Midterm exam

Electrostatics (approx 14 points)

Infinite charged cylinder and cylindric capacitor

We consider an infinite cylinder of radius R and axis Δ and an orthonormal cylindric direct basis $(\overrightarrow{e_r}, \overrightarrow{e_\theta}, \overrightarrow{e_z})$ whose vector $\overrightarrow{e_z}$ is along Oz axis (fig 1). The cylinder surface can be set to an electric potential $V_1 = V(R)$ leading to the existence of a uniform surface density of charge σ (in C/m^2). We consider a point M whose distance to axis Δ is described by parameter r.

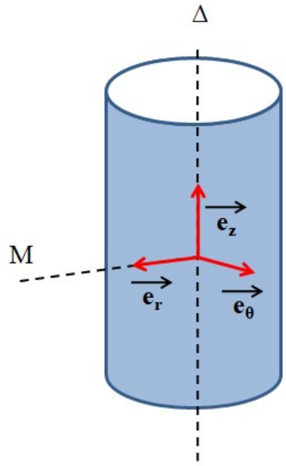

Fig 1

1) Use two symmetric points of the surface to justify that the electric field created at the point M by these two points is perpendicular to axis Δ. You can make a picture.

Calculation of electric field amplitude using Gauss theorem

2) Give the statement of Gauss theorem.

3) Use the Gauss theorem to determine the electric field amplitude **inside** the cylinder $E_{in}(r)$ for a distance $r < R$ by specifying clearly the Gauss volume (or the Gauss closed surface) you use.

4) Use the Gauss theorem to determine the electric field amplitude **outside** the cylinder $E_{out}(r)$ for a distance $r > R$ by specifying clearly the Gauss volume (or the Gauss closed surface) you use.

5) Draw the shape of the electric field amplitude $E(r)$.

6) Write the two relations that exist between the electric field $\overrightarrow{E(r)}$ and the electric potential $V(r)$.

7) Deduce the expression of the electric potential inside and outside the cylinder.

8) Draw the shape of the electric potential $V(r)$.

Application to cylindric capacitor

A cylindric capacitor is made of two coaxial cylinders of axis Δ and height h respectively of radius R_1 and R_2 (with $R_2 > R_1$) . The small cylinder is set at electric potential V_1 with surface charge density σ and the big cylinder is set at electric potential V_2. We assume that the electric field between the two cylinders is given by the following expression:

$$E(r) = \frac{\sigma R_1}{\varepsilon_0 r}$$

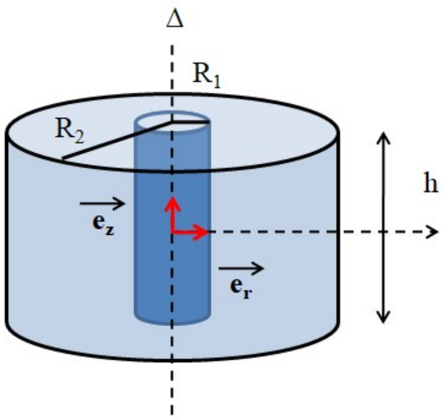

Fig 2

9) Deduce from the last equation the potential difference $\Delta V = V_2 - V_1$

10) Give the general relation between the capacitance C, the voltage difference ΔV and the electric charge Q.

11) Express ΔV as a function of the charge Q set on the small cylinder and show that the capacitance of the coaxial cylinder reads:

$$C = \frac{2\pi h \epsilon_0}{ln\left(\frac{R_1}{R_2}\right)}$$

12) We give $\epsilon_0 = 8.8 \times 10^{-12}$ F/m, $R_1 = 3$ mm, $R_2 = 6$ mm, $h = 1.8$ cm. Calculate the absolute value of the capacitance C.

Magnetostatics (approx 6 points)
Magnetic field created by a rotating electrostatic sphere

We consider a sphere of radius R electrically charged on its surface with uniform surface charge density σ. The sphere is rotating at angular velocity ω about an axis Oz crossing its center. The rotation of the static electric charge generates an electric current on the surface that creates a magnetic field. We will calculate the magnetic field at the center of the sphere.

1) We assume the rotation is about Oz and the sphere rotates through $\varphi > 0$. A point M located at the surface at distance R from O and angular position θ will describe a circle of radius $R \sin\theta$. The motion of the electric charges occur in a crown of surface $dS = 2\pi R^2 \sin\theta \, d\theta$ generating a loop of current (see Fig 4). The current can be written as the ratio of the electric charge in the crown to the period of revolution T

$$dI = \frac{\sigma dS}{T}$$

where σ is the uniform charge density of the sphere having a total charge Q on its surface $4\pi R^2$. Show that the current dI will be written

$$dI = \frac{Q\omega \sin\theta \, d\theta}{4\pi}$$

2) We remember the expression of the magnetic field dB created along the axis of a loop of current dI of radius a at a distance z from the loop. We introduce the angle θ with $a \tan\theta = z$

$$dB = \frac{\mu_0 dI}{2a} \sin^3\theta$$

We assume that the total current on the rotating sphere surface can be seen as a sum of elementary current loops having a radius equal to $R \sin\theta$. Consequently the total magnetic field $B(O)$ created at point O can be seen as the superposition of the elementary fields dB produced by all loops. Show that the expression of $B(O)$ is:

$$B(O) = \frac{\mu_0 Q\omega}{6\pi R}$$

You may use the result of the following integral

$$\int_0^{\pi} dx \sin^3 x = \frac{4}{3}$$

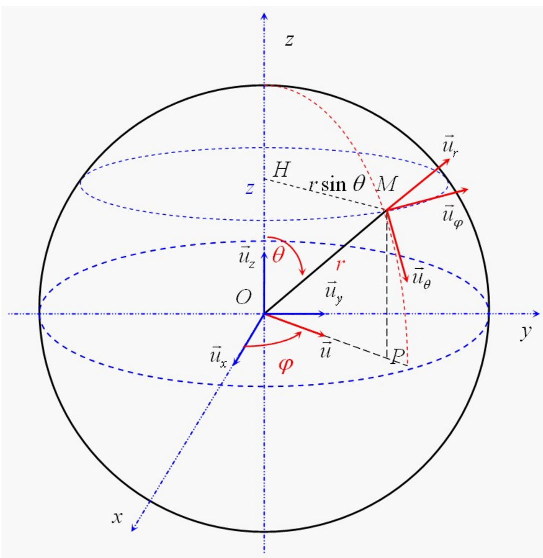

Fig 3

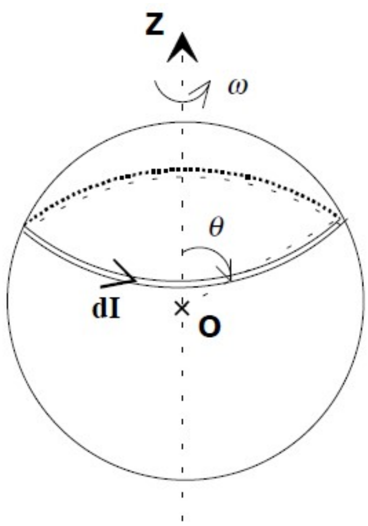

Fig 4