Report on Electrostatics-Magnetostatics exam

The report is organised as follows

1) Some comments are given regarding what we have seen in exam papers
2) We remember the exam subject
3) A correction is proposed

BEFORE APPLYING FOR EXAM CONSULATION, READ CAREFULY THE REMARKS AND THE CORRECTION. THEN, MAKE YOUR CHOICE. WE REMEMBER THAT IT IS HIGHLY RECOMMENDED NOT TO TRY TO NEGOCIATE FOR POINTS. GRADING HAS BEEN GENEROUS, YOU CAN ALSO LOSE POINTS DURING CONSULTATION.

Some remarks about Electrostatic exam

It was allowed to bring all types of document in the exam. We have seen too many badly written exams. You were lucky that we accept to correct all of them. Some teachers could have refused to correct some papers. Think that an exam is like a job interview, if we can't understand you, you will not be hired.

About questions

1) Usually this part done in tutorial was correct even the change of notation as z becoming y.
2) Often students start directly from the expression of the disc (seen in lecture but not asked in exam) and forgot to justify the case of R1.
3) There was no need to make demonstration of Gauss theorem. Obviously no points were given from this especially if students had their notes.
4) The students that calculate the electric field created by a cylinder didn't got any points of course. When doing correctly for the plan often students forgot to justify the relation between surface S,S1 and S2….

5) In that case, the positive plate was in the y-positive part of the y-axis and the negative plate in the negative part of y-axis (opposite case compared to the lecture), consequently the field should have a negative sign.
6) In general answers made by students were catastrophic. They have calculated Va and Vb by using the notation of the tutorial (purely recopying) WHILE here other notations where used. Even if the calculation were correct, no points have been given if the final expression that do not contains E_0 and D. We have accepted some points only when students were able to define the E_0 of the statement with the notations of tutorial.

No special remarks for **7) 8) 9) and 10) 11) 12).** Except no unit, no points

13) Recopying the Taylor Development of the lecture was not a good idea. No sense because other development was given.
14) same remark than **13)**
15) and 16) No unit, no points.

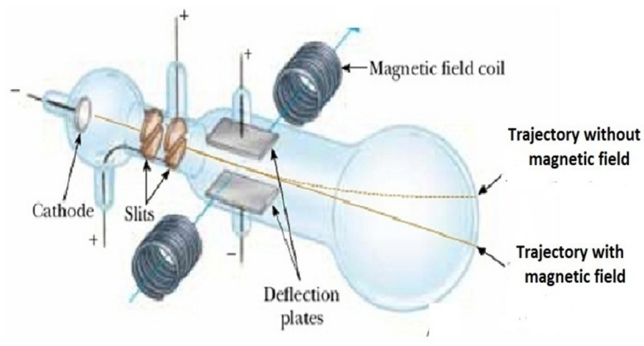
Final Exam: Electrostatics- Magnetostatics- 1.5 hour

Paper documents and calculators are allowed- Computers are forbidden

In 1897 J.J. Thomson managed to measure the charge-to-mass ratio of the electron e/m. He build an apparatus (Fig 1) where the negative particles produced by a cathode with a velocity v_i along Oz were first accelerated and deviated by an electric field $\overrightarrow{E_0}$ over a distance L. The electric field along Oy was produced by two charged rectangular plates (Fig 2). To calculate e/m it is possible to measure the angle of deviation θ and to use the theoretical expression:

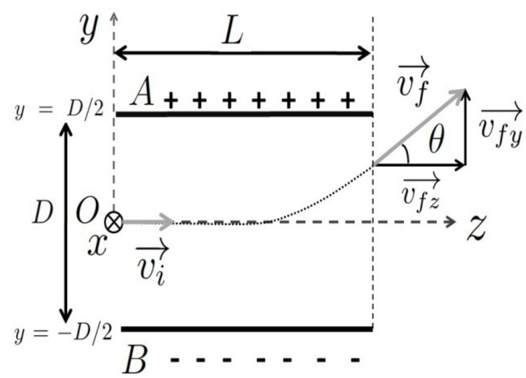
$$\tan\theta = \frac{e}{m} E_0 \frac{L}{v_i^2}.$$

In the above formula, E_0 and L can be determined, but the initial velocity of the electron v_i is unknown. To determine the value of v_i , Thompson added a magnetic field in the Ox direction in order to compensate the deviation due to the electric field. Choosing the appropriate amplitude of the magnetic field one can balance the electric and the magnetic force as: $e\overrightarrow{E_0} + e\overrightarrow{v_i} \times \overrightarrow{B_0} = \vec{0}$. After projection, it leads to $v_i = E_0/B_0$ which permits to calculate e/m. In that experiment, it is important to produce uniform and constant electric and magnetic field.



