

# **Access Control via Belnap Logic: Intuitive, Expressive, and Analyzable Policy Composition**

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## Outline of Access Control talk



- Motivation
- Belnap Logic
- Core Policy Language
- Expressiveness of Core Language
- Policy Analysis
- Conclusions



## Motivation

Access control in IT systems increasingly relies on ability to compose policies

Composition framework should

- be intuitive, formal, expressive, and analyzable
- be independent of application domains but extendable to such domains
- support change management, separation of concerns, and reuse

We develop here such a framework based on Belnap Logic

Belnap Logic used in the past for reasoning in Artificial Intelligence

$$p \vee (\neg(p \vee \neg p) \wedge q)$$

## Belnap Logic

In the 1970ies, Belnap suggested the use of a four-valued logic

- Ordinary truth values for truth and falsity
- A third truth value that expresses lack of knowledge
- And a fourth truth value that expresses inconsistent knowledge

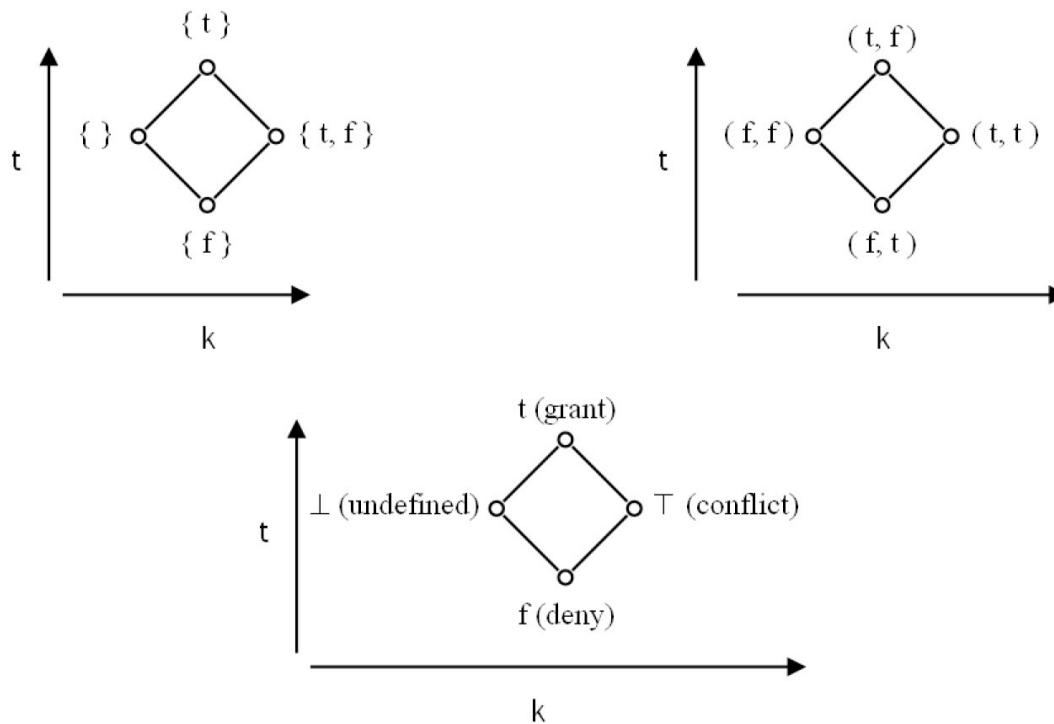
He developed a semantics and a sound and complete Hilbert style proof system for this logic

Belnap logic extends naturally to first- and higher-order logics

Key idea for us: Belnap's evidence-based notion of truth, e.g.

- conjunction of "don't know" and "false" is "false"
- but conjunction of "don't know" and "true" is "don't know"

## Belnap Logic: four values as composition of two



$\{\dots\}$  collects results of policies  $p$  and  $q$ , e.g.  $\{t,f\}$  as conflict

( $\dots$ ) asks ``does single policy  $p$  (grant?, deny?)'', e.g.  $(t,t)$  means  $p$  grants and denies

