**Finite Difference Discretization of First Order Wave Equation by Implicit method, first order in time and second order in space.**

**The accuracy of the solution is first order and unconditionally stable.**

∂u/∂t + c ∂u/∂x = 0.…. (1)

Let subscript i denote space nodes and superscript n denote time nodes. **n+1**

Forward Difference of Time, ∆t

∂u/∂t = (uin+1– uin)/∆t…. (2) **i-1 i i+1 n**

Central Difference of space in implicit approach, ∆x ∆x ∆x

∂u/∂x = (ui+1n+1 – ui-1n+1)/2∆x…. (3)

Substituting in (2) and (3) in (1),

(uin+1– uin)/∆t = -c (ui+1n+1 – ui-1n+1)/2∆x)

Let CFL number be ‘C’ = c∆t/∆x, rearranging known and unknown terms,

(C/2) \* (ui-1n+1) - uin+1 - (C/2) \* (ui+1n+1) = - uin

**For solving numerically,**

(C/2) can be taken as coefficient A,

Unknowns are all the points in time level n+1 on the left-hand side. Time level n is known.

Unknowns are,

ui-1n+1, uin+1 and ui+1n+1.

Known term, - uin.

For convenience if we drop the superscript and at first boundary grid point i=2, we have,

A\*U1 - U2 - A\*U3 = -k2

U represents value at timescale n+1 and k at time scale n.

At boundary term A\*U1 is known.

Denoting, -k2- A\*U1 as k2’.

So, -U2 – A\*U3 = k2’.

At i=3,

A\*U2 -U3 - A\*U4 = -k3 and so on.

At final boundary grid point, i= 42,

A\*U41 -U42 = -k42. Here k44 is known.