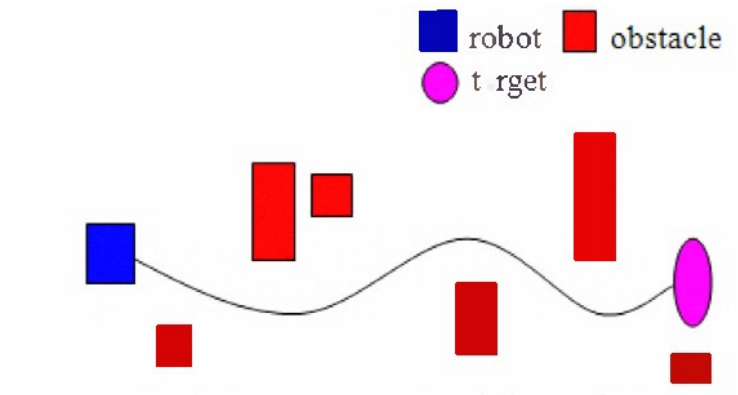


# Path following for mobile robots

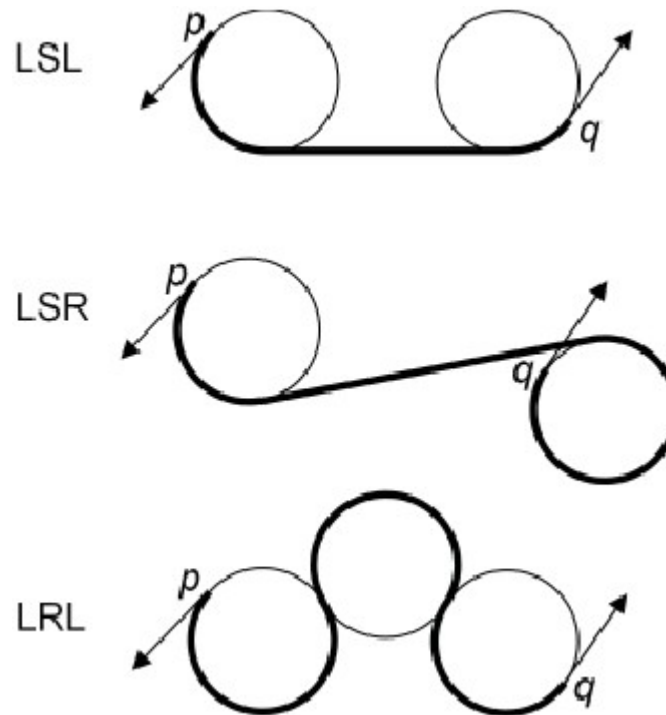
## Guiding Vector Fields (GVF)

Given a desired path, how does a robot follow it?



# Following straight lines and circles

Dubin's path consists of a sequence of lines and circles



# Implicit equations of the path

Circle  $\rightarrow f(x,y) := (x-x_c)^2 + (y-y_c)^2 + r^2 = 0$

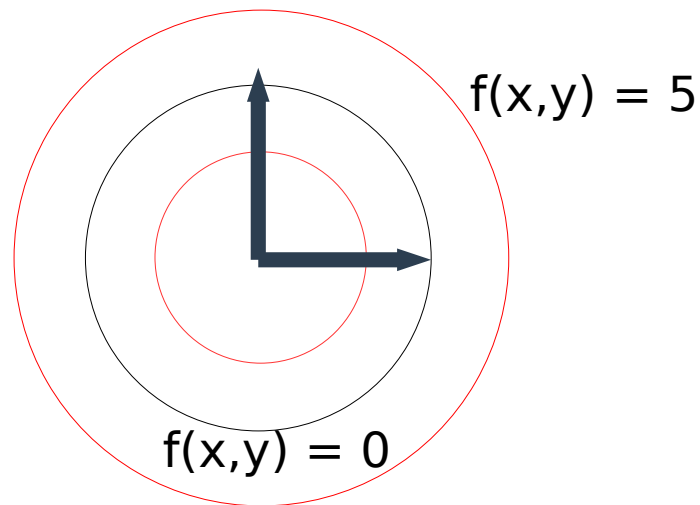
Straight line  $\rightarrow f(x,y) := ax + by + c = 0$