Thomson's apparatus

Fig 1.



Scheme of Deflection plates without magnetic field

Fig 2.

Electrostatics: How to produce a uniform electric field?

We consider a crown of internal radius R_1 and external radius R_2 (Fig 3.). It has a uniform surface charge density σ so that an infinitesimal surface element $dS = r\,dr\,d\theta$ located at distance r from O has a charge $dq = \sigma dS$.

1) Show that the electric field created by a crown along the axis *Oy* going through the center *O* at a distance y from *O* is given by:

$$\vec{E}(y) = \frac{\sigma y}{2\varepsilon_0}\left(\frac{1}{\sqrt{R_1^2 + y^2}} - \frac{1}{\sqrt{R_2^2 + y^2}}\right)\overrightarrow{e_y}.$$

2) Explain how to extrapolate to the case of an infinite plane. Show that we obtain the following expression for the electric field

$$\vec{E}(y) = \frac{\sigma}{2\varepsilon_0}\overrightarrow{e_y}.$$

3) Give the statement of Gauss theorem.

4) Recover the results of question **2)** by applying Gauss theorem in the case of an infinite plane.

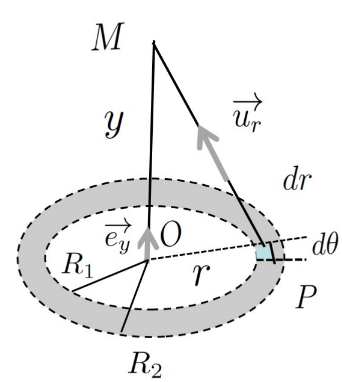


Fig 3.

We consider now the two charged metallic plates in the Oxz plan (Fig 2). The plate A located at $y = D/2$ has a positive electrostatic potential V_A and an electric surface density $\sigma > 0$. The plate B located at $y = -D/2$ has a negative electrostatic potential V_B and an electric surface density $\sigma < 0$. The mathematical expression of the electrostatic potential V between the two plates is given by $V = V(y) = E_0\, y$ where y is the y-position and where E_0 is the amplitude of the electric field.

5) With the result of question **2)**, explain why the electric field is uniform and constant between the two plates. Give its expression.
6) Calculate the potential V_A for $y = D/2$ and the potential V_B for $y = -D/2$. Express the difference of potential (voltage) $\Delta V = V_A - V_B$ as a function of D and E_0.
7) What is the amplitude of the electric field E_0 for a voltage ΔV =1600 V and a distance D=0.04 m?
8) What is the direction of the electric field vector $\vec{E}$ between the two plates?
9) Deduce the value of the electric charge density σ.

Magnetostatics: How to produce a uniform magnetic field?

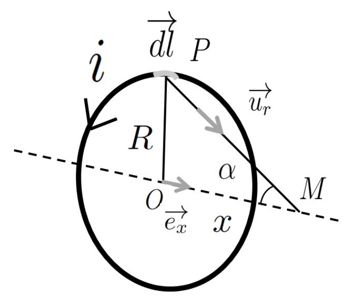


Fig 4.

10) Give the general expression of Biot and Savart law.
11) We consider a loop of current (a coil) of radius R whose axis is parallel to Ox axis (Fig 4). Show that the magnetic field created by the current i onto Ox axis at distance x is given by

$$\vec{B}(x) = \frac{\mu_0\, i}{2}\frac{R^2}{(R^2+x^2)^{\frac{3}{2}}}\vec{e_x}.$$

12) The amplitude of the magnetic field is given by $B(x)$. Show that we can write $B(x) = B(0)\left(1+\frac{x^2}{R^2}\right)^{\alpha}$ and give the expression of $B(0)$ and α. Give the expression of $B(\pm R/2)$ and $B(\pm R)$.
13) Calculate the second-order derivative $B''(x)$ and show its value is zero for $x=\pm R/2$. Then draw the shape of the graph of $B(x)$.