Third graph captures both interpretations abstractly

## Belnap space $(4, \leq_t, \leq_k, \neg)$

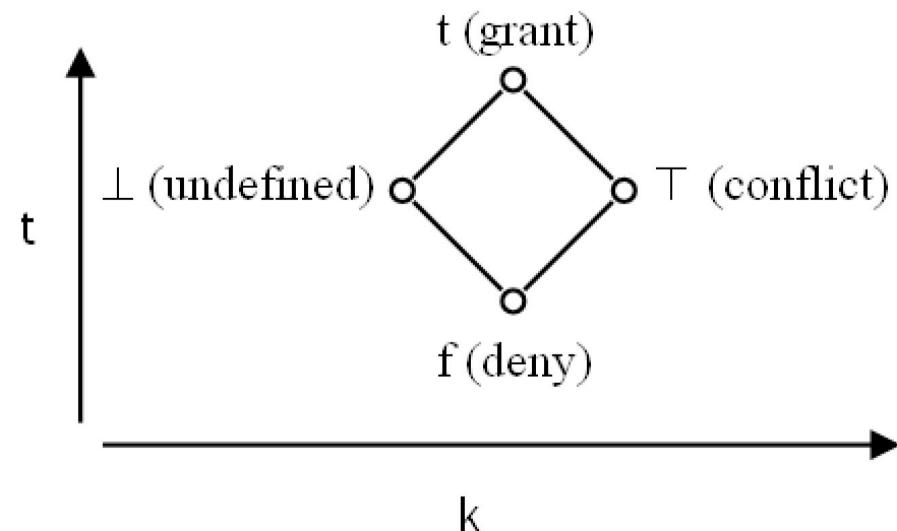
Belnap bilattice over  $4 = \{\mathbf{t}, \mathbf{f}, \top, \perp\}$

Axioms:

$$x \leq_t y \Rightarrow \neg y \leq_t \neg x,$$
$$x \leq_k y \Rightarrow \neg x \leq_k \neg y, \text{ and}$$
$$\neg \neg x = x.$$

Truth negation  $\neg$  swaps denials and grants, and leaves other two values fixed.

x-axis: knowledge ordering  
y-axis: truth ordering



## Belnap space functionally complete

Implication:

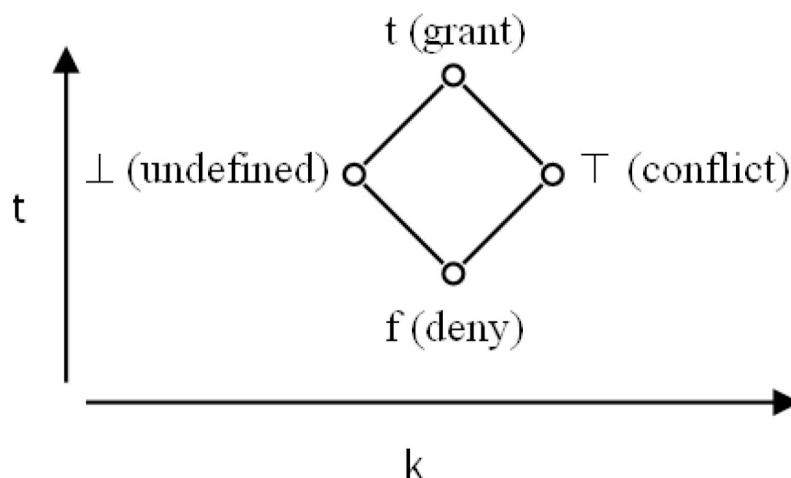
$$a \supset b = b \text{ if } a \in \{t, \top\}$$

$$a \supset b = t \text{ otherwise}$$

Conjunction:

$\wedge$

meet (aka infimum) in truth ordering



## Policy Language

Core language PBel, pronounced “pebble”

$rp ::= Request\ Predicate$	$p, p' ::= Policy$
$a$ Atomic	$b \text{ if } rp$ Basic policy
$true$	$\top$ Conflict
$false$	$\neg p$ Logical negation
	$p \wedge p'$ Logical meet
	$p \supset p'$ Implication

$\top$  overloaded: denotes policy and denotes element of Belnap space

Atomic request predicates  $a$  denote sets of access requests.

$$b \in \{\mathbf{t}, \mathbf{f}\}$$

## Syntactic sugar

PBel is a core language, similar to byte code for higher-level languages.

Convenient policy composition operators map (syntactically and semantically) into this core language, e.g.