# Level sets

$$x^2 + y^2 - r^2 = 5 \quad \text{Outer}$$

$$x^2 + y^2 - r^2 = 0 \quad \text{Target trajectory}$$

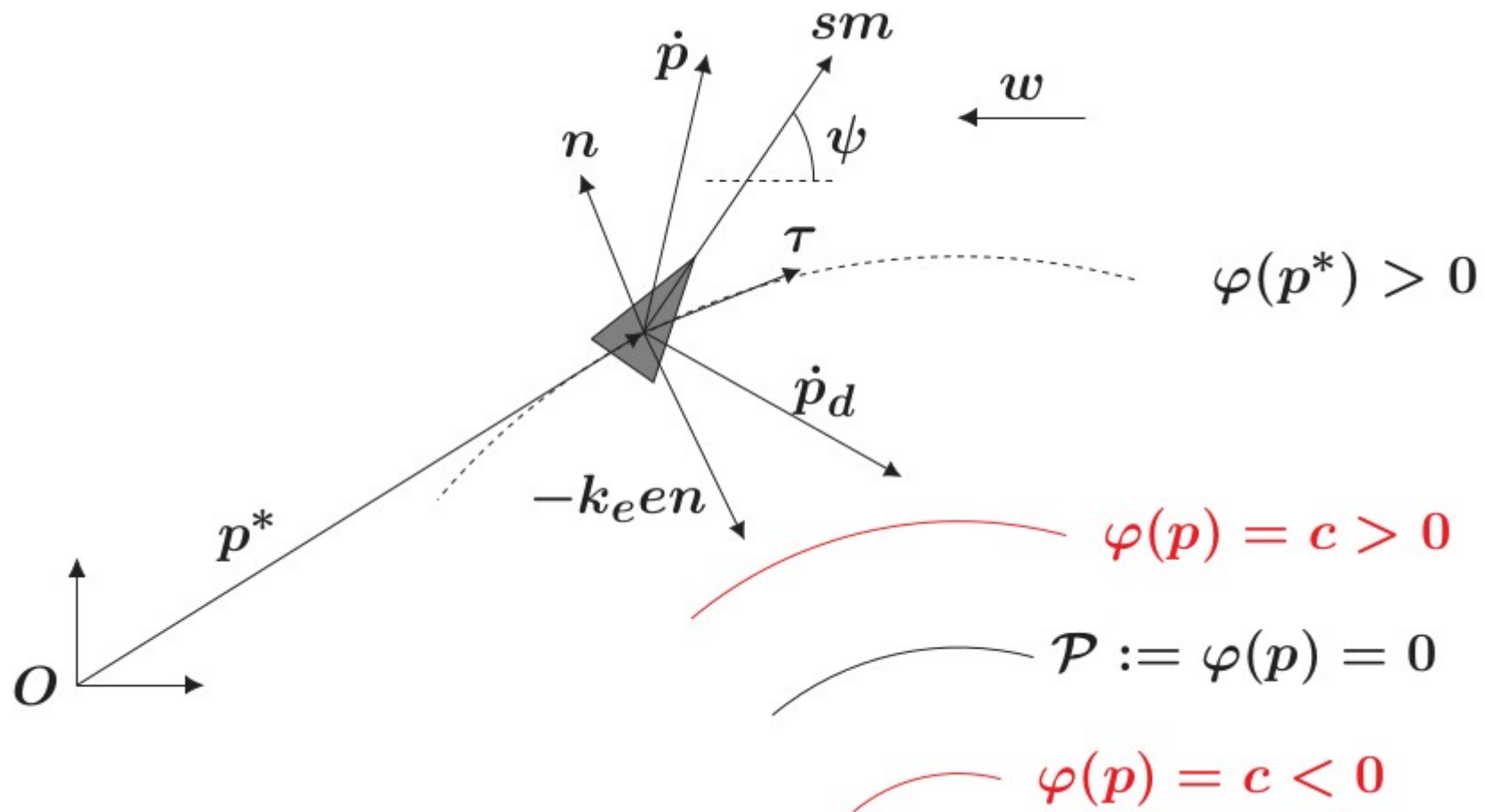
$$x^2 + y^2 - r^2 = -5 \quad \text{Inner}$$



$f(x,y)$  can be used as an error signal!

# How to follow the desired trajectory?

We need the normal and tangent vectors



# We need the Jacobian of the path

Circle  $\rightarrow f(x,y) := (x-x_c)^2 + (y-y_c)^2 + r^2 = 0$

Straight line  $\rightarrow f(x,y) := ax + by + c = 0$

The Jacobian tells us in which direction the level set grows! (normal to the path)

Jacobian of the circle

$\text{Jac} = 2[(x-x_c) \ (y-y_c)]$  is the normal vector to the circle

Jacobian of the straight line

$\text{Jac} = [a \ b]$  is the normal vector to the line

# We need the tangent to the path

It is just the 90 degrees rotation of the Jacobian

$$\text{Tangent} = \text{Rot}(90) \text{ Jac}$$

# The direction to follow is the combination of the normal and tangent

Control action or direction to follow

$e = f(x, y)$  (error signal)

Direction to follow = tangent -  $e \cdot \text{normal}$



# Example with an ellipse