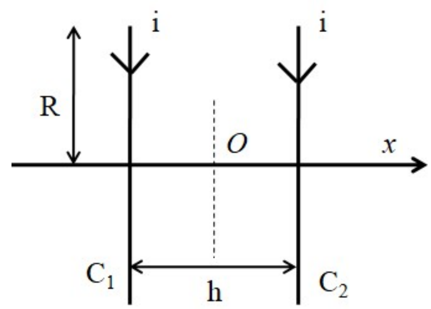
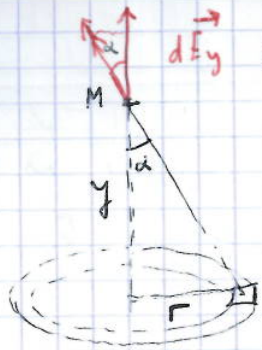


Fig 5.

To obtain a uniform magnetic field, one can use Helmholtz coils (Fig 5). That device is composed of two coils C_1 and C_2 separated by a distance h and located respectively at distance $-h/2$ and $h/2$. The magnetic fields produced by C_1 and C_2 can be written as $B_1(x) = B(x-h/2)$ and $B_2(x) = B(x+h/2)$ where $B(x)$ function is given in question **11)** or **12)**. Using a Taylor development at second order, it is possible to show that the total field onto the x-axis is given by:

$$B_1(x) + B_2(x) = 2B(h/2) + x^2 B''(h/2) + o(x^3).$$

14) What should be the distance h between the two coils C_1 and C_2 to make the second order in the previous formula equal to zero? Calculate the value of the field at O between C_1 and C_2 and compare to the one in the center of each coils. Comments.
15) In Thompson experiment the magnetic field is taken to $B_0 = 6.24\times10^{-5}\times\frac{i}{R}$ with i=1.73 A and R=10 cm. Using the results of **7)** and knowing that L=8 cm and θ=23.1°, deduce the experimental value of $(e/m)_{exp}$.
16) Compare to the tabulated $(e/m)_{tab}$ value knowing that $e = 1.6\times10^{-19}$ C and $m = 9.11\times10^{-31}$kg.

1) 

$$\vec{dE_y} = \frac{dq}{4\pi\varepsilon_0 PM^2}\cos\alpha\,\vec{e_y}$$

$$PM^2 = r^2 + y^2$$

$$\cos\alpha = \frac{y}{\sqrt{r^2+y^2}}$$

$$dq = \sigma\, dS = \sigma\, r\,dr\,d\theta$$

$$\vec{dE_y} = \frac{\sigma\, r\,dr\,d\theta\; y}{4\pi\varepsilon_0\,(r^2+y^2)^{3/2}}\,\vec{e_y}$$

$$\vec{E} = \int_{r=R_1}^{R_2}\int_{\theta=0}^{2\pi}\frac{\sigma\, r\,dr\,d\theta\; y}{4\pi\varepsilon_0\,(r^2+y^2)^{3/2}}\,\vec{e_y}$$

$$= \frac{\sigma y}{4\pi\varepsilon_0}\int_0^{2\pi} d\theta\int_{R_1}^{R_2}\frac{r\,dr}{(r^2+y^2)^{3/2}}\,\vec{e_y}$$

$$= \frac{\sigma y}{4\pi\varepsilon_0}\cdot\left[\theta\right]_0^{2\pi}\left[\frac{-1}{\sqrt{r^2+y^2}}\right]_{R_1}^{R_2}\vec{e_y}$$

$$= \frac{\sigma y}{2\cancel{4\pi}\varepsilon_0}\cdot\cancel{2\pi}\cdot\left[\frac{1}{\sqrt{R_1^2+y^2}} - \frac{1}{\sqrt{R_2^2+y^2}}\right]\vec{e_y}$$