$t$	$= t$ if true	$\perp$	$= t$ if false
$f$	$= \neg t$	$\top$	already in PBel
$p \vee q$	$= \neg(\neg p \wedge \neg q)$	$p \otimes q$	$= (p \wedge \perp) \vee (q \wedge \perp) \vee (p \wedge q)$
$p \oplus q$	$= (p \wedge \top) \vee (q \wedge \top) \vee (p \wedge q)$	$\neg p$	$= (\neg p \supset \perp) \oplus (\neg(p \supset \perp))$
$p[\mathbf{f} \mapsto q]$	$= p \vee (\neg(p \vee \neg p) \wedge q)$	$p[\mathbf{t} \mapsto q]$	$= p \wedge (\neg(p \wedge \neg p) \vee q)$
$p[\perp \mapsto q]$	$= p \oplus (\neg(p \oplus \neg p) \otimes q)$	$p[\top \mapsto q]$	$= p \otimes (\neg(p \otimes \neg p) \oplus q)$
$p \text{ if } rp$	$= p \otimes ((\mathbf{t} \text{ if } rp) \oplus (\mathbf{f} \text{ if } rp))$	$p : q$	$= (p \supset q) \otimes \neg(p \supset \neg q)$
$p \downarrow$	$= p[\top \mapsto \mathbf{f}][\perp \mapsto \mathbf{f}]$	$p \uparrow$	$= p[\top \mapsto \mathbf{t}][\perp \mapsto \mathbf{t}]$

## Models

Access-control model  $\mathcal{M}$

consists of non-empty set of requests  $R_{\mathcal{M}}$

and interpretations of request predicates:  $rp^{\mathcal{M}} \subseteq R_{\mathcal{M}}$

$true^{\mathcal{M}} = R_{\mathcal{M}}$  and  $false^{\mathcal{M}} = \{\}$

Example:

- set of requests as triples of form (subject,object, action, context)
- interpretation of atom *manager* is all triples whose subjects are managers
- interpretation of atom *lowThreat* is all triples whose context represents a low threat level

## Semantics

$$\llbracket b \text{ if } rp \rrbracket_{\mathcal{M}}(r) = \begin{cases} b & \text{if } r \in rp^{\mathcal{M}} \\ \perp & \text{otherwise} \end{cases}$$

$$\llbracket \top \rrbracket_{\mathcal{M}}(r) = \top$$

$$\llbracket \neg p \rrbracket_{\mathcal{M}}(r) = \neg \llbracket p \rrbracket_{\mathcal{M}}(r)$$

$$\llbracket p \wedge q \rrbracket_{\mathcal{M}}(r) = \llbracket p \rrbracket_{\mathcal{M}}(r) \wedge \llbracket q \rrbracket_{\mathcal{M}}(r)$$

$$\llbracket p \supset q \rrbracket_{\mathcal{M}}(r) = \llbracket p \rrbracket_{\mathcal{M}}(r) \supset \llbracket q \rrbracket_{\mathcal{M}}(r)$$

*Meaning of policy  $p$  in model  $M$  maps requests  $r$  to element of Belnap space*

*Key point: policy composition is pointwise extension of Belnap operators*

## Support for composite request predicates

Propositional logic structure on request predicates compiles into PBel, e.g.

$$(\mathbf{t} \text{ if } (\text{Manager} \wedge \text{OnDuty} \wedge \neg \text{Weekend} \wedge \text{ReadPDF})) > \mathbf{f}$$

*is translated into PBel with function T below, for semantics on the left:*

$\  a \ _{\mathcal{M}} = a^{\mathcal{M}}$	$T(\mathbf{t} \text{ if } \neg cp) = T(\mathbf{t} \text{ if } cp) \supset \perp$
$\  \text{true} \ _{\mathcal{M}} = R_{\mathcal{M}}$	$T(\mathbf{t} \text{ if } cp \wedge cp') = T(\mathbf{t} \text{ if } cp) \wedge T(\mathbf{t} \text{ if } cp')$
$\  \text{false} \ _{\mathcal{M}} = \{\}$	$T(\mathbf{t} \text{ if } cp \vee cp') = \neg(T(\mathbf{f} \text{ if } cp) \wedge T(\mathbf{f} \text{ if } cp'))$
$\  \neg cp \ _{\mathcal{M}} = R_{\mathcal{M}} \setminus \  cp \ _{\mathcal{M}}$	
$\  cp \wedge cp' \ _{\mathcal{M}} = \  cp \ _{\mathcal{M}} \cap \  cp' \ _{\mathcal{M}}$	
$\  cp \vee cp' \ _{\mathcal{M}} = \  cp \ _{\mathcal{M}} \cup \  cp' \ _{\mathcal{M}}$	
	$T(\mathbf{f} \text{ if } \neg cp) = \neg(T(\mathbf{t} \text{ if } cp) \supset \perp)$
	$T(\mathbf{f} \text{ if } cp \wedge cp') = \neg(T(\mathbf{t} \text{ if } cp) \wedge T(\mathbf{t} \text{ if } cp'))$
	$T(\mathbf{f} \text{ if } cp \vee cp') = T(\mathbf{f} \text{ if } cp) \wedge T(\mathbf{f} \text{ if } cp')$

