$\boxed{\text{INVERSE CAMERA RESPONSE FUNCTION}}$

$y = f(x)$ MODEL AS POLYNOMIAL WITH N COEFFICIENTS

$$y = c_N x^N + c_{N-1} x^{N-1} + \dots + c_1 x + c_0 = \sum_{n=0}^{N} c_n x^n$$

$1 = f(1)$ CONSTRAINT

$$1 = c_N + c_{N-1} + \cdots + c_1 + c_0 = \sum_{n=0}^{N} c_n = c_N + \sum_{n=0}^{N-1} c_n$$

$$\boxed{c_N = 1 - \sum_{n=0}^{N-1} c_n}$$

$0 = f(0)$ CONSTRAINT

$$0 = c_0 x^0 \qquad (x^0 = 1)$$

$$\boxed{c_0 = 0}$$

GIVEN TWO EXPOSURES e AND e' AND ASSOCIATED CAMERA RESPONSE VALUES x AND x'

$$e = f(x) = \sum_{n=0}^{N} c_n x^n \quad \text{AND} \quad e' = f(x') = \sum_{n=0}^{N} c_n x'^n$$

SOLVE FOR $c_{N-1}, \dots, c_0$

$$1 = 1/e \sum_{n=0}^{N} c_n x^n \quad \text{AND} \quad 1 = 1/e' \sum_{n=0}^{N} c_n x'^n$$

$$0 = 1/e \sum_{n=0}^{N} c_n x^n - 1/e' \sum_{n=0}^{N} c_n x'^n \qquad \text{(SQUARE OF THIS IS ERROR)}$$

$$0 = 1/e \left(c_N x^N + \sum_{n=0}^{N-1} c_n x^n\right) - 1/e' \left(c_N x'^N + \sum_{n=0}^{N-1} c_n x'^n\right)$$

$$0 = 1/e \left(\left(1 - \sum_{n=0}^{N-1} c_n\right) x^N + \sum_{n=0}^{N-1} c_n x^n\right) - 1/e' \left(\left(1 - \sum_{n=0}^{N-1} c_n\right) x'^N + \sum_{n=0}^{N-1} c_n x'^n\right)$$

$$0 = 1/e \left(x^N - \sum_{n=0}^{N-1} c_n x^N + \sum_{n=0}^{N-1} c_n x^n\right) - 1/e' \left(x'^N - \sum_{n=0}^{N-1} c_n x'^N + \sum_{n=0}^{N-1} c_n x'^n\right)$$

$$0 = x^N/e + 1/e \sum_{n=0}^{N-1} c_n (x^n - x^N) - x'^N/e' - 1/e' \sum_{n=0}^{N-1} c_n (x'^n - x'^N)$$

$$1/e \sum_{n=0}^{N-1} c_n (x^n - x^N) - 1/e' \sum_{n=0}^{N-1} c_n (x'^n - x'^N) = x'^N/e' - x^N/e$$

$$\sum_{n=0}^{N-1} c_n \left(1/e\,(x^n - x^N) - 1/e'\,(x'^n - x'^N)\right) = x'^N/e' - x^N/e$$

$$\left(1/e\,(x^{N-1} - x^N) - 1/e'\,(x'^{N-1} - x'^N), \dots, 1/e\,(x^1 - x^N) - 1/e'\,(x'^1 - x'^N)\right) \begin{bmatrix} c_{N-1} \\ \vdots \\ c_1 \end{bmatrix} = x'^N/e' - x^N/e$$

(RECALL $c_0 = 0$)

$$\left(\frac{1}{e}\left(\underline{v}^T - x^N\right) - \frac{1}{e'}\left(\underline{v}'^T - x'^N\right)\right) \begin{bmatrix} c_{N-1} \\ \vdots \\ c_1 \end{bmatrix} = \frac{1}{e'} x'^N - \frac{1}{e} x^N,$$

WHERE $\underline{v} = (x^{N-1}, \dots, x^2, x)^T$ AND $\underline{v}' = (x'^{N-1}, \dots, x'^2, x')^T$

$$\boxed{\begin{aligned} \underline{a}^T \underline{x} &= b \\ (\underline{a}\,\underline{a}^T)\,\underline{x} &= \underline{a}\; b \\ \left(\sum_i \underline{a}_i \underline{a}_i^T\right) \underline{x} &= \sum_i \underline{a}_i b_i \end{aligned}}$$