2) Infinite plane: $R_1 \to 0$ and $R_2 \to \infty$

$$\frac{1}{\sqrt{R_1^2+y^2}} \to \frac{1}{\sqrt{y^2}} = \frac{1}{y}$$

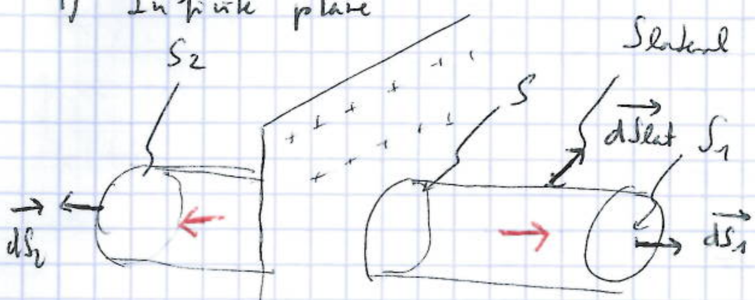
$$\frac{1}{\sqrt{R_2^2+y^2}} \to \frac{1}{\infty} \to 0.$$

and $\vec{E(y)} = \frac{\sigma y}{2\varepsilon_0}\cdot\frac{1}{y}\,\vec{e_y} = \frac{\sigma}{2\varepsilon_0}\,\vec{e_y}.$

3) $$\oiint \vec{E}\cdot\vec{dS} = \frac{Q_{int}}{\varepsilon_0}$$

The flux of electric field through a closed surface is equal to the electric charge internal to the surface limited by ε_0.

4) Infinite plane



$$\oiint \vec{E}\cdot\vec{dS} = \iint \vec{E}\cdot\vec{dS_1} + \iint \vec{E}\cdot\vec{dS_2} + \iint_{lateral} \vec{E}\cdot\vec{dS}$$
$$= E.S_1 + E.S_2 + 0$$

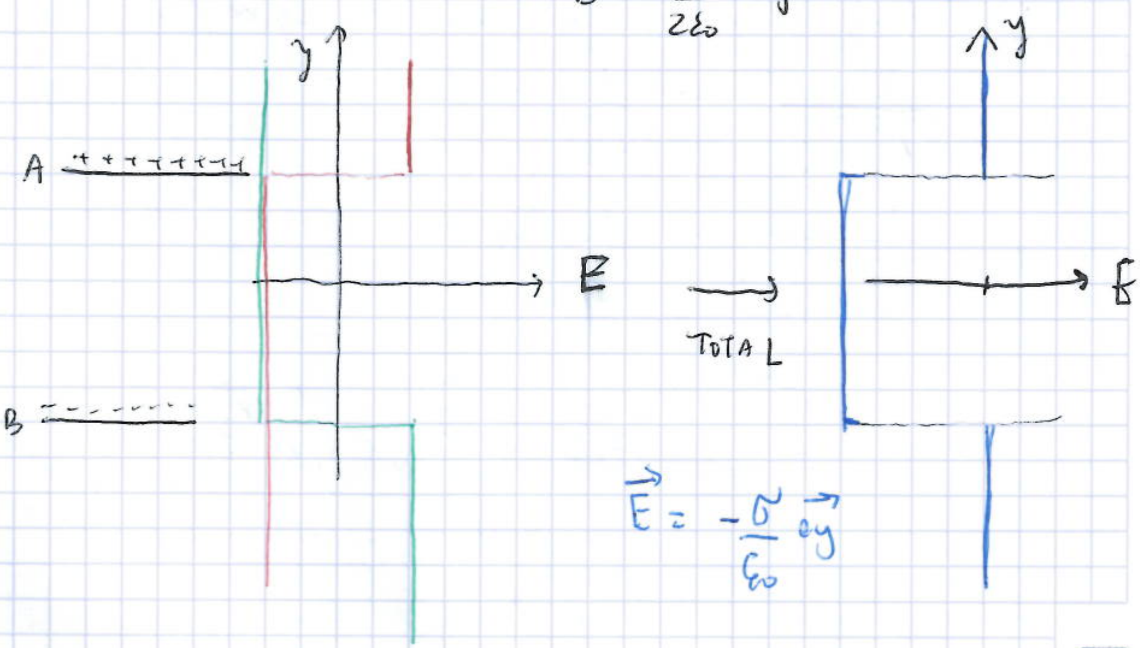
$$\frac{Q_{int}}{\varepsilon_0} = \frac{\sigma S}{\varepsilon_0}$$

and $S_1 = S_2 = S$

$$2E = \frac{\sigma}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

so $\vec{E} = \frac{\sigma}{2\varepsilon_0}\vec{e_y}$

5)



$\vec{E} = -\frac{\sigma}{\varepsilon_0}\vec{e_y}$

6) $V(y) = E_0 y$

$$\left.\begin{aligned} V\left(\frac{D}{2}\right) &= E_0 \frac{D}{2} = V_A \\ V\left(-\frac{D}{2}\right) &= -E_0 \frac{D}{2} = V_B \end{aligned}\right\} \quad \Delta V = V_A - V_B = E_0 \frac{D}{2} - E_0\left(-\frac{D}{2}\right) = E_0 D$$

7) $E_0 = \frac{\Delta V}{D} = \frac{1600}{0{,}04} = \frac{400}{0{,}01} = 40\,000 \text{ V/m}$

8) From $\oplus$ to $\ominus$.