## Safe sublanguages

Policy  $p$  is **conflict-free** if it never returns  $\top$  for any request in any model.

Sublanguage in which all and only conflict-free policies can be written:

$p, q ::= \text{Conflict-free policy}$

$b \text{ if } rp$	Basic policy	$r[\top \mapsto p]$	Conflict resolution
$\perp$	Gap	$p[v \mapsto q]$	Sequential composition
$\neg p$	Logical negation	$p \text{ if } rp$	Generalized basic policy
$p \wedge q$	Logical meet	$p : q$	Guard connective
$\mathbf{r} \supset q$	Implication	$\mathbf{r} \downarrow$	Pessimistic wrapper
$p \vee q$	Logical Join	$\mathbf{r} \uparrow$	Optimistic wrapper
$p \otimes q$	Knowledge meet		

Boldface  $r$  denotes any policy expression of PBel

## Safe sublanguage for gap-freedom

*Policy  $p$  is **gap-free** if it never returns  $\perp$  for any request in any model.*

*Sublanguage in which all and only gap-free policies can be written:*

$p, q ::=$	<i>Gap-free policy</i>		
$b \text{ if true}$	Gap-free basic policy	$p \oplus r$	Right knowledge join
$\top$	Conflict	$r[\perp \mapsto p]$	Gap resolution
$\neg p$	Logical negation	$p[v \mapsto q]$	Sequential composition
$p \wedge q$	Logical meet	$p \text{ if true}$	Generalized basic policy
$r \supset q$	Implication	$r \downarrow$	Pessimistic wrapper
$p \vee q$	Logical join	$r \uparrow$	Optimistic wrapper
$r \oplus q$	Left knowledge join		

*Boldface  $r$  denotes any policy expression of PBel*

## Safe sublanguage for conclusive decisions

*Sublanguage in which only (and all) policies can be written that are gap-free and conflict-free:*

$p, q ::= \text{Conclusive policy}$

$b \text{ if true}$  Gap-free basic policy

$\neg p$  Logical negation

$p \wedge q$  Logical meet

$\mathbf{r} \supset q$  Implication

$p \vee q$  Logical join

$p[v \mapsto q]$  Sequential composition

$p \text{ if } rp$  Generalized basic policy

$\mathbf{r} \downarrow$  Pessimistic wrapper

$\mathbf{r} \uparrow$  Optimistic wrapper

*Boldface  $r$  denotes any policy expression of PBel*

## Retrofitting policies

*Abbreviation for priority composition:*  $p > q = p[\perp \mapsto q]$

*Exceptional override:*  $(\mathbf{f} \text{ if } rp_{exc}) > p$

*Behaves like policy  $p$  except at exceptional request set  $rp_{exc}$  at which it denies*

*Exclusive rights and exclusive prohibitions:*  $((\mathbf{t} \text{ if } rp_1) \oplus (\mathbf{f} \text{ if } rp_2)) > p$

*Behaves like  $p$  except at two (assumed to be disjoint) request sets that encode absolute rights and absolute prohibitions (respectively)*

## Composing request predicates as policies

$$(t \text{ if } ChW) \wedge (t \text{ if } RegisteredAnalyst) > f$$

*Might model that an access is granted if*

- *the requester is a registered analyst*
- *and if the request is compliant with a Chinese Wall policy*

*Otherwise, the request is denied.*

*Note: ChW is a request predicate that presumably stems from a policy.*

*PBel supports such demotions of policies to predicates (see analysis part below).*