HANDLING COLOR

$y_i = f_i(x_i)$ TO SCALE, EACH CHANNEL i IS INDEPENDENT OF OTHER CHANNELS

m CHANNELS $\underline{x} = (x_1, \ldots, x_m)^T$ CAMERA RESPONSE VALUES

$\underline{y} = (y_1, \ldots, y_m)^T$ SCALED RADIANCE VALUES

ASSUME RESPONSE FUNCTIONS PRESERVE CHROMATICITY

COLOR CORRECTED SCALED RADIANCE VALUES $\hat{\underline{y}} = (k_1 y_1, \ldots, k_m y_m)^T$

SOLVE FOR $k_1, \ldots, k_m$ SUCH THAT

$$\frac{\hat{\underline{y}}}{\|\hat{\underline{y}}\|} = \frac{\underline{x}}{\|\underline{x}\|}$$

NOTE $\|\underline{x}\| = (x_1^2 + \cdots + x_m^2)^{1/2}$

$\|\hat{\underline{y}}\| = (\hat{y}_1^2 + \cdots + \hat{y}_m^2)^{1/2} = (k_1^2 y_1^2 + \cdots + k_m^2 \hat{y}_m^2)^{1/2}$

$$\frac{\hat{y}_i}{\|\hat{\underline{y}}\|} = \frac{x_i}{\|\underline{x}\|} \quad \forall i, \text{ SOLVE FOR } k_1, \ldots, k_m$$

$$\frac{\hat{y}_i^2}{\|\hat{\underline{y}}\|^2} = \frac{x_i^2}{\|\underline{x}\|^2}$$

NOTE $\|\underline{x}\|^2 = x_1^2 + \cdots + x_i^2 + \cdots + x_m = \sum_i x_i^2$

$\|\hat{\underline{y}}\|^2 = k_1^2 y_1^2 + \cdots + k_i^2 y_i^2 + \cdots + k_m^2 y_m^2$

$$\hat{y}_i^2 \|\underline{x}\|^2 = x_i^2 \|\hat{\underline{y}}\|^2$$

$$-\hat{y}_i^2 \|\underline{x}\|^2 + x_i^2 \|\hat{\underline{y}}\|^2 = 0$$

$$-k_i^2 y_i^2 (x_1^2 + \cdots + x_i^2 + \cdots + x_m^2) + x_i^2 (k_1^2 y_1^2 + \cdots + k_i^2 y_i^2 + \cdots + k_m^2 y_m^2) = 0$$

$$-k_i^2 y_i^2 x_1^2 - \cdots - k_i^2 y_i^2 x_i^2 - \cdots - k_i^2 y_i^2 x_m^2 + x_i^2 k_1^2 y_1^2 + \cdots + x_i^2 k_i^2 y_i^2 + \cdots + x_i^2 k_m^2 y_m^2 = 0$$

$$\underbrace{\left(\mathrm{diag}(-\|\underline{x}\|^2 (\underline{y} \circ \underline{y})) + (\underline{x} \circ \underline{x})(\underline{y} \circ \underline{y})^T\right)}_{m \times m} \begin{bmatrix} k_1^2 \\ \vdots \\ k_m^2 \end{bmatrix} = \underline{0}$$

HADAMARD PRODUCT

(ELEMENT WISE MULTIPLICATION)

SOLVE FOR $(k_1^2, \ldots, k_m^2)^T$ (TO SCALE)

SHOULD ALL BE POSITIVE NUMBERS

$$\underline{A}\,\underline{x} = \underline{0}$$

$$\underline{A}^T \underline{A}\,\underline{x} = \underline{0}$$

$$\left(\sum_i \underline{A}_i^T \underline{A}_i\right) \underline{x} = \underline{0}$$

$\boxed{\text{DO NOT INCLUDE UNDEREXPOSED OR OVEREXPOSED PIXEL VALUES IN ANY ESTIMATIONS.}}$

$$LIMIT_{MIN} = \frac{2^{b_n} - 1}{2^b - 1} \quad \text{AND} \quad LIMIT_{MAX} = \frac{2^b - 2^{b_n}}{2^b - 1} = 1 - LIMIT_{MIN}$$

WHERE b IS THE NUMBER OF BITS USED TO REPRESENT THE MEASUREMENT
b_n IS THE NUMBER OF THOSE BITS USED TO CHARACTERIZE THE NOISE

TYPICALLY, USE 2-3 BITS OF 8 BITS TO CHARACTERIZE THE NOISE
(I.E., $LIMIT_{MIN} = 0.02$)