9) $\|E\| = \frac{\sigma}{\varepsilon_0} \Rightarrow \sigma = E \times \varepsilon_0$

$= 40.10^{3} \times 8{,}85\ 10^{-12}$

$= 3{,}54\ 10^{-7}\ C/m^2.$

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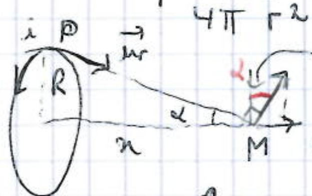
10) $d\vec{B} = \frac{\mu_0 i\ \vec{dl} \times \vec{u_r}}{4\pi r^2}$

11) 

$r = PM = \sqrt{R^2 + x^2}$ $\quad \sin\alpha = \frac{R}{\sqrt{R^2+x^2}}$

$\vec{dB_x} = \frac{\mu_0 i\ dl}{4\pi\ PM^2} \sin\alpha\ \vec{e_x} = \frac{\mu_0 i}{4\pi} \frac{R}{(R^2+x^2)^{3/2}} \vec{e_x}$

$\vec{B} = \oint dl \times \frac{\mu_0 i}{4\pi} \frac{R}{(R^2+x^2)^{3/2}} \vec{e_x}$

$= \frac{2\pi R\ \mu_0 i}{4\pi} \frac{R}{(R^2+x^2)^{3/2}} \vec{e_x}$

$= \frac{\mu_0 i}{2} \frac{R^2}{(R^2+x^2)^{3/2}} \vec{e_x}$

12) $B(x) = \frac{\mu_0 i R^2}{2\ R^{2\,3/2}} \frac{1}{\left(1 + \frac{x^2}{R^2}\right)^{3/2}} = \frac{\mu_0 i}{2R}\left(1 + \frac{x^2}{R^2}\right)^{-3/2}$

$= B(0)\left(1 + \frac{x^2}{R^2}\right)^{\alpha}$ $\qquad B(0) = \frac{\mu_0 i}{2R}$ $\quad \alpha = -3/2$

$\cdot\ B\left(\pm\frac{R}{2}\right) = B(0)\left(1+\left(\frac{R}{2R}\right)^2\right)^{-3/2} = B(0)\left(1+\frac{1}{4}\right)^{-3/2}$

$$= B(0)\left(\frac{5}{4}\right)^{-3/2} \simeq 0{,}72\,B(0)$$

$\cdot\ B(\pm R) = B(0)\left(1+\left(\frac{R}{R}\right)^2\right)^{-3/2} = B(0)\,2^{-3/2} \approx 0{,}36\,B(0).$

13) $B'(x) = B(0)\left(-\frac{3}{2}\right)\times\frac{2x}{R^2}\times\left(1+\frac{x^2}{R^2}\right)^{-5/2}$

$B''(x) = B(0)\left(-\frac{3}{2}\right)\left[\frac{2}{R^2}\left(1+\frac{x^2}{R^2}\right)^{-5/2} + \frac{2x}{R^2}\left(-\frac{5}{2}\right)\left(\frac{2x}{R^2}\right)\left(1+\frac{x^2}{R^2}\right)^{-7/2}\right]$

$$= B(0)\left(-\frac{3}{2}\right)\frac{2}{R^2}\left(1+\frac{x^2}{R^2}\right)^{-5/2}\left[1-\frac{5x^2}{R^2}\left(1+\frac{x^2}{R^2}\right)^{-1}\right]$$