## Expressiveness of PBel: policy functions

*Policy function* for model  $\mathcal{M}$  is total function  $f: R_{\mathcal{M}} \rightarrow 4$

*Policy p expresses policy function f if  $\llbracket p \rrbracket_{\mathcal{M}} = f$*

*Policy functions of form  $\llbracket p \rrbracket_{\mathcal{M}} = f$  have the same output for requests that cannot be distinguished by any request predicate in p*

*It turns out that all functions of that form are expressible as meanings of policies in PBel.*

*These results customize to the safe sublanguages for gap-free, conflict-free, and conclusive policies (not shown in this talk).*

## Expressing data-independent policy functions

Let  $R$  be a set of request predicates, e.g. those occurring in a policy  $p$ .

On each model  $\mathcal{M}$  we define equivalence relation  $\equiv_{\mathcal{M}}^R$  to be  $\{(r, r') \in R_{\mathcal{M}} \times R_{\mathcal{M}} \mid \forall a \in R: r \in a^{\mathcal{M}} \text{ iff } r' \in a^{\mathcal{M}}\}$ .

A policy function  $f: R_{\mathcal{M}} \rightarrow \mathbf{4}$  is data-independent for  $R$  in  $\mathcal{M}$  iff  $r \equiv_{\mathcal{M}}^R r'$  implies  $f(r) = f(r')$ .

We write  $PBel^R$  for the set of  $PBel$  policies that contain only atoms from  $R$ . In particular,  $PBel^{AP}$  equals  $PBel$ .

E.g.  $(t \text{ if } ChW) \wedge (t \text{ if } RegisteredAnalyst) > f$  has four equivalence classes

Result:

For each  $p$  in  $PBel^R$ , policy function  $\llbracket p \rrbracket_{\mathcal{M}}$  is data-independent for  $R$  in  $\mathcal{M}$ .

Conversely, let  $f$  be a policy function that is data-independent for  $R$  in  $\mathcal{M}$  for finite set  $R_{\mathcal{M}}$ . Then there is some  $p$  in  $PBel^R$  with  $f = \llbracket p \rrbracket_{\mathcal{M}}$ .

## Proof that data-independent function $f$ is expressible

*Policy  $p$  expressing  $f$  is knowledge join of a grant and a denial part:*

$$p = p_{\mathbf{t}} \oplus p_{\mathbf{f}}$$

*Each part is the  $n$ -ary knowledge join of (negations of) policies  $p_r$*

$$p_{\mathbf{t}} = \sum_{r \in R_{\mathcal{M}} \mid \mathbf{t} \leq_k f(r)} p_r \quad p_{\mathbf{f}} = \sum_{r \in R_{\mathcal{M}} \mid \mathbf{f} \leq_k f(r)} \neg p_r$$

*Each building block  $p_r$  is a characteristic function that grants on the equivalence class  $\equiv_{\mathcal{M}}^R$  of request  $r$  and is undefined otherwise:*

$$p_r = \left( \bigwedge_{a \in R \mid r \in a^{\mathcal{M}}} t \text{ if } a \right) \wedge \left( \bigwedge_{a \in R \mid r \notin a^{\mathcal{M}}} (t \text{ if } a) \supset \perp \right)$$

## Expressiveness of PBel: policy composition

Example: majority vote of three policies should make as decision  
the majority of decisions

$$G(p_1, p_2, p_3) = (p_1 \wedge p_2) \vee (p_1 \wedge p_3) \vee (p_2 \wedge p_3)$$

*Our pointwise composition means that any such function G is determined by a function*

$$g: 4^n \rightarrow 4$$

*G(p1,...,pn) at request r decides g(v1,...,vn) where vi is decision of pi on r*

## All composition functions are expressible

Free algebra  $\mathcal{A}$   
generated from operators  $\{\top, \neg, \wedge, \supset\}$   
and variables  $X_1, X_2, \dots$

Result:

Let  $n \geq 0$  and  $g \in 4^n \rightarrow 4$

Then there is a term  $t_g \in \mathcal{A}$  such that

$$[\![t_g(p_1, \dots, p_n)]\!]_{\mathcal{M}}(r) = g([\![p_1]\!]_{\mathcal{M}}(r), \dots, [\![p_n]\!]_{\mathcal{M}}(r)) \\ (\forall p_i \in PBel, \mathcal{M}, r \in \mathsf{R}_{\mathcal{M}})$$

Example: term for knowledge join  $\oplus$  is

$$\neg(\neg(X_1 \wedge \top) \wedge \neg(\neg((\top \wedge X_2) \wedge \neg(X_1 \wedge X_2))))$$

## Policy Analysis

We develop a simple query language.

