have the factor of conversion between the time and the distance. After obtaining the ratio turn off the CCD camera.

Figure 7.

To reduce the laser intensity we will use polarizers. The electric field of the laser is non polarized meaning the electric field vector has a random direction in the Oxy plane if we assume the light is propagating in the Oz direction. A polarizer can impose a direction to the electric field vector by selecting only the components parallel to its axis of polarization.
When putting a second polarizer, called analyser, after the first polarizer, it is possible to select another axis of polarization making an angle θ, the amplitude of the outgoing electric field is reduced by a factor cosθ (see Figure) .The intensity of the light thus follows the so-called Malus law:

$$I = I_0 \cos\theta^2$$

Consequently when the angle between the polarizer and the analyser is 90° we can even reduced the light intensity to zero. We called it extinction.

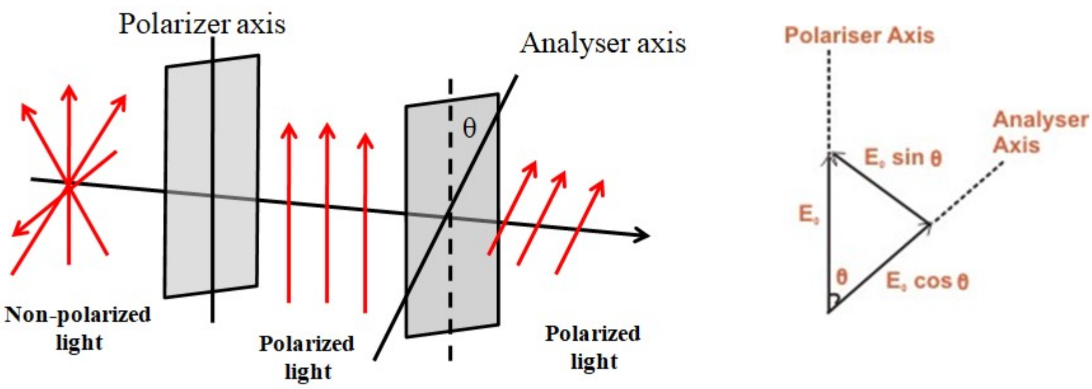


Figure 8.

Experimental observation

Turn on the laser and put really close the first polarizer and just after the second one (the analyser). Set both polarizer and analyzer to zero degree and observe the light spot on a screen or on the wall. Then rotate the axis of polarization of the analyser and observe the decrease of the intensity until the angle of the two axis reach 90° for which the spot light vanishes.

This method will be used in the next part to observe a signal with a CCD camera when studying interferences.

IV. Interferences obtained by a bi-slit: the Young slits experiment

Theoretical part

The interference pattern given by two ponctual sources separatd by a distance b in the plan parrallel to the axis of the sources is given by

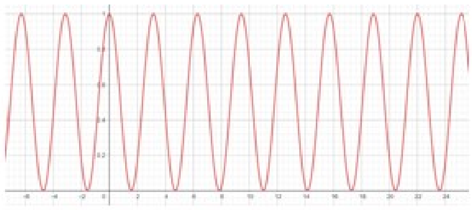
$$I = I_0\left(1 + \cos\frac{2\pi bx}{\lambda D}\right) = I_0\left(\cos\frac{2\pi bx}{\lambda D}\right)^2$$

Where D is the distance between the sources and the observation plane and where x is the distance compared to the central fringe in the observation plane. The distance between two maxima (or two minima) is called the interfringe whose expression is given by:

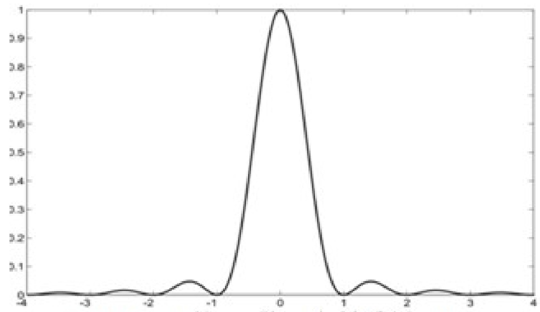
$$i = \frac{\lambda D}{b}$$

According to the expression of the intensity, the interference pattern should be seen from $x = -\propto$ to $x = +\propto$ while it is not the case when working with a finite opening. The transmitted light is limited by the diffraction phenomena. In the case of interferences created by two slits, the intensity of the interference pattern is modulated by the diffraction pattern related to the geometrical form of the slit. The total intensity observed on the screen is thus written as follows:

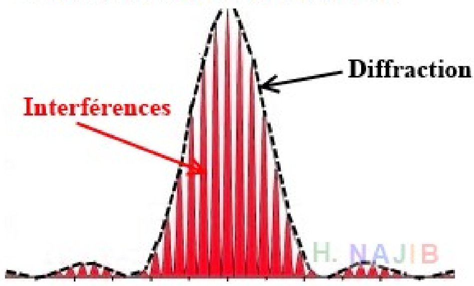
$$I = I_0\left(sinc\left(\frac{\pi xa}{\lambda D}\right)cos\left(\frac{\pi bx}{\lambda D}\right)\right)^2$$



Interference pattern of two ponctual sources



Diffraction pattern given by a slit



Interference pattern of two slits

Figure 9.

We can see on Figure 9. that the interference pattern is included into the diffraction pattern.

Experimental part

We will observe the interference pattern with the CCD camera.

- First take the double slit slide from the diffraction box kit (or multiple slit 2) and set it into an optical holder.
- Then perform the optical set-up by placing in the order: the laser, the polarizer, the analyser (closed to the laser) and then the double slit. Observe the interference pattern on the screen.

- The goal now is to add the CCD camera. To this end the interference pattern has to be projected onto the pixels of the CCD camera. This operation is not easy to perform since you have to align correctly all the optical elements.
- Observe the signal on the oscilloscope. This one is also sensitive to the perpendicular alignment of the slit and the camera with the optical axis. If the signal is saturated, you can increase the angle between the analyser and the polarizer to reduce the intensity of the signal. You can also put the closed box around the CCD camera to reduce the intensity due to parasite light.
- When you observe clean interferences on the oscilloscope screen, use the cursor mode to determine the interfringe. The oscilloscope will give you a temporal value that you will convert in distance with the conversion factor determined in III. The obtained value is the size of the intefringe in the CCD captor
- Then, deduce the value of the distance between the two slits.

V. Extra-manipulations

If you still have time you can perform one or the two following experiments. Just have fun !!

1) With the CCD camera, observe the diffraction pattern given by a hole and determine the size of its aperture using the method developed before.

2) With the CCD camera, check the Malus law when the light crosses the polarizer and then the analyser.