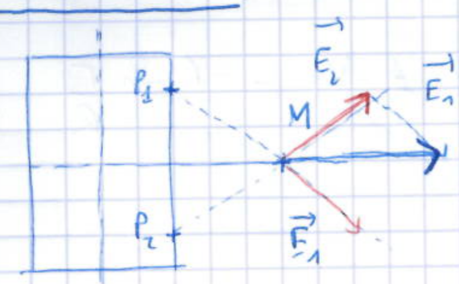
Exercise 1:

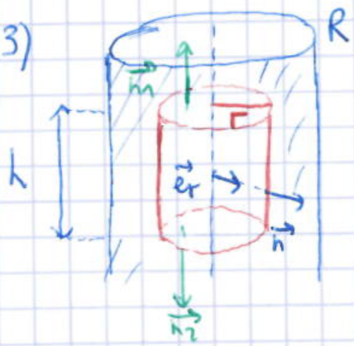
1)



$\vec{E} = \vec{E_1} + \vec{E_2}$ is along $\vec{e_r}$:

2) $$\oiint \vec{E}\cdot\vec{dS} = \frac{Q_{int}}{\varepsilon_0}$$

3)



$\boxed{r < R}$ $\vec{E} \parallel \vec{e_r}$ and $\perp(\vec{m}, \vec{n_2})$

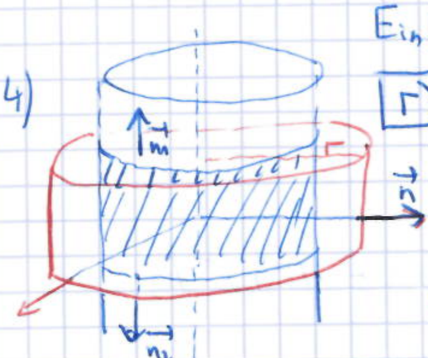
$$\iint \vec{E}\cdot\vec{dS_1} + \iint \vec{E_2}\cdot\vec{dS_2} + \iint \vec{E}(r)\cdot dS\vec{n} = \frac{Q_{int}}{\varepsilon_0}$$

$$E(r)\,2\pi r h = \frac{Q_{int}}{\varepsilon_0}$$

But cylinder has charges on surface so $Q_{int} = 0$

$E_{in}(r) = 0$.

4)



$\boxed{r > R}$

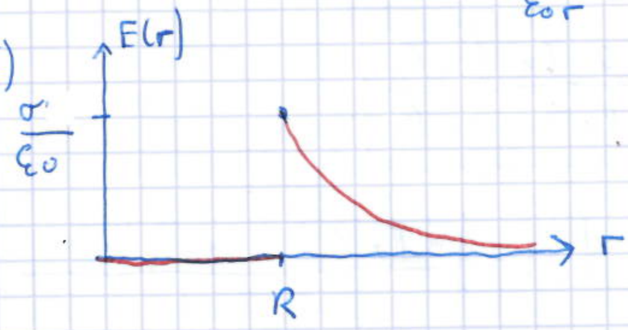
$$\iint \vec{E}\cdot dS\vec{n} + 0 + 0 = \frac{Q_{int}}{\varepsilon_0}$$

$$E(r)\,2\pi r h = \frac{Q_{int}}{\varepsilon_0}$$

$$Q_{int} = \sigma S = \sigma\, 2\pi R h$$

$$E(r) = \frac{\sigma R}{\varepsilon_0 r}$$

5)



6) $\vec{E}(r) = -\vec{\mathrm{grad}}\, V(r)$

$$V(r) = -\int \vec{E}(r)\cdot\vec{dr}$$

7) $r < R$: $V = \text{constante} = V(R) = V_1$

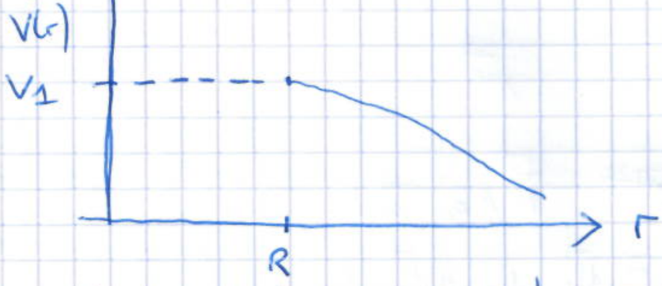
$r > R$ $V(r) = -\int \frac{\sigma R}{\varepsilon_0 r}\vec{e_r}\cdot dr\,\vec{e_r} = -\frac{\sigma R}{\varepsilon_0}\ln r + c^{te}$

and with boundaries $V(R) = V_1 = -\frac{\sigma R}{\varepsilon_0}\ln R + c^{te} \Rightarrow$

$$\Rightarrow cte = V_1 + \frac{\sigma R}{\varepsilon_0} \ln R$$

$$\text{and } \underset{r > R}{V(r)} = \frac{\sigma R}{\varepsilon_0} \ln \frac{R}{r} + V_1 = V_1 - \frac{\sigma R}{\varepsilon_0} \ln \frac{r}{R}$$