Many important policy analyses are expressible as queries in this language.

Here, we ask whether queries hold in all models: validity checking.

Validity checking is appropriate, e.g. for gap and conflict analysis.

Queries can be implications whose antecedents encode domain-specific or other assumptions.

This policy analysis reduces to validity checking of propositional logic.

## Query Language

$cp, cp' ::= \text{Composite Request Predicate}$

a	Atomic	$\neg cp$	Negation
true	Truth	$cp \wedge cp'$	Conjunction
false	Falsity	$cp \vee cp'$	Disjunction

$\phi, \phi' ::= \text{Query}$

$p \leq_t q$	Policy refinement in truth ordering
$p \leq_k q$	Policy refinement in information ordering
$\alpha \Rightarrow \phi$	Assume-guarantee query
$\phi \wedge \phi'$	Conjunction

where **assumption**  $\alpha$  ranges over **composite request predicates**

## Query Examples

*Policy q is more defined and less permissive than p*

$$(p \leq_k q) \wedge (p \leq_t q)$$

*Policy q is more defined but less permissive than p*

$$(p \leq_k q) \wedge (q \leq_t p)$$

*Policies p and q are equivalent*

$$(p \leq_t q) \wedge (q \leq_t p)$$

*Policy p has no gaps*       $p \leq_t p[\perp \mapsto \mathbf{f}]$

*Policy p has no conflicts*     $p \leq_k p[\top \mapsto \mathbf{f}]$

## Query Semantics

- $\mathcal{M} \models p \leq_k q$  iff for all  $r \in R_{\mathcal{M}}$  we have  $\llbracket p \rrbracket_{\mathcal{M}}(r) \leq_k \llbracket q \rrbracket_{\mathcal{M}}(r)$
- $\mathcal{M} \models p \leq_t q$  iff for all  $r \in R_{\mathcal{M}}$  we have  $\llbracket p \rrbracket_{\mathcal{M}}(r) \leq_t \llbracket q \rrbracket_{\mathcal{M}}(r)$
- $\mathcal{M} \models \alpha \Rightarrow \phi$  iff  $\| \alpha \|_{\mathcal{M}} \neq R_{\mathcal{M}}$  or  $\mathcal{M} \models \phi$
- $\mathcal{M} \models \phi \wedge \psi$  iff  $\mathcal{M} \models \phi$  and  $\mathcal{M} \models \psi$

Atomic queries are interpreted **universally**: policy  $p$  is below policy  $q$  for all requests

Implication is modeled **universally**: if all requests of the model satisfy assumption, then the guarantee query has to be true in the model.

Conjunction has standard interpretation.

## Query Analysis

For each query  $\phi$  we generate a composite request predicate  $C(\phi)$  such that

Query  $\phi$  is valid in all models

Iff

$C(\phi)$  is valid when interpreted as formula in propositional logic

Definition of  $C(\phi)$  proceeds in two steps:

1. Capture constraints for grants and denials of policies contained in query
2. Capture the logical structure of the query in terms of these policy constraints

## Constraints for PBel policies

Each request  $r$  in model  $M$  determines model of propositional logic:

$$\rho_r^M(a) = t \text{ if } r \in a^M \quad \rho_r^M(a) = f \text{ if } r \notin a^M$$

Semantics of that model matches that of request model  $M$ :  $\rho_r^M$  satisfies those composite request predicates that contain  $r$  in their interpretation