$B''(x) = 0$

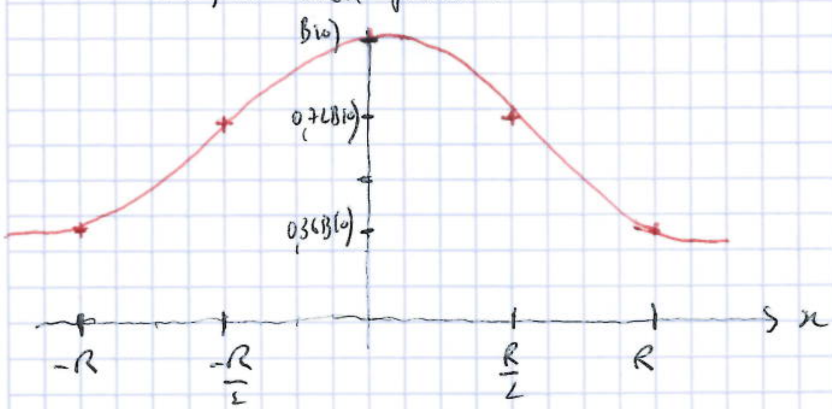
if $1 - 5\frac{x^2}{R^2}\left(1+\frac{x^2}{R^2}\right)^{-1} = 0$

$\Leftrightarrow\ 5x^2 = R^2\left(1+\frac{x^2}{R^2}\right) = R^2 + x^2$

$\Leftrightarrow\ 4x^2 = R^2$

$$\boxed{x = \pm\frac{R}{2}}$$

$B(x)$ is even function



14) $B_1(x) + B_2(x) = 2B\left(\frac{h}{2}\right) + x^2 B''\left(\frac{h}{2}\right) + o(x^3)$

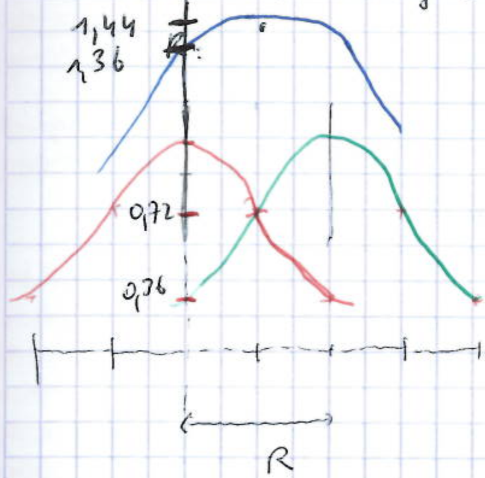
if we put $h = \frac{R}{2}$ we have

$$B_{TOTAL}(x) = 2B\left(\frac{R}{2}\right) + x^2\underbrace{B''\left(\frac{R}{2}\right)}_{0\ !!} + o(x^3)$$

. total field is a constant and so

$$B_{TOTAL} = 2B\left(\frac{R}{2}\right) = 2B(0).\left(\frac{5}{4}\right)^{-3/2} \approx 1{,}44\,B(0)$$

. In the center of a coil we have; since coils are distant from R



B_1 at C_1 and B_2 at C_2

$$B_1 = B(0) + B(R)$$
$$= B(0) + 0{,}36\,B(0) = 1{,}36\,B(0)$$

It makes a difference of:

$$\frac{|1{,}44 - 1{,}36|}{1{,}44} = 0{,}05 \approx 5\%.$$

The magnetic field is almost uniform.

15). $$\underline{B_0} = 6{,}24 \times 10^{-5} \times \frac{i}{R} = 6{,}24 \times 10^{-5} \times \frac{1{,}73}{0{,}1} = \underline{1{,}080\,mT} = 1{,}08\,10^{-3}\,T$$

$$\tan\theta = \frac{e}{m} E_0 . \frac{L}{v_i^2} = \frac{e}{m} . L . \frac{B_0^2}{E_0}$$

and $E_0 = v_i B_0.$
$v_i = \frac{E_0}{B_0.}$

$$\frac{e}{m} = \frac{\tan\theta\, E_0}{L\, B_0^2} = \frac{\tan 23{,}1 \times 40.10^3}{0{,}08 \times (1{,}08\,10^{-3})^2}$$

with $E_0 = 40.10^3\,V$ $= 1{,}92\,10^{11}\,C/kg.$

$$\left(\frac{e}{m}\right)_{exp} = 1{,}92\,10^{11}\,C/kg.$$

16) $$\left(\frac{e}{m}\right)_{Lab} = \frac{1{,}6\,10^{-19}}{9{,}11 . 10^{-31}} = 1{,}75 . 10^{11}\,C/kg.$$

$$\sigma\% = \frac{|1{,}82 - 1{,}75|}{1{,}75} \times 100 = 4\%.$$