8)



<u>Application to cylindric capacitor</u>

9) $$\left[V\right]_{V_1}^{V_2} = -\int_{R_1}^{R_2} \vec{E} \cdot \vec{dr} = -\int_{R_1}^{R_2} \frac{\sigma R_1}{\varepsilon_0 r} \vec{e_r} \cdot dr \vec{e_r}$$

$$\left[V\right]_{V_1}^{V_2} = -\frac{\sigma R_1}{\varepsilon_0} \ln \frac{R_2}{R_1} = V_2 - V_1$$

$$V_2 - V_1 = \boxed{\frac{\sigma R_1}{\varepsilon_0} \ln \frac{R_1}{R_2}}$$

10) $$\Delta V = \frac{Q}{C}$$

11) $$\underset{\text{cylinder 1}}{Q = \sigma S} = \sigma h 2\pi R_1$$

and so:

$$\frac{\sigma R_1}{\varepsilon_0} \ln \frac{R_1}{R_2} = \boxed{\frac{Q \cancel{R_1}}{h 2\pi \cancel{R_1} \varepsilon_0} \ln \frac{R_1}{R_2}} = \Delta V = \frac{Q}{C}$$

$$C = \frac{2\pi h \varepsilon_0}{\ln\left(\frac{R_1}{R_2}\right)}$$

12) $$|C| = \left| \frac{2\pi \times 1,8\, 10^{-2} \times 8,8 \times 10^{-12}}{\ln\left[\frac{3}{6}\right]} \right|$$

$$= 1,44\ 10^{-12}\ F$$

$$= \underline{1,44\ pF}$$

Exercice 2

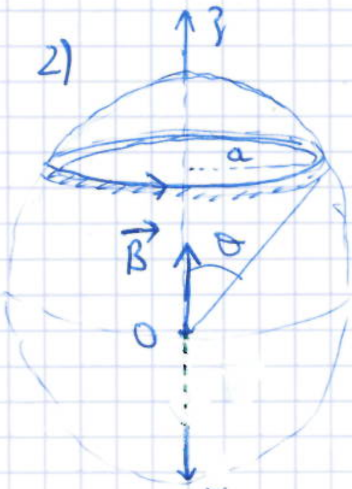
1) $dI = \frac{\sigma\, dS}{T} =$

$\sigma = \frac{Q}{4\pi R^2}$; $dS = 2\pi R^2 \sin\theta\, d\theta$; $T = \frac{2\pi}{\omega}$

$$dI = \frac{Q}{4\pi \cancel{R^2}} \times \cancel{2\pi} \cancel{R^2} \sin\theta\, d\theta \times \frac{\omega}{\cancel{2\pi}}$$

$$dI = \frac{Q\omega \sin\theta\, d\theta}{4\pi}$$

2)



$$d\vec{B} = \frac{\mu_0\, dI}{2a} \sin^3\theta\, \vec{u_z}$$ field created by the loop of radius $a = R\sin\theta$

$$d\vec{B} = \frac{\mu_0}{2R\cancel{\sin\theta}} \times \frac{Q\omega \cancel{\sin\theta}}{4\pi}\, d\theta \times \sin^3\theta\, \vec{u_z}$$

$$dB = \frac{\mu_0 Q\omega \sin^3\theta}{2R \cdot 4\pi}\, d\theta$$

To have contribution of all loops, we integrate

$$B(0) = \int_{\theta=0}^{\pi} \frac{\mu_0 Q\omega}{2R} \frac{\sin^3\theta\, d\theta}{4\pi}$$

$$= \frac{\mu_0 Q\omega}{2R \cdot 4\pi} \times \underbrace{\int_0^{\pi} \sin^3\theta\, d\theta}_{4/3} = \frac{\mu_0 Q\omega}{2R\,\pi\, 3}$$

$$B(0) = \frac{\mu_0 Q\omega}{6\pi R}$$