Define, below, propositional logic formula  $p \uparrow b$  such that

$$\rho_r^M \models p \uparrow b \text{ iff } b \leq_k \llbracket p \rrbracket_M(r)$$

$$(b' \text{ if } rp) \uparrow b = \begin{cases} rp & \text{if } b = b' \\ \text{false} & \text{otherwise} \end{cases}$$

$$\top \uparrow b = \text{true}$$

$$(p \wedge q) \uparrow f = p \uparrow f \vee q \uparrow f$$

$$(p \supset q) \uparrow f = p \uparrow t \wedge q \uparrow f$$

$$(\neg p) \uparrow b = p \uparrow \neg b$$

$$(p \wedge q) \uparrow t = p \uparrow t \wedge q \uparrow t$$

$$(p \supset q) \uparrow t = \neg(p \uparrow t) \vee q \uparrow t$$

## Constraints for Queries

*Query for knowledge ordering uses that truth and falsity are prime elements of the distributive lattice in the knowledge ordering*

*Query for assume-guarantee reasoning translates this into a propositional implication (as done in assume-guarantee reasoning for linear-time temporal logic)*

$$\begin{aligned} C(p \leq_k q) &= (p \uparrow \mathbf{f} \rightarrow q \uparrow \mathbf{f}) \wedge (p \uparrow \mathbf{t} \rightarrow q \uparrow \mathbf{t}) \\ C(\alpha \Rightarrow \phi) &= \alpha \rightarrow C(\phi) \end{aligned}$$

*Query for truth ordering uses characterization of truth ordering in terms of knowledge ordering*

*Query for conjunction is interpreted compositionally (sound for validity checking)*

$$\begin{aligned} C(p \leq_t q) &= (q \uparrow \mathbf{f} \rightarrow p \uparrow \mathbf{f}) \wedge (p \uparrow \mathbf{t} \rightarrow q \uparrow \mathbf{t}) \\ C(\phi \wedge \psi) &= C(\phi) \wedge C(\psi) \end{aligned}$$

## Example Policy Analysis

$$p = (\mathbf{t} \text{ if } rd) \oplus (\mathbf{f} \text{ if } wr)$$

$$q = p[\top \mapsto \mathbf{f}]$$

*Valid query (assumes that no read action is also a write action):*

$$\neg(rd \wedge wr) \Rightarrow (p \leq_t q)$$

*Propositional constraints for policies:*

$$p \uparrow \mathbf{t} = rd \vee \mathbf{false} = rd$$

$$p \uparrow \mathbf{f} = \mathbf{false} \vee wr = wr$$

$$q \uparrow \mathbf{t} = p \uparrow \mathbf{t} \wedge (\neg(p \uparrow \mathbf{f}) \vee \mathbf{f} \uparrow \mathbf{t}) = rd \wedge (\neg wr \vee \mathbf{false}) = rd \wedge \neg wr$$

$$q \uparrow \mathbf{f} = p \uparrow \mathbf{f} \wedge (\neg(p \uparrow \mathbf{t}) \vee \mathbf{t} \uparrow \mathbf{t}) = wr \wedge (\neg rd \vee \mathbf{true}) = wr$$

*Propositional constraint for above query is equivalent to valid formula*

$$rd \vee wr \vee ((\neg rd \vee wr) \wedge (\neg rd \vee \neg wr))$$

## Some Related Work

- Halpern and Weissman 2003: policies specified in first-order logic, access granted if formula logically entails formal permission predicate
- XACML standard: has no formal semantics, its semantic values and policy combination algorithms can be interpreted within PBel for suitable interpretation
- Ni et al. 2009: D-algebras as functionally complete value spaces with algebraic operators, result spaces rather ad-hoc in nature
- Bauer et al. 2005: Polymer, access-control language for untrusted Java applications, policy is query method that returns one of six values for code execution request
- Moffett and Sloman 1994: early work on policy conflict analysis
- Ribeiro et al. 2001: access-control language SPL, three-valued, can be cleanly embedded into PBel

## Things we did but didn't mention here

- Support for attribute language for request predicates
- Clean encoding of policy language SPL in PBel
- Expressiveness results specialized to safe sub-languages, both for policy functions and for policy composition
- Symbolic query analysis, where policies are parameters
- Methods and interface specifications
- Request mappings, e.g. ``grant read access whenever write access is given''

## Conclusions

We developed an access-control policy language based on

- abstract request predicates that encapsulated domain-specific aspects of sets of access requests
- the 4-valued Belnap bilattice whose operators were extended pointwise to our language

This gave us a very expressive core language over which common policy combination idioms can be expressed.

The core language (and so any idioms compiling into it) has a simple query analysis that supports many of the desired policy analyses and reduces them to validity checking of propositional logic.

## Acknowledgments

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